Analysis of Node Localizability in Wireless Ad-hoc Networks

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Abstract

Location awareness is highly critical for wireless ad-hoc and sensor networks. Many efforts have been made to solve the problem of whether or not a network can be localized. Yet two fundamental questions remain unaddressed: First, given a network configuration, whether or not a specific node is localizable? Second, how many nodes in a network can be located and which are them? In this study, we analyze the limitation of previous works and propose a novel concept of node localizability. By deriving the necessary and sufficient conditions for node localizability, t is possible to analyze how many nodes one can expect to locate in sparsely or moderately connected networks.

Keyword

Location, Localizability, wireless ad-hoc

Catalogue

I. Introduction

The ground truth of a network can be modeled by a distance graph G. We assume G is connected and has at least 4 vertices in the following analysis.

1. Terms Explanation

Realization: a function p that maps the vertices of G to points in a Euclidean space (this study assumes 2-dimension space).
Rigid: one cannot continuously deform its realizations while preserving distance constraints.
Globally rigid: A graph is uniquely realizable.
Beacons: some special nodes know their global locations and the rest determine their locations by measuring the Euclidean distances to their neighbors.
Redundant rigidity : figure1 further shows a 3-connected and rigid graph which becomes flexible upon removal of an edge. After the removal of the edge (u, v), a subgraph can swing into a different configuration in which the removed edge constraint is satisfied and then reinserted. This type of ambiguity is eliminated by redundant rigidity, the property that a graph remains rigid upon removal of any single edge.

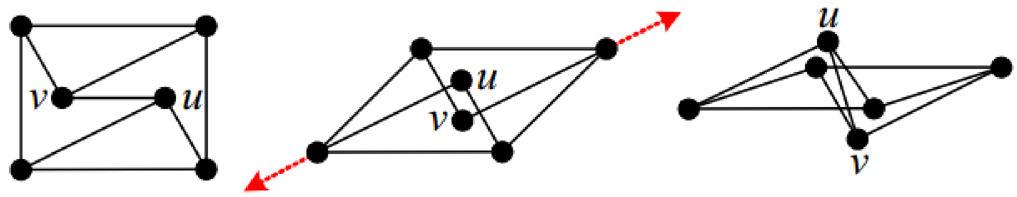


figure 1

2. Network Localizability

As we know, a globally rigid graph can be uniquely determined if fixing any group of 3 vertices to avoid trivial variation in 2D plane, such as translation, rotation, or reflection. Hence, a network with at least 3 beacons is entirely localizable if and only if its distance graph is globally rigid. Therefore, if a distance is globally rigid, any node in this graph is located.

Following the results for network localizability, an obvious solution is to find a localizable subgraph from the distance graph, and identify all the nodes in the subgraph localizable. Unfortunately, such a straightforward attempt misses some localizable nodes and wrongly identifies them as non-localizable, since some conditions essential to network localizability are no longer necessary to node localizability.

However, it's wise to study network localizability firstly.

The necessary and sufficient condition has been provided for global rigidity in the following theorem.

Theorem 1. [1]

A graph with $n \geqslant 4$ vertices is globally rigid in 2 dimensions if and only if it is 3-connected and redundantly rigid.

II. Necessary Conditions

1. 3C

As we know, 3-connectivity conditions is essential to network localizability but not to network localizability. The first non-trivial necessary condition has been proposed: if a vertex is localizable, it has at least 3 vertex-disjoint paths to 3 beacons. We denote such a condition as 3C for short.

Proof: If the node has 1 vertex-disjoint path to a beacon, it can be any point in a circle just as figure 2(a); If the node has 2 vertex-disjoint paths to 2 beacons, It definitely suffers from a potential flip ambiguity by reflecting along the line of a pair of cut vertices just as figure 2(b).

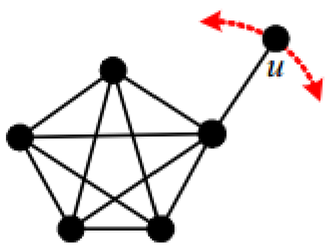


figure 2(a)

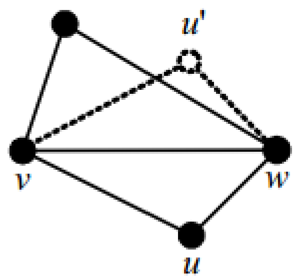


figure 2(b)

2. RR

Although a distance graph is rigid but not globally, there are still some localizable nodes in this graph just like u in figure 3.

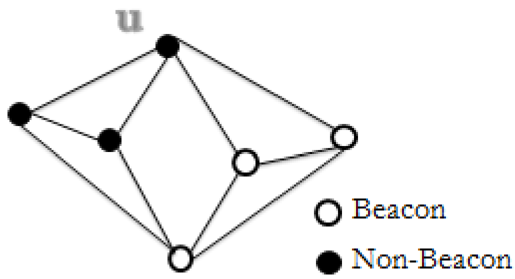


figure 3

We try to provide a theorem about Necessity of redundant rigidity.

Theorem 2.[1]

In a distance graph G = (V, E) with a set B⊂V of k ⩾3 vertices at known locations, if a vertex is localizable, it is included in the redundantly rigid component that contains B.

Proof: This theorem can be proved by contradiction. Assume the special case that G is rigid but not redundantly rigid. Suppose R is the redundantly rigid component containing B and a vertex u ∉ R. There is an edge e = (v, w) whose removal results in u and B belonging to different rigid components in G-e. Accordingly, there is a continuous flexing in which u changes its location relative to B. The distance between v and w will be a multi-valued function for almost every point on this circle. Hence, there exists another realization of G-e that keeps the distance value unchanged according to the generic graph assumption. Adding e back, it forms a realization of G in which the location of u is changed. Therefore, u is non-localizable.

Now we have obtained a better necessary condition for node localizability by combining 3C (3 vertex-disjoint paths) and Theorem 2 (redundant rigidity), which we call RR3C for short.

III. Sufficient Conditions

1. 3P

As we know, an obvious sufficient condition to node localizability is as follows: if a vertex belongs to the globally rigid subgraph of G that contains at least 3 beacon vertices, it is uniquely localizable.

2. RR

Note that a localizable vertex does not necessarily satisfy RRT, as shown in Figure 3. The graph consists of 3 beacon vertices (denoted by white circles) and 3 non-beacon vertices (denoted by black ones). It is clear that u is not in the 3-connected component of 3 beacon vertices. However u's location can be uniquely determined under the configuration.

The possible reason is the distance between u and v is actually fixed although no edge connects them. If we add the edge (u, v) to G, u can be easily identified as localizable since the distances from u to 3 beacon vertices are available. This observation leads us to explore the implicit edges for identifying localizable vertices.

Therefore, For a distance graph G = (V, E), its extended distance graph is G` = (V, E∪E`) where E` is a set of implicit edges of G.

So, combining Theorem 1 and the concept of implicit edges, we achieve the following theorem.

Theorem 3. [2]

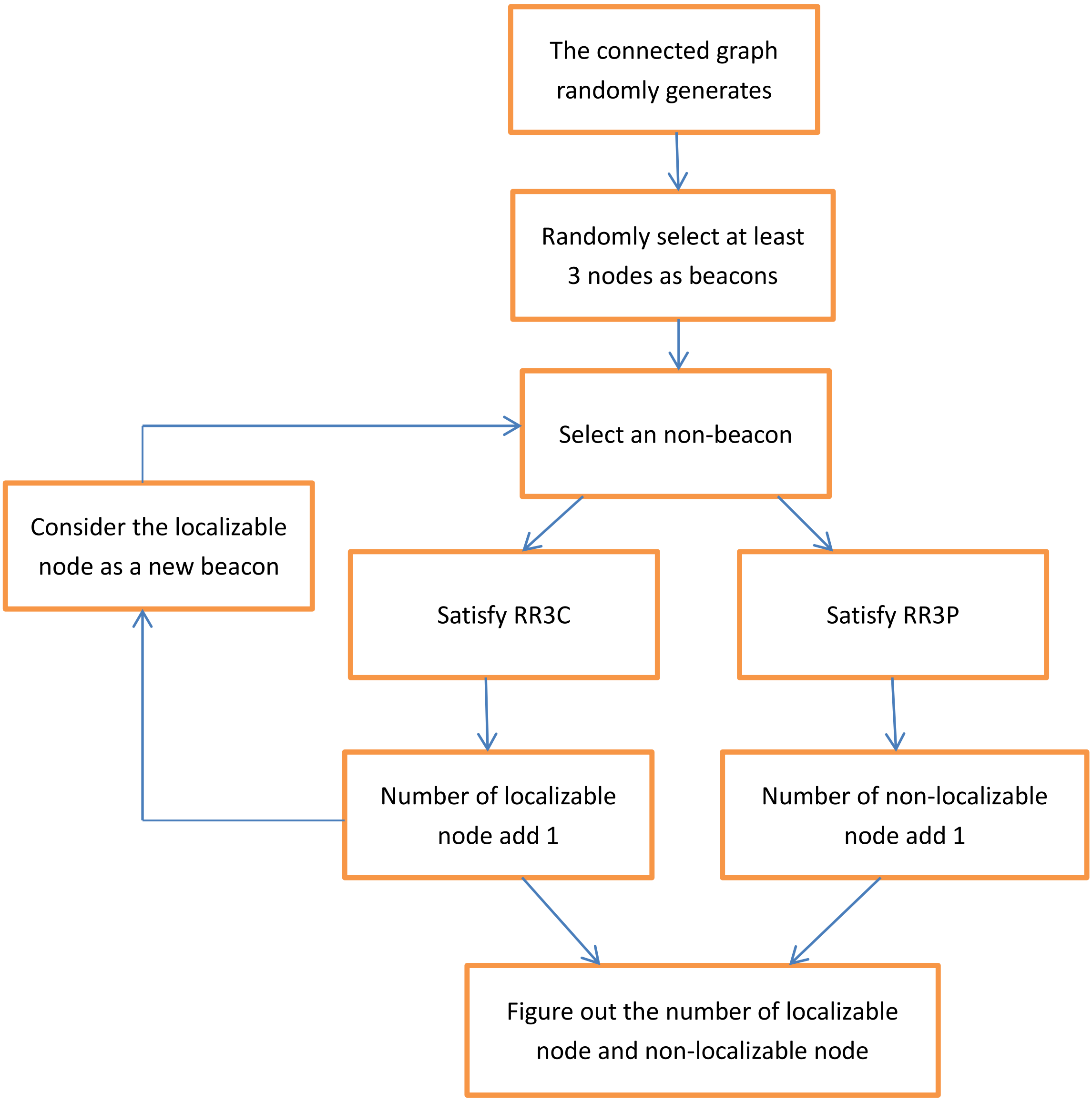
Let G` denote the extended distance graph of G = (V, E) which has a set $B \subset V$ of $k \geqslant 3$ vertices at known locations. If a vertex belongs to a globally rigid subgraph of G` that contains at least 3 vertices in B, it is uniquely localizable.

We denote it RR for short, so we obtain the sufficient condition RR3P.

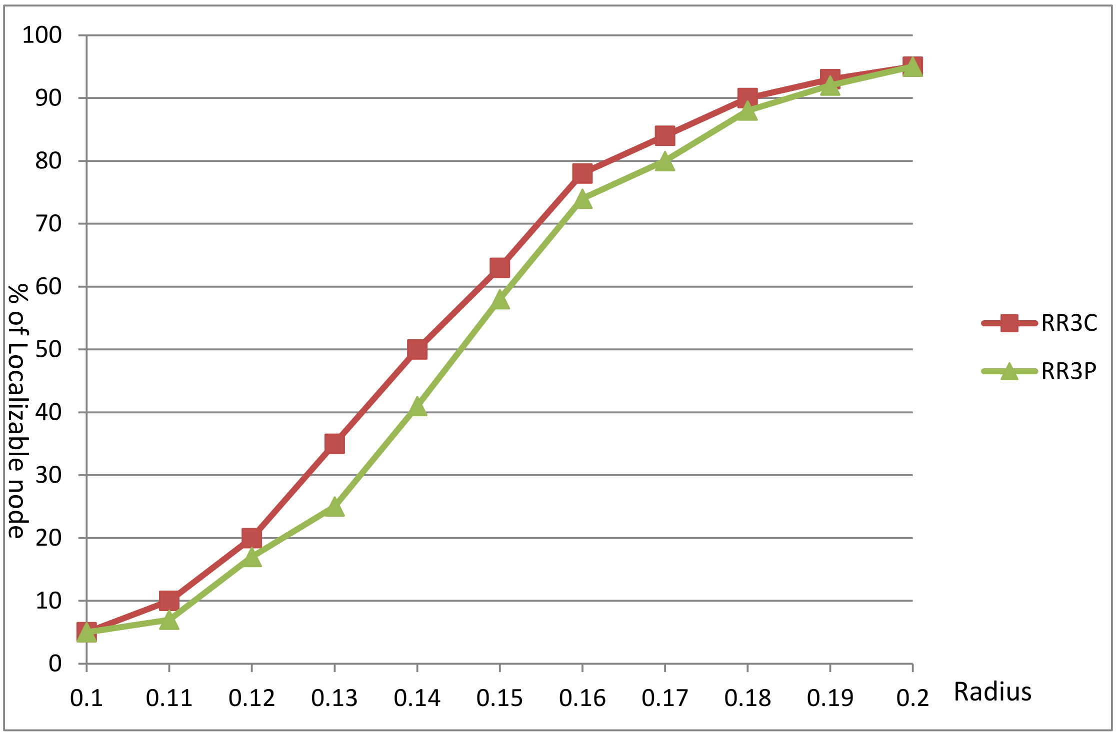
IV. Simulation

From the analysis above, we can get that if a node satisfies the RR3P condition, it is localizable; if a node, on the other hand, does not satisfy the RR3C condition, it is non-localizable. I use the language to simulate the model. I randomly generate networks of 400 nodes, uniformly deployed in a unit square. The unit disk model with a radius is adopted for communication and distance ranging.

1. Flow Chart



2. Results



We can see, nodes above the curve of RR3P are non-localizable while those below the curve of RR3C are localizable.

Clearly, two curves are close to each other and the gap between them is always narrow, indicating a small number of nodes whose localizability cannot be determined.

V. Reference

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