Delay-Throughput Tradeoff with Correlated Mobility in Ad-Hoc Networks

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***Abstract*—In this paper, we investigate the delay-throughput tradeoffs in correlated mobility ad-hoc network. We consider three regimes under correlated mobility: 1)cluster sparse regime, 2)cluster dense regime, and 3)cluster critical regime. Given a delay constraint D, we characterize the maximum throughput the whole network can sustain and find that heterogeneous can increase the delay-throughput tradeoffs.**

I. INTRODUCTION

During the last few years, there has been tremendous interests in the researches about mobile ad-hoc networks (MANETs) by the store-carry-forward transmission paradigm , due to their future requests. The applications include non-real-time and delay-tolerated servers in vehicle network, mobile machine-to-machine network, mobile device-to-device network, Internet of things, smart phone network, etc..

In these MANETs where the store-carry-forward paradigm is adopted, nodes receive data from other nodes, store them in the physical storage medium, carry them by moving spatially and forward them to other suitable nodes. A lot of wireless resources are consumed when the data are transferred via multiple hops. However data are transferred by nodes mobilities by the store-carry-forward paradigm. So the store-carry-forward paradigm can improve the throughput at the cost of the delay. Thus for non-real-time and delay-tolerated applications, it is a good solution to transmit by store-carry-forward paradigm.

In the work [2], the concept of correlated mobility was first demonstrated. Three models cluster sparse regime, cluster dense regime, and cluster critical regime are introduced. In their work, it has shown us the maximum throughput and the corresponding delay of cluster sparse and dense regime, and found that cluster sparse may increase the delay-throughput tradeoff, leading to that correlated mobility can help to increase the network performance. But in their work, the detailed tradeoff haven't been shown, the model of cluster critical regime haven't been considered, and the cluster dense network shown not perform better than the uniformly distributed network.

In this paper, we interested in the detailed tradeoff under correlated mobility model. We try to reveal the tradeoff under three regimes cluster sparse, cluster dense. The main contributions of this paper consist of the following parts:

1) We provide the detailed delay-throughput tradeoff under cluster sparse, cluster dense, and cluster critical regime, not only the maximum throughput.
2) We found that the cluster dense and cluster critical regime can also help to better the delay-throughput tradeoff performance, not only the cluster sparse one.
3) We show that under the cluster dense regime, the delay-throughput tradeoff can be totally better than the uniformly distributed one if we choose some certain system parameters.

II. NETWORK AND MOBILITY MODEL

A. Network Topology

We consider n nodes moving over a square with area n, the n nodes, however, are divided into $m = \Theta(n^v)$ $(0 \le v < 1)$ groups. Each group covers a circular area with radius $R = \Theta(n^\beta)$ $(0 \le \beta \le 1/2)$.

We assume time is divided into time slots of unit duration. At each time slot, the position of nodes are assumed to be static and the mobility of each node comply correlated mobility, which is first introduced in [2]. We describe the mobility for node i in cluster j with two steps: i) the center of cluster j's position are i.i.d. and are uniformly chosen among the whole network area at each time slot; ii) node i's position are i.i.d and are uniformly chosen among the area cluster j covered. The above two steps are called group movement and node movement respectively; the combination of them describe the correlated mobility in our work.

We also assume slow mobility model during our data transmission; radio transmission is much faster than node mobility, that is multihop schedule can be operated within a single time slot.

TABLE I: System Parameter.

n	number of nodes
m	number of cluster
v	Growth exponent of $m : m = \Theta(n^v), 0 < v \le 1$
q	Average number of node per cluster, $q = n/m = \Theta(n^{1-v})$
R	cluster radius
β	growth exponent of R: $R = \Theta(n^\beta)$

B. Transmission Protocol

To limit the interference, we adopt the protocol model proposed in [1]. Let X_i denote the position of node i $(i = 1, ..., n)$ and $|X_i - X_j|$ denote the Euclidean distance between i and

j. A sender i can transmit at W bit/second successfully to a destination j when

$$|X_j - X_k| \leq (1+\Delta)|X_i - X_j|$$

for any other simultaneously active transmitters k, where Δ is a positive number.

C. Traffic Model

We assume all sources communicate with their destinations at same rate λ and $\bar{D}$ denote the average delay over all messages among all source-destination pairs.

Definition of Asymptotic Capacity and Delay: Let λ_i ($i = 1, ..., n$) denote the sustainable rate of data flow for node i and D_b ($b = 1, ..., \lambda nT$) denote the sustainable data delay for message b at time T. Assume that $\lambda = \min\{\lambda_1, \lambda_2, ..., \lambda_{n-1}, \lambda_n\}$ and $\bar{D} = \sum_{b=1}^{\lambda nT} D_b/\lambda nT$. Then $\lambda = \Theta(f(n))$ is defined as the asymptotic throughput if there exist constant $c > c' > 0$, that

$$\lim_{n\to\infty} \mathbf{Pr}(\lambda = cf(n) \text{ is achievable}) < 1,$$
$$\lim_{n\to\infty} \mathbf{Pr}(\lambda = c'f(n) \text{ is achievable}) = 1.$$

And $\bar{D} = \Theta(g(n))$ is defined as the asymptotic delay as well

III. Upper bound of the cluster sparse regime

We divide our model into three regimes. Cluster sparse regime when $v + 2\beta < 1$ (*i.e.*, $mR^2 = o(n)$), cluster dense regime when $v + 2\beta > 1$, and cluster critical regime when $v + 2\beta = 1$. In this section we consider the tradeoff upper bound under cluster sparse regime.

Under cluster sparse regime, m clusters only cover a negligible fraction of whole network area. In the other aspect, density in the cluster is relatively high (the density is about $n/(mR^2) = \omega(1)$) and overlaps between different clusters are sporadic.

A. Scheduling policy

In this section, we will first design a scheduling policy generalizing the policy in [2], which refer to some scheduling parameters. We then propose several lemmas to exclude the parameters don't affect the asymptotic throughput and delay.

Nodes in different cluster have little chance to communicate, owing to the cluster sparse regime. For a traffic stream $s \to d$, we denote C_s as the cluster containing s and C_d as the cluster containing d. We assume $C_s \neq C_d$, which maximize the character of correlated mobility. Our original scheduling policy is shown as follow:

1) s create R_s duplication nodes as relays in C_s with muticast.
2) When relays meet a cluster C_k ($k = 1, ..., {R_c}^s$, where ${R_c}^s$ is the maximum number of clusters containing relay) not containing duplication node, a duplication will be created in C_k with one-hop unicast.
3) New-created relay in C_k create R_k duplication nodes in C_k with broadcast.
4) When a relay meet C_d, a duplication will be created in C_d with one-hop unicast.
5) New-created relay in C_d create ${R_d}^s$ duplication nodes in C_d with broadcast.
6) When R_d relays are captured by the destination with range l^s, the message will be transmitted to destination with a h^s-hop mutihop transmission

Our policy can be divided into two parts: sending message form C_s to C_d and sending message within C_d. The network topology of the second part is similar as uniform distribution like [3], so we should focus on the first part which reflect the character of correlated mobility.

In the first part, there exist many duplication nodes. We use term "inter-cluster duplication" to denote the cluster containing duplication nodes; we use term "intra-cluster duplication" to denote the duplication nodes in a certain cluster. In our original policy the number of inter-cluster duplication is ${R_c}^s$ and the number of intra-cluster duplication is a set $\{R_s, R_1, ..., R_{{R_c}^s}, R_d\}$. As radio resource is needed to create duplication node, Lemma 3.1 will help us determine some duplication value.

Lemma 3.1: Under cluster sparse regime, the most intra-cluster duplication $\{R_s, R_1, ..., R_{{R_c}^s}, R_d\}$ (not including R_d) will decrease the asymptotic throughput without decreasing the asymptotic delay.

We will show detailed prove of Lemma 3.1 in Lemma 6.2. Now we just let it as something we have already proved. So $R_k = 1$ ($k = s, 1, 2, ..., {R_c}^d$). We can particularly use "intra-cluster duplication" to denote duplication in C_d, and our scheduling policy become:

1) When s and relays meet a cluster C_k ($k = 1, ..., {R_c}^s$, where ${R_c}^s$ is the number of inter-cluster duplication) not containing duplication node, a duplication will be created in C_k with one-hop unicast.
2) When a relay meet C_d, a duplication will be created in C_d with one-hop unicast.
3) New-created relay in C_d create ${R_d}^s$ duplication nodes in C_d with broadcast.
4) When ${R_d}^s$ relays are captured by the destination with range l^s, the message will be transmitted to destination with a h^s-hop mutihop transmission

B. Tradeoff for delay

This section will prove a fundamental tradeoff about delay, which is one of the cornerstone for deriving the upper bound of delay-throughput tradeoff. We will first divide whole scheduling policy into three parts corresponding to three parts of delay. We will then find which part dominate the delay. At last, we will get the tradeoff between delay and related scheduling parameters.

Our scheduling policy can be divided into three parts. D_I^s stands for the delay of creating ${R_c}^s$ inter-cluster duplications, D_{II}^s stands for the delay of ${R_c}^s$ inter-cluster duplications transmitting message to C_d, and D_{III}^s stands for the delay of transmission within C_d.

As for D_I^s, we assume that $D_I^s = \sum_{k=1}^{{R_c}^s} D_{Ik}^s$, where D_{Ik}^s stands for the delay of creating kth inter-cluster duplication. We denote $P_I^s(k)$ as the probability that, when we have already created $k-1$ inter-cluster duplications, inter-cluster duplications meet a cluster not containing duplication. From Fig. 1, we can get that

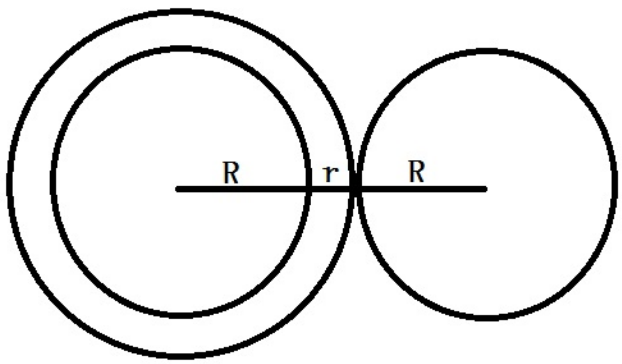

Fig. 1: Transmission between two clusters.

$$P_I^s(k) = 1 - \Big(1 - \frac{\pi k(2R+r)^2}{n}\Big)^{m-k} \tag{1}$$

Then it's easy to get $D_{Ik}^s = 1/P_I^s(k)$, which leads to

$$\begin{aligned} D_I^s &= \sum_{k=1}^{{R_c}^s} \frac{1}{1-(1-\pi k(2R+r)^2/n)^{m-k}} \\ &\le \sum_{k=1}^{{R_c}^s} \frac{n}{\pi k(m-k)(2R+r)^2} \\ &= \frac{n}{\pi m(2R+r)^2} \sum_{k=1}^{{R_c}^s} \frac{1}{k} + \frac{1}{m-k} \\ &\le \Theta\Big(\frac{n}{mR^2}\big(\ln \frac{{R_c}^s m}{m-{R_c}^s} + \gamma\big)\Big) \end{aligned}$$

where γ is the Euler constant and r is the transmission range for a single hop.

As for D_{II}^k, the delay needed for R_c inter-cluster duplication transmitting data to C_d can be formulated by

$$\begin{aligned} D_{II}^s &= \frac{1}{1-(1-\pi(R-r)^2/n)^{{R_c}^s}} \\ &\le \Theta\big(\frac{n}{{R_c}^s R^2}\big) \end{aligned}$$

Similarly,

$$D_{III}^s = \frac{R^2}{{R_d}^s l^2}$$

Considering these three delays D_I^s, D_{II}^s, and D_{III}^s, the total delay D^s under cluster sparse regime is $\max\{D_I^s, D_{II}^s, D_{III}^s\}$. However $D_{I\max}^s = \Theta(n\log m/mR^2)$ and $D_{II\min}^s = \Theta(n/mR^2)$, which means D_I^s will not exceed D_{II}^s by a logarithmic factor. Omitting the logarithmic factor, we get

$$D_b^s = \max\{D_{IIb}^s, D_{IIIb}^s\}$$

where b stands for a particular bit.

A more sophisticated strategy is "opportunistic duplication scheme" that at each time slot t, if one of the relays get a chance to communicate with C_d or destination node, message will be transmitted to C_d or destination node from the relay. Otherwise, duplication will be created as normal. This scheme may get a better result for $D_I^s + D_{II}^s$ and D_{III}^s. However the following lemma show that this scheme can only improve the delay with a $\log n$ factor.

Lemma 3.2: Under the cluster sparse regime, the delay for a particular bit b and its scheduling parameters comply the following inequality

$$c_1^s \log n \mathbb{E}[D_b^s] \le \max\left\{\frac{n}{R^2\mathbb{E}[{R_c}_b^s]}, \frac{R^2}{\mathbb{E}[{R_d}_b^s]\mathbb{E}[l_b^s + \frac{mR^2}{n^2}]^2}\right\} \tag{2}$$

where c_1^s is a positive constant and variable X_b^s denote the variable X under cluster sparse regime for a particular bit b. The proof of Lemma 3.2 is reported in Appendix A.

C. Tradeoff for radio resource

This section will prove another fundamental tradeoff about radio resource. We will first recall the disjoint disk. We will then focus on some special property for cluster sparse regime. At last, we will get the tradeoff between delay and related scheduling parameters.

As we use protocol model as our communication model, disjoint disk is a specific model describing limited radio resource, which is first proved in [1].

Consider that nodes i, j directly transmit to nodes k and l, respectively, at the same time. Then, according to the interference constraint:

$$\begin{aligned} |X_j - X_k| \ge (1+\Delta)|X_i - X_k| \\ |X_i - X_l| \ge (1+\Delta)|X_j - X_l| \end{aligned}$$

Hence,

$$\begin{aligned} |X_j - X_i| &\ge |X_j - X_k| - |X_i - X_k| \\ &\ge \Delta|X_i - X_k| \end{aligned}$$

Therefore,

$$|X_j - X_i| \ge \frac{\Delta}{2}(|X_i - X_k| + |X_j - X_l|)$$

A disks of radius $\Delta|X_i - X_k|/2$, where i, j is a sending-receiving pair, centering at sender are disjoint from each other.

Under the cluster sparse cluster, a phenomenon is that nodes only cover a small part of network area at each time slot, and this phenomenon leads to two properties we need to notice as we derive the tradeoff.

One is that the area of radio resource we use is only $\Theta(mR^2)$, not $\Theta(n)$ as the uniform distributed one. The other is that [2] have proved that a certain cluster has only a probability of mR^2/n to meet other clusters. When creating a inter-cluster duplication, $n/(mR^2)$ chances are needed to operate successfully. Creating ${R_c}_b^s$ inter cluster duplications is equivalent to transmitting $n{R_c}_b^s/(mR^2)$ times.

Lemma 3.3: Under cluster sparse regime and concerning radio resource, the throughput for a particular bit b and its scheduling parameters comply the following inequality

$$\sum_{b=1}^{\lambda^s nT} \frac{\Delta^2}{4} \frac{\mathbb{E}[R_{d_b^s}]}{n} + \mathbb{E}[\sum_{b=1}^{\lambda^s nT} \sum_{h=1}^{h_b^s + \frac{nR_{c_b^s}}{mR^2}} \frac{\pi\Delta^2}{4} \frac{{r_b^h}^2}{mR^2}] \leq c_2^s WT \log n \tag{3}$$

where c_2^s is a positive number, h_b^s is the number of transmission hops after message being captured by destination node, and r_b^h is the transmission range of each hop, $h = 1, ..., h_b^s$.

Since no node can transmit and receive

Proof is similar to Appendix B in [3], so we omit it for simplification.

D. Tradeoff for Half Duplex and Mutihop

Since no node can transmit and receive at the same time and over same frequency, the following inequality holds,

Lemma 3.4: The following inequality holds,

$$\sum_{b=1}^{\lambda^s nT} \sum_{h=1}^{h_b^s + \frac{nR_{c_b^s}}{mR^2}} 1 \leq \frac{WT}{2} n \tag{4}$$

The following inequality holds for the nature of multihop.

Lemma 3.5: The following inequality holds,

$$\sum_{b=1}^{\lambda^s nT} \sum_{h=1}^{h_b^s} r_b^h \geq l_b^s \tag{5}$$

E. Upper bound on delay-throughput tradeoff

The upper bound under cluster sparse regime can be derived from the basic tradeoff we have proven. In this section, we will separate our proof into two parts. One is $D_{III}^s \geq D_{II}^s$ and the other is $D_{III}^s < D_{II}^s$.

Lemma 3.6: Under cluster sparse regime, when $D_{III}^s \geq D_{II}^s$, let $\bar{D}^s$ denote the mean delay averaged over all bits and let λ^s be the throughput of each source-destination pair. The following upper bound holds,

$$(\lambda^s)^3 \leq \Theta(\frac{m\bar{D}^s}{n} \log^3 n)$$

Proof: From Lemma 3.2, when $D_{III}^s \geq D_{II}^s$, we have

$$\begin{aligned}
c_1^s \log n \mathbb{E}[D_b^s] &\leq \frac{R^2}{\mathbb{E}[R_{d_b^s}](\mathbb{E}[l_b^s] + \frac{mR^2}{n^2})^2} \\
\sum_{b=1}^{\lambda^s nT} \mathbb{E}[R_{d_b^s}] &\geq \frac{1}{c_1^s \log n} \sum_{b=1}^{\lambda^s nT} \frac{R^2}{(\mathbb{E}[l_b^s] + \frac{mR^2}{n^2})^2 \mathbb{E}[D_b^s]} \\
&\geq \frac{R^2}{c_1^s \log n} \frac{\sum_{b=1}^{\lambda^s nT} 1}{\sum_{b=1}^{\lambda^s nT} \mathbb{E}[D_b^s]} \\
&\quad \times \frac{(\sum_{b=1}^{\lambda^s nT} 1)^3}{(\sum_{b=1}^{\lambda^s nT} (\mathbb{E}[l_b^s] + \frac{mR^2}{n^2}))^2} \qquad (6) \\
&= \frac{R^2}{c_1^s \log n} \frac{(\sum_{b=1}^{\lambda^s nT} 1)^3}{\bar{D}^s(\sum_{b=1}^{\lambda^s nT} (\mathbb{E}[l_b^s] + \frac{mR^2}{n^2}))^2}
\end{aligned}$$

Inequality (6) is deduced by using Jensen's Inequality and Hölder's Inequality. From Lemma 3.3 and Cauchy-Schwartz inequality, we get

$$\frac{\pi\Delta^2}{2WTnmR^2} (\sum_{b=1}^{\lambda^s nT} \mathbb{E}[\sum_{h=1}^{h_b^s + \frac{nR_{c_b^s}}{mR^2}} r_b^h])^2 + \sum_{b=1}^{\lambda^s nT} \frac{\Delta^2}{4} \frac{\mathbb{E}[R_{d_b^s}]}{n} \leq c_2^s WT \log n$$

Case 1: when $h_b^s \geq \frac{nR_{c_b^s}}{mR^2}$, then

$$\sum_{b=1}^{\lambda^s nT} \frac{\Delta^2}{4} \frac{\mathbb{E}[R_{d_b^s}]}{n} + \frac{\pi\Delta^2}{2WTnmR^2} (\sum_{b=1}^{\lambda^s nT} \mathbb{E}[l_b^s])^2 \leq c_2^s WT \log n$$

$$\frac{\Delta^2 R^2}{4c_1^s n \log n} \frac{(\sum_{b=1}^{\lambda^s nT} 1)^3}{\bar{D}^s(\sum_{b=1}^{\lambda^s nT} (\mathbb{E}[l_b^s] + \frac{mR^2}{n^2}))^2} + \frac{\pi\Delta^2}{2WTnmR^2} (\sum_{b=1}^{\lambda^s nT} \mathbb{E}[l_b^s])^2 \leq c_2^s WT \log n$$

If $\sum_{b=1}^{\lambda^s nT} [l_b^s] < \lambda^s mR^2 T/n$,

$$\frac{\Delta^2 R^2}{4c_1^s n \log n} \frac{(\lambda^s nT)^3 n^2}{\bar{D}^s(\lambda^s mR^2 T)^2} \leq c_2^s WT \log n$$

$$\frac{\Delta^2 \lambda^s n^4 T}{4c_1^s \bar{D}^s m^2 R^2 \log n} \leq c_2^s WT \log n$$

$$\lambda^s \leq \frac{4c_1^s c_2^s WT \bar{D}^s m^2 R^2 \log^2 n}{\Delta^2 n^4 T} \tag{7}$$

If $\sum_{b=1}^{\lambda^s nT} [l_b^s] \geq \lambda^s mR^2 T/n$,

$$\frac{\Delta^2 R^2}{4c_1^s n \log n} \frac{(\sum_{b=1}^{\lambda^s nT} 1)^3}{\bar{D}^s(\sum_{b=1}^{\lambda^s nT} (\mathbb{E}[l_b^s])} + \frac{\pi\Delta^2}{2WTnmR^2} (\sum_{b=1}^{\lambda^s nT} \mathbb{E}[l_b^s])^2 \leq c_2^s WT \log n$$

$$\sqrt{\frac{\pi\Delta^2 T^2}{8c_1^s W \log n} \frac{(\lambda^s)^3 n}{m\bar{D}^s}} \leq c_2^s WT \log n \tag{8}$$

$$(\lambda^s)^3 \leq \frac{8c_1^s (c_2^s)^2 W^3 m\bar{D}^s \log^3 n}{\pi\Delta^2 n} \tag{9}$$

Case 2: when $h_b^s \leq \frac{nR_{c_b^s}}{mR^2}$, then

The hop number h_b^s for each bit will not consume the radio resource asymptoticly, it will, however, decrease the capture range and increase the delay. So we assume $h_b^s = \Theta(nR_{c_b^s}/(mR^2))$; all h_b^s and $nR_{c_b^s}/(mR^2)$ in the above Lemmas are interchangeable, as we consider asymptotic capacity and delay.

Finally compare the two inequalities (7) and (9). Inequality (9) be the upper bound for delay-throughput tradeoff when $D_{III}^s \geq D_{II}^s$.

$$(\lambda^s)^3 \leq \Theta(\frac{m\bar{D}^s}{n} \log^3 n)$$

■

Lemma 3.7: Under cluster sparse regime, when $D^s_{III} < D^s_{II}$, let $\bar{D}^s$ denote the mean delay averaged over all bits and let λ^s be the throughput of each source-destination pair. The following upper bound holds,

$$\lambda^s \leq \Theta(\frac{mR^4\bar{D}^s}{n^2}\log^3 n)$$

Proof: From Lemma 3.2, when $D^s_{III} < D^s_{II}$, we have

$$c_1^s \log n \mathbb{E}[D_b^s] \leq \frac{n}{R^2\mathbb{E}[R_{cb}^s]}$$
$$\sum_{b=1}^{\lambda^s nT}\mathbb{E}[R_{cb}^s] \geq \frac{1}{c_1^s\log n}\sum_{b=1}^{\lambda^s nT}\frac{n}{R^2\mathbb{E}[D_b^s]}$$
$$\geq \frac{n}{c_1^s \log n R^2}\frac{(\sum_{b=1}^{\lambda^s nT}1)^2}{\sum_{b=1}^{\lambda^s nT}\mathbb{E}[D_b^s]} \quad (10)$$
$$= \frac{n(\sum_{b=1}^{\lambda^s nT}1)}{c_1^s\log n R^2 \bar{D}^s} \quad (11)$$

Inequality 10 is deduced using Jensen's Inequality. From Lemma 3.3 and assume $h_b^s = n^\gamma n R_{cb}^s/(mR^2)$, $(1 \leq h_b^s \leq n/m)$ we get

$$\frac{\pi\Delta^2}{4mnR^2}\sum_{b=1}^{\lambda^s nT}\mathbb{E}[\sum_{h=1}^{\frac{(1+n^\gamma)nR_{cb}^s}{mR^2}} n{r_b^h}^2] + \sum_{b=1}^{\lambda^s nT}\frac{\Delta^2}{4}\frac{\mathbb{E}[R_{db}^s]}{n} \leq c_2^s WT\log n$$

$$\frac{\pi\Delta^2 n}{4m^2R^4}\sum_{b=1}^{\lambda^s nT}\frac{(1+n^\gamma)\mathbb{E}[R_{cb}^s{r_b^h}^2]}{\log n} + \sum_{b=1}^{\lambda^s nT}\frac{\Delta^2}{4}\frac{\mathbb{E}[R_{db}^s]}{n} \leq c_2^s WT\log n \quad (12)$$

$$\frac{\pi\Delta^2 n}{4m^2R^4}\sum_{b=1}^{\lambda^s nT}\frac{(1+n^\gamma)\mathbb{E}[R_{cb}^s]\mathbb{E}[r_b^h]^2}{\log n} + \sum_{b=1}^{\lambda^s nT}\frac{\Delta^2}{4}\frac{\mathbb{E}[R_{db}^s]}{n} \leq c_2^s WT\log n \quad (13)$$

Inequality (12) using Chernoff bound and Inequality (13) using Hölder's Inequality. If the first term in Inequality (13) domain, using Inequality (11)

$$\frac{\pi\Delta^2 n(1+n^\gamma)}{4m^2R^4\log^2 n}\frac{n\lambda^s nT}{c_1^s R^2\bar{D}^s}\mathbb{E}[r_b^h]^2 \leq c_2^s WT\log n$$
$$\lambda^s \leq \frac{4c_1^sc_2^sWT}{\pi\Delta^2}\frac{m^2R^6\bar{D}^s}{n^3} \times\frac{\log^3 n}{(1+n^\gamma)\mathbb{E}[r_b^h]} \quad (14)$$

Less the $\mathbb{E}[r_b^s]$ and γ are, better the tradeoff will be, $\mathbb{E}[r_b^s]$, however has a minimum $\Theta(\sqrt{m/n}R)$. Because small $\mathbb{E}[r_b^s]$ will cause connectivity problem [1]. The Inequality (14) become

$$\lambda^s \leq \frac{4c_1^sc_2^sWT}{\pi\Delta^2}\frac{mR^4\bar{D}^s\log^3 n}{n^2}$$
$$\lambda^s \leq \Theta(\frac{mR^4\bar{D}^s}{n^2}\log^3 n) \quad (15)$$

If the second term in Inequality (13) domain, it is easy to get $\lambda^s \leq o(\frac{mR^4\bar{D}^s}{n^2}\log^3 n)$. Then we get the result Inequality (15) ■

Theorem 3.1: Under cluster sparse regime, let $\bar{D}^s$ denote the mean delay averaged over all bits and let λ^s be the throughput of each source-destination pair. The following upper bound holds,

$$\begin{cases} (\lambda^s)^3 \leq \Theta(\frac{m\bar{D}^s}{n}\log^3 n) & D^s_{III} \geq D^s_{II} \\ \lambda^s \leq \Theta(\frac{mR^4\bar{D}^s}{n^2}\log^3 n) & D^s_{III} < D^s_{II} \end{cases}$$

Proof: Using Lemma 3.6 and Lemma 3.7, we can get the Theorem directly ■

IV. Detailed upper bound of the cluster sparse regime

In this section, we will develop a achievable lower bound that is close to upper bound. The study in upper bound help us achieve this target.

A. *Optimal values of key parameters*

We assume that the mean delay is $\Theta(n^d)$. By Theorem 3.1, we will have

TABLE II: The order of the optimal values of the parameters under cluster sparse regime when $D^s_{III} \geq D^s_{II}$.

${R_d}_b^s$: # of Intra-cluster duplications of C_d	$\Theta(n^{\frac{1-d-v}{3}})$
${R_c}_b^s$: # of Inter-cluster duplications	$\Theta(n^{1-d-2\beta}/\log n)$
l_b^s: Capture Range	$\Theta(n^{\frac{v+6\beta-2d-1}{6}}/\log^{\frac{1}{2}} n)$
h_b^s: # of Hops	$\Theta(n^{\frac{1-v-d}{3}}/\log n)$
r_b^h: Transmission range of Each Hop	$\Theta(n^{\frac{v-1+2\beta}{2}}\log^{\frac{1}{2}} n)$

TABLE III: The order of the optimal values of the parameters under cluster sparse regime when $D^s_{III} < D^s_{II}$.

${R_d}_b^s$: # of Intra-cluster duplications of C_d	$\Theta(n^{2-v-4\beta-d}/\log^3 n)$
${R_c}_b^s$: # of Inter-cluster duplications	$\Theta(n^{1-d-2\beta}/\log n)$
l_b^s: Capture Range	$\Theta(\min\{R, n^{\frac{3-v-6\beta-2d}{2}}/\log n)$
h_b^s: # of Hops	$\Theta(\min\{n^{\frac{1-v}{2}}, n^{2-v-4\beta-d}/\log n\})$
r_b^h: Transmission range of Each Hop	$\Theta(n^{\frac{v-1+2\beta}{2}})$

In order to get the tight upper bound of tradeoff, Inequality (2), (4), (5) and (8) should get equality, and some constrains with l_b^s and r_b^h should be considered. By solving these equations making inequalities tight with constraints, we can get the optimal value for key parameters. As these process are trivial, we omit it for simplification and show Table II and Table III directly.

B. Detailed tradeoff with optimal values

In this section, we will get a detailed picture about the tradeoff with the optimal value of key parameters. As a blurry separation ($D_{III}^s \geq D_{II}^s$ and $D_{III}^s < D_{II}^s$) is used in Theorem 3.1, which isn't a intuitional expression, we will use the value of key parameter to decide the precise separation of our upper bound.

The key parameters suffer some common constraints, $R_{cb}^{\ s} \leq m$, $R_{db}^{\ s} \leq q$, $l_b^s \leq R$ and $h_b^s \geq 1$. Other constraints are different for two situations $D_{III}^s \geq D_{II}^s$ and $D_{III}^s < D_{II}^s$, so we will discuss them separately.

Case 1: When $D_{III}^s \geq D_{II}^s$

We solve the common constraints with the value of key parameters in Table II, discarding the meaningless result. We have $d \geq -v - 2\beta$ and $d \leq 1 - v$, which are equivalent to $\bar{D^s} \geq n/(mR^2)$ and $\bar{D^s} \leq n/m$. These two are the nature lower bound [2] and upper bound of $\bar{D^s}$ for our cluster spare regime.

However there exist two other constraints. One is $D_{III}^s \geq D_{II}^s$ and the other is $h_b^s \geq nR_{cb}^{\ s}/(mR^2)$. The first come also into the nature bound $\bar{D^s} \geq n/(mR^2)$ and the second one come into $d \geq 5/2 - v - 6\beta$, which is one of our targets. So our tradeoff can be partly written as

$$(\lambda^s)^3 \leq \Theta(\frac{m\bar{D^s}}{n}\log^3 n) \quad d \geq \frac{5}{2} - v - 6\beta \tag{16}$$

Case 2: when $D_{III}^s < D_{II}^s$

We omit the solution of common constraint, which become the nature bound of our network. Another constraint is $D_{III}^s < D_{II}^s$, which become $d < 5/2 - v - 6\beta$. Hence

$$\lambda^s \leq \Theta(\frac{mR^2\bar{D^s}}{n^2}\log^3 n) \quad d < \frac{5}{2} - v - 6\beta \tag{17}$$

Theorem 4.1: Under cluster sparse regime, let $\bar{D^s}$ denote the mean delay averaged over all bits and let λ^s be the throughput of each source-destination pair. Assume all the key parameters are same for all bits. The following upper bound holds,

$$\begin{cases} (\lambda^s)^3 \leq \Theta(\frac{m\bar{D^s}}{n}\log^3 n) & d \geq \frac{5}{2} - v - 6\beta \\ \lambda^s \leq \Theta(\frac{mR^4\bar{D^s}}{n^2}\log^3 n) & d < \frac{5}{2} - v - 6\beta \end{cases}$$

Proof: Using Inequality (16) and Inequality (17), we can get the Theorem directly ∎

V. Lower bound of the cluster sparse regime

We have get the upper bound and the optimal value of key parameters, so will construct a achieving scheme and prove the scheme can achieve our upper bound by only a logarithmic factor.

Tradeoff achieving scheme: We will divide our normal time slot into three subsolts. The operation of each slot are shown below.

1) The nodes (source node or duplication) create inter-cluster duplications and the destination cluster C_d receive data from inter-cluster duplication, using one hop transmission manner with transmission range r_b^h.
2) $R_{db}^{\ s}$ Intra-cluster duplications is created during this subslot, using multicast manner.
3) Intra-cluster is captured by a range l_b^s and transmit to the destination, using h_b^s-hop multihop manner.

The key parameters in our scheme using the optimal value in Table II and Table III. The operation in each slot are similar to the scheduling policy in our upper bound.

In each subslot, we tessellate the network into several cells. For each cast, we employ a cellular time-division multi-access (TDMA) transmission scheme such that each cell is scheduled to be active regularly according to cell time-slots. When a cell is activated, nodes within it are allowed to transmit to nodes inside the same cell or neighbouring cells. The TDMA transmission scheme allow each cell have a $1/c_3^s$ amount of time to transmit, where c_3^s is a constant being independent of the tessellation information. We describe how our scheme achieve the tradeoff then.

1) In the 1st subslot, we divide each cluster $\Theta(R^2)$ into $\mathbb{T}_2 = q = n^{1-v}$ equal-area cells. Assume that each message has a length of $\lambda^s/\log^2 n \leq mR^2/(nR_{cb}^{\ s})$, and all transmission are employed by one-hop unicast. So each node can transmit at least $nR_{cb}^{\ s}/(mR^2)$ messages when it has a chance to transmit Each cluster have at least a chance of $\Theta(mR^2/(n\log n))$ per time slot to communicate with other clusters, which indicates at least $R_{cb}^{\ s}/\log n$ messages can be sent per slot and network can sustain $\lambda^s/\log^2 n$ per slot throughput. If each time the network cannot sustain mR^2/n per-node throughput of inter-cluster communication, we call this *Error*$_I^s$. If a message cannot be sent to its C_d during $\Theta(D_{II}^s)$ time slots, we call this *Error*$_{II}^s$.

2) & 3) In the 2rd and 3th subslot, all messages are transmitted in their C_d. Nodes in a certain cluster follow the uniform distribution.The achievable lower bound under uniform condition have been studied widely that the network can achieve $\Theta(\lambda^s/\log n)$ throughput with $\Theta(\bar{D^s})$ delay. However, exists a problem. If different clusters overlap at a certain area, they will take turns to transmit. *Error*$_{III}^s$ denote more that c_4^s overlap at a certain area, where c_4^s is a positive number.

We start to prove three errors *Error*$_I^s$, *Error*$_{II}^s$, and *Error*$_{III}^s$ come to 0 as $n \to \infty$

Lemma 5.1: the network can sustain $mR^2/(n\log n)$ per-node throughput of inter-cluster communication as $n \to \infty$, which indicates $\mathbf{P}[Error_I^s] \to 0$, as $n \to \infty$

Proof: Let $\Lambda_i \quad (i = 1, 2, ..., n^2/(mR^2))$ be the amount of data can be transmit under a cell with area of mR^2/n in the network. And $\Lambda = \sum_{i=1}^{n^2/(mR^2)} \Lambda_i$.

The probability that at least two nodes from different clusters staying in the same cell area:

$$E[\Lambda_i] = \big(1 - (1 - \frac{R^2}{n})^m\big)\big(1 - (1 - \frac{R^2}{n})^{m-1}\big)\big(1 - (1 - \frac{r^2}{R^2})^q\big)^2$$
$$= \frac{m^2R^4}{n^2}$$

By Chernoff bound, we can get that:

$$\mathbf{P}[\Lambda < \frac{mR^2}{\log n}] \leq \frac{1}{e^{mR^2/4}}$$

When $n \to \infty$, $\mathbf{P}[\Lambda < mR^2/\log n] \to 0$, which means that our network can at least sustain a per-node throughput of $mR^2/(n \log n)$ of inter-cluster communication. Leads to $\mathbf{P}[Error^s_{III}] \to 0$, as $n \to \infty$ ■

Lemma 5.2: Under the cluster sparse regime. A message can be sent to its C_d with delay $D^s_{II} \leq 2n/(R_c R^2)$, which indicates $\mathbf{P}[Error^s_{II}] \to 0$, as $n \to \infty$.

Proof: We have already proven that $E[D^s_{II}] = n/(R_{cb}^{\ s} R^2)$ in Lemma 3.2. Assume X^d_i be the independent random variable taking on values 0 or 1 with probability $1R_{cb}^{\ s}R^2/n$ to be 1. $X^d = \sum_{i=1}^{n/(R_{cb}^{\ s}R^2)} X^d_i$. By using multiplicative form of Chernoff bound,

$$\mathbf{P}[X^d > \frac{2n}{R_{cb}^{\ s}R^2}] < (\frac{e^2}{27})^{\frac{n}{R_{cb}^{\ s}R^2}}$$

Therefore

$$\mathbf{P}[D^s_{II} > \frac{2n}{R_{cb}^{\ s}R^2}] < (\frac{e^2}{27})^{n^{1-v-2\beta}}$$

When $n \to \infty$, $\mathbf{Pr}[D^s_{II} > 2n/(R_{cb}^{\ s}R^2)] \to 0$, which means a message can be sent to its C_d with delay $D^s_{II} \leq 2n/(R_{cb}^{\ s}R^2)$, indicating $\mathbf{P}[Error^s_{III}] \to 0$, as $n \to \infty$ ■

Lemma 5.3: Under cluster-sparse regime, each time slot network have no more than c^s_4 number of cluster overlap, where c^s_4 is a constant being independent of system parameter, which indicates $\mathbf{P}[Error^s_{III}] \to 0$, as $n \to \infty$.

Proof: From Fig 1, we know that if two cluster have a overlap part, their cluster center must stay in an circle with radius R. Using Chernoff's bound, let $X^o = \sum_{i=1}^m X^o_i$ be a random variable, with parameter m and R^2/n (the probability of success of each X^o_i).

$$\mathbf{P}[X^o > c_2] < e^{-\frac{c_4^s 2n}{2mR^2}}$$

With $n \to \infty$, $\mathbf{P}[X^o > c^s_4] \to 0$. The overlap in cluster sparse regime only affect the tradeoff with a constant factor, which indicates $\mathbf{P}[Error^s_{III}] \to 0$, as $n \to \infty$. ■

$\mathbf{P}[Error^s_I]$, $\mathbf{P}[Error^s_{II}]$, and $\mathbf{P}[Error^s_{III}]$ all come to 0, as $n \to \infty$. The following theorem holds.

Theorem 5.1: The above scheme allows each node get a throughput of $\Theta(\lambda^s/\log^2 n)$ with delay $\Theta(\bar{D}^s)$. The probability come to 1, as $n \to \infty$.

VI. Upper bound of the cluster dense regime

Cluster dense regime (i.e. $v + 2\beta > 1$) shows a different property from that under cluster sparse regime. The clusters here have a high probability to overlap. [4] tells us that every point in the network is covered by $\Theta(mR^2/n) = \Theta(n^{v+2\beta-1})$ clusters w.h.p, which indicates that nodes are almost distributed uniformly over the network domain. So the mean distance between two closet nodes is $\Theta(1)$. However it isn't truly uniform distribution indeed. The correlated mobility model show some special phenomena which improve the delay-throughput tradeoff.

A. Scheduling policy

In this section, we will first show some special phenomena under cluster dense regime. Then we will use these phenomena to design our scheduling policy.

The phenomena that every point in the network is covered by $\Theta(mR^2/n) = \Theta(n^{v+2\beta-1})$ clusters w.h.p seems is helpful for us to create inter-cluster duplications under cluster dense regime. However it is not.

Lemma 6.1: Under cluster dense regime, an area of $\Theta(R^2)$ is covered by $\Theta(mR^2/n)$ cluster.

Proof: We assume X^{oc}_i denote the event that a cluster and a certain area $\Theta(R^2)$ in the network overlap.and $X^{oc} = \sum_{i=1}^m X^{oc}_i$. From Fig. 1, we can get

$$\mathbf{P}[X^{oc}_i = 1] = \frac{(2R+r)^2}{n} = \Theta(\frac{R^2}{n})$$

Using the multiplicative form of chernoff bound, we have

$$\mathbf{P}[X^{oc} > \frac{2mR^2}{n}] < (\frac{e}{27})^{\frac{mR^2}{n}}$$
$$\mathbf{P}[X^{oc} < \frac{\frac{mR^2}{2}}{n}] < e^{-\frac{mR^2}{8n}}$$

An area of $\Theta(R^2)$ is covered by $\Theta(mR^2/n)$ cluster come to probability 1 when $n \to \infty$ ■

Lemma 6.2: The probability that a source cluster send a message to a certain cluster is independent of the number of nodes in source cluster containing the message, assuming transmitting range $r = o(R)$.

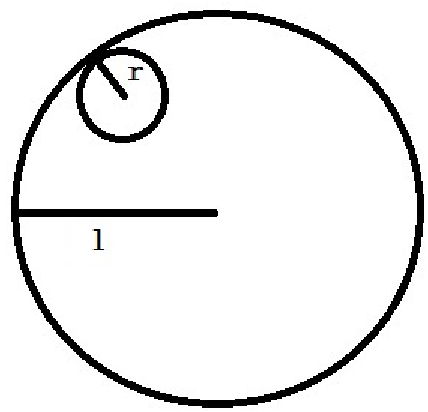

Fig. 2: Upper bound of inter-cluster transmission when only one node in source cluster contain message.

Proof: Fig.1 shows the situation where all nodes in source cluster contain message. The probability become $\Theta((2R + r)^2/n) = \Theta(R^2/n)$

Fig.2 shows the situation where only node in source cluster contains message. The probability become $\Theta((R-r)^2/n) = \Theta(R^2/n)$ ■

These two properties discourage us if we want to create inter-cluster duplication with traditional broadcast or one-hop unicast manner. The latter one fail to utilize the cluster overlap under cluster dense regime. The former one perform bad if we want to create more than mR^2/n inter-cluster duplications. Unless we set the broadcast range larger than R, which is obviously a kind of wasting radio resource, we can only mR^2/n inter-cluster duplications.

So we use u time broadcast with broadcast area $A_d \in [1, mR^2/n]$. This operation perform well under cluster dense regime.

Lemma 6.3: If we have already created inter-cluster duplications in $R_x \le \Theta(m)$ cluster, each point will still be covered by at least $mR^2/(2n)$ clusters not containing duplication.

Proof: Assume $X^{ec} = \sum_{i=1}^{mR^2/n} X_i^{ec}$ is the number of cluster not containing duplications cover a certain point in the network, as there exists still $m - R_x = \Theta(m)$ cluster not containing duplication. With chernoff's bound,

$$\mathbf{Pr}[X^{ec} < \frac{mR^2}{2n}] < e^{-\frac{mR^2}{8n}}$$

So each point will still be covered by at least $mR^2/(2n)$ clusters not containing duplication, as $n \to \infty$. ■

We may create at most $\Theta(m)$ inter-cluster duplications, so u times broadcast will be effectively operated. Each time when doing broadcast, duplications and source node can cooperate to create duplications. So we can create $\Theta(A_d^u)$ inter-cluster duplications during u times broadcast, where A_d is the broadcast area.

Now we introduce the scheduling policy under cluster-dense regime:

1) Nodes containing a certain message create inter-cluster duplications with u times broadcast until it is captured .
2) When inter-cluster relays are captured by any node in C_d with range ${l_1}^d$, message will be transmitted to the node with a ${h_1}^d$-hop multihop transmission.
3) New-created relay in C_d create ${R_d}^d$ duplications in C_d with broadcast.
4) When ${R_d}^d$ relays are captured by the destination with range ${l_2}^d$, the message will be transmitted to destination with a ${l_2}^d$-hop multihop transmission.

In the following analysis, we divide our schedule into two parts. One is 1)-2) (Part I) and the other is 3)-4) (Part II). We will analyse them respectively.

B. Tradeoff for delay of Part I

In this section, we will first divide the process of Part I into two parts and then find the tradeoff between delay and related scheduling parameter respectively.

Our scheduling policy can be divided into two parts by the scheduling step. D_{I1}^d stands for the delay of creating ${R_c}^d$ inter-cluster duplications, D_{II1}^d stands for the delay of ${R_c}^d$ inter-cluster duplications transmitting message to node in C_d.

D_{I1}^d is the delay for u times broadcast. ${R_c}^s = {A_d}^u$ will be created within u times ($A_d \in [1, mR^2/n]$). When ${R_c}^d = \omega(1) = n^a$ and $A_d = n^\alpha$, where a and α are two constants greater than 0. Hence

$$u = \frac{\log n^a}{\log n^\alpha} = \frac{a}{\alpha} = \Theta(1)$$

When ${R_c}^d = \Theta(1)$, we let broadcast area $A_d = \Theta({R_c}^d)$, so D_{I1}^d is still bounded by $\Theta(1)$, which indicates the D_{I1}^d is negligible.

D_{II}^d is the delay for nodes in C_d catch one of the relays, so

$$\begin{aligned} D_{II1}^d &= \frac{1}{(1-(1-\frac{R^2}{n})^{{R_c}^d})\frac{{l_1^d}^2 n/m}{R^2}} \\ &\le \frac{m}{{R_c}^d {l_1^d}^2} \end{aligned}$$

Even we use the "opportunistic duplication scheme", delay can only be improved with a $\log n$ factor.

Lemma 6.4: Under the cluster dense regime, the delay for a particular bit b of Part I and its scheduling parameters comply the following inequality

$$c_1^d \log n \mathbb{E}[D_{b1}^s] \le \frac{m}{\mathbb{E}[{R_c}_b^d]\mathbb{E}[{h_1}_b^d + \frac{1}{n^2}]^2} \quad (18)$$

where c_1^d is a positive constant and variable X_b^d denote the variable X under cluster dense regime for a particular bit b. The proof of Lemma 6.4 is similar to Appendix A, so we omit it for simplification.

C. Tradeoff for Radio Resource, Half Duplex and Mutihop of Part I

The radio resource of u times broadcast is similar to traditional broadcast. So we can get the tradeoff as that under cluster sparse regime.

Lemma 6.5: Under cluster dense regime and concerning radio resource, the following inequality holds

$$\sum_{b=1}^{\lambda_1^d nT} \frac{\Delta^2}{4}\frac{\mathbb{E}[{R_c}_b^d]}{n} + \mathbb{E}[\sum_{b=1}^{\lambda_1^d nT}\sum_{h=1}^{{h_1}_b^d} \frac{\pi\Delta^2}{4}\frac{{r_b^h}^2}{n}] \le c_2^d WT \log n \quad (19)$$

where c_2^d is a positive number, ${h_1}_b^d$ is the number of transmission hops after message being captured by the node in C_d, and r_b^h is the transmission range of each hop.

Since no node can transmit and receive at the same time and over same frequency, the following inequality holds,

Lemma 6.6: The following inequality holds,

$$\sum_{b=1}^{\lambda_1^d nT}\sum_{h=1}^{{h_1}_b^d} 1 \le \frac{WT}{2} n \quad (20)$$

The following inequality holds for the nature of multihop.

Lemma 6.7: The following inequality holds,

$$\sum_{b=1}^{\lambda_1^d nT}\sum_{h=1}^{{h_1}_b^d} r_b^h \ge {l_1}_b^d \quad (21)$$

D. Detailed upper bound on delay-throughput tradeoff of Part I

In this section, we will first derive the one part of tradeoff on the basis of fundamental tradeoff. We then get the optimal value of key parameters. We finally get the other part of tradeoff by optimal value and constraint of the key parameters.

From Lemma 6.4, we have

$$\begin{aligned}\sum_{b=1}^{\lambda_1^d nT}\mathbb{E}[R_{cb}^{\ d}] &\geq \frac{1}{c_1^d\log n}\sum_{b=1}^{\lambda_1^d nT}\frac{m}{(\mathbb{E}[l_{1b}^{\ s}]+\frac{1}{n^2})^2\mathbb{E}[D_{1b}^d]}\\ &\geq \frac{m}{c_1^d\log n}\frac{\sum_{b=1}^{\lambda_1^d nT}1}{\sum_{b=1}^{\lambda_1^d nT}\mathbb{E}[D_{1b}^d]}\\ &\quad\times\frac{(\sum_{b=1}^{\lambda_1^d nT}1)^3}{(\sum_{b=1}^{\lambda_1^d nT}(\mathbb{E}[l_{1b}^{\ d}]+\frac{1}{n^2}))^2} \qquad (22)\\ &= \frac{m}{c_1^d\log n}\frac{(\sum_{b=1}^{\lambda_1^d nT}1)^3}{\bar{D}_1^s(\sum_{b=1}^{\lambda_1^s nT}(\mathbb{E}[l_{1b}^{\ s}]+\frac{1}{n^2}))^2}\end{aligned}$$

Inequality (22) is deduced by using Jensen's Inequality and Hölder's Inequality. From Lemma 6.5 and Cauchy-Schwartz inequality, we get

$$\sum_{b=1}^{\lambda_1^d nT}\frac{\Delta^2}{4}\frac{\mathbb{E}[R_{cb}^{\ d}]}{n}+\frac{\pi\Delta^2}{2WTn^2}(\sum_{b=1}^{\lambda_1^d nT}\mathbb{E}[l_{1b}^{\ d}])^2 \leq c_2^d WT\log n$$

$$\frac{\Delta^2 m}{4c_1^d n\log n}\frac{(\sum_{b=1}^{\lambda_1^d nT}1)^3}{\bar{D}_1^d(\sum_{b=1}^{\lambda_1^d nT}(\mathbb{E}[l_{1b}^{\ d}]+\frac{1}{n^2}))^2}+\frac{\pi\Delta^2}{2WTn^2}(\sum_{b=1}^{\lambda_1^d nT}\mathbb{E}[l_{1b}^{\ d}])^2 \leq c_2^s WT\log n$$

If $\sum_{b=1}^{\lambda_1^d nT}[l_{1b}^{\ d}] < \lambda_1^d T/n$,

$$\frac{\Delta^2 m}{4c_1^d n\log n}\frac{(\lambda_1^d nT)^3 n^2}{\bar{D}_1^d(\lambda_1^d T)^2} \leq c_2^d WT\log n$$

$$\lambda_1^d \leq \frac{4c_1^d c_2^d WT\bar{D}_1^d\log^2 n}{\Delta^2 n^5 mT} \qquad (23)$$

If $\sum_{b=1}^{\lambda_1^d nT}[l_{1b}^{\ d}] \geq \lambda_1^d T/n$,

$$\frac{\Delta^2 m}{4c_1^d n\log n}\frac{(\sum_{b=1}^{\lambda_1^d nT}1)^3}{\bar{D}_1^d(\sum_{b=1}^{\lambda_1^d nT}(\mathbb{E}[l_{1b}^{\ d}])^2}+\frac{\pi\Delta^2}{2WTn^2}(\sum_{b=1}^{\lambda_1^d nT}\mathbb{E}[l_{1b}^{\ d}])^2 \leq c_2^d WT\log n$$

$$\sqrt{\frac{\pi\Delta^2 T^2}{8c_1^d W\log n}\frac{(\lambda_1^d)^3 m}{\bar{D}_1^d}} \leq c_2^d WT\log n \qquad (24)$$

$$(\lambda_1^d)^3 \leq \frac{8c_1^d {c_2^d}^2 W^3\bar{D}_1^s\log^3 n}{\pi\Delta^2 m} \qquad (25)$$

Compare the two inequalities (23) and (25), hence

$$(\lambda_1^d)^3 \leq \Theta(\frac{\bar{D}_1^d}{m}\log^3 n)$$

But it is the final result for delay-throughput tradeoff under cluster dense regime. We assume that the mean delay is $\Theta(n^d)$. In order to get the tight upper bound of the tradeoff, Inequality (18), (20), (21) and (24) should get equality.

TABLE IV: The order of the optimal values of the parameters under Part I of cluster dense regime I.

$R_{cb}^{\ s}$: # of Inter-cluster duplications	$\Theta(n^{\frac{v-d}{3}}/\log n)$
$l_{1b}^{\ s}$: Capture Range	$\Theta(n^{\frac{v-d}{3}}/\log^{\frac{1}{2}} n)$
$h_{1b}^{\ s}$: # of Hops	$\Theta(n^{\frac{v-d}{3}}/\log n)$
r_b^h: Transmission range of Each Hop	$\Theta(1\log^{\frac{1}{2}} n)$

There exists two constraints for our key parameters $1 \leq R_{cb}^{\ d} \leq m$ and $1 \leq l_{1b}^{\ d} \leq \sqrt{mR^2/n}$. By using optimal value in Table V and omitting the logarithmic factor, we get $d \geq (3-v-6\beta)/2$. That means that if we reach $d < (3-v-6\beta)/2$, $l_{1b}^{\ d} = \sqrt{mR^2/n}$ should maintain. Then Lemma 6.4 become

$$c_1^d\log n\mathbb{E}[D_{b1}^s] \leq \frac{n}{\mathbb{E}[R_{cb}^{\ d}]R^2}$$

And Lemma 6.5 comes into

$$\sum_{b=1}^{\lambda_1^d nT}\frac{\Delta^2}{4}\frac{\mathbb{E}[R_{cb}^{\ d}]}{n}+\frac{\pi\Delta^2}{2WTn^2}(\sum_{b=1}^{\lambda_1^d nT}\sqrt{\frac{mR^2}{n}})^2 \leq c_2^d WT\log n$$

$$\Theta(\frac{\lambda_1^d n}{R^2\bar{D}_1^d\log^2 n}+\frac{(\lambda_1^d)^2 mR^2}{n\log^2 n}) \leq \log n$$

$$\lambda_1^d \leq \frac{R^2\bar{D}_1^d}{n}\log^3 n$$

TABLE V: The order of the optimal values of the parameters under Part I of cluster dense regime II.

$R_{cb}^{\ s}$: # of Inter-cluster duplications	$\Theta(n^{1-2\beta-d}/\log n)$
$l_{1b}^{\ s}$: Capture Range	$\Theta(n^{\frac{v+2\beta-1}{2}}/\log^{\frac{1}{2}} n)$
$h_{1b}^{\ s}$: # of Hops	$\Theta(n^{\frac{v+2\beta-1}{2}}/\log n)$
r_b^h: Transmission range of Each Hop	$\Theta(1\log^{\frac{1}{2}} n)$

Theorem 6.1: Under cluster sparse regime, let $\bar{D}_1^d$ denote the mean delay averaged over all bits and let λ_1^d be the throughput of each source-destination pair. Assume all the key parameters are same for all bits. In Part I, the following upper bound holds,

$$\begin{cases}(\lambda_1^d)^3 \leq \Theta(\frac{\bar{D}_1^d}{m}\log^3 n) & d \geq \frac{3-v-6\beta}{2}\beta\\ \lambda_1^d \leq \Theta(\frac{R^2\bar{D}_1^d}{n}\log^3 n) & d < \frac{3-v-6\beta}{2}\beta\end{cases}$$

E. Tradeoff for delay of Part II

D_{I2}^d is the delay for R_d^d intra-cluster duplications being captured by the destination with range l_2^d. We can get

$$D_{I2}^d = \frac{1}{(1-(1-\frac{{l_2^d}^2}{R^2})^{{R_d}^d})} \le \frac{R^2}{{R_d}^d {l_2^d}^2}$$

Even we use the opportunistic duplication scheme, delay can only be improved with a $\log n$ factor.

Lemma 6.8: Under the cluster dense regime, the delay for a particular bit b of Part II and its scheduling parameters comply the following inequality

$$c_3^d \log n \mathbb{E}[D_{b2}^s] \le \frac{R^2}{\mathbb{E}[{R_d}_b^d]\mathbb{E}[{l_2}_b^d + \frac{1}{n^2}]^2} \quad (26)$$

where c_1^d is a positive constant and variable X_b^s denote the variable X under cluster dense regime for a particular bit b.

F. Tradeoff for Radio Resource, Half Duplex and Mutihop of Part II

The radio resource and some key parameters follow the tradeoff

Lemma 6.9: Under cluster dense regime and concerning radio resource, the following inequality holds

$$\sum_{b=1}^{\lambda_2^d nT} \frac{\Delta^2}{4} \frac{\frac{mR^2}{n}\mathbb{E}[{R_d}_b^d]}{n} + \mathbb{E}[\sum_{b=1}^{\lambda_2^d nT}\sum_{h=1}^{{h_2}_b^d} \frac{\pi\Delta^2}{4}\frac{{r_b^h}^2}{n}] \le c_4^d WT \log n \quad (27)$$

where c_4^d is a positive number, ${h_2}_b^s$ is the number of transmission hops after message being captured by the destination, and r_b^h is the transmission range of each hop.

Since no node can transmit and receive at the same time and over same frequency, the following inequality holds,

Lemma 6.10: The following inequality holds,

$$\sum_{b=1}^{\lambda_2^d nT}\sum_{h=1}^{{h_2}_b^d} 1 \le \frac{WT}{2} n \quad (28)$$

The following inequality holds for the nature of multihop.

Lemma 6.11: The following inequality holds,

$$\sum_{b=1}^{\lambda_2^d nT}\sum_{h=1}^{{h_2}_b^d} r_b^h \ge {l_2}_b^d \quad (29)$$

G. Detailed upper bound on delay-throughput tradeoff of Part II

In this section, we will first derive the one part of tradeoff on the basis of fundamental tradeoff. We then get the optimal value of key parameters. We finally get the other part of tradeoff by optimal value and constraint of the key parameters.

From Lemma 6.8, we have

$$\sum_{b=1}^{\lambda_2^d nT} \mathbb{E}[{R_d}_b^d] \ge \frac{1}{c_3^d \log n}\sum_{b=1}^{\lambda_2^d nT} \frac{R^2}{(\mathbb{E}[{l_2}_b^s]+\frac{1}{n^2})^2 \mathbb{E}[D_{2b}^d]} \ge \frac{R^2}{c_3^d \log n}\frac{(\sum_{b=1}^{\lambda_2^d nT} 1)^3}{\bar{D}_2^s(\sum_{b=1}^{\lambda_2^s nT}(\mathbb{E}[{l_2}_b^s]+\frac{1}{n^2}))^2}$$

From Lemma 6.9 and Cauchy-Schwartz inequality, we get

$$\sum_{b=1}^{\lambda_2^d nT}\frac{\Delta^2}{4}\frac{mR^2\mathbb{E}[{R_d}_b^d]}{n^2} + \frac{\pi\Delta^2}{2WTn^2}(\sum_{b=1}^{\lambda_2^d nT}\mathbb{E}[{l_2}_b^d])^2 \le c_4^d WT\log n$$

$$\frac{\Delta^2 mR^4}{4c_3^d n^2 \log n}\frac{(\sum_{b=1}^{\lambda_2^d nT} 1)^3}{\bar{D}_2^d(\sum_{b=1}^{\lambda_2^d nT}(\mathbb{E}[{l_2}_b^d]+\frac{1}{n^2}))^2} + \frac{\pi\Delta^2}{2WTn^2}(\sum_{b=1}^{\lambda_2^d nT}\mathbb{E}[{l_2}_b^d])^2 \le c_4^s WT\log n$$

If $\sum_{b=1}^{\lambda_2^d nT}[{l_2}_b^d] < \lambda_2^d T/n$,

$$\frac{\Delta^2 mR^4}{4c_3^d n^2 \log n}\frac{(\lambda_2^d nT)^3 n^2}{\bar{D}_2^d(\lambda^d T)^2} \le c_4^d WT \log n$$

$$\lambda_2^d \le \frac{4c_3^d c_4^d WT\bar{D}_1^d \log^2 n}{\Delta^2 n^4 mR^2 T} \quad (30)$$

If $\sum_{b=1}^{\lambda_2^d nT}[{l_2}_b^d] \ge \lambda_2^d T/n$,

$$\frac{\Delta^2 mR^2}{4c_1^d n^2 \log n}\frac{(\sum_{b=1}^{\lambda_2^d nT} 1)^3}{\bar{D}_2^d(\sum_{b=1}^{\lambda_2^d nT}(\mathbb{E}[{l_2}_b^d])^2} + \frac{\pi\Delta^2}{2WTn^2}(\sum_{b=1}^{\lambda_2^d nT}\mathbb{E}[{l_2}_b^d])^2 \le c_4^d WT\log n$$

$$\sqrt{\frac{\pi\Delta^2 T^2}{8c_3^d W\log n}\frac{(\lambda_2^d)^3 mR^2}{n\bar{D}_2^d}} \le c_4^d WT\log n \quad (31)$$

$$(\lambda_2^d)^3 \le \frac{8c_3^d {c_4^d}^2 W^3 n\bar{D}_2^s \log^3 n}{\pi\Delta^2 mR^2} \quad (32)$$

Compare the two inequalities (30) and (32), hence

$$(\lambda_1^d)^3 \le \Theta(\frac{n\bar{D}_1^d}{mR^2}\log^3 n)$$

But it is the final result for delay-throughput tradeoff under cluster dense regime. We assume that the mean delay is $\Theta(n^d)$. In order to get the tight upper bound of the tradeoff, Inequality (26), (28), (29) and (31) should get equality.

There exists two constraints for our key parameters $1 \le {R_d}_b^d \le n/m$ and $1 \le {l_2}_b^d \le R$. By using optimal value in Table VI and omitting the logarithmic factor, we get $d \le 2-2v-2\beta$. That means that if we reach $d > 2-2v-2\beta$, ${R_d}_b^d = 1$ should maintain. Then Lemma 6.8 become

$$c_3^d \log n\mathbb{E}[D_{b2}^s] \le \frac{R^2}{\mathbb{E}[{l_2}_b^d]^2}$$

And Lemma 6.9 comes into

$$\sum_{b=1}^{\lambda_2^d nT} \frac{\Delta^2}{4}\frac{mR^2}{n^2} + \frac{\pi\Delta^2}{2WTn^2}(\sum_{b=1}^{\lambda_2^d nT} l_{2b}^{h})^2 \leq c_4^d WT\log n$$

$$\Theta(\frac{(\lambda_2^d)^2R^2}{\bar{D}_1^d\log^2 n} + \frac{\lambda_2^d mR^2}{n\log^2 n}) \leq \log n$$

$$(\lambda_2^d)^2 \leq \frac{\bar{D}_2^d}{R^2}\log^3 n$$

TABLE VI: The order of the optimal values of the parameters under Part II of cluster dense regime I.

${R_d}_b^s$: # of Inter-cluster duplications	$\Theta(n^{\frac{2-2v-2\beta-d}{3}}/\log n)$
${l_2}_b^s$: Capture Range	$\Theta(n^{\frac{v+4\beta-1-d}{3}}/\log^{\frac{1}{2}} n)$
${h_2}_b^s$: # of Hops	$\Theta(n^{\frac{v+4\beta-1-d}{3}}/\log n)$
r_b^h: Transmission range of Each Hop	$\Theta(1\log^{\frac{1}{2}} n)$

TABLE VII: The order of the optimal values of the parameters under Part II of cluster dense regime II.

${R_d}_b^s$: # of Inter-cluster duplications	$\Theta(1)$
${l_2}_b^s$: Capture Range	$\Theta(n^{\frac{2\beta-d}{2}}/\log^{\frac{1}{2}} n)$
${h_2}_b^s$: # of Hops	$\Theta(n^{\frac{2\beta-d}{2}}/\log n)$
r_b^h: Transmission range of Each Hop	$\Theta(1\log^{\frac{1}{2}} n)$

Theorem 6.2: Under cluster sparse regime, let $\bar{D}_2^d$ denote the mean delay averaged over all bits and let λ_2^d be the throughput of each source-destination pair. Assume all the key parameters are same for all bits. In Part II, the following upper bound holds,

$$\begin{cases} (\lambda_2^d)^2 \leq \Theta(\frac{\bar{D}_2^d}{R^2}\log^3 n) & d \geq 2-2v-2\beta \\ (\lambda_2^d)^3 \leq \Theta(\frac{n\bar{D}_1^d}{mR^4}\log^3 n) & d < 2-2v-2\beta \end{cases}$$

H. Overall Upper bound of delay-throughput tradeoff

Theorem 6.1 and 6.2 show us the tradeoff of Part I and Part II. We assume $\bar{D}_1^d = \bar{D}_2^d = \bar{D}^d$. Then the overall upper bound can be derived easily.

Theorem 6.3: Under cluster sparse regime, let $\bar{D}^d$ denote the mean delay averaged over all bits and let λ^d be the throughput of each source-destination pair. Assume all the key parameters are same for all bits. Assume $\bar{D}_1^d = \bar{D}_2^d = \bar{D}^d$. The following upper bound holds,

$$\lambda^d = \min\{\lambda_1^d, \lambda_2^d\}$$

We can see that if we choose the system parameter v and β carefully, the correlated mobility can perform better than the uniformly distributed one. Fig 3 show us an example of cluster-dense regime where $v = 4/9$ and $\beta = 1/3$. The blue line is λ_2^d, red line is λ_1^d, and green one is the tradeoff in [3].

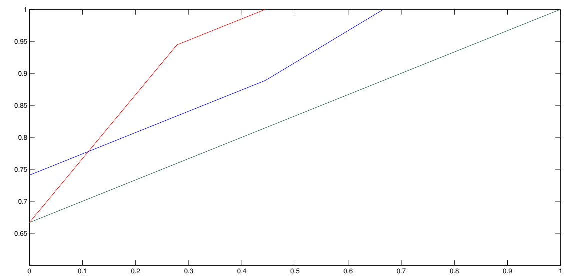

Fig. 3: Tradeoff of cluster dense communication when $v = 4/9$ $\beta = 1/3$

APPENDIX A
PROOF OF LEMMA 3.2

To simplify our proof process, some notation are needed. Under the cluster sparse regime, let $X_i^s(t)$ denote the position of node i at time slot t. Let b denote a bit message in our network. Let ${t_0}_b^s$ denote time when bit b is generated. Let $\hat{l_b^s(t)}$ denote the minimum distance form the edge of cluster containing duplication nodes (inter cluster duplication) to the edge of C_d at time slot t, and $\hat{l_b^s(t)}$ can be negative if inter cluster duplication and C_d overlap. Let $\hat{L_b^s(t)} = \max\{0, \hat{l_b^s(t)}\}$. Let ${R_c}_b^s(t)$ denote the number of inter cluster duplications at slot t. Let ${t_s}_b^s$ denote the time when bit b is captured by C_d.

We focus on the transmission of sending bit b from source to its C_d and $\mathbb{I}_A$ be the indicator function on set A

$$\mathbb{E}\left[\frac{n}{(R+\hat{L_b^s(t)})^2}\right] = \mathbb{E}\left[\frac{n}{R^2}\mathbb{I}_{\hat{L_b^s(t)}\leq 0}\right] + \mathbb{E}\left[\frac{n}{(R+\hat{l_b^s(t)})^2}\mathbb{I}_{\hat{L_b^s(t)}>0}\right]$$

Since the definition of expectation,

$$\mathbb{E}\left[\frac{n}{(R+\hat{l_b^s(t)})^2}\mathbb{I}_{\hat{L_b^s(t)}>0}\right]$$
$$= \int_0^{\sqrt{n}} \frac{n}{(R+u)^2} d\mathbf{P}[\hat{l_b^s(t)}) \leq u]$$
$$= 1 - \frac{n}{R^2}\mathbf{P}[\hat{l_b^s(t)} \leq 0] + \int_0^{\sqrt{n}} \frac{2n}{(R+u)^3}\mathbf{P}[\hat{l_b^s(t)} \leq u]du$$

Therefore,

$$\mathbb{E}\left[\frac{n}{(R+\hat{L_b^s(t)})^2}\right] = 1 + \int_0^{\sqrt{n}} \frac{2n}{u'^3}\mathbf{P}[R+\hat{l_b^s(t)} \leq u]du'$$
$$= 1 + \int_R^{\sqrt{n}} 2\pi {R_c}_b^s(t)\frac{(R+u')^2}{u'^3}du'$$
$$\leq 1 + 6\pi {R_c}_b^s(t)\int_R^{\sqrt{n}} \frac{1}{u'}du'$$
$$= 1 + 6\pi {R_c}_b^s(t)\log\frac{\sqrt{n}}{R}$$
$$\leq 6\pi {R_c}_b^s(t)\log n$$

We let

$$W(t) = 6\pi \log n[t - t_{0_b}^s] - \sum_{t_{0_b}^s+1}^{t} \mathbb{E}\left[\frac{n}{(R + \hat{l_b^s(t)})^2 R_{c_b}^s(t)} \mathbb{I}_{t=t_{s_b}^s}\right]$$

Then

$$\begin{aligned} &\mathbb{E}[W(t) - W(t-1)] \\ &= 6\pi \log n - \mathbb{E}\left[\frac{n}{(R + \hat{l_b^s(t)})^2 R_{c_b}^s(t)} \mathbb{I}_{t=t_{s_b}^s}\right] \\ &\leq 6\pi \log n - \mathbb{E}\left[\frac{n}{(R + \hat{l_b^s(t)})^2 R_{c_b}^s(t)}\right] \\ &\leq 0 \end{aligned}$$

which means that $W(t)$ is a sub-martingale. By the Optional Stopping Theorem [5]. We get

$$6\pi \log n \mathbb{E}[D_{II_b}^s] \geq \mathbb{E}\left[\frac{n}{(R + \hat{l_b^s(t)})^2 R_{c_b}^s}\right]$$

By Hölder's Inequality [5]

$$\begin{aligned} 6\pi \log n \mathbb{E}[D_{II_b}^s] &\geq \frac{n}{\mathbb{E}^2[R + \hat{l_b^s(t)}]\mathbb{E}[R_{c_b}^s]} \\ &\geq \frac{n}{(2R + r_b^s)^2 \mathbb{E}[R_{c_b}^s]} \end{aligned}$$

Therefore,

$$54\pi \log n \mathbb{E}[D_{II_b}^s] \geq \frac{n}{R^2 \mathbb{E}[R_{c_b}^s]} \tag{33}$$

The part for $D_{III_b}^s$ is similar as [3], so we directly give the result:

$$8\pi \log n \mathbb{E}[D_{III_b}^s] \geq \frac{R^2}{\mathbb{E}[R_{d_b}^s]\mathbb{E}[l_b^s + \frac{mR^2}{n^2}]^2} \tag{34}$$

Inequality (33) and (34) lead to the Lemma 3.2 directly

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