Group Multicast Capacity in Large Scale Wireless Networks

Xican Yang[1,2], Jinbei Zhang[1], Jian Li[1]

[1]Department of Electronic Engineering, Shanghai Jiao Tong University
[2]The State Key Laboratory of Integrated Services Networks, Xidian University, China
Email: [1]{09420902150876, abelchina, fighting0818}@sjtu.edu.cn

Abstract. In this paper, we investigate the impact of group multicast on the capacity of large-scale random wireless networks. n nodes are randomly distributed in the networks, among which n^s nodes are selected as sources and n^d destined nodes are chosen for each. Specifically, we consider two different scenarios, i.e., (1) regular distribution scenario, and (2) random distribution scenario. The upper bound capacity of group multicast is derived for the network. Furthermore, we propose the corresponding capacity-achieving communication schemes to achieve the upper bound. Moreover, our study is the first attempt to understand how group multicast may impact on large scale network capacity from a theoretical perspective.

1 Introduction

Since the seminal work by Kumar et al. [1], which showed that the optimal unicast per-node capacity is $O(1/\sqrt{n})$, the fundamental capacity research about wireless ad hoc networks has drawn tremendous interest. Later on, Grossglauser and Tse [2] proposed a 2-hop relaying algorithm in which nodes are allowed to move, and they demonstrated that $\Theta(1)$ capacity per source-destination is achievable but packets have to endure a larger delay of $\Omega(n)$. Since then, how to improve the network performance, in terms of the capacity and delay, has become a critical issue.

The analysis on how to improve the network performance has arised in recent years. Some works [3], [4], [5], [6], [7] investigated the improvement by introducing different kinds of mobility into the networks. Other works attempt to improve capacity by introducing base stations as infrastructure support [8], [9], [10]. Besides, there has been impressive recent works on the characterization of delay and capacity tradeoff in wireless ad hoc networks [11], [12].

However, all the above researches relay on the unicast traffic pattern. As the demand of information sharing increases rapidly, multicast traffic which genenalized both the unicast and broadcast traffic are proposed. Multicast capacity for large-scale wireless ad hoc network was first analyzed in [13]. It shows that when the number of destination nodes $k = O\left(n/\log n\right)$, the per-node multicast capacity is $\Theta(\frac{1}{\sqrt{n\log n}}\frac{W}{\sqrt{k}})$; when $k = \Omega\left(n/\log n\right)$, the per-node capacity is $\Theta\left(W/n\right)$, which is equivalent to the broadcast case. The result also implies that the per-node capacity decreases to zero as n goes to infinity. This means static ad hoc network is not scalable under unicast, multicast and broadcast traffic model. Wang et al. [14] generalized the result to anycast traffic pattern and Mao et al. [15] studied multicast networks with infrastructure support.

While unicast and multicast traffic pattern have been extensively studied in previous work, group multicast is still a relatively new concept and under active research. Group multicast refers to a traffic pattern in which data is delivered from a source to multiple destinations originazed in a multicast group, which differs from multicast in that its destinations are located in a more centralized area. Recently, many new applications appeared such as Introstate Television (TV), Stadium TV that impose stringent broadband services on group multicast.

In this paper, we have studied the theoretical group multicast capacity of wireless ad hoc networks. More precisely, we consider a wireless ad hoc network that consists of n nodes, among which n^s nodes are selected as sources and n^d destination nodes are chosen for each. Thus, n^s multicast sessions are formed. Furthermore, we assume that there are a set of n^{1-d} multicast groups $A = \{A_1, A_2, \ldots, A_{n^{1-d}}\}$. We study two kinds of common transmission scenarios, i.e., regular distribution scenario and random distribution scenario. The first type represents that n^{1-d} groups are regularly distributed in the network and each group covers the whole cell without intersections. While in the latter pattern these groups are randomly distributed in the network, there will inevitably be interferences due to the intersections between groups. The analysis method can also be applied to large scale wireless networks [16], [17], [18].

Our main contributions can be summarized as follows:

– Under the regular distribution scenario with group multicast, the capacity is $\Theta(\frac{1}{\sqrt{n^s}})$ when $s+d \leq 1$, and $\min\{\Theta(\frac{1}{\sqrt{n^s}}), \Theta(\frac{1}{n^{s+d-1}})\}$ when $s + d > 1$. While under the random distribution scenario, the capacity of wireless ad hoc network stays the same.
– To our best knowledge, this paper is the first work that characterizes the impact of group multicast on network capacity from a theoretical perspective.

The rest of this paper is organized as follows. In Section II, we describe the wireless network model. In Section III and IV, we investigate the capacity of regular distribution and random distribution group multicast respectively, and we gives the aggregated capacity of the wireless network and analyzes the results. Finally, we conclude our paper in Section V.

2 Wireless Network Model

2.1 Network Architecture

We employ the extended network model. We assume that there is a set $V = \{V_1, V_2, \cdots, V_n\}$ of n normal wireless nodes uniformly deployed in a square region with side length $\sqrt{n}$. All the wireless nodes have the same transmission power, and hence have the same transmission range r.

We further assume that there are a set of n^{1-d} multicast groups $A = \{A_1, A_2, \cdots, A_{n^{1-d}}\}$. These n^{1-d} regularly distributed groups divide the network region into n^{1-d} cells. Each group covers the whole cell and has n^d nodes in the group. When these groups are randomly distributed in the network, there will inevitably be interferences due to the intersections between the groups, making it difficult to schedule the model. We first optimally place the groups in the network region, as shown in Figure 1, to avoid the interference among the groups. And then we will discuss the random-distributed situation.

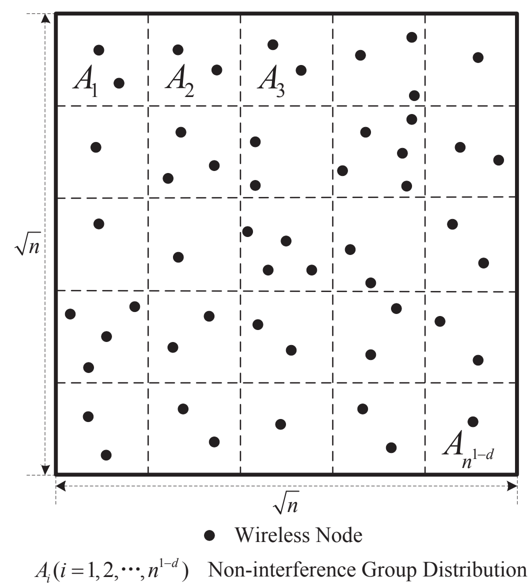

Fig. 1. Network model

2.2 Multicast Traffic Pattern

Among the n wireless nodes, a total of n^s source nodes are randomly selected, and each source node chooses a distinct group which has n^d destination nodes as a multicast session. Note that a particular group may be included by different multicast sessions as destination.

2.3 Interference Model

We employ the traditional *protocol model* in [1] as the interference model. All nodes employ a common range r for all their transmissions. When node V_i transmits to a node V_j, this transmission is successfully received by V_j if

1) The distance between V_i and V_j is no more than r, i.e.,

$$|V_i - V_j| \leq r.$$

2) For every other node V_k simultaneously transmitting,

$$|V_k - V_j| \geq (1 + \Delta)r.$$

The quantity $\Delta > 0$ is a guard zone specified by the protocol to prevent a neighboring node from transmitting at the same time.

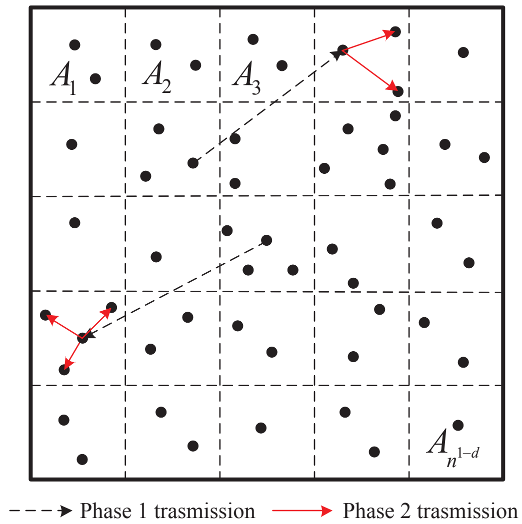

Fig. 2. Transmission mode

2.4 Definition of Capacity

Throughput: λ_i bits/sec that can be transmitted by source node V_i to its chosen n^d destination nodes on average is called the *per-node throughput*. The sum of per-node throughput over all the nodes, $\Lambda(n) = \lambda_1 + \lambda_2 + \cdots + \lambda_{n^s}$, is defined as the *throughput of the network*.

Feasible Throughput: A multicast rate vector $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)$ bits/sec is feasible if there is a spatial and temporal scheme for scheduling transmissions such that every node V_i can send λ_i bits/sec on average to its destinations. An aggregated multicast throughput $\Lambda(n)$ bits/sec is feasible if there is a feasible rate vector $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)$.

Capacity of The Network: The per-node throughput capacity is of order $O(f(n))$ bits/sec if there is a deterministic constant $c < +\infty$ such that

$$\liminf_{n\to\infty} Pr\left(\lambda(n) = cf(n) \text{ is feasible}\right) < 1$$

and is of order $\Theta(f(n))$ bits/sec if there are deterministic constants $c > 0$ and $c < c' < +\infty$ such that

$$\lim_{n\to\infty} Pr\left(\lambda(n) = cf(n) \text{ is feasible}\right) = 1$$

$$\liminf_{n\to\infty} Pr\left(\lambda(n) = c'f(n) \text{ is feasible}\right) < 1$$

3 Capacity of Regular Distribution Group Multicast

Any group A_i will be chosen by a certain source node as a destination with the probability $p = n^{d-1}$. Let N_i be the number of times when the group A_i is chosen as a destination, then clearly N_i follows the binomial distribution with parameters n^s and p. Easily we can derive

$$Pr(N_i = k) = \binom{n^s}{k} p^k (1-p)^{n^s - k},$$

and the expected value of N_i is $E[N_i] = n^s p = n^{s+d-1}$. Thus we will employ different approaches to investigate the network capacity, according to the various circumstances of $s + d$.

We can randomly choose only one node inside a group as the destination node, thus the destination nodes are less than or included in the original group nodes. We define the procedure that source nodes transmit packets to destination nodes as $phase\ one\ transmission$, then we will calculate the network capacity, which is similar to the unicast capacity. When the number of destination nodes increases, just like original situation, network performance will be inevitably worse compared to phase one. We define the process that packets are transmitted from the destination node to the whole group as $phase\ two\ transmission$, and we will study capacity of phase two transmission which is similar to the broadcast capacity. We cam easily know that the network capacity is upper-bounded both by the per-flow unicast capacity and the per-flow broadcast capacity. Thus, combining the two phases, we can get the upper bound of wireless network capacity. See Figure 2 for illustration.

3.1 When $s + d < 1$

The expected value of N_i diminishes to zero as the number of nodes is increased. Since the number of nodes goes to infinity while the product $n^s p$ remains fixed, the binomial distribution converges towards the Poisson distribution. Therefore the Poisson distribution with parameter $\lambda = n^s p = n^c$ can be used as an approximation to the binomial distribution here.

Capacity of Phase One Transmission Taking consideration of the source-destination pairs, there are totally n^s source nodes transmitting packets to their destinations. According to Kumar et al. [1], we can get the capacity of phase one transmission as

$$\lambda_1(n) = O(\frac{1}{\sqrt{n^s}}).$$

Capacity of Phase Two Transmission We will take the similar approach in [19], [20], which is similar to the well-known maximum-flow and minimum-cut theorem. Considering a time interval T which is large enough, since each node can send data at $\lambda(n)$, the total amount of packets to be delivered between all source-destination pairs during T is $c_P k\lambda(n)T$, where k is the number of simultaneous transmission pairs and positive constant $1/c_P$ is the average number of bits per packet. Besides, the total wireless channel bandwidth is fixed to W bits per second, then the total number of packets the wireless network can provide is $c_p WT$. We can derive

$$c_P k\lambda(n)T \leq c_p WT,$$

and thus the capacity of the wireless network is

$$\lambda(n) = O(\frac{W}{k}).$$

To determine the maximum flow, k, we derive the following lemma.

Lemma 1. *The number of times when any group is selected as a destination is at most constant when $s + d < 1$.*

Proof. Let K be a positive integer. As N_i follows the Poisson distribution with parameter $\lambda = n^s p = n^c$, then we can have

$$\begin{aligned} Pr(N_i \geq K) &= \sum_{k \geq K} e^{-\lambda}\frac{\lambda^k}{k!} \\ &\leq \sum_{k \geq K} e^{-\lambda}\lambda^k \\ &\leq \frac{e^{-\lambda}\lambda^K}{1-\lambda}, \end{aligned}$$

$$\begin{aligned} Pr(\max_i N_i \geq K) &\leq \sum_i Pr(N_i \geq K) \\ &\leq \frac{ne^{-\lambda}\lambda^K}{1-\lambda}. \end{aligned}$$

Since $c = s + d - 1 < 0$, $\lambda = n^c$ goes to zero when the number of nodes n goes to infinity. Then we can derive

$$\lim_{n\to\infty} Pr(\max_i N_i \geq K) \leq \lim_{n\to\infty} \frac{n^{cK+1}e^{-\lambda}}{1-\lambda} = 0$$

as long as $cK + 1 < 0$. We choose $K = 1 + [\frac{1}{1-s-d}]$.

Based on Lemma 1, we can easily get the capacity

$$\lambda_2(n) = O(\frac{W}{K}) = O(1).$$

Since the network capacity is upper-bounded both by the per-flow unicast capacity and the per-flow broadcast capacity, combining phase one transmission and phase two transmission, we can derive the capacity of regular distribution scenario as

$$\begin{aligned}\lambda(n) &= O(\min\{\Theta(\frac{1}{\sqrt{n^s}}), \Theta(1)\}) \\ &= O(\frac{1}{\sqrt{n^s}}).\end{aligned}$$

Next, we will show that the bound is achievable. To achieve this goal, we design a scheduling scheme that satisfies the following two propositions.

1. For each source node V_i, we randomly and independently select a group A_i as its destination, then packets are transmitted, through ad hoc mode, from V_i to any node $V_{i,d}$ in the group A_i.
2. $V_{i,d}$ transmits the packets to all other nodes in the group A_i.

To meet the first proposition, we follow the schedule of Franceschetti et al. [21], adopting percolation theory in routing.

To meet the second proposition, we employ flooding algorithm. Flooding, where packets from a source node is delivered to all other nodes, has extensive applicability in ad hoc wireless networks. The traditional flooding scheme generates excessive packet retransmissions, resource contention, and collisions since every node forwards the packet at least once. Recently, several flooding schemes have been proposed to avoid these problems. Kim et al. [22] proposed $PriorityForwarding$, for efficient and fast flooding operations in wireless ad hoc networks. They demonstrated that with priority checking, the host closest to the coverage perimeter of a flooding packet would forward the packet immediately without delay. Obviously, the schedule can achieve the bound, and we have

Theorem 1. *The capacity of regular distribution group multicast is $\Theta(\frac{1}{\sqrt{n^s}})$, when $s + d < 1$.*

Proof. As stated above, we design a scheduling scheme which ensures the transmissions of all nodes by a time-division multi-access (TDMA) manner such that all nodes will be able to transmit at least once in every time $\sqrt{n^s} + K$ slots, thus the capacity can achieve

$$\begin{aligned}\lambda(n) &= \Omega(\frac{1}{\sqrt{n^s} + K}) \\ &= \Omega(\frac{1}{\sqrt{n^s}}).\end{aligned}$$

3.2 When $s + d > 1$

Since the expected value of N_i goes to infinity when the number of nodes increases, the methods above is not applicable anymore. Previously each group will receive information from K source nodes at most, while when $s + d > 1$ each group has to receive packets from more than constant source nodes, then the packets will be broadcast in the group. If we select a representative node similar to the previous case, it is difficult to understand the topology of the network and devise effective routing scheme since the representative node will be chosen as a destination node for many times, which is similar to the convergecast traffic pattern.

Thus we modify the transmission mode. As shown in Figure 3, we choose n^{s+d-1} nodes in each group as representatives. Firstly packets are transmitted from each source node to any representative node in a randomly selected group, and then representative nodes employ a celluar TDMA transmission scheme, broadcasting the packets to all other nodes in the group.

Capacity of Phase One Transmission There are totally n^s source nodes transmitting packets to their destinations and the transmission mode is the same as that of Kumar et al. [1], thus the capacity of phase one transmission is

$$\lambda_1(n) = O(\frac{1}{\sqrt{n^s}}).$$

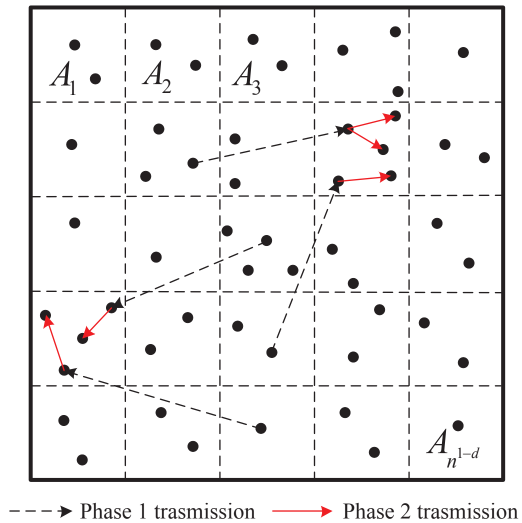

Fig. 3. Transmission mode when $s + d > 1$

Capacity of Phase Two Transmission From the above derivation we have $E[N_i] = n^s p = n^{s+d-1}$, which means that every group will be chosen as a destination n^{s+d-1} times on average. In other words, $k = n^{s+d-1}$, it follows,

$$\lambda_2(n) = O(\frac{W}{n^{s+d-1}}) = O(\frac{1}{n^{s+d-1}}).$$

Combining phase one transmission and phase two transmission, we can derive the upper bound of the capacity of regular distribution scenario as

$$\lambda(n) = O(\min\{\Theta(\frac{1}{\sqrt{n^s}}), \Theta(\frac{1}{n^{s+d-1}})\}).$$

Using similar scheduling scheme as described previously, we can achieve the following theorem.

Theorem 2. *The capacity of regular distribution group multicast is* $\min\{\Theta(\frac{1}{\sqrt{n^s}}), \Theta(\frac{1}{n^{s+d-1}})\}$*, when* $s + d > 1$*, which is shown in Figure 4.*

Proof. We choose n^{s+d-1} nodes in each group as representatives. Firstly packets are transmitted from each source node to any representative node in a randomly selected group through ad hoc mode adopting percolation theory in routing, and then representative nodes employ the flooding scheme, broadcasting the packets to all other nodes in the group. This can be accomplished by adopting a TDMA scheme so that the time-slot is further divided into sub packet time-slots, and each node can be scheduled to transmit at least once in every $\sqrt{n^s} + n^{s+d-1}$ time-slots, thus the capacity can achieve

$$\lambda(n) = \Omega(\frac{1}{\sqrt{n^s} + n^{s+d-1}}) = \Omega(\min\{\Theta(\frac{1}{\sqrt{n^s}}), \Theta(\frac{1}{n^{s+d-1}})\}).$$

3.3 when $s + d = 1$

Since $E[N_i] = n^s p = n^{s+d-1} = 1$, clearly N_i follows the Poisson distribution with parameters $\lambda = 1$ as the number of nodes goes to infinity. Thus we can get that the number of times when any group is appointed as a destination is at most $\log n^s$.

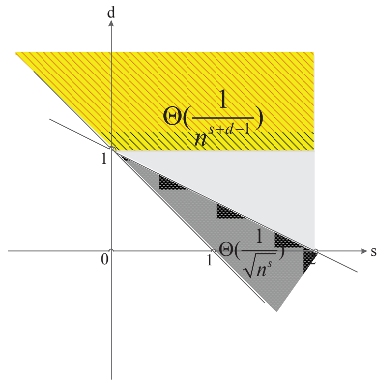

Fig. 4. Capacity of regular distribution group multicast when $s + d > 1$

Similarly, we have

$$\begin{aligned}\lambda(n) &= \min\{\Theta(\frac{1}{\sqrt{n^s}}), \Theta(\frac{1}{\log n^s})\} \\ &= \Theta(\frac{1}{\sqrt{n^s}}).\end{aligned}$$

4 Capacity of Random Distribution Group Multicast

In this section, we discuss the bounds for random group distribution wireless networks. As shown in Figure 5, when these groups are randomly distributed in the network, there will inevitably be interferences due to the intersections between the groups, making it difficult to schedule the model. Unlike previous analysis, we mainly discuss the random scenario capacity from the perspective of destination nodes.

Any node V_i in the network will be selected by a certain source node as its destination node with the probability $p = n^{d-1}$. Let N_i be the number of times when the node V_i is chosen as a destination node, then clearly N_i follows the binomial distribution with parameters n^s and p. Easily we can derive

$$Pr(N_i = k) = \binom{n^s}{k} p^k (1-p)^{n^s - k},$$

and the expected value of N_i is $E[N_i] = n^s p = n^{s+d-1}$. Similarly according to the various circumstances of $s + d$, we will employ different approaches to investigate the network capacity.

4.1 When $s + d < 1$

As the number of nodes goes to infinity while the product $n^s p$ remains fixed, the binomial distribution converges towards the Poisson distribution. Therefore the Poisson distribution with parameter $\lambda = n^s p = n^c$ can be used as an approximation to the binomial distribution here. Consequently, we can derive the following lemma.

Lemma 2. *The number of times when any node is appointed as a destination node under random distribution scenario is also at most constant when $s + d < 1$.*

Proof. Let K be a positive integer. Using the same technique described previously, we can have

$$\begin{aligned}Pr(N_i \geq K) &= \sum_{k \geq K} e^{-\lambda} \frac{\lambda^k}{k!} \\ &\leq \sum_{k \geq K} e^{-\lambda} \lambda^k \\ &\leq \frac{e^{-\lambda} \lambda^K}{1 - \lambda},\end{aligned}$$

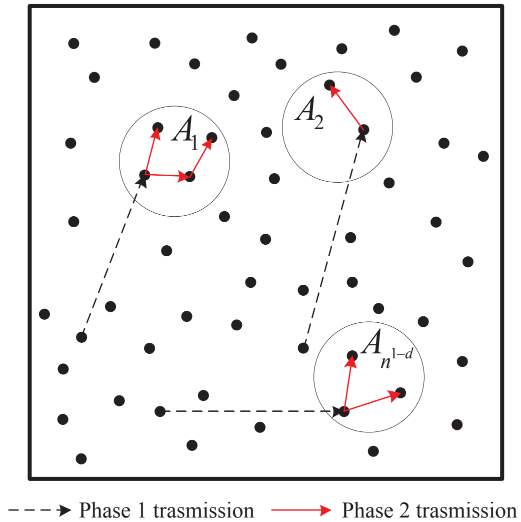

Fig. 5. Transmission mode when groups are randomly distributed

$$
\begin{aligned}
Pr(\max_i N_i \geq K) &\leq \sum_i Pr(N_i \geq K) \\
&\leq \frac{ne^{-\lambda}\lambda^K}{1-\lambda},
\end{aligned}
$$

$$\lim_{n\to\infty} Pr(\max_i N_i \geq K) \leq \lim_{n\to\infty} \frac{n^{cK+1}e^{-\lambda}}{1-n^c} = 0$$

as long as $cK + 1 < 0$. We choose $K = 1 + [\frac{1}{1-s-d}]$.

Based on Lemma 2, we have

$$
\begin{aligned}
\lambda(n) &= O(\min\{\Theta(\frac{1}{\sqrt{n^s}}), \Theta(\frac{W}{[\frac{1}{1-s-d}]+1})\}) \\
&= O(\frac{1}{\sqrt{n^s}}).
\end{aligned}
$$

Similarly, we design the following schedule to achieve the capacity bound.

1. Each source node transmits packets, through ad hoc mode, to a destination node $V_{i,d}$ in a randomly selected group.
2. $V_{i,d}$ transmits the packets to all other nodes in the group.

We follow the schedule of Franceschetti et al. [21], adopting percolation theory in routing, in order to meet the first requirement. To meet the second requirement, we employ flooding algorithm. We should design the process such that every node acts as both a transmitter and a receiver, and each node tries to transmit packages to every one of its neighbors except the source node. In addition, the frequency that a node is transmitting should be the same as the frequency that it is receiving. Obviously, the bound can be achieved through the schedule, and we have,

Theorem 3. *The capacity of random-distribution group multicast is $\Theta(\frac{1}{\sqrt{n^s}})$, when $s + d < 1$.*

Proof. The proof is similar to that of Theorem 1, which employs the TDMA scheme.

4.2 When $s+d>1$

According to the previous derivation, we need to establish the maximum flow, k, of the network. Firstly we consider how close two adjacent crossing pairs can be under the scheme to obtain the minimum distance between adjacent crossing pairs, thus to determine the maximum flow. Then we intend to utilize the properties of binomial distribution. However, we do not work out the problem as expected.

Finally we succeed analyzing the capacity of wireless networks from the perspective of destination nodes instead of groups. Let's recall some results from the appendix of Vasudevan et al. [23], as shown in Lemma 3.

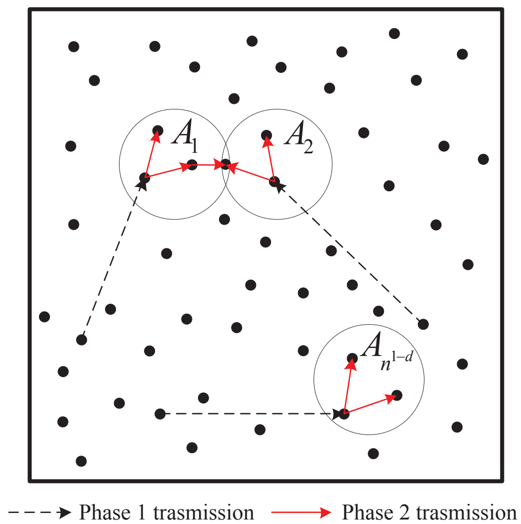

Fig. 6. Transmission mode when groups are randomly distributed and $s+d>1$

Lemma 3. *Let X be a Poisson random variable with parameter λ.*

1. *If $x>\lambda$, then $Pr(X\geq x)\leq e^{-\lambda}(e\lambda)^x/x^x$.*
2. *If $x<\lambda$, then $Pr(X\leq x)\leq e^{-\lambda}(e\lambda)^x/x^x$.*

Performing some algebraic manipulations, we obtain

Lemma 4. *The number of times when any node is appointed as a destination node under random distribution scenario is at most $2n^{s+d-1}$ when $s+d>1$.*

Proof. As N_i follows the Poisson distribution with parameter $\lambda=n^s p=n^{s+d-1}$, then we can have

$$\begin{aligned} Pr(N_i\geq 2\lambda) &\leq e^{-\lambda}(\frac{e}{2})^{2\lambda} \\ &= (\sqrt{e})^{-2\lambda}(\frac{e}{2})^{2\lambda} \\ &= (\frac{\sqrt{e}}{2})^{2\lambda}, \end{aligned}$$

$$\begin{aligned} Pr(\max_i N_i\geq 2\lambda) &\leq \sum_i Pr(N_i\geq 2\lambda) \\ &\leq n(\frac{\sqrt{e}}{2})^{2\lambda}. \end{aligned}$$

Since $\frac{\sqrt{e}}{2} < 1$, $\lambda = n^{s+d-1}$ goes to infinity when the number of nodes n increases, then we have

$$\lim_{n\to\infty} Pr(\max_i N_i \geq 2\lambda) \leq \lim_{n\to\infty} n(\frac{\sqrt{e}}{2})^{2n^{s+d-1}} = 0.$$

From the above derivation we have $k = 2\lambda = 2n^{s+d-1}$, it follows,

$$\begin{aligned}\lambda\,(n) &= O(\min\{\Theta(\frac{1}{\sqrt{n^s}}), \Theta(\frac{W}{2n^{s+d-1}})\}) \\ &= O(\min\{\Theta(\frac{1}{\sqrt{n^s}}), \Theta(\frac{1}{n^{s+d-1}})\}).\end{aligned}$$

Using the same schedule described previously, we can then prove that the bound can be achieved, and that the capacity of the random distribution scenario is

$$\lambda(n) = \Omega(\frac{1}{\sqrt{n^s} + 2n^{s+d-1}}) = \Omega(\min\{\Theta(\frac{1}{\sqrt{n^s}}), \Theta(\frac{1}{n^{s+d-1}})\}).$$

With the above discussion, we can prove the following Theorem.

Theorem 4. *The capacity of random-distribution group multicast is* $\min\{\Theta(\frac{1}{\sqrt{n^s}}), \Theta(\frac{1}{n^{s+d-1}})\}$*, when* $s + d > 1$.

Proof. The proof is similar to that of Theorem 2, and the capacity is displayed in Figure 4.

4.3 When $s + d = 1$

This is identical to the situation when $s + d = 1$ under regular distribution scenario, and we need to establish the maximum flow of the network.

Lemma 5. *The number of times when any node is appointed as a destination node under random distribution scenario is at most* $\log n$ *when* $s + d = 1$.

Proof. When $s + d = 1$, N_i follows the Poisson distribution with parameter $\lambda = n^{s+d-1} = 1$,

$$\begin{aligned}Pr(N_i \geq \log n) &\leq e^{-1}(\frac{e}{\log n})^{\log n} \\ &\leq e^{-1}(\frac{1}{e^2})^{\log n} \\ &= \frac{1}{en^2},\end{aligned}$$

$$\begin{aligned}Pr(\max_i N_i \geq \log n) &\leq \sum_i Pr(N_i \geq \log n) \\ &\leq \frac{1}{en}.\end{aligned}$$

When the number of nodes n goes to infinity , we have

$$\lim_{n\to\infty} Pr(\max_i N_i \geq \log n) \leq \lim_{n\to\infty} \frac{1}{en} = 0.$$

Thus, we have

$$\begin{aligned}\lambda(n) &= \min\{\Theta(\frac{1}{\sqrt{n^s}}), \Theta(\frac{1}{\log n^s})\} \\ &= \Theta(\frac{1}{\sqrt{n^s}}).\end{aligned}$$

5 Conclusion

We have studied the theoretical group multicast capacity of wireless ad hoc network. In particular, we have investigated wireless networks using both regular distribution and random distribution models. Our results can well unify the previous multicast results in wireless ad hoc networks. What's more, our study is the first attempt to understand how group multicast may impact on network capacity from a theoretical perspective.

We are also interested in how to improve the throughput capacity of wireless ad hoc network by adopting multiple groups as destinations and using physical model. Although group multicast provides gain in certain wireless ad hoc network, an interesting question is what is the impact of combining group multicast and social network. And this question is also important in realistic cellular network.

Acknowledgment

This paper is supported by National Fundamental Research Grant (No. 2010CB731803); NSF China (No. 60832005); China Ministry of Education New Century Excellent Talent (No. NCET-10-0580); China Ministry of Education Fok Ying Tung Fund (No. 122002); Qualcomm Research Grant; Shanghai Basic Research Key Project (No. 11JC1405100); National key project of China (2012ZX03001009, 2010ZX03003-001-01); China Postdoctoral Science Foundation Grant (No. 2011M500774).

References

1. P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks," *in IEEE Trans. on Inform. Theory*, vol. 46, no. 2, pp. 388-404, March 2000.
2. M. Grossglauser and D. N. C. Tse, "Mobility increases the capacity of ad-hoc wireless networks," *in IEEE/ACM Trans. on Networking*, vol. 10, pp. 477-486, August 2002.
3. X. Lin, G. Sharma, R. R. Mazumdar and N. B. Shroff, "Degenerate delay-capacity tradeoffs in ad-hoc networks with Brownian mobility," *in IEEE Trans. on Inform. Theory*, vol. 52, no. 6, pp. 277-2784, June 2006.
4. M. Neely and E. Modiano, "Capacity and Delay Tradeoffs for Ad-Hoc Mobile Networks," *in IEEE Trans. on Inform. Theory*, vol. 51, no. 6, pp. 1917-1937, 2005.
5. X. Lin and N. B. Shroff, "The Fundamental Capacity-Delay Tradeoff in Large Mobile Ad Hoc Networks," *in Proc. Third Annu. Mediterranean Ad Hoc Netw. Workshop*, 2004.
6. L. Ying, S. Yang and R. Srikant, "Optimal delay-throughput trade-offs in mobile ad-hoc networks," *in IEEE Trans. on Inform. Theory*, vol. 9, no. 54, pp. 4119-4143, September 2008.
7. Pan Li, Yuguang Fang, Jie Li, and Xiaoxia Huang, "Smooth Trade-offs Between Throughput and Delay in Mobile Ad Hoc Networks," *in IEEE Trans. on Mobile Computing*, Vol. 11, No. 3, pp. 427-438, March 2012.
8. U. Kozat and L. Tassiulas, "Throughput Capacity of Random Ad Hoc Networks with Infrastructure Support," *in Proc. ACM Mobicom*, San Diego, CA, USA, June 2003.
9. B. Liu, Z. Liu and D. Towsley, "On the Capacity of Hybrid Wireless Networks," *in Proc. IEEE INFOCOM*, San Francisco, CA, USA, March 2003.
10. W. Huang, X. Wang, Q. Zhang, "Capacity Scaling in Mobile Wireless Ad Hoc Network with Infrastructure Support," *in Proc. of IEEE ICDCS 2010*, Genoa, Italy, 2010.
11. X. Wang, W. Huang, S. Wang, J. Zhang, C. Hu, "Delay and Capacity Tradeoff Analysis for MotionCast," *in IEEE/ACM Trans. on Networking*, Vol. 19, no. 5, pp. 1354-1367, Oct 2011.
12. L. Fu, Y. Qin, X. Wang, X. Liu, "Throughput and Delay Analysis for Convergecast with MIMO in Wireless Networks," *in IEEE Trans. on Parallel and Distributed Systems*, Vol. 23, no. 4, pp. 768-775, Apr 2012.
13. X. -Y. Li, "Multicast Capacity of Wireless Ad Hoc Networks," *in IEEE/ACM Trans. on Networking*, vol.17, no.3, pp.950-961, June 2009.
14. Z. Wang, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, "A unifying perspective on the capacity of wireless ad hoc networks," *in Proc. IEEE INFOCOM*, 2008, pp. 211-215.
15. X. Mao, X. -Y. Li and S. Tang, "Multicast capacity for hybrid wireless networks," *in Proc. ACM Mobihoc*, New York, NY, USA, 2008, pp. 189-198.
16. Pan Li, Chi Zhang, and Yuguang Fang, "The Capacity of Wireless Ad Hoc Networks Using Directional Antennas," *in IEEE Trans. on Mobile Computing*, Vol. 10, No. 10, pp. 1374-1387, October 2011.
17. Tao Jing, Xiuying Chen, Yan Huo, and Xiuzhen Cheng, "Achievable Transmission Capacity of Cognitive Mesh Networks With Different Media Access Control," *in Proc. IEEE INFOCOM*, Orlando, Florida USA, March 25-30, 2012.
18. Yuan Le, Liran Ma, Wei Cheng, Xiuzhen Cheng, and Biao Chen, "Maximizing Throughput When Achieving Time Fairness in Multi-Rate Wireless LANs," *in Proc. IEEE INFOCOM Mini-Conference*, Orlando, Florida USA, March 25-30, 2012.
19. M. Grossglauser and D. Tse, "Mobility increases the capacity of ad-hoc wireless networks," *in Proc. INFOCOM 2001*, vol. 3, pp. 1360-1369, 2001.
20. J. Liu, D. Goeckel, D. Towsley, "Bounds on the Gain of Network Coding and Broadcasting in Wireless Networks," *in Proc. INFOCOM 2007*, pp. 724-732, May 2007.

21. M. Franceschetti, O. Dousse, D. Tse and P. Thiran, “Closing the Gap in the Capacity of Wireless Networks Via Percolation Theory,” *in IEEE Trans. on Inform. Theory*, vol. 53, no. 3, pp. 1009-1018, March 2007.
22. K. Kim, Y. Cai, W. Tavanapong, “A priority forwarding technique for efficient and fast flooding in wireless ad hoc networks,” *in Proc. ICCCN 2005*, pp. 223- 228, October 2005.
23. S. Vasudevan, D. Goeckel and D. F. Towsley, “Security-capacity tradeoff in large wireless networks using keyless secrecy,” *in Proc. ACM Mobihoc 2010*, pp. 21-30, New York, NY, USA, 2010.