FINAL REPORT ON PROJECT 15 BY GROUP 7

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ABSTRACT

Under the protocol interference model, in which we assume that n wireless nodes are uniformly randomly distributed in a square region with side-length a and all nodes have the uniform transmission range r and uniform interference range $R > r$. What's more, we assume that each node can transmit or receive at W bits/second over a common channel. In this paper we will show you some upper bounds and lower bounds on capacity of the unicast, broadcast and multicast.

Keywords: protocol interference model, capacity, unicast, broadcast, multicast.

INTRODUCTION

AD HOC networks play a vital role in the evolution of wireless communication. For instance, self-organized ad hoc networks of PDAs or laptops are used in disaster relief, conference, and battlefield environments. For the ad hoc networks, it also has some traditional problems of wireless and mobile communications, such as power control and transmission-quality enhancement. Furthermore, the multihop mode and possible lack of a fixed infrastructure,although ad hoc networks promise convenient infrastructure-free communication, bring some new challenges, network configuration as well as ad hoc addressing and etc. included, to the researches.

There is an all-time discussed part of the wireless networks, which is called capacity. With respect to capacity of ad hoc networks, Gupta and Kumar had made a particular analysis, see [1] and [2]. After this, more and more researchers devoted to this area. There are so many achievements on the study of broadcast capacity and multicast capacity constantly. The research on this field is getting more and more mature.

MODEL

- ## UNICAST CAPACITY

  Assumed that there are *n* nodes locating in an area of $1m^2$ in which each node can transmit W bits/sec over a common wireless channel, see Fig[1]. What's more, it is inessential to separate W into several subchannel of capacity W_k, where $k = 1, 2..., N$, as long as $\sum_{k=1}^{N} W_k = W$. In the following we will discuss under what conditions the transmission will be successful.

  Consider two types of networks including:
  - *Arbitrary Networks*— nodes locations, destinations of sources and traffic demands are all arbitrary.
  - *Random Networks*— nodes locations, destinations of sources and traffic demands are all random.

  Under the protocol model, Kumar gave us the following conclusions:

Model	Node Locations	Constraint on Power/Range	Power Attenuation	Major Results	Chap-ters	Note
Protocol model (Defined in terms of geometry)	Arbitrary in a region of area A	Arbitrary	No assumption	T(n) = $\Theta(W\sqrt{An})$ bit*meter/sec	2,3	W is the maximum single link rate
	Randomly distributed in a finite domain	Common transmission range *r*	No assumption	$\lambda(n) = \Theta(\frac{W}{\sqrt{n\log n}})$ bit/sec	5	Every node has a randomly chosen destination node
	Poisson process with density n in a unit domain	Different ranges are allowed	No assumption	$\lambda(n) = \Theta(W/\sqrt{n})$	7	Every node is only one destination node

  ### FUNDAMENTAL

  - Suppose that node X_i transmits to the node $X_{R(i)}$ over the *m*th subchannel. The condition of successful transmission is that:

    $$|X_k - X_{R(i)}| \geq (1 + \Delta)|X_i - X_{R(i)}|.$$

    for every other nodes X_k that transmit simultaneously over the same subchannel.

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$$|X_k - X_{R(i)}| \geq (1 + \Delta)|X_k - X_{R(k)}|.$$

  for every other nodes X_k that transmit to the node $X_{R(k)}$ simultaneously over the same subchannel.
- Suppose that associated with each transmitter-receiver pair$(X_k, X_{R(k)})$ there is an area I_k, such that for a transmission from X_i to $X_{R(i)}$ to be successful, it is necessary that

$$X_{R(i)} \notin I_k, \forall k \in \{Active\, transmitters\}, k \neq i.$$

  Such an area I_k will be called an $Interference\ Region$. The expression of I_k is $I_k := \{X : |X - X_k| < (1 + \Delta)|X_k - X_{R(i)}|\}$.

MODEL

With respect to arbitrary protocol model, we analyze the bounds they have.

As showed in the above figure, in the Arbitrary networks mode, the capacity is $\Theta\left(W\sqrt{n}\right)$ bit-meter per second. Furthermore, the bound $\sqrt{\frac{8}{\pi}\frac{W}{\Delta}}\sqrt{n}$ bit-meter per second can be achieved when the nodes, traffic patterns and the schedules of transmissions are appropriately chosen. Since there are n nodes, if the capacity equally divided between them, then each node get the capacity of $\Theta\left(\frac{W}{\sqrt{n}}\right)$ bit-meter per second.

In the condition of random networks mode, the throughput capacity is of order $\Theta\left(\frac{W}{\sqrt{n\log n}}\right)$ bits per second.

- **BROADCAST CAPACITY**

In [3] Bulent Tavli's work, he considered an ad hoc wireless network with channel capacity C bits per second, area A m^2, constant node density n_0(nodes/m^2) and a total of n nodes. The upper bound on the single-hop equivalent aggregate bandwidth of a nulti-hop network in broadcasting as a function of $n, W^{ag}\left(n\right)$, is expressed as

$$W^{ag}\left(n\right) = \mathrm{O}\left(n\right) \times \mathrm{O}\left(1/n\right) \times C$$

in which $\mathrm{O}\left(n\right)$ stands for spatial reuse, $\mathrm{O}\left(1/n\right)$ stands for the multi-hop relaying and C stands for the channel capacity. Thus we can get that

$$W^{ag}\left(n\right) = \mathrm{O}\left(C\right)$$

$$W^{pn}\left(n\right)\left(per\ node\ capacity\right) = W^{ag}\left(n\right)/n = \mathrm{O}\left(C/n\right)$$

Here is the proof:

The coverage area of each node is $A_0 = \pi r_0^2$ where r_0 is the transmit radius. Thus, there are at most $A_0 n_0$ nodes receives the transmission. In order to cover the whole network, at least $\frac{An_0}{A_0 n_0} = \frac{A}{A_0}$ is required. To make sure that all the nodes can receive the message perfectly, the distance between two transmitters should be at least $2r_0$. Take consideration of the possibility of some nodes maybe in the corner, the number of the transmission can be at most $A/\left(A_0/4\right)$. Thus we can obtain the capacity:

$$C[A/(A_0/4)]/(A/A_0) = 4C$$

Per node broadcast capacity is $4C/n = \mathrm{O}\left(1/n\right)$.

Also in the protocol model, [4] gives us another conclusion: the bound of the capacity is $\Theta\left(W/\max\left(1, \Delta^d\right)\right)$ (W is the channel capacity, Δ is interference parameter and d is the number of dimensions of space in which the network lies). Unlike unicast, the changes of the number of nodes, radio range and network area do not affect the broadcast capacity remarkably. Moreover, the mobility can also not significantly increase the broadcast capacity while the unicast capacity can be increased by at most $\Omega\left(\sqrt{n}\right)$. In the process of analyzing and calculating, the conception of MCDS (Minimum Connected Dominating Set) makes the description clear and comprehensible.

- **MULTICAST CAPACITY**

  In [5], assume that the transmission range r is a constant and the side-length a is a function of n, we can get the conclusion according to the different value of k:
  When the network is connected with high probability and $a/r = \Theta\left(\sqrt{n/\log n}\right)$, the multicast capacity is:

$$\Lambda_k\left(n\right) = \begin{cases} \Theta\left(\sqrt{\frac{n}{\log n}} \cdot \frac{W}{\sqrt{k}}\right), & k = \mathrm{O}\left(\frac{n}{\log n}\right) \\ \Theta\left(W\right), & k = \Omega\left(\frac{n}{\log n}\right) \end{cases}$$

  So the per-flow capacity of multicast session is:

$$\lambda_k\left(n\right) = \begin{cases} \Theta\left(\sqrt{\frac{1}{n\log n}} \cdot \frac{W}{\sqrt{k}}\right), & k = \mathrm{O}\left(\frac{n}{\log n}\right) \\ \Theta\left(W/n\right), & k = \Omega\left(\frac{n}{\log n}\right) \end{cases}$$

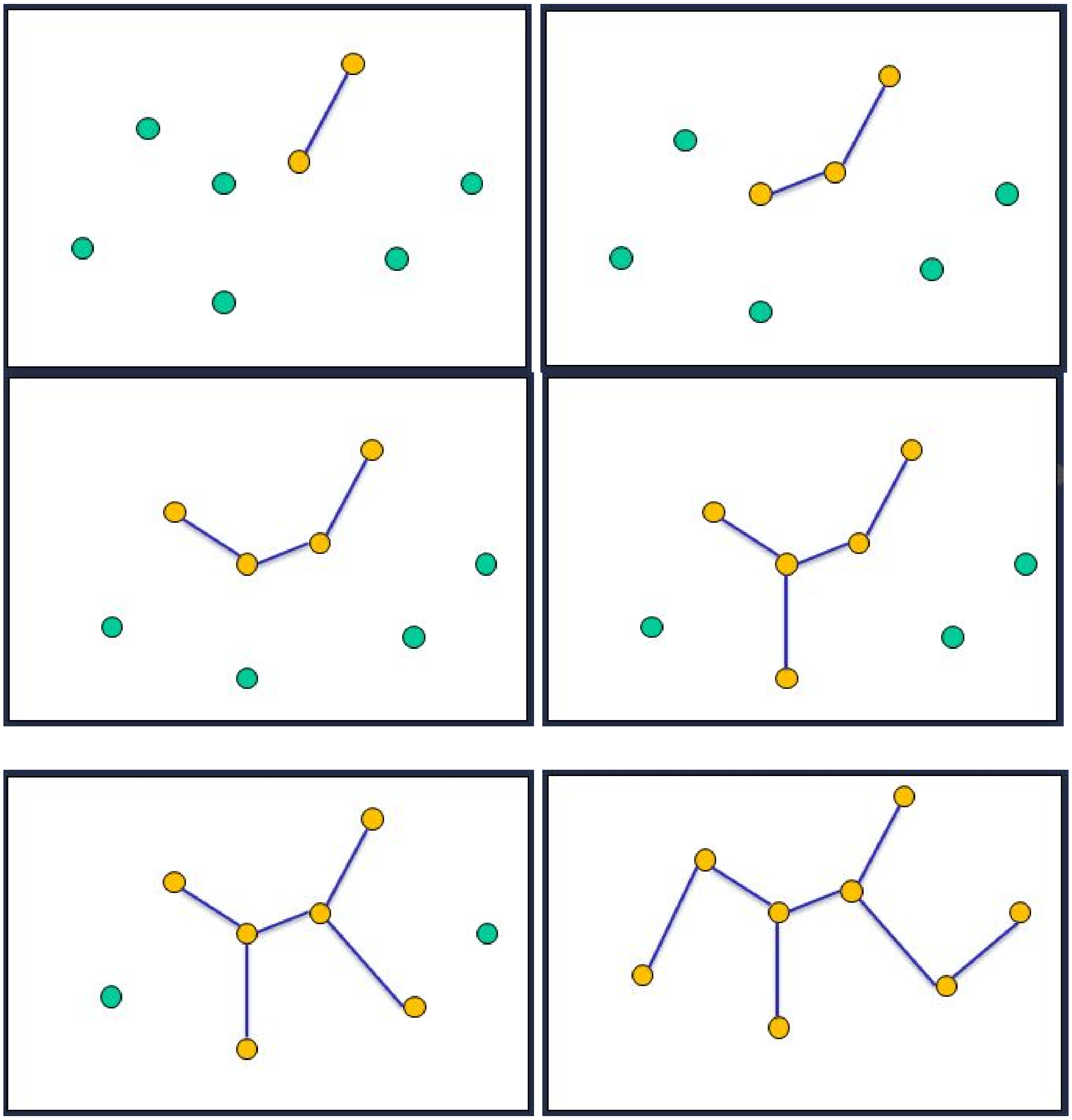
  To prove this conclusion, the essential thought of the author is the calculating of the multicast tree EMST(Euclidean minimum spanning tree). And also this part has early been presented in [6].

CONTRIBUTION

My work focus on the ways to find out the EMST. As follows, I will introduce you two methods:

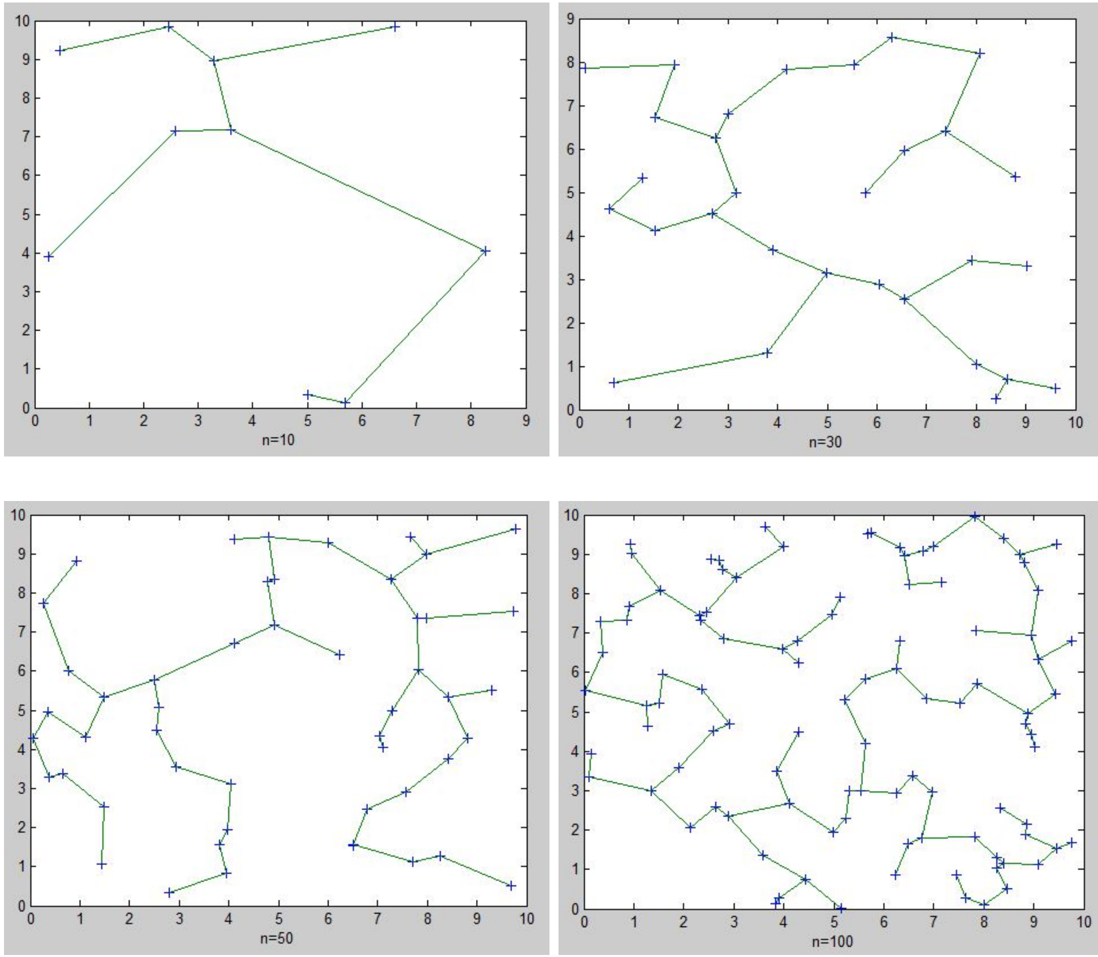
- Method of Congregation:
  Assume that there are n nodes in the transmission field. First, we randomly choose a node, and put it into congregation 1, then put the other nodes into congregation 2. Second, we calculate the distance between every node from congregation 1 and every

node from congregation 2. Select the shortest one, line the two nodes and make the nodes in congregation 1. Repeat the second step and finally we will find that all the nodes are in congregation 1 and a tree is established. And that's the EMST. To make it more clear, here is the illustration.



- Method of Loop-judging:
  With the same assumptions, we calculate every two nodes' distance and list them from small to big. Pick the smallest one(except the lined two nodes) and line the two nodes. If this action make the figure exists a loop, then erase it. Repeat above procedures till we establish $n-1$ links. Then the EMST is finished.

- Simulation result:



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