

Compressive Sensing

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1 Introduction

Conventional approaches to sampling signals or images follow Shannons celebrated theorem: the sampling rate must be at least twice the maximum frequency present in the signal (the Nyquist rate). In fact, this principle underlies nearly all signal acquisition protocols used in consumer audio and visual electronics, medical imaging devices, radio receivers, and so on. However, it has been proved that this theorem can be substituted when the problem deals with sparse signals. Consider the signal below:

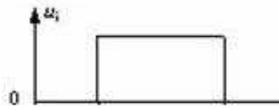


Figure 1

Due to Shannons theorem, we need an extremely high sampling rate although the shape of this signal is simple and most values of the samples are zero. However, recent breakthroughs in compressed sensing have shown that merely M samples ($M \ll$ required number of sampling points according to Shannons theory) can reconstruct the origin signal successfully. Compressive Sampling (CS), also known as Compressed Sensing, is a generalization of conventional point sampling where observations are inner products between an unknown signal and a set of user-defined test vectors. Recent theoretical results show that, for certain ensembles of test vectors, CS projections provide an effective method of encoding the salient information in any sparse (or nearly sparse) signal. Further, these projection samples can be used to obtain a consistent estimate of the unknown signal even in the presence of noise. These results are remarkable because the number of samples required for low-distortion reconstruction is on the order of the number of relevant signal coefficients, which is often far fewer than the ambient dimension

in which the signal is observed. This huge reduction in sampling makes CS a practical and viable option in many resource constrained applications. The whole process is illustrated by figure 2 and figure 3:



Figure 2



Figure 3

1.1 Sensing of signals

Next we will discuss some main characteristics of compressive sensing in detail.

1.1.1 Sparsity

Definition: If X_i is all zero but K entries, the vector is called k -sparse.

Consider a general linear measurement process that computes $M < N$ inner products between x and a collection of vectors $\{\phi_j\}_{j=1}^M$ as in $y_i = \langle x, \phi_j \rangle$. Arrange the measurements y_j in an $M \times 1$ vector y and the measurement vectors ϕ_j^T as rows in an $M \times N$ matrix ϕ . It can be proved that when data is sparse, we can directly acquire a condensed representation with no/little information loss through dimensionality reduction: $y = \phi * x$, where $k < M \ll N$, to a more precise degree, $M = O(K \log N)$.

$$\begin{array}{c}
 \begin{array}{c} M \times 1 \\ \text{measurements} \end{array} \begin{array}{c} y \\ \color{red}\square \color{green}\square \color{blue}\square \color{yellow}\square \color{cyan}\square \color{magenta}\square \color{black}\square \end{array} = \begin{array}{c} \phi \\ M \times N \end{array} \begin{array}{c} \color{red}\square \color{green}\square \color{blue}\square \color{yellow}\square \color{cyan}\square \color{magenta}\square \color{black}\square \\ x \\ N \times 1 \\ \text{sparse} \\ \text{signal} \end{array} \\
 \\
 M = O(K \log(N/K)) \quad \begin{array}{c} \color{red}\square \color{green}\square \color{blue}\square \color{yellow}\square \color{cyan}\square \color{magenta}\square \color{black}\square \\ K \\ \text{nonzero} \\ \text{entries} \end{array}
 \end{array}$$

Figure 4: Compressive Data Acquisition. If x is an $N \times 1$ sparse signal with only K nonzero entries, then it can be projected by an $M \times N$ matrix, to form an $M \times 1$ vector. In addition, M is a little larger than K and much smaller than N . It

means although x seems to need a lot of samples, its sparsity indicate that it can be measured just with M measurements. P.S. a random projection will work quite well.

Further studies have extended $y = \phi x$ to non-sparse signals. Suppose the observed signal x is not sparse, but instead a suitably transformed version of it is. That is, if T is a transformation matrix then $\alpha = \Psi^{-1}x$ is sparse. The CS observations can be written as $y = \phi\Psi\alpha$. This property highlights the universality of compressive sensing.

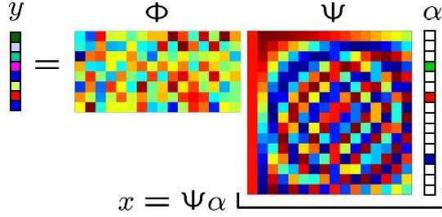


Figure 5: Assuming that x is not sparse itself, but it can still be represented by a sparse signal in certain basis, compressive sensing work appropriately as well.

1.1.2 Incoherence

Suppose we are given a pair (ϕ, Ψ) of orthobases of R^n . The first basis ϕ is used for sensing the object and the second is used to represent f . The restriction to pairs of orthobases is not essential and will merely simplify our treatment.

Definition: the coherence between the sensing basis ϕ and the representation basis Ψ is

$$\mu(\Phi, \Psi) = \sqrt{n} \cdot \max_{1 \leq k, j \leq n} |\langle \phi_k, \psi_j \rangle|$$

The coherence measures the largest correlation between any two elements of Ψ and Φ ; If Φ and Ψ contain correlated elements, the coherence is large. Otherwise, it is small. As for how large and how small, it follows from linear algebra that $(\Psi, \phi) \in [1, \sqrt{n}]$.

Compressive sampling is mainly concerned with low coherence pairs, the reason for which we will explain below.

Due to the theorem presented by E. Cands and J. Romberg, if we fix $f \in R^n$ and suppose that the coefficient sequence x of f in

the basis Ψ is S -sparse. Select m measurements in the ϕ domain uniformly at random. Then if

$$m \geq C \cdot \mu^2(\Phi, \Psi) \cdot S \cdot \log n$$

for some positive constant C , the solution to reconstruct x is exact with overwhelming probability. Considering what we have said above, it is apparently that the smaller the coherence, the fewer samples are needed, hence our emphasis on low coherence systems in the previous section.

Random matrices are largely incoherent with any fixed basis Ψ . Select an orthobasis ϕ uniformly at random, which can be done by orthonormalizing n vectors sampled independently and uniformly on the unit sphere. Then with high probability, the coherence between ϕ and Ψ is about $\sqrt{2 \log n}$. By extension, random waveforms with independent identically distributed entries, e.g., Gaussian or 1 binary entries, will also exhibit a very low coherence with any fixed representation ϕ . If sensing with incoherent systems is good, then efficient mechanisms ought to acquire correlations with random waveforms, e.g., white noise.

1.2 Reconstruction of signals

Sparse Recovery It is necessary to introduce **Restricted Isometry Property** in advance, known as RIP.

Let δ_k be the smallest number such that:

$$(1 - \delta_k) \|X\|_2^2 \geq \|\phi_x\|_2^2 \geq (1 + \delta_k) \|X\|_2^2$$

for all k -sparse vectors x in R^n where

$$\phi = [\phi_1 \dots \phi_n] \in R^{m \times n}$$

The theorem presented by E. J. Cands tells us that:

If $\delta_{2k} < \sqrt{2} - 1$, then for all k -sparse vectors x such that $\phi_x = b$, the solution of (l_1) is equal to the solution of (l_0) .

$$\text{Here, } l_1 = \min \|x\|_1 : \phi_x = b, x \in R^n$$

$$l_0 = \min \|x\|_0 : \phi_x = b, x \in R^n$$

The same compressed data could be generated by many n -dimensional vectors, but we have to find the sparsest one, i.e. the vector whose number of nonzero data is smallest. This might seem to require that any

reconstruction algorithm must exhaustively search over all sparse vectors. However, this procedure is impossible, just to provide a way to measure whether the vector we find is appropriate or not. But fortunately, applying the RIP we have just discussed above, we can use the l_1 norm as a proxy for sparsity instead of l_0 norm so that the process is much more tractable. Given a vector of (noise-free) observations $y = \theta x$, the unknown k -sparse signal x can be recovered exactly as the unique solution to

$$\min \|x\|_1 \text{ subject to } y = \phi x$$

which is known as l_1 minimization.

Of course, there are other effective recovery techniques for CS, such as matching pursuit, iterative thresholding and total variation minimization, but the coverage of them is beyond this article. Let us look at the l_1 minimization from a geometrical point of view. The line denotes all the x vectors which satisfy the equation $\phi x = b$, so that they all have the possibility to be reconstructed. The diamond represents $\|x\|_1$, all the points on the edge of this diamond have an equal $\|x\|_1$, and the points in the inner space of this diamond have a smaller $\|x\|_1$, and vice versa.

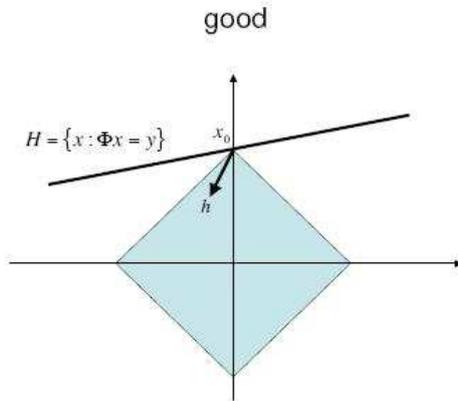


Figure 6: This picture obeys RIP, meaning that finding x_0 equals finding the vector which obeys l_1 minimization. So that it is good for applying l_1 minimization to reconstruct x , since the line has only one point of intersection with the diamond, which determines the uniqueness of

reconstruction. The other vectors on the line all have a larger l_1 .

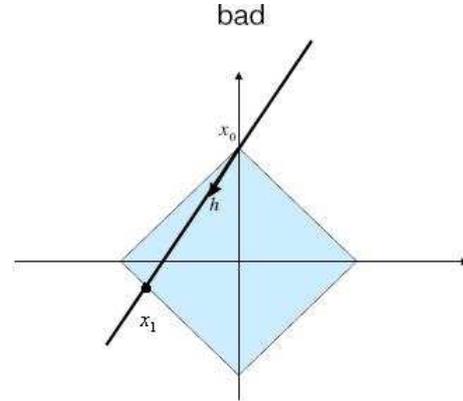


Figure 7: This picture does not obey RIP. It is bad for applying l_1 minimization to reconstruct x , since the line has more than one point of intersection with the diamond. Some other vectors rather than x_0 on the line lie in the inner space of the diamond, which have a smaller l_1 . So that l_1 minimization will find x_1 instead of x_0 .

In addition, RIP also acts as a stable embedding, which means that if x_1 is close to x_2 in R^n , then when they are projected by ϕ , ϕx_1 is close to ϕx_2 as well. It can be proved by the inequation:

$$(1 - \delta_{2k}) \geq \frac{\|\phi x_1 - \phi x_2\|_2^2}{\|x_1 - x_2\|_2^2} \geq (1 + \delta_{2k})$$

Furthermore, if δ_{2k} is less than 0.41, then tractable recovery, robust recovery and stable recovery are ensured.

Compressed sensing remains quite effective even when the samples are corrupted by additive noise, which is important from a practical point of view since any real system will be subjected to measurement inaccuracies. We present noisy measurement as:

$$y = \phi' \alpha_0 + e, \quad \|e\|_2 \geq \varepsilon$$

A variety of reconstruction methods have been proposed to recover (an approximation of) x when observations are corrupted by noise. The fundamental solution is to relax the recovery program, i.e. solve

$$\min \|\alpha\|_{l_1} \text{ subject to } \|\phi' \alpha - y\| \geq \varepsilon$$

Out of doubt, since we have relaxed the condition, there exist some errors generated by this relaxation. However, the recovery error obeys:

$$\|\alpha_0 - \alpha^*\|_2 \geq \sqrt{\frac{N}{M}} \cdot \epsilon + \frac{\|\alpha_0 - \alpha_{0,k}\|_{L_1}}{\sqrt{K}}$$

The first part of the right is called measurement error, for it correlates to the number of measurements M . Moreover, we can see easily that the larger M is, which means we acquire more samples, the smaller the error becomes. The second part is called approximation and α_0, K stands for best K -term approximation.

1.3 Reconstruction algorithm

1.3.1 BP

One of the simplest solutions to pursue a accurate recovery of a sparse signal is a linear program called basis pursuit which relies on conventional linear programming techniques. The computational complexity of this algorithm is polynomial in N while the number of measurements generally required for adequate reconstruction is given by $M=cK$ for $c \geq 1$. The constant, c , refers to an oversampling factor whose value is inversely dependent on sparsity.

1.3.2 OMP

A compressed sensing solver that seeks to compute the minimum l_0 norm solution is the Orthogonal Matching Pursuit (OMP) algorithm. This algorithm attempts to determine which columns of the pseudo random matrix $\Phi_{M \times N}$ are most correlated to the measurement matrix Y . The column with the largest correlation is likely the largest coefficient of S . During an iteration, the column of $\Phi_{M \times N}$ with the largest correlation to Y is found and its contribution to Y is subtracted. The resulting coefficient of S is determined and the process repeats until Y disappears or has a value smaller than some threshold of acceptable error. This algorithm should only need to iterate K times to successfully reconstruct S . It has been

shown that with $M \geq cK \ln \frac{N}{k}$ it is possible to reconstruct every K sparse with a probability exceeding $1 - e^{-MN}$. There has been some critique that OMP cannot produce accurate results except in the simplest (noiseless) circumstances. This leads to reason that the effectiveness of the algorithm may degrade swiftly in the presence of noise.

1.3.3 TMP

Consider x_0 in the wavelet basis:

$$x_0 = \sum_k \beta_{j_0,k} \phi_{j_0,k} + \sum_{j=j_0}^{j_1} \sum_k \alpha_{j,k} \psi_{j,k}$$

where j_0 is some specified coarse scale, j_1 is the finest scale, $\phi_{j_0,k}$ are male wavelets at coarse scale and $\psi_{j_0,k}$ are fine scale female wavelets. Let $\alpha = (\alpha_{j,k} : j_0 \leq j \leq j_1, 0 \leq k \leq 2^j)$ denote the grouping together of all wavelet coefficients, and let $\beta = (\beta_{j_0,k} : 0 \leq k \leq 2^{j_0})$ denote the male coefficients. Notice the multiscale nesting structure of the wavelet atoms the support of each $\psi_{j,k}$ contains the supports of $\psi_{j+1,2k-1}$ and $\psi_{j+1,2k}$ induces a binary tree structure in the wavelet coefficients.

TMP is an adaptation of the Matching Pursuit algorithms to achieve faster reconstruction for piecewise smooth signals, which exploits the tree structure of the wavelet coefficients.

2 Application: Exploiting CS in WSNs

2.1 Introduction of WSN

Due to recent technological advances, the manufacturing of small and low cost sensors became technically and economically feasible. The sensing electronics measure ambient conditions related to the environment surrounding the sensor and transform them into an electric signal. Processing such a signal reveals some properties about objects located and/or events happening in

the vicinity of the sensor. A large number of these disposable sensors can be networked in many applications that require unattended operations, which later develops to the wireless sensor networks. Nowadays, wireless sensor networks (WSNs) have been used for numerous applications including military surveillance, facility monitoring and environmental monitoring. Typically WSNs have a large number of sensor nodes with the ability to communicate among themselves and also to an external sink or a base-station. The sensors could be scattered randomly in harsh environments such as a battlefield or placed at specified locations. The sensors coordinate among themselves to form a communication network such as a single multi-hop network or a hierarchical organization with several clusters and cluster heads. The sensors periodically sense the data, process it and transmit it to the base station.

2.2 CS for WSNs

A typical wireless sensor network, consists of a large number of wireless sensor nodes, spatially distributed over a region of interest, that can sense (and potentially actuate) the physical environment in a variety of modalities, including acoustic, seismic, thermal, and infrared. A wide range of applications of sensor networks are being envisioned in a number of areas, including geographical monitoring, inventory management, homeland security, and health care. The essential task in many applications of sensor networks is to extract some relevant information from distributed data and wirelessly deliver it to a distant destination (the sink node). While this task can be accomplished in a number of ways, one particularly attractive technique leverages the theory of CS and corresponds to delivering random projections of the sensor network data to the sink. In contrast to classical approaches, where the data is first compressed and then transmitted to a given destina-

tion, with CS the compression phase can be jointly executed with data transmission. This is important for WSNs as compressing the data before the transmission to the data gathering point (hereafter called the sink) requires to know in advance the correlation properties of the input signal over the entire network (or over a large part of it) and this implies high transmission costs. With CS, the content of packets can be mixed as they are routed towards the sink. Under certain conditions, CS allows to reconstruct all sensor readings of the network using much fewer transmissions than routing or aggregation schemes. When we utilize CS at the sink node, we obtain more valuable information, and the received values are linear random combinations of the sensor nodes. Nevertheless, there still remain certain problems critical for evaluating the performance of CS for WSNs. (1) How to choose two matrices ϕ and Ψ in the data gathering protocol since the sparsity requirements and the incoherence ought to be met. (2) The energy consumption of transmitting the random combined data of the sensor nodes in the process of CS should be taken into consideration due to the energy limitation of WSNs. (3) The robustness of CS in WSNs is also a problem because of the node fails and the harsh environment.

One preliminary idea presented in [2] is that, according to the fundamental encoding $y = \phi x$, each of the n sensors, $j = 1, \dots, n$, locally computes the term $\phi_{i,j} x_j$ by multiplying its data with the corresponding element of the compressing matrix. The compressing matrix can be generated in a distributed fashion by letting each node locally generate a realization of $\phi_{i,j}$ using a pseudo-random number generator seeded with its identifier which can be easily reconstructed. Then the local terms $\phi_{i,j} x_j$ are simultaneously aggregated and distributed across the network using randomized gossip, which is a simple iterative decentralized algorithm for computing linear functions such as $y_i =$

$\sum_{j=1}^n \phi_{i,j} x_j$. Because each node only exchanges information with its immediate neighbors in the network, gossip algorithms are resilient to failures or changes in the network topology. Moreover, when randomized gossip terminates, the value of y_i is available at every node in the network, so the network data cannot be compromised by eliminating a single server or fusion center. However, this raw thought only realize the application of CS to the network sensing, we dont see many advantages since the gossip routing protocol is not energy efficient. Moreover, each node contains the final projected result is a waste of energy and storage. Also, in many cases, explicit routing information is difficult to obtain and maintain. Due to these characteristics, there develops another application of CS to WSN called random sampling with CS.

2.2.1 Random sampling

We would like to apply CS to wireless network sensing. There exists an interesting method to construct the measuring matrix Φ , see [3]. In that paper, a synthetic signal is generated so as to serve as a comparison to the real sensor networks to measure the effectiveness of the CS application. A common approach is called Random Sampling (RS). In a nutshell, it randomly chooses M nodes from all N nodes and gets their readings, then reconstructs the original signal by interpolation of the collected value. A new method brought by CS is called Random Sampling with CS (RS-CS). As above each node becomes a source with probability M/N . Again, each of these source nodes transmits a packet containing the reading of its own sensor. As this packet travels towards the sink, we combine the value contained therein with that of any other node that is encountered along the path. Specifically, let v_i^m with $i = 1, 2, \dots, l_m$ be the readings of the sensors along the path from node m to the sink, where v_1^m is the read-

ing of the node itself and l_m is the length of the path. Node m sends a packet containing the value $y_1^m = \alpha_1 v_1^m$ as well as the combination coefficient α_1 , where α_1 is a value chosen uniformly at random either from $(0, 1]$ or from the set $\{1, +1\}$. We proceed with these random combinations, where in general node $i+1$ sends out $y_{i+1}^m = y_i^m + \alpha_{i+1} v_{i+1}^m$ until the packet finally reaches

the sink. The sink extracts $y_{l_m}^m = \sum_{i=1}^{l_m} \alpha_i v_i^m$, together with the vector of coefficients that were used along the route. These coefficients, form the m th row of matrix Φ , referred to as φ_m . Note that some optimizations are possible. First, if we know in advance the network topology, we can assign combination coefficients at setup time to all nodes, rather than including them in the packets. We can further use the same pseudo-random number generator at the nodes and the sink and synchronize the seeds. Finally, the sink can build a system of the form

$$y = \begin{pmatrix} y_{\lambda_1}^1 \\ y_{\lambda_2}^1 \\ \dots \\ y_{\lambda_m}^1 \end{pmatrix} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \dots \\ \varphi_m \end{pmatrix} X = \Phi X$$

where the $y_{\lambda_r}^r$ with $r = 1, 2, \dots, m$ are the combined values that were received by the sink in the packet that traversed the r th path, Φ is an $m \times N$ matrix whose generic row r , φ_r , contains the vector of coefficients included in the packet. Note that, in general, some of these coefficients might be equal to zero, because all the nodes cannot be reached on a single path. There are four kinds of choosing the entry value of Φ discussed (See detail in [3]). The measuring matrix Φ generated by this routing method is interesting and close to practice. But whether it works well is what we really consider. As we know, when we reconstruct a signal after we apply CS, not only the signal must be represented as a sparse one in the representing matrix φ , but also the measuring matrix Φ and φ should be incoherent. And the smaller the coherence, the

fewer samples are needed.

From the incoherence discussed and simulation presented in the paper, we see that RS-CS does not outperform RS, even if when the data is pre-distributed. Only when the number of transmissions is high enough and only for certain representing matrix is the reconstruction error smaller than RS. In our opinion, this is because that there are too many zeros in Φ , which leads to the loss of the original signal, also the coherence of Φ and φ contributes to this dissatisfaction. So that the application of CS for real sensor data is not straightforward and needs further exploit, but we think that the method and the way of thinking is a good inspiration.

In the later part, there emerges another kind of application of CS to wireless network sensing named CWS, which we will explain carefully.

2.2.2 Compressive wireless sensing

Recent research has introduced an implementation of CS for sensor networks called Compressive Wireless Sensing (CWS) in which a central base station retrieves wireless sensor network data from a randomly distributed grid of transducers. Much of this work has been focused on the power - distortion - latency relationship for a projection of distributed sensor network data onto an under-determined basis.

The distributed communication architecture has been illustrated in the following Figure 8.

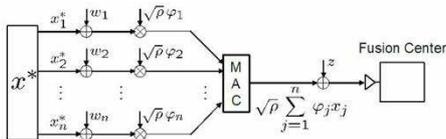


Figure 8

This model involves phase-coherent, low-power, analog transmission of weighted sample values direct from the nodes in the network to the FC via narrow band AWGN

channel. In this Figure, x represents the data sensed by the nodes and w is the Gaussian noise added on the data after the transmission from sensors to the MAC. $\rho > 0$ is a scaling factor used to meet the sensors transmit power constraint. φ is the orthonormal basis where x_i is projected.

According to the theory of CWS, when given sufficient knowledge of φ , that is knowing the basis in which x_i is compressible and the ordering of the coefficients of x_i in the basis, we can discard the $n-k$ coefficients which is smaller than the other k ones and obtain the best k -term approximation of x_i in terms of $x^{(k)} = \sum_{i=1}^k \theta_i \varphi_i$, where θ is the coefficients of x_i . Accordingly, the power-distortion-latency trade-off for this case is given by: $D \sim P_{tot}^{-2\alpha} \sim L^{-2\alpha}$.

However, even though the destination knows the basis in which x_i is compressible, it is not likely that the precise ordering of the coefficients of x_i in this basis is available. One viable approach to this problem could be to use the distributed scheme that resort to random transform domain sampling, that is, the compressive wireless sensing scheme. After k random projections, the observations at the fusion center take the form of $y_i = \sum_{j=1}^n \Phi_{ij}(x_j + w_j) + \tilde{z}_j$,

where $\{w_j\}_{j=1}^n$ and $\{\tilde{z}_j\}_{j=1}^k$ are zero mean Gaussian random variables. Therefore, the power-distortion-latency trade-off can be presented by: $D \sim P_{tot}^{-2\alpha/(2\alpha+1)} \sim L^{-2\alpha/(2\alpha+1)}$.

The goal of this research was to discuss a theoretical model for the compressive sampling of wireless sensor network data. The effects of sensor measurement error, electromagnetic interference present at the base station, and channel phase estimation error were discussed as far as they pertain to accurate determination of the values of Y .

Our research considers a similar distributed grid of sensors (transducers) that measure some physical data (e.g. temperature, pressure) and wirelessly transmit these measurements to a central base station simul-

taneously and phase coherently. A typical sensor network is shown in Figure 9. Here, each black dot represents a sensor with a wireless transmitter communicating sensor readings simultaneously to a base station. In order to simplify the following discussion, we consider an ordered grid to simulate the sensor network.

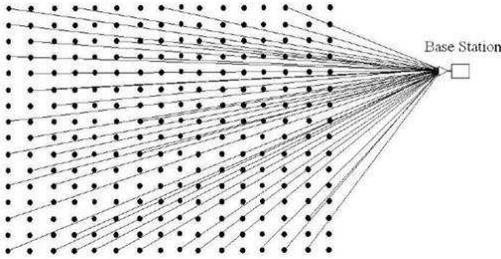


Figure 9

The novelty of the concept of CWS arises from the fact that projections of sensor network data can be naturally added at the base station because the sensors transmit their data phase coherently. Disregarding the effects of noise, the p th observation of the sensor network data which corresponds to the p th element of Y is given by $y_p = \sum_i \Phi_{i,p} x_i$.

Here, ρ_i is the phase dependent gain associated with the i th sensor in the grid which is multiplied by the corresponding element in X . It is assumed that the value of ρ_i is known for each sensor and does not change during the M observations (it is independent of p). The value $\Phi_{i,p}$ is an element of a Rademacher random matrix that enables random sampling of sensor network data, where $\Phi_{i,p} = \Phi_i$ for $\forall j$. This assumes that an entire compressive reading of network data is completed before fading changes. The random matrix, $\Phi_{M \times N}$ for CWS is constructed as a matrix of Rademacher random variables. The pmf of Rademacher variable is presented in the following:

$$f(k) = \begin{cases} 1/2 & \text{if } k=-1 \\ 1/2 & \text{if } k=+1 \\ 0 & \text{otherwise} \end{cases}$$

To normalized the total energy of the sensors, we can modulate k from ± 1 to $\pm \frac{1}{\sqrt{n}}$. This is useful because it allows each of the sensors to locally determine their own vector of Rademacher random variables using their address as a seed value. The base station is then able to construct $\Phi_{M \times N}$ from the seed values given by the appropriate addresses. Thus, the above equation can be given by this: $Y = \Phi_{M \times N} \rho_{N \times N} \Psi_{N \times N}$.

An important focus of this experiment is to examine the capabilities of current compressed sensing reconstruction methods for noisy measurements. It is important to discuss how this noise is modeled in our simulated network. There are two different types of error that are analyzed. The first type is error due to noise present at the base station receiver. This is given by

$$Y = \Phi_{M \times N} \rho_{N \times N} (\Psi_{N \times N} S + W) + Z = \Phi_{M \times N} \rho_{N \times N} \Psi_{N \times N} S + \eta$$

Here Z and W represents an $M \times 1$ and $N \times 1$ vector of zero mean Gaussian random variables respectively. The second type of error present in our analysis arises from fading coefficient estimation error that is introduced when the channel gain from each sensor to the base station is incorrectly estimated.

3 Further thoughts

As increasing small node become available, large scale sensor network are likely to perform significant in a myriad of tasks (thousands of nodes are considered in earthquake). Because the potential number of the sensor nodes may be large, there exist certain disadvantages of the above method of compressive sensing. Firstly, we need M channels for measuring the signal, where M is proportional to N . Secondly, the long distance between sensor nodes and FC can cause high energy dissipation for CWS maintain all the N nodes keep transmitting sig-

nals to FC. Taking the above considerations into account, one way to solve these problems is to partition the whole sensor network into several cells. Figure 10 gives the model of this method. We will develop this model in our further work.

In this model, there remain certain problems to consider:

The size of cell affects the potential degree of frequency reuse in networks. The smaller the size, the more the frequency reuse, and the larger the capacity.

The size of cell affects the efficiency of such local coordination as data aggregation and load balancing.

The performance of the partitioned sensor network while using CWS should be evaluated.

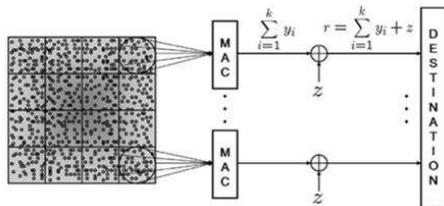


Figure 10

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