The Capacity Gain of Ad Hoc Wireless Network Employing Network Coding(Report II)

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Abstract

Wireless Ad Hoc network suffers from a lot of problems such as low throughput and hot and dead spots. Gupta and Kumar have established the throughput of ad hoc networks only for the unicast pattern which is not effective in application. And they did not take the possible network coding and broadcast and multicast pattern into consideration. There has been interest in applying network coding in ad hoc networks. In this paper, we explore the upper bound of capacity of ad hoc networks for different network models.

1 Introduction

Wireless networks consist of a number of nodes communicating with each other over wireless channels. The architecture can be roughly divided into two categories, the cellular paradigm and ad hoc paradigm.

In recent years most of the wireless communication relies on base stations which is responsible of controlling all the transmissions in the certain region and forwarding data to the prospective receivers. Base stations restrict the area of communication service so people are studying a new technology that is Ad Hoc, in which all the nodes have the same responsibility in transmissions. Basically there are two major advantages of this kind of network. First, no base station is needed. Second, in this network there is no centralized control node so if part of the network break down, the system can still work. In Ad Hoc network, two mobile nodes can reach each other in the range of communication. But if they are far from each other, they must use another node between them to help communicate.Since the range of communication is limited, route consists of many hops.

The characteristics of Ad Hoc network are shown in the following.

1. Independence
   No basic hardware devices are necessary to support this network. So it is easy to setup this mobile network. That is meaningful for rescue or communication in remote places.
2. Changeable topological structure
   Mobile nodes can travel in this network freely. Thus, links between nodes will absolutely change.
3. Limited bandwidth
   Due to the physical characteristics of wireless channels, it is impossible to provide large bandwidth.
4. Limited node energy supply
   In this networks, nodes are almost some mobile devices such as PDA or portable computers whose energy is from battery.
5. Distribution characteristic
   Once some nodes in the network break down, the rest nodes can still work well.
6. Short period of existence
   Ad Hoc is used for temporary communication so its period of existence is short.
7. Limited physical safety
   Mobile network is easier to be attacked in ways such as interception.

Multi-hop wireless network has been intensively studied recently. Due to its flexibility, it can be deployed randomly in the geographic regions to obtain a large scale of information and provide network services. Therefore, the capacity and connectivity of such network are of great interest.

Gupta and Kumar[1] first come up with the idea of the capacity of wireless network. The main conclusion is that when n identical randomly located nodes, each capable of transmitting at W bits per second and using a fixed range, form a wireless network, the throughput $\lambda(n)$obtainable by each node for a randomly chosen destination is $\Theta(\frac{W}{\sqrt{nlogn}})$ bits per second under a noninterference protocol.If the nodes are optimally placed in a disk of unit area, traffic patterns are optimally assigned, and each transmissions range is optimally chosen, the bit-distance product that can be transported by the network per second is $\Theta(W\sqrt{n})$ bit-meters per second. Thus even under optimal circumstances, the throughput is only $\Theta(\frac{W}{\sqrt{n}})$.

It shows that for a single-channel single-interface scenario, in a randomly deployed network, per-flow capacity scales as $\Theta(\frac{W}{\sqrt{nlogn}})$bits/s under a Protocol Model of interference, and that if the available bandwidth W is split into c channels, with each node having a dedicated interface per channel, the results remain the same.V Bhandari, NH Vaidya[8] proposed present a lower bound construction that matches the previous upper bound with capacity as$\Theta(W\sqrt{\frac{p_{rnd}}{nlogn}})$ under multi-channel model.

In the Gupta and Kumar's[1] seminal work, they proposed a model for studying the capacity of fixed ad hoc networks, where nodes are randomly located but are immobile. They have shown that for a random ad hoc network of size n(nodes per unit area), the per node throughput capacity is $\Theta(\frac{1}{\sqrt{nlogn}})$. This is a negative result as it implies that ad hoc networks might not be scalable. In order to increase the capacity of wireless network, Grossglauser and Tse[2] proposed a 2-hop relaying algorithm and showed that it can achieve a throughput capacity of per node by adding mobility. However, they did not consider the delay. Delay limited capacity of ad hoc networks has recently been addressed in [3]. The authors have obtained an approximate expression for the achievable throughput capacity under a maximum delay constraint. In order to solve the trade-offs of mobility, delay and capacity, G Sharma, R Mazumdar, NB Shroff[4] have proposed two different classes of mobility models and showed that they both exhibit critical delays which is inversely proportional to the characteristic path length. Bansal and Liu[5] considered a wireless network consisting of static sender-destination(S-D) pairs and mobile relays, and proposed a geographic routing scheme that achieves a near optimal capacity $O(Wm/n)$and studied its delay performance. Q Dai, L Rong, H Hu[6] derived the analytical expression of mathematical expect value on the capacity in hybrid wireless networks with delay and mobility. A Ozgur, O Leveque[7] proposed hierarchical cooperation to achieve better throughput scaling than classical multihop schemes in static wireless networks. Right now, the core problem of capacity associated with delay and mobility is far from finishing.

Besides, recent work by Fragouli and Katabi[9] introduced the concept of network coding, and there are tremendous interest results when employing network coding to wireless network. [9] shows that the upper bound can be increased by a factor of 2 for the multicast pattern, which is rather satisfactory.

The main concept of [9] is to broadcast coded information through intersecting flows and the next hop of each relay is able to decode the information with the received coded information based on all the former broadcasts and local information. In this case, each node in the relay can broadcast the information to its neighbor nodes through only one transmission instead of multiple data flows. The Figure 1 provides an example of this concept. Assume two sources S1 and S2 are transmitting information both taking the way of node R which serves as a router, to the destination D1 and D2 respectively. With the assumption of broadcast pattern, D1 can receive packets from S2, and D2 can hear from S1 in the same case. The node R can combine the packets from both sources with the method of XORing them. The XORed version of packets is useful for both destination nodes. The respectively information can be easily obtained when XORing again the XORed packets and local ones heard from neighbor source. This pattern is the so-called "X" topology. We can also employ this concept to other patterns in which router nodes exit.

The intuition is that network coding can significantly improve the throughput of ad hoc network because it allows the router node to compress the multiple packets into a single XORed version, so that the number of transmissions is reduced.

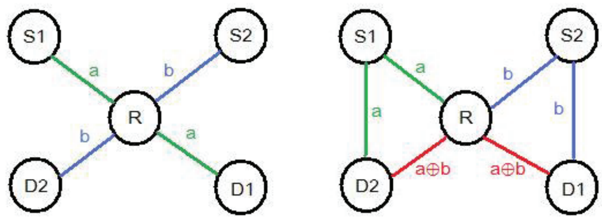

Figure 1: X topology

S. Katti and H. Rahul[10] also proposed the mechanism COPE, a new forwarding architecture that substantially improve the capacity of stationary wireless networks, by inserting a code shim between the IP and MAC layers. The concept of COPE has been put forward by a lot of research work. S. Sengupta and S. Rayanchu[11] employed the COPE-type network coding in traditional unicast network and provide a method for computing source-destination routes and utilizing the best coding opportunities from available ones to maximize the throughput. However, this kind of design often ignore the underlying Physical layer capability and algorithms. Therefore, these schemes are so greedy that it might in fact reduce the throughput of wireless network. P. Chaporkar and A. Proutiere[12] proposed a general method to develop optimal and adaptive joint network coding ana employed it to both COPE-type and XOR-Sym architectures.

2 Model Formulation

In this section, we give some proof and explanations on the

capacity which are mainly about the Gupta and Kumar's work. We list them here just for convenience. In report 1, we only deal with the upper bound of arbitrary networks in both protocol and physical model.

Arbitrary Networks: In the arbitrary network setting, n nodes are located in a certain region, and each node has a destination chosen in advance. That means the location of nodes and traffic patterns can be controlled.

Random Networks: In the random network setting, nodes' location are randomly chosen, i.e. independently and uniformly distributed, in a certain region. Each node also has a randomly chosen destination, which might be the one nearest to the randomly located node.

2.1 Important Assumptions

The conclusions of the report are based on the following assumptions which are very important for the following proof. The assumptions are as follows:

1. There are n nodes arbitrarily located in a disk of unit area on the plane. (The results carry over to any domain of unit area in R^2 which is the closure of its interior.)

2. The network transports λnT bits over T seconds.

3. The average distance between the source and destination of a bit is $\overline{L}$ Note that, together with (2), this implies that a transport capacity of $\lambda n\overline{L}$ bit-meters per second is achieved.

4. Each node can transmit over any subset of M subchannels with capacities W_m bits per second, $1 \leqslant m \leqslant M$, where $\sum_{m=1}^{M} W_m = W$.

5. Transmissions are slotted into synchronized slots of length τ seconds. (This assumption can be eliminated, but makes the exposition easier.)

6. While retaining the restriction (2) for the case of the Physical Model, we can either retain (1) in the Protocol Model or consider an alternate restriction as follows: If a node X_i transmits to another node X_j located at a distance of units on a certain subchannel in a certain slot, then there can be no other receiver within a radius $\triangle r$ of around X_j on the same subchannel in the same slot. This alternate restriction addresses situations where the transmissions are not omnidirectional, but nevertheless there is some dispersion in the neighborhood of the receiver.

2.2 Protocol Model

The Definition of Protocol Model

Suppose node X_i transmits over the m th subchannel to a node X_j. Then this transmission is successfully received by node X_j if

$$|X_k - X_i| \geq (1+\triangle)|X_i - X_j| \tag{1}$$

for every other node X_k simultaneously transmitting over the same subchannel.The quantity $\triangle > 0$ models situations where a guard zone is specified by the protocol to prevent a neighboring node from transmitting on the same subchannel at the same time. It also allows for imprecision in the achieved range of transmissions.

Main Results and Proof

result(1): In the protocol Model, the transport capacity $\lambda n\overline{L}$ is bounded as follows:

$$\lambda n\overline{L} \leq \frac{\sqrt{8}}{\pi}\frac{1}{\triangle}W\sqrt{n}$$

bits per second.

$proof$:Comsider bit b ($1 \leq b \leq \lambda nT$).Suppose it that it moves from the origin to its destination in a sequence of $h(b)$ hops, where the hth hop transvers a distance of r_b^h. Then from the assumption(iii), we can get that

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} r_b^h \geq \lambda nT\overline{L}$$

(Because the bit b is counted for at least once.)

Note that in any slot at most $n/2$ nodes can transmit, for if there are more than $n/2$ nodes which transmit, there will be less than $n/2$ nodes that receive.Hence for any subchannel m and any slot s

$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} 1$(The hth hop of bit b is over subchannel m in slot s)$\leq \frac{W_m \tau n}{2}$

Summing over the subchannels and the slots, and noting that there can be no more than $\frac{T}{\tau}$ slots in T seconds, thus

$$H := \sum_{b=1}^{\lambda nT} h(b) \leq \frac{WTn}{2}$$

.

Suppose thatX_j is receiving a transmission from X_i over the m th subchannel at the same time that X_l is receiving a transmission from X_k over the same subchannel. Then from the triangle inequality and the definition of the Protocol Model.

$$|X_j - X_l| \geq |X_j - X_k| - |X_l - X_k| \geq (1+\triangle)|X_i - X_j| - |X_l - X_k|$$

Similarly,

$$|X_l - X_j| \geq (1+\triangle)|X_k - X_l| - |X_j - X_i|$$

Adding together, we can get:

$$|X_l - X_j| \geq \frac{\triangle}{2}(|X_k - X_l| + |X_i - X_j|)$$

Hence disks of radius $\frac{\triangle}{2}$ times the lengths of hops centered at the receivers over the same subchannel in the same slot are essentially disjoint.

Note that at least a quarter of such a disk is within the domain.(for the minimum situation is that the origin is located at one of the four edges of the square).Since at most $W_m\tau$ bits can be carried in slot from a receiver to a transmitter over the m th subchannel, we have
$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} 1$(The hth hop of bit b is over subchannel m in slot s)$\frac{\pi\triangle^2}{16}(r_b^h)^2 \leq W_m\tau$
Summing over the subchannels and the slots gives

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{\pi\triangle^2}{16}(r_b^h)^2 \leq WT$$

Because

$$H := \sum_{b=1}^{\lambda nT} h(b) \leq \frac{WTn}{2}$$

We can get

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{1}{H}(r_b^h)^2 \leq \frac{16WT}{\pi\triangle^2 H}$$

Note that the quadratic function is convex.Hence

$$(\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{1}{H}r_b^h)^2 \leq \sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{1}{H}(r_b^h)^2$$

Thus

$$\lambda nT\overline{L} \leq \sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} r_b^h \leq \sqrt{\frac{16WTH}{\pi\triangle^2}}$$

And

$$H := \sum_{b=1}^{\lambda nT} h(b)$$

We can get the conclusion:

$$\lambda n\overline{L} \leq \frac{\sqrt{8}}{\pi}\frac{1}{\triangle}W\sqrt{n}$$

2.3 Physical Model

The Definition of Physical Model

In the physical model, the model is mainly constructed by the power level between transmission and receiver nodes compared with the protocol model which is set up on the distance. let $\{X_k; k \in T\}$ be the subset of nodes simultaneously transmitting at some time instant over a certain subchannel. Let P_k be the power level chosen by node $X_k, fork \in T$.Then the transmission from a node $X_i, i \in T$, is successfully received by a node X_j if

$$\frac{\frac{P_i}{|X_i - X_j|^\alpha}}{N + \sum_{k\in T, k\neq i}\frac{P_i}{|X_k - X_j|^\alpha}} \geq \beta$$

This models a situation where a minimum signal-to-interference ratio (SIR) of β is necessary for successful receptions, the ambient noise power level is , and signal power decays with distance γ as $\frac{1}{\gamma^\alpha}$.

Main Results and Proof

result(2): In the physical Model,

$$\lambda n\overline{L} \leq (\frac{2\beta+2}{\beta})^{1/\alpha}\frac{1}{\sqrt{\pi}}Wn^{\alpha-1/\alpha}$$

bit-meters per second.

result(3): If the ratio $\frac{P_{max}}{P_{min}}$ between the maximum and minimum powers that transmitters can employ is strictly bounded above by β, then

$$\lambda n\overline{L} \leq \sqrt{\frac{8}{\pi}}\frac{1}{(\frac{\beta P_{min}}{P_{max}})^{1/\alpha} - 1}W\sqrt{n}$$

bit-meters per second.
proof: Including the signal power X_i also in the denominator, the SIR must be a figure less than 1. In order to get the equation of β, let SIR denotes $\frac{\beta}{\beta+1}$. Therefore

$$\frac{\frac{P_i}{|X_i - X_{j(i)}|^\alpha}}{N + \sum_{k\in T}\frac{P_i}{|X_k - X_{j(i)}|^\alpha}} \geq \frac{\beta}{\beta+1}$$

Because the maximum distance between X_k and $X_{j(i)}$ is the diameter of the circle(ie. $|X_k - X_{j(i)}| \leq \frac{2}{\sqrt{\pi}}$), there is the following inequality

$$|X_i - X_{j(i)}|^\alpha \leq \frac{\beta+1}{\beta}\frac{P_i}{N + \sum_{k\in T}\frac{P_k}{|X_k - X_{j(i)}|^\alpha}}$$
$$\leq \frac{\beta+1}{\beta}\frac{P_i}{N + (\frac{\pi}{4})^{\alpha/2}\sum_{k\in T}P_k}$$

Summing over all transmitter-receiver pairs

$$\sum_{i\in T}|X_i - X_{j(i)}|^\alpha \leq \frac{\beta+1}{\beta}\frac{\sum_{i\in T}P_i}{N + (\frac{\pi}{4})^{\alpha/2}\sum_{k\in T}P_k}$$
$$\leq \frac{\beta+1}{\beta}\frac{\sum_{i\in T}P_i}{(\frac{\pi}{4})^{\alpha/2}\sum_{k\in T}P_k} \leq 2^\alpha\pi^{-(\alpha/2)}\frac{\beta+1}{\beta}$$

Noting that $|X_i - X_{j(i)}|$ denotes $\gamma^\alpha(h,b)$, summing over all slots and subchannels gives

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}\gamma^\alpha(h,b) \le 2^\alpha \pi^{-(\alpha/2)}\frac{\beta+1}{\beta}WT$$

Similar to the Protocol Model, invoking the convexity of γ^α instead of γ^2,

$$(\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}\frac{1}{H}\gamma(h,b))^\alpha \le \sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}\frac{1}{H}\gamma^\alpha(h,b)$$

Because $H \le \frac{WTn}{2}$, we get the conclusion

$$\lambda n\overline{L} \le (\frac{2\beta+2}{\beta})^{1/\alpha}\frac{1}{\sqrt{\pi}}Wn^{\alpha-1/\alpha}$$

If X_i is transmitting to X_j at the same time that $X_k$is transmitting to X_l, both over the same subchannel, then $\frac{\frac{P_i}{|X_i-X_j|^\alpha}}{\frac{P_k}{|X_k-X_j|^\alpha}} \ge \beta$, thus

$$|X_k - X_j| \ge (\frac{\beta P_{min}}{P_{max}})^{\frac{1}{\alpha}}|X_i - X_j| = (1+\Delta)|X_i - X_j|$$

where $\Delta := (\frac{\beta P_{min}}{P_{max}})^{\frac{1}{\alpha}} - 1$. Thus the same upper bound as for the Protocol Model carries over with Δ defined as above and leads to the main result(3).

3 Capacity employing network coding

Due to the character of network coding which means the router can broadcast XORed packets, the transmission times can be reduced. Intuitively, the capacity of network will increase to some extent. [13] proposed a method on physical layer that the time slot used can be reduced to two compared with the original four. This is our motivation to employ network coding on different types of traffic pattern. **In report 2, we have a further discuss on the throughput improvement on both order and quantity.**

The throughput capacity in [14]is denoted as $\lambda_F(n)$ for flow schemes and $\lambda_C(n)$ for coding schemes, the throughput benefit ratio of the coding scheme is denoted $\alpha(n) = \frac{\lambda_F(n)}{\lambda_C(n)}$.

3.1 throughput order of coding scheme

Gupta and Kumar[1] established that the per node throughput of ad hoc networks with multi-pair unicast traffic scales as $\lambda(n) = \Theta(\frac{1}{\sqrt{nlogn}})$. By employing network coding, there is only constant factor improvement in the throughput without order change.

A. 2D case

The main result is proved for the 2D unit square case. [1] proposed that in order to keep connectivity, the transmission radius r(n)=$\frac{\sqrt{logn}}{\sqrt{n}}$ and disks of radius $\frac{\Delta r(n)}{2}$ centered at each receiver are disjoint where Δ is a guard zone. First of all, some definitions are introduced here. Look at figure 2. a **cut** Γ is defined as a partition of the nodes in a graph, the **cut capacity** is the sum of the links' bandwidths crossing the cut, and the **sparsity cut** is a cut where the cut capacity divided by the traffic demand is the minimum over all cuts. The **cut capacity** is defined as $\Lambda_{\Gamma 1,2}$,where $\Lambda_{\Gamma 1,2}$ equals the transmission bandwidth W times the maximum possible number of simultaneous transmissions across the cut from Γ_1 to Γ_2. It is easy to see that all of the direct receivers of transmissions across a cut Γ in one direction lie in the shaded rectangle region with area $l_\Gamma \times r(n)$

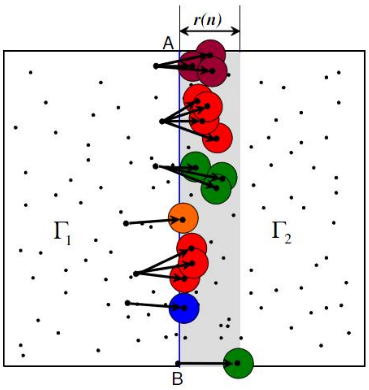

Figure 2: Cut Capacity in 2D

The main proof thought is based on using capacity cut to evaluate the throughput of whole network.

Observation 1: The union of disks (with radius$\frac{\Delta r(n)}{2}$)centered at the receivers of one transmission should be disjoint from the union of disks centered at the receivers of another transmission.

Figure 3 gives us an explicit graph illustration on observation 1.

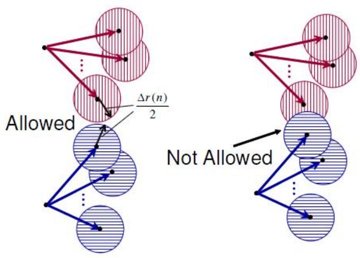

Figure 3: Interference of network coding and broadcasting schemes in 2D

Lemma 1: The capacity of a cut Γ for a 2D region has an upper bound of $\frac{c_\Delta l_\Gamma W}{r(n)}$,where $c_\Delta = max(\frac{16}{\pi\Delta^2}, \frac{\sqrt{3}}{\Delta})$

Proof: Because the transmission range r(n) and the disjoint radius $\frac{\Delta r(n)}{2}$ has some correlation in figure 2, $\Delta = 2$ is a division line to discuss. When $\Delta < 2$, Observation 1 means each transmission across the cut consumes at least an area $\frac{1}{4}\pi(\frac{\Delta r(n)}{2})^2$ of the shaded region in figure 2, which is achieved when the node lies on the corner of rectangle. Thus, the maximum number of simultaneous transmissions across the cut is upper bounded by the area of the shaded region $l_\Gamma \times r(n)$ divided by $\frac{1}{4}\pi(\frac{\Delta r(n)}{2})^2$ which is $\frac{16 l_\Gamma}{\pi \Delta^2 r(n)}$. When $\Delta \geq 2$, as shown in figure 4. Any two receivers of two different transmissions require a $\frac{\sqrt{3}}{2}\Delta$r(n) difference in their coordinates along the cut line. Thus there can be at most $\frac{l_\Gamma}{\frac{\sqrt{3}}{2}\Delta r(n)} + 1 \leq \frac{\sqrt{3} l_\Gamma}{\Delta r(n)}$ simultaneous transmissions across the cut.

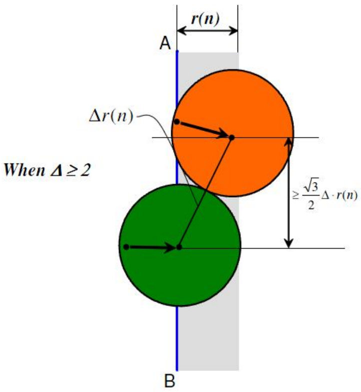

Figure 4: Cut capacity $\Delta \geq 2$case

Since each transmission is able to send W bits/sec, combining the two cases above, the cut capacity is upper bounded by $\Lambda_{\Gamma_{1,2}} \leq \frac{c_\Delta l_\Gamma W}{r(n)}$,where $c_\Delta = max(\frac{16}{\pi \Delta^2}, \frac{\sqrt{3}}{\Delta})$.If we rotate the cut line, there has to be at least one sparsity cut with cut length $l_\Gamma \leq 1$, therefore the sparsity cut capacity of a 2D random network has an upper bound of $\frac{c_\Delta W}{r(n)}$.

Now we have proved the upper bound of the cut capacity although it's not so tight, then we will derive an upper bound for the throughput coding scheme.

Theorem 1: The throughput of coding schemes in a 2D random network is upper bounded by $\Theta(\frac{W}{nr(n)}) = \Theta(\frac{W}{\sqrt{nlogn}})$.

Proof: Assume the coding throughput of the n node random network is $\lambda_c(n)$, distinct messages it receives from the left side of AB as message M.We denote the number of bits of M as B_M, message M can be arbitrarily coded but with only one coding constraint: by Shannon's data compression theory. In order for the right side destination nodes to decode the original data from the left side sources, M has to satisfy $B_M \geq Tn_{\Gamma_{1,2}}\lambda_c(n)$ or $B_M \geq T\Theta(n)\lambda_c(n)$. Because for a sparsity cut Γ_{AB} in the middle, by a Chernoff bound argument it is easy to see that w.h.p. there are $\Theta(n)$ pairs of source-destination nodes that need to cross Γ_{AB} in one direction, or $n_{\Gamma_{1,2}} = n_{\Gamma_{2,1}} = \Theta(n)$w.h.p. Besides,$B_M \leq \frac{c_\Delta W}{r(n)}T$, we derive $\lambda_C(n) \leq \frac{c_\Delta W}{\Theta(n)r(n)} = \frac{c_\Delta W}{\sqrt{nlogn}}$; Thus it has the same order of flow scheme, the benefit ratio $\alpha(n) = \Theta(1)$. It means that network coding can not improve the order of throughput except for a constant factor improvement.

B. 1D case

1D differs from 2D in that the transmission radius does not affect the order of throughput.

Lemma2:The throughput of the coding scheme on a 1D random network is upper bounded by

$$\lambda_C(n) \leq \frac{2W}{n}$$

Brief proof: Using a chernoff bound, the number of sources that need to send data from left to right is larger than $(1-\varepsilon)\frac{n}{4}$ w.h.p. Then $\lambda_C(n)2(1-\varepsilon)\frac{n}{4} \leq W$ which yields the desired results.

Lemma3:The throughput of the coding scheme on a 1D random network is lower bounded by

$$\lambda_F(n) \geq \frac{c_{\Delta_2} W}{n}$$

where $c_{\Delta_2} = min(\frac{1}{2\Delta+2.75}, \frac{1}{4})$

Brief proof: We choose a transmission radius $r(n) = \frac{40logn}{n}$, divide the line deployed region into bins each of length $\frac{r(n)}{2} = \frac{20logn}{n}$, thus yielding $\frac{n}{20logn}$ bins. Each bin contains at least one node. There is a time schedule scheme that on average allows each bin a chance to transmit W bits to each of its two neighboring bins every $\frac{4}{c\Delta_2}$ seconds, according to the graph coloring theory, we can make sure that this schedule scheme can avoid collision and make successful transmission.

Theorem 2: The 1D throughput improvement of the coding scheme over the flow scheme is at most a constant factor;

$$\alpha(n) \leq \frac{2}{c_{\Delta_2}}$$

3.2 Bounds on the throughput benefit ratio α

First we discuss that the network coding can not change the order of throughput. However, we want a definite quantity to have an intuitive view on throughput benefit ratio[15]. For the 1D case, we already have an upper bound of 8 for α. Let's get a tighter bound.

A. 1D case

Theorem 3: The throughput of the flow scheme on a 1D random network is upper bounded by

$$\lambda_F(n) \leq \frac{2W}{(1+\Delta)n}$$

Proof: $\forall \varepsilon > 0$, we need to prove $\lambda_F(n) \leq \frac{2W}{(1+\Delta)n(1-\varepsilon)}$ for large n. W.h.p there are at least $(1-\varepsilon)\frac{n}{4}$ sources that need to cross the sparsity cut from left to right. We evaluate the cut capacity usage for traffic from these sources and show the usage rate to be upper bounded by $\frac{1}{1+\Delta}$. For $\Delta < 1$, in order to successfully transmit from different sources, the receives must have a distance of $\frac{\Delta r(n)}{2}$ apart. For the 1D case, there must be a gap $\Delta r(n)$ as shown in figure 5. Since we consider sources a certain distance away from the cut, the flow conservation constraint implies that these gaps must distribute evenly (averaged over time) along the line around the cut point for the considered traffic. Thus, there will be one silent slot for the cut every (at most) $1 + \frac{1}{\Delta}$ slots.

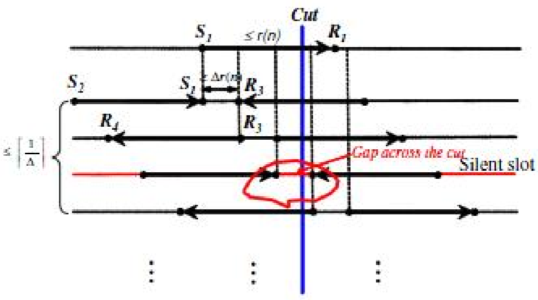

Figure 5: Bound of 1D flow scheme

For $\Delta \geq 1$, The only difference is that now the gap is larger and we drift the transmissions to fill the gap. The cut usage is bounded as at most one transmission can across it every $1 + \Delta$ slots. Again, This results in a usable bandwidth of $\frac{W}{1+\Delta}$ across the sparsity cut.

Theorem 4: The throughput of the coding scheme on a 1D random network is upper bounded by

$$\lambda_C(n) \leq \frac{2W}{(1 + \frac{\Delta}{2})n}$$

Proof: In a coding scheme, the node can broadcast its packets to anyone within the transmission range. For a 1D case, the node can send to both left and right at the same time. Therefore, as we see in figure 6, the frequency of silent slots is decreased. Now, we have one gap cross the cut every (at most) $1 + 2\frac{1}{\Delta}$ slots when $\Delta < 1$ because of the bidirectional transmission. Similarly we derive the same bound for the case of $\Delta \geq 1$.

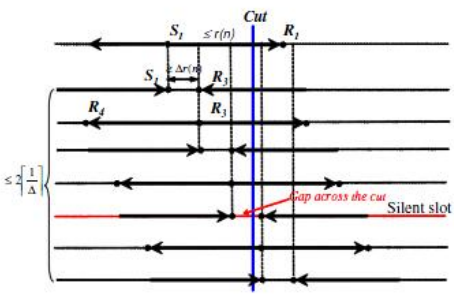

Figure 6: Bound of 1D coding scheme

Theorem 5: The throughput of the flow scheme on a 1D random network is

$$\lambda_F(n) = \frac{2W}{(1 + \Delta)n}$$

Brief proof: $\forall \varepsilon > 0$, we need to prove $\lambda_F(n) \leq \frac{2W}{(1+\Delta)n(1+\varepsilon)}$, using the same binning technique and graph coloring theory and scheduling phase and routing phase, we can derive the answer.

Theorem 6: The throughput of the coding scheme on a 1D random network is

$$\lambda_F(n) = \frac{2W}{(1 + \frac{\Delta}{2})n}$$

Theorem 7: The throughput benefit ratio on a 1D random network is

$$\alpha(n) = \frac{1 + \Delta}{1 + \frac{\Delta}{2}}$$

From the above statement, we derive a specific formula for benefit ratio $\alpha(n)$ in 1D case. It prove again that a constant improvement can be achieved by network coding.

B. 2D case

For the 2D improvement on benefit ratio of throughput, some main conclusions are given and some results are discussed.

Theorem 8: The throughput of the coding scheme on a 2D square random network is upper bounded by

$$\lambda_c(n) \leq \frac{2W}{n}(\frac{1}{\Delta r(n)} + 1)$$

Theorem 9: The throughput of the flow scheme on a 2D square random network is upper bounded by

$$\lambda_F(n) \geq \frac{W}{c_\Delta \sqrt{\pi}(1 + \Delta)nr(n)}$$

where$c_\Delta = max(2, \sqrt{\Delta^2 + 2\Delta})$

Theorem 10: The throughput improvement on a 2D square random network is upper bounded by

$$\alpha(n) \leq 2c_\Delta \sqrt{\pi}\frac{1 + \Delta}{\Delta}$$

From the results above, a upper bound is given in 2D case. But we still haven't got a tight formula for definite $\alpha(n)$.

3.3 Physical Model

What we have discussed above belongs to the protocol model. For the physical Model, some main results are shown and we find that the order is still the same and a constant ratio improvement is achieved by employing network coding. According to [1], physical model is identical to protocol model when denotes $\Delta = \beta^{\frac{1}{\alpha}} - 1$, which is used in the same way to prove the following theorems.

Theorem11: The throughput benefit ratio of coding schemes for 1D random network under the physical model is a constant factor: $\alpha(n) = \Theta(1)$ and $\lambda_F^p = \lambda_c^p = \Theta(\frac{W}{n})$

Theorem12: The throughput benefit ratio of coding schemes for 2D random network under the physical model is a constant factor and $\lambda_F^p = \lambda_c^p = \Theta(\frac{W}{\sqrt{n}})$

4 Future work

1. Since the network coding is considered under Gupta and Kumar[1]'s model, Xue Feng[16] proposed a generalized(Gaussian) physical model. Under this model, the most concern on successful reception is built on SINR. In this case, the data rate from transmitter X_k to its receiver $X_{R(k)}$ is assumed to be

$$W_k = H_m log(1 + \frac{\frac{P_k}{|X_k - X_{R(k)}|^\alpha}}{NH_m + \sum_{i\in T, i\neq k} \frac{P_k}{|X_k - X_{R(R(k))}|^\alpha}}).$$

Our motivation is to make a definite expression for benefit ratio which is a function of path loss exponent α. However, this work is very tough because in this area few questions have been discussed.

2. We will change the traffic pattern into multicast[17] and employ network coding on it to see its effect on capacity, mobility and delay.

3. The study of Liu Junning[14][15] is based on the network layer while some other researchers have focused on the physical layer which is also a direction to try.

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