Throughput Gain of Network Coding in Different Wireless Networks
Group 12 Report 2

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***Abstract*—This report focus on the gain of network coding for different ad hoc networks, unicast, multicast and converge-cast. And for each type of networks, we consider both stationary and mobile situation. For stationary ad hoc networks with unicast, [1] shows that the per-node throughput is $\lambda(n) = \Theta(1/\sqrt{n \log n})$, and [2] proved that the NC gain is a constant $\Theta(1)$. [8] shows the situation of unicast MANETs with 2 hop and multi-hop, the NC gain of former is still a constant, but $\Theta(\log n)$ for the latter. In this report, we try to apply NC to multicast and converge-cast for both stationary and mobility one, and compare with the results of [3] and [9]. The gain of multicast or converge-cast is expected to unify the previous results.**

I. Introduction

Capacity of ad hoc networks and mobile ad hoc networks (MANETs) is a hot topic in wireless networks researching. In [1], Gupta and Kumar found the pre-node throughput of the random static unicast network is $\Theta(1/\sqrt{n \log n})$ in protocol model, which means the capacity for each node declines as the number of nodes increases. To improve the scheme, two main methods comes out. First, Mobility is introduced to the networks in [6], with the overhead of large delay, which shows that the per-node capacity can reach $\Theta(1)$. The result leads to the research of capacity-delay tradeoffs. In [5], three redundancy-based schemes is proposed, namely, 2-hop relay without replicas, 2-hop relay with replicas, multi-hop with replicas, to achieve the capacity of $\Theta(1)$, $\Theta(1/\sqrt{n})$, $\Theta(1/n \log n)$, with the delay of $\Theta(n)$, $\Theta(\sqrt{n})$, and $\Theta(\log n)$, respectively. Second, the concept of network coding is applied to the networks, as [7] and [8]. It is not a long time for using network coding to get higher capacity of unicast networks, in [2] and [3], it shows the network coding can not improve the order of capacity in static unicast networks. And to combine these two approaches, as in [9], it shows that RLC (random linear coding) still cannot improve the order of throughput in MANETs, but changes the throughput-delay tradeoffs significantly.

All above discussion is relay on the model of unicast network, static or mobile. However, the multicast and converge-cast networks are important extension of unicast network. For multicast, in [3] it shows the capacity of static ad hoc network is $\Theta(1/\sqrt{kn \log n})$, where k is the number of the destination for each session, and here $k = O(n/\log n)$. For converge-cast networks, in [10], the work shows both the situation of static and mobile networks, which is very similar to the unicast and multicast, and the results of capacity for different wireless networks without network coding can be summarized as the Table I :

TABLE I
CAPACITY COMPARISON

Mobility	Traffic pattern	Hops	Capacity
stationary	unicast	$\Theta(\sqrt{\frac{n}{\log n}})$	$\Theta(\sqrt{\frac{1}{n \log n}})$
stationary	multicast	$\Theta(\sqrt{\frac{n}{\log n}})$	$\Theta(\sqrt{\frac{1}{nk \log n}})$
stationary	converge-cast	$\Theta(\sqrt{\frac{n}{\log n}})$	$\Theta(\sqrt{\frac{1}{n \log n}})$
i.i.d	unicast	1	$\Theta(1)$
i.i.d	unicast	2	$\Theta(\sqrt{\frac{1}{n}})$
i.i.d	unicast	$\Theta(\log n)$	$\Theta(\frac{1}{n \log n})$
i.i.d	multicast	1	$\Theta(\frac{1}{k})$
i.i.d	multicast	2	$\Theta(\frac{1}{\sqrt{n \log k + k}})$
i.i.d	converge-cast	1	$\Theta(1)$
i.i.d	converge-cast	2	$\Theta(\sqrt{\frac{1}{n \log k}} + \frac{k}{n \log k})$

When the researches of network coding gain of unicast network are nearly finished, the coding gain of multicast and converge-cast is still a new topic. Our task is to exploit the results of the benefit by applying the network coding to these two kind of networks, with and without mobile. To make the statement simply, all the research work we did is under the protocol model in [1]. Here, we present some intuitive senor of the results we expects to:

Since we know that the network coding does not provide order improvement in static unicast, and we also notice that the result is obtained only after applying the coding scheme, but also on the assumption of using broadcasting. Therefore, we aggressively assumes that the coding gain of static multicast or converge-cast networks is also a constant $\Theta(1)$, provide no order improvement. The rough reasoning can be state as follows: First, the result should be able to unify the unicast situation, namely when $k = 1$, the result should still fits, then the gain is at most $\Theta(f(k))$. Second, when adding the constrains of using broadcasting, the gain for unicast should lager than multicast. Last, the coding scheme will not degenerate the capacity, for it eliminate the replicas in the networks. For these three reason, $\Theta(1)$ is most reasonable.

II. Network Model

In this section, we introduce the network models for multicast and converge-cast networks and state some important definitions for further research works.

A. Network model for static ad hoc networks

1) Protocol model for successful transmission: For characterizing the condition for a successful transmission, we assume that all nodes use a common range r_c for their transmissions, and a transmission from node i to node j is successful if and only if $d_{ij} \leq r_c$ and $d_{kj} \geq (1+\triangle)r_c$ for any other simultaneous transmitter, say node k. Here, d_{ij} is the distance between nodes i and j, and $\triangle$ is a positive constant independent of n. During a successful transmission, nodes send data at a constant rate of W bits per second. In the physical model, a transmission is successful if the SINR is greater than some constant. It is well known that with a fading factor $\alpha > 2$, the protocol model is equivalent to the physical model.

2) Model for multicast and converge-cast: The models is plotted as Figure 1 and 1

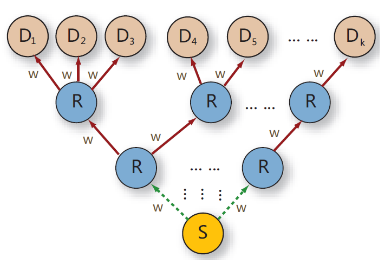

Fig. 1. Data flow diagram for Multicast

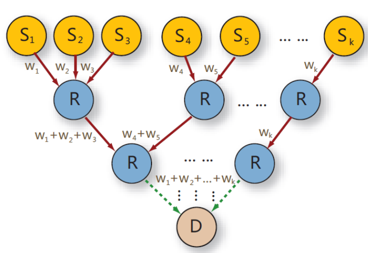

Fig. 2. Data flow diagram for Converge-cast

B. Network model for MANETs with RLC-based relay scheme

We use the models in [9] to apply RLC on both MANETs.

1) Concurrently transmiting cells: In this part, we will define the transmission range and schedule. We choose r_n in such a way that any node in a cell can always directly transmit to any other node in the same cell using the smallest common range of transmission. Obviously, $r_c = \sqrt{2}s_n = \sqrt{2A_n/m} = \Theta(\sqrt{A_n/n})$.

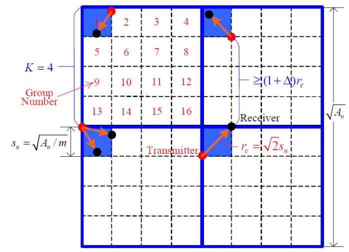

Fig. 3. Cell transmission scheduling. Cells are divided into K_2 groups for the case of $K = 4$. All the cells in group 1 transmit in the same timeslot. In the next timeslot all the cells in group 2 transmit and so on

Time is slotted for packetized transmission. We assume only $\Theta(1)$ packets can be transmitted per cell per timeslot. We adopt the cell scheduling scheme with the following proposition. Under the Protocol model, there exists an interference-free schedule such that each cell becomes active regularly once in K^2 timeslots and it does not interfere with any other simultaneously transmitting cells. Here K depends only on $\triangle$, and is independent of n.

2) 2-hop Relay with RLC:

1) k original packets in each source node will be grouped into one generation. Each source will send $m = (1+\epsilon)k$ coded packets for each generation, where is a constant.
2) Coded packets for each generation will have the same timestamp t_p. The value of t_p is the time the rst coded packet of that generation leaves the source. All coded packets of a generation will be deleted from the relay buffer at the timeslot t if $t - t_p > th_p$, where the threshold th_p depends on $D(n)$ of the scheme and will be sufciently larger than $D(n)$.
3) Each cell becomes active once in every K^2 timeslots. In an active cell, transmission is always between nodes within the same cell.
4) For an active cell with at least two nodes, a random transmitter-receiver pair is selected, with uniform probability over all possible node pairs in the cell. With probability 1/2, the transmitter is scheduled to operate in either Source-to-Relay or Relay-to-Destination mode, described as follows:

- Source-to-Relay Mode: The transmitter sends a coded packet of its current generation, and does so upon every transmission opportunity while it is in source-to-relay mode until m coded packets have been delivered to distinct nodes. If all other nodes in the cell already have one coded packet for that generation, the source will begin to transmit coded packets from the next generation. Every node stores a single packet per S-D pair per generation. When the node receives a new packet, a relay linearly combines the incoming packet with the stored one, and replaces the stored packet with the result. Note that the nodes operate in broadcast mode, i.e., every node will hear every transmission in its range, and update the packet storage as described above.

- Relay-to-Destination Mode: If the designated transmitter has a coded packet in its relay buffer for the destination node, and the rank of coded packets of that generation in the receiver is smaller than k, the coded packet is transmitted to the designated receiver.

3) Multi-hop Relay with RLC:

1) k original packets in each source node will be grouped into one generation. Each source will send $m = (1+\epsilon)k$ coded packets for each generation, where is a constant. Two timestamps for each generation are used. One is called the generating time t_g , based on the time for k original packets to be grouped into a generation in the source. Another is called transmission time t_p , based on the time the rst coded packet of that generation is transmitted by the source.
2) Each cell becomes active once in every K^2 timeslots. In an active cell, transmission is always between nodes within the same cell.
3) For an active cell with at least two nodes, perform the following: among all packets contained in at least one node of the cell and which have useful information for some other node in the same cell, choose the packet with the smallest generating time t_g . If there are ties, choose the packet from the S-D pair i which maximizes $(t_g + i)\ mod\ n$. Transmit this packet to all other nodes in the cell. If the selected packet is in the source, then the source will transmit the linear combination of its k original packets of the same generation, instead of a particular packet belonging to that generation.
4) Every node stores a single packet per S-D pair per generation. When the node receives a new packet, a relay linearly combines the incoming packet with the stored one, and replaces the stored packet with the result.
5) All coded packets of a generation will be deleted from the relay buffer at the timeslot t if $t - t_p > th_p$, where the threshold th_p depends on D(n) of the scheme and should be sufciently larger than D(n).

III. Gain of Network Coding for Static Multicast Ad Hoc Network

The throughput of coding schemes in a random static multicast network is upper bounded by $\Theta(\frac{W}{nkr(n)}) = \Theta(\frac{W}{\sqrt{nk\log n}})$

The proof of the constant gain of network coding for static multicast ad hoc network is almost the same as Theorem 1 in [2], except the pairs of source-destination nodes that need to cross Γ_{AB} in one direction is no longer $\Theta(n)$, but is $\Theta(nk)$.

In [3], the throughput of flooding schemes in a random static multicast network is $\Theta(\frac{W}{\sqrt{nk\log n}})$

thus, the throughput benefit ratio of a random static multicast network is upper bounded by a constant $\alpha(n) = \Theta(1)$

IV. Gain of Network Coding for Static Converge-Cast Ad Hoc Network

V. Gain of Network Coding for Multicast MANETs

VI. Gain of Network Coding for Converge-Cast MANETs

VII. Conclusion

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