Resource Allocation for Cognitive Radio Networks Report

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Chapter 1

Introduction

1.1 Cognitive Radio: A Future Radio System

The term cognitive radio was firstly introduced to public in an article [1] by Joseph Mitola III where it was defined as:
"The point in which wireless personal digital assistants (PDAs) and the related networks are sufficiently computationally intelligent about radio resources and related computer-to-computer communications to detect user communications needs as a function of use context, and to provide radio resources and wireless services most appropriate to those needs."
CR is actually a development from software-defined radio (SDR), which can reconfigure itself to operate on different frequencies by software programming. Recently, the exact definition of CR by IEEE SCC41's P1900.1 working group is as follows:[2]
"a. A type of Radio in which communication systems are aware of their environment and internal state and can make decisions about their radio operating behavior based on that information and predefined objectives. NOTE: The environmental information may or may not include location information related to communication systems.
b. Cognitive Radio (as defined in a.) that utilizes radio, adaptive radio, and other technologies to automatically adjust its behavior or operations to achieve desired objectives. "
To extend this conception, two significant aspects about CR are worthy of note:

1. *Adaptivity*: With the help of sensing component, CR may obtain a relatively enough awareness of the environment, thus to make the possible maximum utilization of the limited frequency band and achieve the harmony of current users. In other words, CR is adaptive to the outer circumstance.

2. *Dynamic Spectrum Access*: Cognitive radio techniques provide the capability to use or share the spectrum in an opportunistic manner. Dynamic spectrum access techniques allow the cognitive radio to operate in the best available channel.[3]

So,CR can promise to improve the utilization of radio frequencies making room for new and additional commercial data, emergency, and military communications services [4].In this report,we focus on the application of dynamic spectrum access of cognitive radio.

1.2   Components in a CR system

After defining what CR is, then we may start to investigate how to let a CR system run.
First, to collect the information of the environment, a sensing component with finer sensibility is essential.
Then, once aggregating circumstantial data, the system need to decide what configuration should be applied. So we introduce a policy database which includes the presetted policy and a learning and reasoning component that can figure out which policy to be made, grounded on presetted ones and experience.
Furthermore, as the circumstance changes rapidly, system should reconfigure itself accordingly. That's why a reconfiguration radio is included.
Finally, for the sake of safety, we also need a configuration database to storage the current configuration.
A CR system usually contains five parts:[5]

- *Sensing*: A sensing engine can accept inputs from the radio components such as the radio frequency (RF), but possibly other sources such as data sources on the internet or other networked nodes. Data exchanged may include geolocation data.

- *Learning and Reasoning*: A reasoning engine can accept inputs from the sensing engine and policy data base and determines an appropriate configuration for the radio components. The reasoning engine may be capable of learning based on experience.

- *Configuration Database*: A configuration database is required to maintain the current configuration of the radio components.

- *Reconfiguration Radio*: A CR system may have a single reconfigurable radio component with a reasoning engine accepting sensing information from local node but not from external data sources.

- *Policy Database*: a policy data base may exist that determines what behavior is acceptable in what circumstances.

1.3   Dynamic Spectrum Access

As listed above, the components of a CR system can detect the RF environment. So we may wonder, can these components be distributed across multiple protocol layer and devices in a network? The key of this question is "Dynamic Spectrum Access", or simply noted as DSA. The specific content about DSA is[3]:

1. determine which portions of the spectrum is available and detect the presence of licensed users when a user operates in a licensed band (spectrum sensing),

2. select the best available channel (spectrum management)

3. coordinate access to this channel with other users (spectrum sharing)

4. vacate the channel when a licensed user is detected (spectrum mobility).

Networks that built on this idea is called Dynamic Spectrum Access Networks(DSANs), also known as NeXt Generation (xG) communication networks. The mainfuction of a xG networks is actually a extension of DSA, as follow[3]:

- *Spectrum sensing*: Detecting unused spectrum and sharing the spectrum without harmful interference with other users.

- *Spectrum management*: Capturing the best available spectrum to meet user communication requirements.

- *Spectrum mobility*: Maintaining seamless communication requirements during the transition to better spectrum.

- *Spectrum sharing*: Providing the fair spectrum scheduling method among coexisting xG users.

In our report, we mainly focus on how to obtain the sharing part in an effective way, because this part is directly related to the interest of network users.
There usually exist two kinds of users–licensed users or refered as primary users and unlicensed users or secondary users. While ensuring the communication activities of the primary users, we should distribute the spectrum band not in use now to the secondary ones, but in a precondition that it will not interfere the primary users' benifit. So, we'll go further to show you our thoughts about how to share the spectrum band in a fair and economical method.

Chapter 2

Related Works

2.1 Opportunistic Spectrum Access

With a large increasing number of wireless devices, Frequency spectrum is becoming more and more scarce. For wireless communications, most spectrum has been allocated to licensed users by the regulatory agencies statically. In fact, these spectrum have not been used all the time or sufficiently utilized at some time, which means most spectrum have been wasted as "white space" in existing bands. To exploit these spectrum, a spectrum policy has been employed by cognitive radio which enables secondary users to share unemployed spectrum with primary users opportunistically.[7],[8].

In order to improve the efficiency of spectrum sharing, several methods are proposed, in [7], non-intrusive spectrum access schemes is focused on, which do not require primary users to alter their existing hardware or behavior. It introduces two metrics to protect primary performance, namely collision probability and overlapping time and presents two spectrum access schemes using different sensing, back-off, and transmission mechanisms.

In [8], it uses the techniques of adaptive queueing and Lyapunov Optimization to design an online flow control, scheduling and resource allocation algorithm for a cognitive network that maximizes the throughput utility of the secondary users subject to a maximum rate of collisions with the primary users. This algorithm operates without knowing the mobility pattern of the secondary users and provides explicit performance bounds.

In [20], it does not assume the user has a priori knowledge regarding the statistics of channel states. The main goal of the work is to design robust strategies that decide, based only on knowledge of the channel bandwidths/data rates, which channels to probe. They derives optimal strategies that maximize the total expected bandwidth/data rate in the worst-case, via a performance measure in the form of a competitive regret (ratio) between the average performance of a strategy and a genie (or omniscient observer). This formulation can also be viewed as a two-player zero-sum game between the user and an adversary which chooses the channel state that minimizes the user's gain.

In [11][12], Yongle Wu develop a cheat-proof etiquette for unlicensed spectrum sharing by modeling the distributed spectrum access as a repeated game, which enforces the competing users cooperate with each other honestly. In order to enforce the competing users to cooperate with each other, a certain punishment which will be triggered if any user deviates

from cooperation is set. Also, two proof strategies, based on the maximum total throughput and proportional fairness criteria respectively, are developed to provide the player with the incentive to be honest and to make cheating nearly unprofitable.

In[13], Beibei Wang propose a self-learning repeated game framework for distributed primary-prioritized dynamic spectrum access. In this framework, there are multiple selfish secondary users and constantly access some temporarily unused licensed spectrum band. With the proposed distributed repeated game approach, the optimization of secondary users' access to the spectrum dynamics are available and better fairness among dissimilar secondary users with higher spectrum efficiency is provided.

2.2 Power Allocation

For wireless networks, power allocation is an important factor influencing the efficiency of communication, because power allocation will influence the interference among different channels and the consumption of energy. In cognitive radio networks, a well-organized power allocation is necessary with large numbers of secondary users communicating with each other. In[18], Pan Zhou introduces a novel utility function for the proposed non-cooperative joint power and rate control game with interference power pricing. It uses pricing to study the problem of interference power caused by SUs and the net utilities experienced by SUs will be largely improved. Also, in the numeral results part, it proposes an algorithm to find the best pricing factors.

In[17], Fan Wang designs a channel/power/rate allocation scheme that overcomes the inefficiency of iterative water-filling and yet can be implemented in a distributed fashion. It proposes a price-based iterative water-filling (PIWF) algorithm, and shows that this algorithm maintains the simplicity and distributed operation of the IWF algorithm while achieving better bandwidth efficiency (i.e., higher sum-rate).

And [25] uses the game theory to analyze the issue of cooperative selecting channel and power in the cognitive radio network. It builds a model as N secondary user pairs(SU) with strategy[si, pi] and one primary user with the strategy which is fix when it is accessible to the network. It defines exact potential game (EPG) and proves the existence of nash equilibrium, and the convergence ,then defines a stackelberg game,in which when SU is cognitive of the existence of NE, SU will give up. Besides,it also defines a stackelberg game,in which when SU is cognitive of the existence of NE, and define two primary users' strategy of pricing.

In[26], a Space-Time Block Codes (STBC) MC-CDMA system and a hand-off technique is proposed to choose highly reliable communication and ensure the unimpaired operation of licensed users, and A noncooperative power game (NPCG) based on SIR is designed to enhance the system performance by sharing spectrum resource. For the same power consumption, the total throughput is significant increase. So it could achieve better system performance and meet the needs between licensed users and secondary users

2.3   Utility Optimization

A major goal of cognitive radio is to improve the utilization of radio frequency spectrum in wireless networks. The problem of spectrum sharing among primary users and secondary users could be formulated as an oligopoly market competition and use a noncooperative game to obtain the spectrum allocation for secondary users.[14],[15]

In fact,game theory has been applied to wireless communication: the decision makers in the game are rational users or networks operators who control their communication devices. These devices have to cope with a limited transmission resource that imposes a conflict of interests[6].In [14],a dynamic game in which the selection of strategy by a secondary user is solely based on the pricing information has been used for each secondary user to adjust the size of its spectrum.

To select the secondary users who are allowed to share a channel,the Game theoretic axiom of fairness, i.e., Nash Bargaining Solutions(NBS) also can contribute to the approach of utility optimization.[16] assumes the primary system is a cellular OFDM-based network. and develop the optimum resource allocation strategies which guarantee a level of QoS, defined by minimum rate and the target Bit Error Rate (BER), for the primary system.

In order to get utility optimization,market-driven dynamic spectrum auctions can make contributions.[9] proposes VERITAS ,a truthful and efficient spectrum auction to support dynamic spectrum market.It allows wireless users to obtain and pay for the spectrum based on their demands, and enables spectrum owners to maximize their revenues by assigning spectrum to the bidders who truly value it the most.However,it only considers one-sided auction which means one seller and multiple buyers.To extend it to double auction,the TRUST [10] has come out,which enables spectrum reuse to significantly improve spectrum utilization.

In [19], it models the spectrum allocation in wireless networks with multiple selfish legacy spectrum holders and unlicensed users as multi-stage dynamic games. In order to combat user collusion, A pricing-based collusion-resistant dynamic spectrum allocation approach is proposed to optimize overall spectrum efficiency, while not only keeping the participating incentives of the selfish users but also combating possible user collusion.

In [24], a selfish spectrum sharing problem in CR networks is proposed, by taking into account the potential noncooperativeness. It investigates user communication session via a cross-layer, optimization approach, with joint consideration of power control, scheduling, and routing. In terms of spectrum access opportunities, an equilibrium pricing scheme is presented to show that it is close to optimal in most scenarios. The proposed Game-theoretic CR Spectrum Sharing algorithm (GCSS) highlights the trend of spectrum pricing design that it is not necessarily bad for the network users to behave selfishly.

Chapter 3

System Models and Analysis

3.1 Graph Coloring Model of Cognitive Radio

As the mobiles in a network can be regarded as a vertex in a graph, using a specific method, the spectrum allocation can evolve to a problem of providing each vertex a unique color under the conditions that no conflicts exist and also performance can be enhanced,[27]. In this point of view, before further investigation on this graph coloring model, we should define some important parameters in this model.

3.1.1 Parameters in Graph Model

We consider a network where N users from 0 to N-1 and they competes for M spectrum bands, which are marked from 0 to M-1.

- *Channel availability*:
$$L = \{l_{n,m} | l_{n,m} \in \{0, 1\}\}_{N \times M} \tag{3.1}$$

  L is a N by M binary matrix representing the channel availability. If channel m is available for user n, then $l_{n,m} = 1$, otherwise $l_{n,m} = 1$.

- *Reward Matrix*:
$$B = \{b_{n,m}\}_{N \times M} \tag{3.2}$$

  B is called as reward matrix, each element $b_{n,m}$ of which represents the maximum bandwidth can be obtained when channel m is occupied by user n.

- *Interference Matrix*:
$$C = \{c_{n,k,m} | c_{n,k,m} \in \{0, 1\}\}_{N \times N \times M} \tag{3.3}$$

  Interference matrix C indicates the interference relation between different users. If $c_{n,k,m} = 1$, it means that when user n and k transmit on channel m at the same time, interference will arise. It should be noted that $c_{n,k,m} < l_{n,m} \times l_{k,m}$, $c_{n,n,m} = 1 - l_{n,m}$ and even two users might conflicts with each other on one channel, but may not on another one.

- *Non-interference Channel Allocation Matrix*:

$$A = \{a_{n,m} | a_{n,m} \in \{0, 1\}, a_{n,m} \leq l_{n,m}\}_{N \times M} \tag{3.4}$$

This matrix A is the result of distribution channel, in which $a_{n,m} = 1$ suggests that channel m is allocated to user n. And we can see that if and only if $l_{n,m} = 1$ could $a_{n,m}$ be set to 1. Also, when the network works in a conflict free pattern, the following inequation should be satisfied:

$$a_{n,m} + a_{k,m} \leq 1, if c_{n,k,m} = 1, \forall n, k < N, m < M \tag{3.5}$$

- *User-reward*:

$$\beta_n = \sum_{m=0}^{M-1} a_{n,m} \cdot b_{n,m} \tag{3.6}$$

β_n is a N by 1 reward vector which characterizes what each users get under a given channel spectrum. And let $\bar{\beta}_n$ denote the average reward of users.

3.1.2 Allocation Policy

After defining the fundamental factors related to spectrum assignment, then we should focus on the objective of assignment. This paper is concerned with the following three utility functions that describes the policy of how we choose the proper plan in respect of average reward $\bar{\beta}_n$:

- **Max Average Reward**:

$$U_{sum} = \sum_{n=0}^{N-1} \bar{\beta}_n / N \tag{3.7}$$

The utility function is to maximize the spectrum utilization.

- **Max Min Reward**:

$$U_{min} = \min_{0 \leq n < N} \bar{\beta}_n \tag{3.8}$$

The utility function is to maximize the reward of bottleneck users.

- **Max Proportional Fair Reward**:

$$U_{fair} = (\prod_{n=0}^{N-1} (\bar{\beta}_n + 1e - 4))^{\frac{1}{N}} \tag{3.9}$$

This utility function considers the proportionality of all the users in the distribution.

3.1.3 Color-Sensitive Graph Coloring

We present a bidirectional graph G = (V,L,E), where V is a set of vertexes denoting the users, L is the available channel or the color list at each vertex, and E is a set of undirected edges between vertexes representing interference between any two vertices. For any two vertices $u, v \in V$, a m-colored edge exists between u and v if $c_{u,v,m} = 1$. E depend on the Interference Matrix C (see subsection 3.1.1).

Now, the spectrum allocation problem has been transferred into coloring each vertex using a number of colors from its color list to maximize system utility. The coloring scheme is constrained by that if a m colored edge exists between any two vertexes, color m cannot be assigned to them simultaneously. This problem is called color-sensitive graph coloring (CSGC).

3.1.4 Approach to CSGC

As CSGC is a NP-hard, we need introduce some special idea to solve this problem efficiently. The one we are concerned about is greedy method.

In details, this method is consist of these steps:

1. Provide each vertex a label associated with a color under a given utility policy.

2. Choose the vertex with the highest label and assign the corresponding color to it.

3. Delete this vertex from the graph, and also the assigned color from the color list of the neighbors constrained by this color.

4. Continue the former steps until all colors has been distributed.

It could be seen that this algorithm always consider the utility first, and that is why we say it is greedy.

And now, let's consider the policies listed in subsection 3.1.2. And it is noteworthy the word 'collaborate' means that the rules take the interference between users into consideration, while the word 'non-collaborate' means that by the rules the users only care his own interests. So, each rule is formed by two kinds of policies:

- **Max-sum Reward Rules**:

    1. *Collaborative-Max-Sum Reward (CSUM) rule*:

    $$label_n = \max_{m \in l_n} b_{n,m}/(D_{n,m} + 1)$$

    $$color_n = \arg \max_{m \in l_n} b_{n,m}/(D_{n,m} + 1) \qquad (3.10)$$

    $D_{n,m}$ is the numbers of users which have interference with the user n in the channel m, and l_n represents the available channels in this stage of the spectrum distribution. The CSUM rule not only improves the over-all utilization but also deals with the neighbor's interference, so we say that it is a collaborative one.

2. *Non-collaborative-Max-Sum Reward (NSUM) rule*:

$$label_n = \max_{m \in l_n} b_{n,m}$$

$$color_n = \arg\max_{m \in l_n} b_{n,m} \tag{3.11}$$

Compared with the CSUM rule, NSUM is selfish as it stands, by which reason we say it is non-collaborative. The main objective of this rule is to increase the system utilization, while without taking into account the interference.

- **Max-min Reward Rules**:

  1. *Collaborative-Max-Min Reward (CMIN) rule*:

  $$label_n = -\bar{\beta}_n$$

  $$color_n = \arg\max_{m \in l_n} b_{n,m}/(D_{n,m} + 1) \tag{3.12}$$

  By this rule, the user with the least cumulative reward will be able to obtain the max reward. And CMIN also considers interference from other users.

  2. *Non-collaborative-Max-Min Reward (NMIN) rule*:

  $$label_n = -\bar{\beta}_n$$

  $$color_n = \arg\max_{m \in l_n} b_{n,m} \tag{3.13}$$

  NMIN is similar with CMIN, but the former doesn't take the interference into consideration.

- **Max-proportional-fair Reward Rules**:

  1. *Collaborative-Max-Proportional-Fair Reward (CFAIR) rule*:

  $$label_n = \frac{\max_{m \in l_n} b_{n,m}/(D_{n,m} + 1)}{\bar{\beta}_n}$$

  $$color_n = \arg\max_{m \in l_n} b_{n,m}/(D_{n,m} + 1) \tag{3.14}$$

  The channel is always allocated to the user who has the max value of $r_n/\hat{R}_n$ in the system by the proportional way, where r_n represents the obtained reward in the current allocation stage and $\hat{R}_n$ represents the average reward which user n gets in the past.

  2. *Non-collaborative-Max-Proportional-Fair Reward (NFAIR) rule*:

  $$label_n = \frac{\max_{m \in l_n} b_{n,m}}{\bar{\beta}_n}$$

  $$color_n = \arg\max_{m \in l_n} b_{n,m} \tag{3.15}$$

  This rule works exactly the same as CFAIR except that it does not consider the interference.

While there exists a central station that decides the policy of spectrum assignment, the collaborative rules are more reasonable. But when on the contrary, the non-collaborative ones may prove to be more efficient.

3.2 Network Model for opportunistic scheduling in Cognitive Radio

This model develops opportunistic scheduling policies for cognitive radio networks that maximize the throughput utility of the secondary users subject to maximum collision constraints with the primary users. It uses the technique of Lyapunov Optimization to design an online flow control, scheduling and resource allocation algorithm for a cognitive network with static primary users and potentially mobile secondary users.

3.2.1 Underlying Assumptions

The cognitive radio network consisting of M primary users and N secondary users as shown in Fig. 1. Each primary user has a unique licensed channel and these are orthogonal in frequency and/or space. The secondary users do not have any such channels and opportunistically try to send their data to the access points by utilizing idle primary channels. The network we talk about here is a time-slotted model. The primary users are assumed to be static while the secondary users could be mobile so that the set of channels they can access can change over time. But we assume that the topological pattern of the network remains the same during one time slot. Exactly one unique channel is assigned to every licensed user. And all these channels are orthogonal to each other. In order to make things simple and clear, exactly one packet can be transmitted over any channel during a time slot. We also make the assumption that channel state information and channel accessibility of secondary users are Markovian process. Finally, the network here is a distributed one which means that no user knows a whole picture of the network.
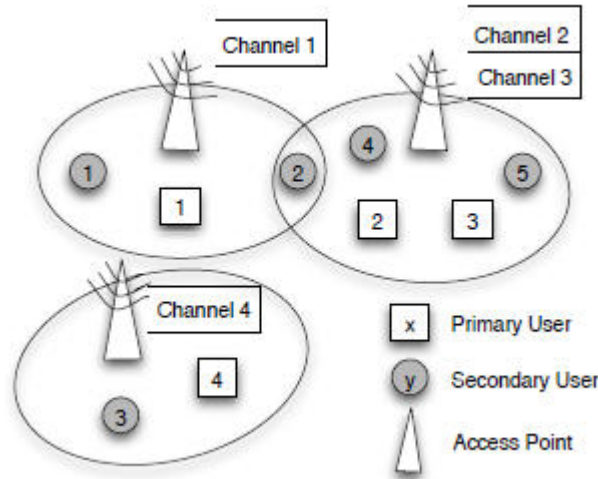

Figure 3.1: Network structure of the cognitive network

3.2.2 Aim of this model

Throughput under the constraint of collision and interference is evidently the ultimate goal of cognitive radio designing. Let Rn be the number of new packets admitted into this queue in slot t. Let rn denote the time average rate of admitted data for secondary user n that means:

$$r_n = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} R_n(\tau) \tag{3.16}$$

Let r=$(r_1,...,r_N)$ denote the vector of these time average rates of these N secondary users. Under a specific but common situation, the weight of these N secondary users are the same, so the throughput should be defined as $\frac{1}{n} \sum r_n$.But for a more general purpose, let $\{\theta_1,...,\theta_n\}$ be a collection of positive weights for N secondary users, then the aim of the model is to design a flow control and scheduling policy that yields a r that maximize$\sum_{n=1}^{N} r_n \theta_n$ while subject to some constraints.

3.2.3 Important Definition and Variables

- Channel accessibility matrix
  H(t)=$\{h_{nm}\}_{N \times M}$

$$h_{nm}(t) = \begin{cases} 1 & if\, sec.\, user\, n\, can\, access\, channel\, m\, in\, slot\, t \\ 0 & else \end{cases} \tag{3.17}$$

  As we have mentioned above, the H(t) process is Markovian and has a well dened steady state distribution.

- Channel occupancy
  Let S(t)=$(S_1(t),S_2(t),...,S_M(t))$ represent the current primary user occupancy state of the M channels. $S_i(t)$=0 if channel i is occupied by primary user i in time slot t and $S_i(t)$=1 if i is idle in time slot t. Because we only have two states(occupied or idle) over a channel and the number of primary user is finite, S(t) evolves according to a finite state ergodic Markov chain on the space $\{0;1\}^M$. Due to some limitation in carrier sensing, the exact channel state may nit be available to the secondary users. The channel state available to secondary user is described by a probability vector P(t) discussed below.

- Channel state probability vector.
  P(t)=$(P_1(t),P_2(t),...,P_M)$ where P_i is the probability that channel i is idle in time slot t. This vector can be obtained through a knowledge of the traffic statistics of primary users. The statistic nature of P(t) leads to the inherent sensing measurement errors that no primary transmission detection algorithm could solve. As collision is inevitable, our goal is to constrain it under a pre-given constant ρ_m.

- Channel set of interference
  These M channels mentioned above may notbe orthogonal to secondary users, so variables are needed to characterize the interference between secondary users. We define I_{nm} as the set of channels that secondary user n interferes with when it uses channel m. A indicator variable is further defined as:

$$I_{nm} = \begin{cases} 1 & if k \in I_{nm} \\ 0 & else \end{cases}$$ (3.18)

  Clearly, if there is no interference between secondary users, then $I_{nm} = \{m\} \forall n$

- Data receiving process
  Each secondary user n receives data according to an i.i.d arrival process $A_n(t)$ which is upper bounded by a constant value A_{max} that has rate λ_n packet/slot. We will show later that this A_{max} guarantees the worst performance of this model which is very important in practical scenario.

- Backlog queue in network layer
  U_n is defined as the backlog queue of secondary user n at the beginning of time slot t.

- Virtual collision queue
  We define $X_m(t)$ to track the amount by which the number of collisions suffered by a primary user m exceeds its time average collision constraint rate ρ_m.

- New packets admitted
  R_n is the number of new packets admitted into this queue in slot t.

- Number of attempted transmission
  Let $\mu_{nm}(t)$ be the number of attempted packet transmission when a control action allocates channel m to n.

- Collision variable

$$C_{nm}(t) = \begin{cases} 1 & if there was a collison with the primary user in channel m in time slot t \\ 0 & else \end{cases}$$

(3.19)

  Let $c_m(t) = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} C_m(\tau)$

- Control variable
  Control variable V offered by the algorithm that we will discuss later enables an explicit trade-o between the average throughput utility and delay.

3.2.4   Modeling the Network with Queuing Dynamics

There are two kinds of queues involved in this model. One is the backlog queue in network layer of secondary users as we described above and the other is the virtual collision queue which is maintained in software.

- The queuing dynamics of the secondary user n is described by:

$$U_n(t+1) = max[U_n(t) - \sum_{m=1}^{M} \mu_{nm}(t)S_m, 0] + R_n(t) \qquad (3.20)$$

Which means that the backlog at the beginning of time slot t + 1 equals to the remaining backlog of time slot t plus the number of new packets admitted in the queue during time slot t. And the constraints are:

  - Constraint on transmission rate: $\mu_{nm}(t) \in 0, 1 \forall m, n$
  - Idle channel: $\mu_{nm}(t) \leq h_{nm}(t) \forall m, n$
  - Allocation constraint: $0 \leq \sum_{m=1}^{M} \mu_{nm}(t) \leq 1 \forall n$
  - Successful transmission: $\mu_{nm}(t) = 1 \Longleftrightarrow \sum_{j=1}^{M} \sum_{i=1, j \neq n}^{N} I_{ij}^{m} m \mu_{ij}(t) = 0 \forall m, n$
  - Data rate constraint: $0 \leq R_n(t) \leq A_n(t)$

When the channels are orthogonal for secondary users, these constraints simplifies to $0 \leq \sum_{n=1}^{N} \mu_{nm}(t) \leq 1$

- The queuing dynamics for virtual collision queue $X_m(t)$ is:

$$X_m(t+1) = max[X_m(t) - \rho - m, 0] + C_m(t) \qquad (3.21)$$

The whole system is rate stable only when $c_m \leq \rho_m$, but the value of queuing dynamics lies in that we can turn the time average constraint into queuing problems if our flow control and resource allocation policies to stabilize all collision queue.

3.2.5 An online algorithm to achieve maximized throughput

This algorithm is a cross-layer strategy which contains two aspects.

- Flow control:
  We aim at minimizing $R_n(t)[U_n(t) - V\theta_n]$ under the constraint of $0 \leq R_n(t) \leq A_n(t)$. We can easily affect the performance/delay tradeoff by changing the parameter V.

- Resource allocation:
  We choose an allocation that maximize $\sum_{n,m} \mu_{nm}(t)[U_n(t)P_m(t) - \sum_{k=1}^{m} X_k(t)(1 - P_k(t))I_{nm}^k]$ This is the difference between the current queue backlog $U_n(t)$ weighted by the probability that primary user m is idle and the weighted sum of all collision queue backlog for the channels that user n interferes with if it uses channel m.

The two maximization requires solving the Maximum Weight Match(MWM) problem on an $N \times M$ bipartite graph of N secondary users and M channels which is presented in [8]

3.3 Game Theory Model in Cognitive Radio Network

In this section, we present the model defined in [14] to solve the competitive spectrum sharing problem. As frequency spectrum is really a scarce resource for wireless communications and it may be congested to accommodate diverse type of users in the next generation wireless networks, so it is important to allow as many users as possible to use the network with a proper spectrum sharing mechanism. For the sake of the need of frequency spectrum, we model spectrum sharing as a noncooperative game between secondary users and the main objective of this noncooperative game is to maximize the profit of all secondary users, which ensures that the revenue of the primary users/service provider can be maximized as well.

3.3.1 Definition in the Model and Assumption

- *Game Theory* : Game theory is a branch of applied mathematics that is used in the social sciences, biology, political science, computer science and philosophy. Game theory attempts to mathematically capture behavior in strategic situations, in which an individual's success in making choices depends on the choices of others. While initially developed to analyze competitions in which one individual does better at another's expense (zero sum games), it has been expanded to treat a wide class of interactions, which are classified according to several criteria.

- *Nash Equilibrium* : In game theory, the Nash equilibrium is a solution concept of a game involving two or more players, in which no player has anything to gain by changing only his or her own strategy unilaterally. If each player has chosen a strategy and no player can benefit by changing his or her strategy while the other players keep theirs unchanged, then the current set of strategy choices and the corresponding payoffs constitute a Nash equilibrium.

- *Primary and Secondary Users* : We consider a spectrum overlay-based cognitive radio wireless system with one primary user and N secondary users (Fig. 1). The primary user is willing to share some portion of the spectrum (b_i) with secondary user i. The primary user charges a secondary user for the spectrum at a rate of c per unit bandwidth, where c is a function of the total size of spectrum available for sharing by the secondary users. After allocation, the secondary users transmit in the allocated spectrum by using adaptive modulation to enhance the transmission performance. The revenue of secondary user i is denoted by r_i per unit of achievable transmission rate.

- *Wireless Transmission Model* : We assume a wireless transmission model based on adaptive modulation and coding (AMC) where the transmission rate can be dynamically adjusted based on channel quality. With AMC, the signal-to-noise ratio (SNR) at the receiver γ is partitioned into $S + 1$ non-overlapping intervals with threshold denoted by Γ_s, where S denotes the number of transmission modes . So the probability of using transmission mode s, i.e., $P_r(s)$ and average packet error rate $\overline{PER}_s$ for a given average SNR $\overline{\gamma}$ are obtainable. Then, the average transmission rate $k_i = \sum_{s=1}^{S} I_s P_r(s)(1 - \overline{PER}_s)$, I_s is the spectral efficiency of transmission mode s.
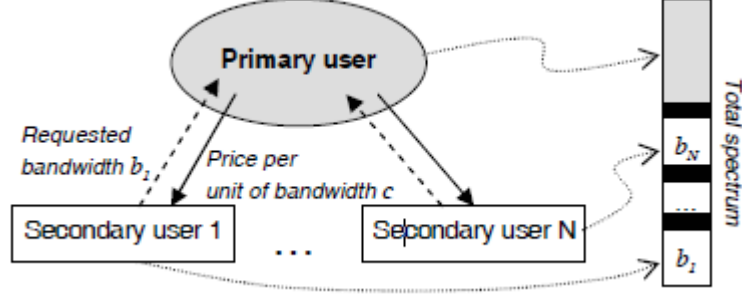
Figure 3.2: System Model for Spectrum Sharing

- *Oligopoly Market Competition and Noncooperative Game* : The spectrum sharing problem can be modeled as an oligopoly market competition. In a general noncooperative game model for oligopoly market, all the firms compete in terms of product quantity to achieve the highest profit. In the spectrum sharing model considered here, the secondary users compete with each other to share the bandwidth offered by the primary user. The competition among the secondary users here is in terms of the requested spectrum size (i.e., the product quantity). The profit of a secondary user can be computed based on the price charged by the primary user and the benefits gained from utilizing the allocated spectrum.

3.3.2 Static Game Model

In this section, we will use a static game model to solve the competitive spectrum sharing problem, where all secondary users can completely observe the strategies and the payoffs of other secondary users.

Based on the system model described above, a noncooperative game can be formulated as follows. The players in this game are the secondary users. The strategy of each of the players is the requested/allocated spectrum size (denoted by b_i for secondary user i) which is non-negative. The payoff for each player is the profit (i.e., revenue minus cost) of secondary user i (denoted by π_i) in sharing the spectrum with the primary user and other secondary users. The commodity of this oligopoly market is the frequency spectrum.

First, there is an assumption that the pricing function used by the primary user to charge the secondary users is :

$$c(\mathbf{B}) = x + y \left(\sum_{b_j \in \mathbf{B}} b_j \right)^{\tau} \tag{3.22}$$

The variables of this equation are :

- x, y, *and* τ : non-negative constants and $\tau \geq 1$.

- $\mathbf{B}$: $\mathbf{B} = \{b_1, \cdots b_N\}$, presenting the strategies of all secondary users.

We assume that the primary user charges all of the secondary users at the same price, and w denotes the worth of the spectrum for the primary user. To ensure that the primary

user has the incentive to share spectrum of size $b = \sum_{b_j \in \mathbf{B}} b_j$ with the secondary users, there must be an inequation $c(\mathbf{B}) > w$.

As the cost of secondary user i is $b_i c(\mathbf{B})$, and the revenue of secondary user i is $r_i \times k_i \times b_i$; so the profit of secondary user i is:

$$\pi_i(\mathbf{B}) = r_i k_i b_i - b_i \left[x + y \left(\sum_{b_j \in \mathbf{B}} b_j \right)^\tau \right] \tag{3.23}$$

The marginal profit function for secondary user i can be obtained from

$$\frac{\partial \pi_i(\mathbf{B})}{\partial b_i} = r_i k_i - x - y \left(\sum_{b_j \in \mathbf{B}} b_j \right)^\tau - y b_i \tau \left(\sum_{b_j \in \mathbf{B}} b_j \right)^{\tau-1} \tag{3.24}$$

Let $\mathbf{B}_{-i}$ denote the set of strategies adopted by all except secondary user i. In this case, the optimal size of allocated spectrum to one secondary user depends on the strategies of other secondary users. *Nash equilibrium* is considered as the solution of the game to ensure that all secondary users are satisfied with the solution. By definition, Nash equilibrium is obtained by using the best response function of one player given others' strategies.

In this static game, all the other players' actions are given, so the best response function of secondary user i is:
$$BR_i(\mathbf{B}_{-i}) = \arg \max_{b_i} \pi_i(\mathbf{B}_{-i} \cup \{b_i\}) \tag{3.25}$$

The set $\mathbf{B}^* = \{b_1^*, \cdots b_N^*\}$ denotes the Nash equilibrium of the game if and only if $b_i^* = BR_i(\mathbf{B}_{-i}^*)$ for any i. $\mathbf{B}_{-i}^*$ denotes the set of best responses for secondary users j for $j \neq i$.

So there are N equations for $i = 1, \cdots N$:

$$\frac{\partial \pi_i(\mathbf{B})}{\partial b_i} = r_i k_i - x - y \left(\sum_{b_j \in \mathbf{B}} b_j \right)^\tau - y b_i \tau \left(\sum_{b_j \in \mathbf{B}} b_j \right)^{\tau-1} = 0 \tag{3.26}$$

We can obtain the Nash equilibrium by solving the above set of equations.

3.3.3 Dynamic Game Model

In a static game,the secondary users have to know the strategies and the payoffs of other secondary users,which is a centralized spectrum sharing scenario.However,in the practical cognitive radio environment,secondary users may only be able to observe the pricing information from the primary user but not the strategies and profits of other secondary users.Therefore,to relax the assumption of static game,the dynamic game model is presented.Since all secondary users are rational to maximize their profits,they can adjust the size of the requested spectrum b_i based on the marginal profit function.

That means,each secondary user can communicate with the primary user to obtain the differentiated pricing function for different strategies.The adjustment of spectrum size can be modeled as a dynamic game as follows:

$$b_i(t+1) = Q(b_i(t)) = b_i(t) + \alpha_i b_i(t) \frac{\partial \pi_i(\mathbf{B})}{\partial b_i(t)} \tag{3.27}$$

where $b_i(t)$ is the allocated spectrum size at time t and α_i is the adjustment speed parameter of secondary user i.Note that the initial strategy is denoted by $b_i(0) = b_i^{(0)}$ for each secondary user (i.e.,each i).

In an actual system,the value of $\frac{\partial \pi_i(\mathbf{B})}{\partial b_i(t)}$ can be estimated by the secondary users.In particular,a secondary user inquires the primary user of the pricing function at time t by submitting the spectrum sizes $b_i(t) \pm \epsilon$,where ϵ is a enough small number.Then ,the secondary user observes the response price c^- and c^+ for each spectrum size respectively ,and then computes the profits π_i^- and π_i^+.Therefore,the marginal profit can be estimated from $\frac{\partial \pi_i}{\partial b_i(t)} \approx \frac{\pi_i^+ - \pi_i^-}{2\epsilon}$

It is noted that in this strategy,the players could not immediately adjust their strategies to the optimal quantity which could be obtained by solving a profit maximization problem as in static game.However,it will change the strategy based on the current pricing information.

To make this strategy easy to understand, let us consider the case where $\tau = 1$.With this ,the dynamic game can be expressed in matrix form as follows:

$$\mathbf{b}(t+1) = Q(\mathbf{b}(t)) \tag{3.28}$$

At the equilibrium,we have $\mathbf{b}(t+1) = \mathbf{b}(t) = \mathbf{b}$.With the function (3.24) and (3.27),the equilibrium point $\mathbf{b}$ can be obtained by solving the following set of equations:

$$\alpha_i b_i \left(r_i k_i - x - 2b_i y - y \sum_{j \neq i} b_j \right) = 0, \forall i. \tag{3.29}$$

Take the condition that there are only two secondary users in the environment,we have fixed points as follows by solving the equations above :

$$\mathbf{b}_0 = (0,0), \mathbf{b}_1 = \left(\frac{r_1 k_1 - x}{2y}, 0 \right), \mathbf{b}_2 = \left(0, \frac{r_2 k_2}{2y} \right)$$

$$\mathbf{b}_3 = \left(\frac{r_2 k_2 - 2(r_1 k_1) + x}{1 - 4y}, \frac{r_1 k_1 - 2(r_2 k_2) + x}{1 - 4y} \right) \tag{3.30}$$

where $\mathbf{b}_3$ is the Nash equilibrium.To analyze local stability of this spectrum sharing ,the fixed point is stable if and only if the eigenvalues λ_i are all inside the unit circle of the complex plane(i.e.,$|\lambda| < 1$).With two secondary users,the Jacobian matrix can be expressed as follows:

$$J(b_1, b_2) = \begin{bmatrix} 1 + \alpha_1(r_1 k_1 - x - 4yb_1 - yb_2) & -y\alpha_1 b_1 \\ -y\alpha_2 b_2 & 1 + \alpha_2(r_2 k_2 - x - 4yb_2 - yb_1) \end{bmatrix} \tag{3.31}$$

Then,To investigate the stability condition at $\mathbf{b}_0$,we have

$$J(0,0) = \begin{bmatrix} 1 + \alpha_1(r_1 k_1 - x) & 0 \\ 0 & 1 + \alpha_2(r_2 k_2 - x) \end{bmatrix} \tag{3.32}$$

The $\mathbf{b}_0$ will be stable if and only if $|1 + \alpha_1(r_1 k_1 - x)| < 1$ and $|1 + \alpha_2(r_2 k_2 - x)| < 1$.Therefore,there are two cases which the first case is

$$\alpha_1(r_1 k_1 - x) > -2, \alpha_2(r_2 k_2 - x) > -2 \tag{3.33}$$

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which are possible.The second case is

$$r_1k_1 < x and r_2k_2 < x \tag{3.34}$$

These conditions imply that when none of the secondary users is willing to share the spectrum with the primary user,that is, when the cost of the spectrum is hgher than the revenue gained from allocated spectrum,the secondary user will not share the spectrum.

For fixed points $\mathbf{b}_1$ and $\mathbf{b}_2$,the Jacobian matrix expressed as follows:

$$J\left(\frac{r_1k_1 - x}{2y}, 0\right) = \begin{bmatrix} 1 + \alpha_1(-r_1k_1 + x) & \alpha_1 \frac{r_1k_1 - x}{2} \\ 0 & 1 + \alpha_2\frac{1}{2}(2r_2k_2 - r_1k_1 - x) \end{bmatrix} \tag{3.35}$$

$$J\left(0, \frac{r_2k_2}{2y}\right) = \begin{bmatrix} 1 + \alpha_1(-r_1k_1 + x) & 0 \\ \alpha_2\frac{r_2k_2 - x}{2} & 1 + \alpha_2\frac{1}{2}(2r_2k_2 - r_1k_1 - x) \end{bmatrix} \tag{3.36}$$

With these matrix ,for the first eigenvalue,$r_1k_1 > x$ is the common condition for stability of both $\mathbf{b}_1$ and $\mathbf{b}_2$.For the second eigenvalue,we can arrive at two conditions for each of the points:

- $\mathbf{b}_1$:

  1. $2r_2k_2 < r_1k_1 + x$:In this case,the $\mathbf{b}_1$ is stable.That means,if the value of r_1k_1 in the revenue function for the first secondary user is much larger than that for the other secondary user,this point can be obtained by this strategy.
  2. $2r_2k_2 \geq r_1k_1 + x$:In this case,the point $\mathbf{b}_1$ is never stable.

- $\mathbf{b}_2$:

  1. $2r_1k_1 < r_2k_2 + x$:In this case,the $\mathbf{b}_2$ is stable.That means,if the value of r_2k_2 in the revenue function for the second secondary user is much larger than that for the other secondary user,this point can be obtained by this strategy.
  2. $2r_1k_1 \geq r_2k_2 + x$:In this case,the point $\mathbf{b}_2$ is never stable.

For the fixed point $\mathbf{b}_3$,which is the Nash equilibrium,the Jacobian matrix can be expressed as:

$$J\left(\frac{r_2k_2 - 2(r_1k_1) + x}{1 - 4y}, \frac{r_1k_1 - 2(r_2k_2) + x}{1 - 4y}\right) = \begin{bmatrix} j_{1,1} & j_{1,2} \\ j_{2,1} & j_{2,2} \end{bmatrix} \tag{3.37}$$

where

$$j_{1,1} = 1 + \alpha_1\left(\frac{r_1k_1 - 2yr_2k_2 + 3yr_1k_1 - yx - x}{1 - 4y}\right)$$

$$j_{1,2} = -y\left(\frac{r_2k_2 - 2r_1k_1 + x}{1 - 4y}\right)$$

$$j_{2,1} = -y\left(\frac{r_1k_1 - 2r_2k_2 + x}{1 - 4y}\right)$$

$$j_{2,2} = 1 + \alpha_2\left(\frac{r_2k_2 - 2yr_1k_1 + 3yr_2k_2 - yx - x}{1 - 4y}\right)$$

To obtain the eigenvalues , solve the equation as follow:$\lambda^2 - \lambda(j_{1,1} + j_{2,2}) + (j_{1,1}j_{2,2} - j_{1,2}j_{2,1}) = 0$
Therefore,

$$(\lambda_1, \lambda_2) = \frac{(j_{1,1} + j_{2,2}) \pm \sqrt{4j_{1,2}j2, 1 + (j_{1,1} - j_{2,2})^2}}{2} \tag{3.38}$$

There is no doubt that we can obtain the relationship between α_1 and α_2 for that the Nash equilibrium is stable.That is to say,the profit of the secondary users cannot be increased by altering the allocated spectrum size.

In the case that there are more than two secondary users in the system,the Jacobian matrix of the fixed point of the Nash equilibrium is given as follows:

$$J = \begin{bmatrix} 1 + \alpha_1(r_1k_1 - x - 4yb_1 - y\sum_{j\neq 1} b_j) & \cdots & -y\alpha_1 b_1 \\ \vdots & \ddots & \vdots \\ -y\alpha_N b_N & \cdots & 1 + \alpha_N(r_Nk_N - x - 4yb_N - y\sum_{j\neq N} b_j) \end{bmatrix} \tag{3.39}$$

Then,the same condition for the local stability as that for the two user case is applicable.That is to say,the fixed point of the Nash equilibrium is stable if and only if all eigenvalues of the corresponding Jacobian matrix are inside the unit circle of the complex plane.

Chapter 4

Potential Research Direction(to be revised)

This chapter is a temporary chapter only to record what we consider and may research.In fact ,it will not be included in final report of our work.

Among the papers we have read,we figure out several points that we want to or could research further:

1. In utility optimization, auction theory is really an interesting and useful method to solve problems.After we read over [9][10],we actually want to research something further in spectrum sharing of cognitive radio with auction model.Besides, we find several issues: First,how auction model can be used in CR networks? We assume that primary owners may be auctioneer,primary users may be sellers and secondary users may be buyers.Then the double auction model might be used in these condition;Secondly,in double auction model as referred in [10],there are additional economic properties that could be considered to increase the utility such as tradeoff between efficiency and economic robustness.

2. As far as I consider, there are two restrictions in[11]. First, it chooses the orthogonal channel allocation as cooperation rules, which is only beneficial when the interference level among selfish players is medium to high; so we may try to find out a proper cooperation rules fitting in low interference level. Also, the detection of deviating behavior is conducted by the current channel user; as the "punish-and-forgive" strategy will hurt not only the player who deviates the rule but also other players, current channel user do not have enough incentive to report the deviating behavior to the system and start the "punish-and-forgive" strategy up. So we may develop a strategy to enforce the current channel user to report honestly.

3. We are interested in Stackelberg game model,perhaps we will do more research on that.

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