OR-Play: An Optimal Relay Placement Scheme for High-Quality Wireless Network Services

Ailun Song, Xiaofeng Gao*, Fan Wu, Linghe Kong, Guihai Chen
Shanghai Key Laboratory of Scalable Computing and Systems, Department of Computer Science and Engineering,
Shanghai Jiao Tong University, Shanghai, 200240, China
Email: {salvatore, linghe.kong}@sjtu.edu.cn, {gao-xf, fwu, gchen}@cs.sjtu.edu.cn

***Abstract*—With the development of wireless communication and social network, wireless network service demands have increased rapidly in recent years. To amplify the wireless signals and expand the coverage of wireless networks, wireless relay nodes are introduced. This paper addresses the problem of finding an optimal deployment of access points and wireless relay nodes in an arbitrary environment to provide all the potential users with higher quality wireless network services. Our ambition is to maximize the coverage rate and to minimize the energy consumption of the relay nodes.**

Correspondingly, we design a scheme named *OR-Play: an optimal relay placement scheme* to provide high-quality wireless services, which consists of three phases. First, OR-Play provides an area coverage for an arbitrary area. We use the virtual force model to determine the positions of wireless devices, including access points and relay nodes, and thus extend the network lifetime. In the second phase, OR-Play selects access points by a 2-approximation algorithm for the metric k-center problem. In the third phase, we define a new problem: k-minimum energy broadcasting trees. We design a distributed greedy strategy to determine the broadcasting trees, based on which the power of relay nodes are precisely assigned. Finally, the simulation results validate the effectiveness and efficiency of OR-Play.

I. Introduction

With the development of mobile technology, wireless network has gradually become a necessary part in our daily life. However, it could be hard to communicate directly between source nodes and destination nodes in reality because of the limitation of signal propagation distance, the perturbation and collision phenomena, or the limited transporting capacity of wireless devices. Correspondingly, wireless relay nodes are introduced to amplify and relay the signal so that the coverage area could be expanded, as illustrated in Fig. 1. In this figure, wireless signal is sent from an access point (AP), amplified through Relay Node 1, and then received by a mobile user. According to their mobility property, wireless relay nodes are widely used to construct wireless distributed systems in public area since the network topology can be easily modified.

Introducing relay nodes would bring up some interesting but difficult issues, including where the nodes should be deployed to maximize the coverage rate, and how the nodes should relay information to minimize the consumption of energy. To provide area coverage, the accurate optimized positions of nodes are calculated in [1]–[4]. However, their areas of interest are either infinite or regular, while the reality is more complicated. We should design a robust method suitable for an arbitrary area.

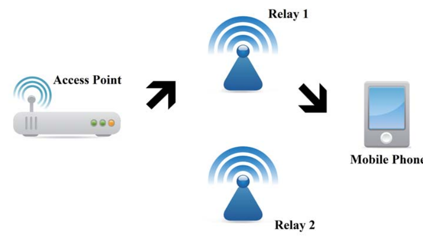


Fig. 1. Expand the wireless coverage area by relay nodes

To provide area coverage in an arbitrary area, a common solution [5]–[8] is to spread the nodes randomly in the area and then heal the coverage hole. However, when a relay node fails, such solution keeps healing the coverage hole by moving the existing nodes, which would cost more energy from the residual nodes. It implies that we should design a better schedule to prolong the network's lifetime.

To minimize energy consumption of nodes, minimum energy broadcasting problem [9] is proposed. Omri et al. [10] considered the problem of relay selection as a broadcasting tree to minimize the signal-to-noise ratio (SNR). However, the models in these papers have only considered the situation with a single source. We should extend it into multi-sources to correspond with the reality.

Additionally, Ma et al. [11] considered relay node placement in two-tiered wireless sensor networks with resource constraint to extend the network lifetime. They converted the relay node placement problem into set cover problem with limited hop counts. However, in practice we should relax such hop count constraint in case that the network is in large scale and the number of access point is limited.

In all, in this paper our ambition is to maximize the coverage rate and minimize the energy consumption in wireless network by introducing wireless relay nodes. Correspondingly, we design a scheme named *OR-Play: an optimal relay placement scheme* to provide high-quality wireless services, which consists of three phases. First, OR-Play provides area coverage for an arbitrary area. We would like to maximize the coverage rate by deploying the nodes. We use the virtual force model to determine the positions of wireless devices, including access

* corresponding author

1521-9097/16 $31.00 © 2016 IEEE
DOI 10.1109/ICPADS.2016.19

points and relay nodes, and thus extend the network lifetime. In the second phase, we would like to select several access points as clusters to minimize the energy consumption. OR-Play selects access points by a 2-approximation algorithm for the metric k-center problem. In the third phase, we would like to determine the propagation pathes. We define a new problem: k-minimum energy broadcasting trees. We then design a distributed greedy strategy to determine the broadcasting trees, based on which the power of relay nodes are precisely assigned. Finally, the simulation results validate the effectiveness and efficiency of OR-Play.

To summarize, our contributions mainly include:

1) We develop a comprehensive scheme OR-Play to place access points and relay nodes and better improve the network quality. OR-Play has many novel designs, including the optimal node deployment for an arbitrary environment, the position selections of access points among the candidates, and the construction of broadcasting trees for power allocation.
2) To provide a robust coverage, we extend the virtual force model into higher dimension. A working schedule can be generated automatically according to the different layers. The network lifetime can be extended because of the existence of backup relay nodes.
3) We introduce a new problem, k-minimum energy broadcasting problem. We propose a distributed greedy strategy to construct k-minimum energy broadcasting trees.

The remaining of our paper is organized as follows: The related works are listed in Sec. II. We formally declare our problems and propose our solution, OR-Play scheme, in Sec. III. The simulating result is reported in Sec. IV. We conclude our paper in Sec. V.

II. Related Works

Wireless relay nodes can effectively enhance the signal strength and improve the network quality while they also involve new challenges. The problems introduced by wireless relay nodes include packet loss and decrease of bandwidth. Packet loss means that one or more packets fail to reach the destination during the packet transmission process. Choi et al. [12] studied the application of the hybrid automatic repeat request (HARQ) protocol to provide reliable packet transmissions and relieve the packet loss phenomena. Theoretical analysis [13] showed that wireless systems with multi-hop relay using conventional bandwidth allocation will seriously suffer from low bandwidth utilization. Liu et al. [13] then proposed a spectrum efficiency based adaptive resource allocation algorithm to deal with this problem.

There are several methods to maximize the coverage rate by deploying the nodes: The first one is to calculate precisely the optimized position. In an infinite plane, Kershner [1] proved that the triangular tessellation achieves a full coverage with an asymptotic minimum number of sensors. Yun et al. [2] studied the deployment patterns to achieve k-coverage, where $k \leq 6$, in a boundaryless area. Bai et al. [3] studied the optimized deployment pattern in an infinite space. Khoufi et al. [4] studied a projection-based method to deploy nodes in an arbitrary polygon. However in reality, the boundary of an area are usually irregular, which makes the above methods useless.

A more common method is to initialize the nodes randomly, and then fill in the coverage hole generated by the initialization. So the kernel is to fill in the existing coverage hole. Amgoth et al. [5] studied how to find a coverage hole and how to heal it. Hashim et al. [6] proposed an algorithm based on Artificial Bee Colony to heal the initial communication hole in wireless sensor network. The authors constructed the network backbone by applying minimum spanning tree. Zou et al. [7] proposed the virtual force algorithm as a sensor deployment strategy to enhance the coverage after an initial random placement of sensors. Lin et al. [8] proposed the virtual force model with boundary to heal the coverage hole.

There are several algorithms to classify the nodes and select the clusters. Dasgupta et al. [14] proposed an approximation algorithm, k-clustering, to minimize the maximum length of two points in each class. Gupta et al. [15] proposed a fuzzy logic approach to select clusters in wireless sensor network. Hartigan et al. [16] proposed the k-means algorithm. Du et al. [17] proposed a 2-approximation algorithm for the metric k-center problem.

There are several researches discussing the minimum energy broadcasting problem. Čagalj et al. [18] proved that the minimum energy broadcasting problem is NP-hard. Wieselthier et al. [9] proposed the Broadcast Incremental Power (BIP) algorithm based on Prim's algorithm. The approximation ratio of BIP algorithm is from 6 to 12. Du et al. [17] provided an 8-approximation solution for the minimum energy broadcasting problem. Navarra [19] proposed a 6-approximation algorithm and proved that it is a tight bound.

There also exist some novel applications of relays. Michalopoulos et al. [20] considered relay nodes as a tool to simultaneously deliver information and energy. They proposed an algorithm to balance the efficiency of the information transferred to the receiver and the amount of energy transferred to the energy harvesters. Luo et al. [21] considered the deployment of Energy Harvesting (EH) relay and the broadcasting schedule to minimize the source transmit power. Min et al. [22] considered a deployment problem for relay robots by using Genetic Algorithm and Particle Swarm Optimization. Ruby et al. [23] considered relay selection and power allocation in wireless relay network. They proposed a Stackelberg game theory method. Chen et al. [24] considered relay selection and resource allocation in cognitive radio networks. They convert the optimization problem into max-matching and solve it by Hungary algorithm.

Chen et al. [25] considered how to allocate the energy of information forwarding and the energy of harvesting for each relay to maximize the achieving rate. They proposed a game theory method using Nash Equilibrium. Bhattacharya et al. [26] designed a multi-hop wireless network for interconnecting sensors to Base Station by deploying a minimum number of relay nodes at a subset of given potential locations. Mezzavilla et al. [27] extended the coverage by enabling

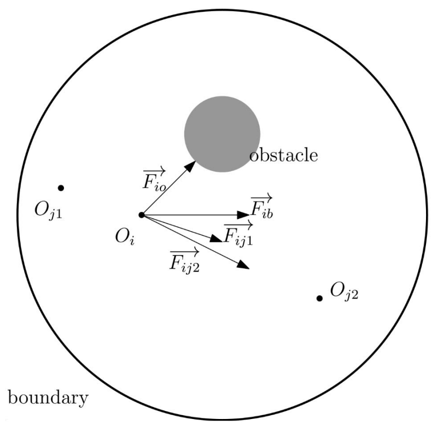


Fig. 2. Illustration of virtual force model

nearby idle users' equipment to serve as relays. In all, we summarize the most up-to-date works on relays in Table I, depicting their characteristics and methodologies.

III. Problem Formulation and Solution

We design a novel scheme named *OR-Play*: an optimal relay node placement scheme to better improve the wireless network services. OR-Play solves three sub-problems: maximum coverage rate, metric k-center, and the multi-minimum energy broadcasting tree. Correspondingly, the first sub-section will study the deployment of nodes given the area and the number of devices. The second subsection will discuss the selection of access points given the position of nodes and the power function. The third section will introduce the path of broadcasting trees and the allocation of power given the set of access points and wireless relays.

A. Maximum area coverage

In this phase, we determine the potential locations for the access points and relay nodes. To provide better services for the users, a coverage with high coverage rate should be achieved.

Definition 1 (Maximum area coverage problem)**.** *Assume that there are N nodes with a uniform coverage radius L, in an arbitrary area Ω. The set of nodes are noted as $V = \{O_1, \ldots, O_N\}$. We define that a point P is covered by a node O_i if $d(P, O_i) \leq L$. The coverage area is defined by $\Omega_c = \{P \in \Omega \mid \min_i d(P, O_i) \leq L\}$. The maximum area coverage problem is to determine the locations of nodes in the set V so as to maximize the coverage area, i.e.*

$$\max \eta = \frac{Area(\Omega_c)}{Area(\Omega)}, \tag{1}$$

where function $Area(\Omega)$ represents the area of Ω.

In this subsection, we improve the virtual force method in [7] to provide full coverage for Ω.The whole system of virtual force model is illustrated in Fig. 2. We denote the location of node O_i at moment t as $O_i(t) = (x_i(t), y_i(t))$. We define the virtual force suffered by node O_i at moment t as $\overrightarrow{F_i}(t) = \Sigma_{j \neq i} \overrightarrow{F_{ij}}(t) + \overrightarrow{F_{ib}}(t) + \overrightarrow{F_{io}}(t)$, where $\overrightarrow{F_{ij}}(t)$ is the virtual force of node $O_j(t)$ acting on node $O_i(t)$. Then we have:

$$\overrightarrow{F_{ij}}(t) = k_{ij} \frac{d(O_i(t), O_j(t)) - r}{d(O_i(t), O_j(t))} \overrightarrow{O_i(t) O_j(t)} \tag{2}$$

where

$$k_{ij} = \begin{cases} w_{attractive}, & d(S_i, S_j) \geq r \\ w_{repulsive}, & \text{otherwise.} \end{cases} \tag{3}$$

Meanwhile, $\overrightarrow{F_{ib}}(t)$ is the virtual force of the boundary acting on node $O_i(t)$. We have:

$$\overrightarrow{F_{ib}}(t) = \oint w_{boundary} \frac{\overrightarrow{PO_i(t)}}{d(P, O_i(t))} dl \tag{4}$$

And $\overrightarrow{F_{io}}(t)$ is the virtual force of the obstacles acting on node $O_i(t)$. We have:

$$\overrightarrow{F_{io}}(t) = \Sigma_P w_{obstacle} \frac{\overrightarrow{PO_i(t)}}{d(P, O_i(t))} \tag{5}$$

In which, $w_{attractive}$ represents the coefficient of the attractive virtual force. $w_{replusive}$ represents the coefficient of the repulsive virtual force. $w_{boudary}$ represents the coefficient of the virtual force given by the boundary. $w_{obstacle}$ represents the coefficient of the virtual force given by the obstacle.

The locations of nodes are initialized by deploying them randomly in area Ω, which are $O_1(0), \ldots, O_N(0)$. We use a recurrence relation to infer the subsequent locations:

$$x_i(t+1) = \begin{cases} x_i(t) - 1, & \overrightarrow{F_i}(t) \cdot \overrightarrow{e_x} < -threshold \\ x_i(t) + 1, & \overrightarrow{F_i}(t) \cdot \overrightarrow{e_x} > threshold \\ x_i(t), & \text{otherwise} \end{cases} \tag{6}$$

and

$$y_i(t+1) = \begin{cases} y_i(t) - 1, & \overrightarrow{F_i}(t) \cdot \overrightarrow{e_y} < -threshold \\ y_i(t) + 1, & \overrightarrow{F_i}(t) \cdot \overrightarrow{e_y} > threshold \\ y_i(t), & \text{otherwise} \end{cases} \tag{7}$$

where $threshold$ represents the criteria of moving or not. The locations of nodes will converge after sufficient rounds of iterations. We consider the result as the location of the deployment of devices.

The virtual force model is effective to offer full coverage. However since the energy of a wireless relay is limited, the failure of node will cause the appearance of coverage hole which will decrease the Quality of Service (QoS). Although the coverage hole can be healed by moving other relays [8], it is not a good choice since motion of relay would cost extra energy which would lead to an accelerated distinction of the whole network.

In order to provide a stable and robust wireless network, we consider the virtual force model in higher dimension.

TABLE I
SUMMARY OF RECENT WORKS ON RELAYS

Reference	Year	Problem Type	Objective	Model/Method
[24]	2013	Relay Selection, Power Allocation	Minimize the outage percentage	Max-Matching
[26]	2014	Relay Placement	Minimize the Relay Number	Shortest Path Tree
[27]	2014	Relay Selection	Maximize the utility of user rates	Idle User Equipment Serves as Relay
[20]	2015	Relay Selection	Balance Capacity-Energy Trade-off	Time-Sharing Selection, Threshold-Checking Selection
[22]	2015	Relay Placement	Minimize the moving length	Genetic Algorithm, Particle Swarm Optimization
[23]	2015	Relay Selection, Energy Allocation	Minimize the total power	Geometric Programming, Game Theory
[25]	2015	Power Allocation	Maximize the Achieving Rate	Game Theory
[11]	2015	Relay Placement	Extend the Network Lifetime	Geometric Disc Covering
[21]	2016	Relay Placement	Extend the Network Lifetime	Linear Programming

We record the information of environment in two dimensions as

$$Map_2(x,y)=\begin{cases}1, & \text{if a node can be deployed on point } (x,y)\\ 0, & \text{otherwise.}\end{cases} \tag{8}$$

We extend the environment into three dimensions by adding a z-axis when satisfying

$$Map_3(x,y,z)=Map_2(x,y). \tag{9}$$

We apply the virtual force model in the three dimensions environment, $Map_3(x,y,z)$, to determine the location of node O_i =(x_i,y_i,z_i), where an extra recurrence relation should be added:

$$z_i(t+1)=\begin{cases}z_i(t)-1, & \overrightarrow{F_i}(t)\cdot\overrightarrow{e_z}<-threshold\\ z_i(t)+1, & \overrightarrow{F_i}(t)\cdot\overrightarrow{e_z}>threshold\\ z_i(t), & \text{otherwise.}\end{cases} \tag{10}$$

We divide the points into different layers according to the information on z-axis. Then we project all the points on the plan Oxy when maintaining the information of layers. Fig. 3 illustrates the process of projection of virtual force model from 3D to 2D. Three different colors stand for three different layers. At first, we utilize the nodes in the blue layer. If a node in the blue layer fails, the nodes around in the other layers would wake up to heal the coverage hole. This method extends the network lifetime while keeping the complexity of calculation.

B. Metric k-center problem

Now we have some representative points to cover the whole area and we should select some points as the access points. The access points should be the center of relay nodes to decrease the hop count. So the metric k-center problem is considered.

Definition 2 (Metric k-center problem)**.** *Assume that* $G=(V,E)$ *is a complete graph with a weight function* $d:E\rightarrow\mathbb{R}^+$ *which satisfies the triangle inequality:*

$$\forall A,B,C\in V^3, d(A,C)\leq d(A,B)+d(B,C) \tag{11}$$

Our ambition is to determine $S\subseteq V$ *so as to minimize the maximal distance from a node* $v\in V$ *to* S*. The distance from a node* $v\in V$ *to* S *is defined as the minimal distance from* $v\in V$ *to a node* $s\in S$*. The problem can be formulated in the form of:*

$$\min\max_{v\in V}\min_{s\in S} d(v,s) \tag{12}$$

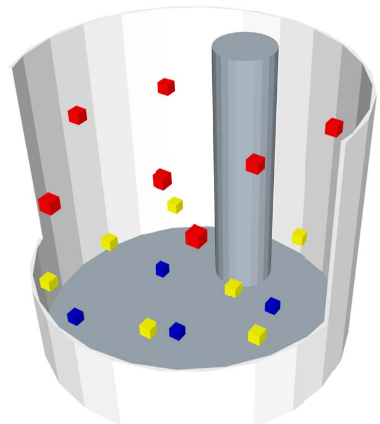


Fig. 3. Projection of virtual force model from 3D to 2D

Here, we consider the Euclidean distance as the weight function. In this phase, we choose k nodes as access points. Vazirani et al. [28] converted the metric k-center problem into the dominate set problem, which is NP-hard. So we use a 2-approximation algorithm introduced by [28] to solve the metric k-center problem. The algorithm in [28] needs to calculate the maximum independent set of a graph, which is usually picked by greedy algorithm at random or by Monte-Carlo method according to [29]. The randomness in picking the maximum independent will affect the result we get. To provide a better result, we run the algorithm for sufficient times which does not change the complexity. We also improve the algorithm by decreasing the searching times to $O(\log n)$ by binary search. Algorithm 1 indicates the detailed procedure where

$$G_i^2=(V,(u,v)|\exists w,(u,w)\in E(G_i),(w,v)\in E(G_i)) \tag{13}$$

C. Power allocation

In this phase, we consider how to allocate the power. To increase the quality of service, we introduce a competition

Algorithm 1: 2-approximation for metric k-center

input : A complete graph, $G = (V, E)$
The power function, $d : E \rightarrow \mathbb{R}^+$
The number of access points, k
output: The set of access points, S

Sort the edge of G *in order of* $d(e_1) \leq d(e_2) \leq \cdots \leq d(e_{card(E)})$;
Set two auxiliary variables: left$\leftarrow 0$,$right \leftarrow card(G)$;
while $\neg(|left - right| \leq 1)$ **do**
  $i \leftarrow [\frac{l+r}{2}]$;
  $G_i = (V, \{e_1, e_2, \ldots, e_i\})$;
  Calculate G_i^2;
  Calculate a maximum independent set of G_i^2*:* M_i *for sufficient times (say 100)*;
  Pick a M_i *which minimizes* $card(M_i)$*, note it as* S;
  if $card(S) \leq k$ **then**
    $r \leftarrow i$;
  else
    $l \leftarrow i$;
return S;

mechanism that a relay node would select a best source to relay the information.

We consider the path loss of power during the propagation. According to Friss formula [30], if information is transferred from node O_i to node O_j , we have:

$$P_{receive}(O_j) = \frac{k \times P_{transmit}(O_i)}{d(O_i, O_j)^2} \tag{14}$$

where $P_{receive}(O_j)$ represents the power received by node O_j and $P_{transmit}(O_i)$ represents the transmit power of node O_i.

The allocation should satisfy the following conditions:

1) Each node should receive a sufficient power $T(O_j)$ to transfer the information correctly , i.e. $P_{receive}(O_j) \geq T(O_j)$.
2) Especially, if the node is an access point, i.e.$O_j \in S$, we define $T(O_j) = 0$.
3) To assure the connectivity, all nodes should satisfy their children's needs of power, i.e.
$$P_{transmit}(O_i) = \max_j \frac{T(O_j)d(O_i, O_j)^2}{k} \tag{15}$$
where O_j is a child of O_i.
4) Notice that the transmit power of a wireless relay is limited while the transmit power should also be larger then a threshold to provide service for the users even if it does not have any children. So there is a constraint: $P_{min} \leq P_{transmit}(O_i) \leq P_{max}$.

Finally, we have an expression of transmit power for each node:

$$P^*(O_i) = \max(\min(\max_j \frac{T(O_j)d(O_i, O_j)^2}{k}, P_{max}), P_{min}) \tag{16}$$

where O_j is a child of O_i.

Definition 3 (k-minimum energy broadcasting tree problem)**.** *Given a finite set of point* V *and a set of source* $S \subseteq V$*, we should determine a series of broadcasting trees to minimize the total power consumed by the wireless devices. i.e.*

$$\min \sum P^*(O_i), O_i \in V \tag{17}$$

To simplify, we suppose that $P_{min} = 0$ and $P_{max} = \infty$.

We consider the critical version of k-minimum energy broadcasting tree problem. We consider that the function T here is defined as

$$T(O_j) = \begin{cases} 0, O_j \in S \\ 1, O_j \in V/S \end{cases} \tag{18}$$

Meanwhile, we also add a virtual point O_0 with the property as follow:

$$d(O_0, O_j) = \begin{cases} 0, O_j \in S \\ \infty, O_j \in V/S \end{cases} \tag{19}$$

In this case, we define the critical version of k-minimum energy broadcasting tree problem as follow:

Definition 4 (critical version of Definition. 17)**.** *Given a finite set of point* $V \cup O_0$ *and the root* O_0*, we should determine a series of broadcasting trees,* $E^{'}_{min}$ *to minimize the total power consumed by the wireless devices. i.e.*

$$\min c \sum \max_j d(O_i, O_j)^2 \tag{20}$$

where $O_i \in V$*,* $O_j \in V$*,* O_j *is a child of* O_i *and* c *is a constant.*

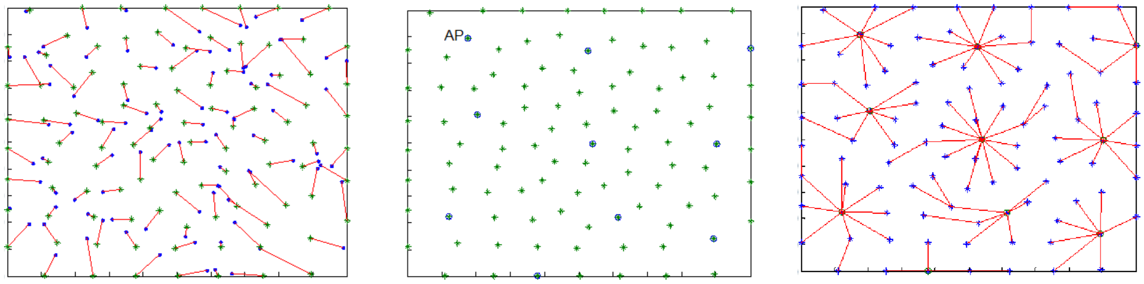
According to [18], the minimum-energy broadcasting problem is NP-hard. So k-minimum energy broadcasting problem is NP-hard. According to [19], the minimum spanning tree (MST) is a 6-approximate solution. We note the minimum spanning tree of $V \cup O_0$ as E_m. The result can be easily modified to a reasonable solution for k-minimum energy broadcasting tree with an homogeneous T if we delete the virtual point O_0.

When T is heterogeneous, we define a new weight function:

$$d'(O_i, O_j) = \begin{cases} 0, O_j \in S, i = 0 \\ \infty, O_j \in S, i \neq 0 \\ T(O_j)^2 d(O_i, O_j)^4, O_j \in V/S \end{cases} \tag{21}$$

Suppose that we have constructed a series of broadcasting trees noted as $G_s = (V_s, E_s)$. We note V_l as the set of leaves in V_s. According to Formula. 15, there should be exactly $A = card(V_s) - card(V_l)$ terms in Formula. 17. By Cauchy-Schwarz inequality, we have

$$\sum \max_j T(O_j)d(O_i, O_j)^2 \leq \sqrt{A \sum T(O_j)^2 d(O_i, O_j)^4} \tag{22}$$



(a) Deployment obtained by virtual force model

(b) Selection of access points obtained by an approximate algorithm of metric k-center

(c) Broadcasting tree obtained by greedy strategy

Fig. 4. A square environment

The equation indicates that $\min \sum P^*(O_i)$ has an upper bound of

$$\min k^{-1} \sqrt{(card(V_s) - card(V_l)) \sum T(O_j)^2 d(O_i, O_j)^4} \tag{23}$$

We note the minimum directed spanning tree (MDST) of the directed graph, $G' = (V \cup O_0, \{(O_i, O_j) \mid d'(O_i, O_j) < \infty\}, d')$ as E_{MDST}. E_{MDST}, which minimize $\sum T(O_j)^2 d(O_i, O_j)^4$, can be calculated by Edmond's algorithm [31]. Similarly, we delete the virtual point O_0 to get a solution for k-minimum energy broadcasting tree problem.

To minimize A i.e. maximize $card(V_l)$, we consider to optimize hop count in the network.

Definition 5 (k-minimum hop count broadcasting tree problem)**.** *Given a finite set of point V and a set of source $S \subseteq V$, we should determine a series of broadcasting trees, E_{min} to minimize the maximum hop count which a user would take to connect to an access point.*

In fact, if we limit the maximum of hop number as n ($n \geq 2$), we can select access points differently and construct a different series of broadcasting trees by calculating the dominate set of $G_i^n = (V, E_n)$, where $E_n =$

$$\{(u, v) | \exists k \leq n, \exists S = \{s_0, \ldots, s_{k-1}\}, \forall i, (s_i, s_{i+1}) \in E(G_i)\} \tag{24}$$

where s_0 stands for node u, s_k stands for node v and $i \in [0, k-1]$. The characteristic of G_i^n will promise that the approximate rate of the optimal length of the longest edge is n. However this method can hardly be applied since the number of access points are determined at the very beginning.

To minimize the maximum hop-count, we consider a distributed greedy strategy to determine the broadcasting tree. To accelerate the network's scanning speed, we restrict the communication range of nodes: A node can only communicate with its neighbors. Neighbors of a node are defined by $N(O_j) = \{O_i \mid d'(O_i, O_j) \leq L_{max}\}$, where L_{max} is a constant which represents the maximal communication capacity.

Now we have the broadcasting trees, E_{min}. According to Algorithm 2, there should be $card(S)$ connected components in (V, E_{min}). In each components, there exists exactly an access point as the root of a broadcasting tree.

Algorithm 2: Greedy strategy for k-minimum hop count broadcasting tree

input : The set of nodes, V
The set of access points, S
The maximal communication distance, L_{max}
output: The broadcasting tree, E_{min}

```
∀s ∈ S, Set status(s) ← 1;
∀s ∈ V-S, Set status(s) ← 0;
Set level ← 1;
Set E_min = ∅;
while ¬(∀s ∈ V, status(s) ≠ 0) do
    ∀s ∈ status^{-1}(0),;
    if ∃O_j ∈ N(s), status(O_j = level) then
        E_min = E_min ∪ (s, O_{argmin d(s,O_j)});
        status(s) ← level + 1;
    level ← level + 1;
return E_min;
```

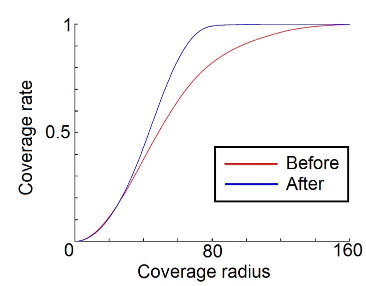
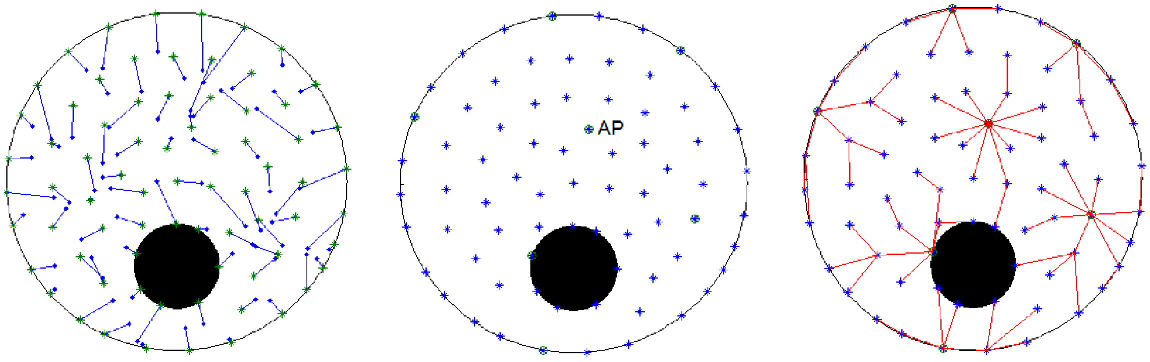


Fig. 5. Comparison of coverage rate with different coverage radius

IV. Simulation

In this section, we use MATLAB R2014a to simulate the result. We consider a simple situation. We assume that Ω is a square with the range of 1000×1000, $N = 100$.

First of all, we should know how to deploy the wireless devices in the area Ω. Fig. 4(a) illustrates the position of

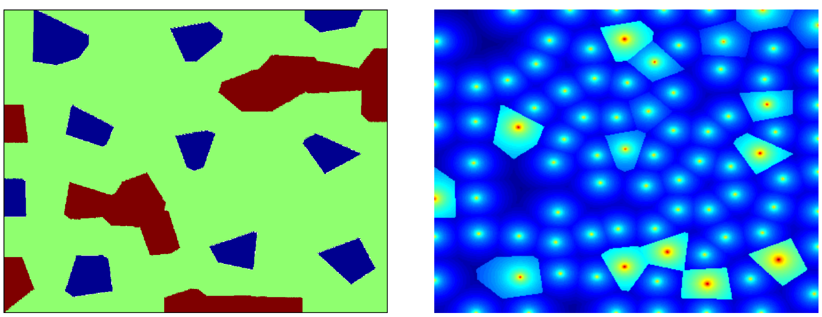


(a) Deployment obtained by virtual force model

(b) Selection of access points by an approximate algorithm of metric k-center

(c) Broadcasting tree obtained by greedy strategy

Fig. 6. An environment with obstacle



(a) The distribution of hop count in the square environment

(b) The distribution of signal strength obtained by user in the square environment

Fig. 7. The distribution of hop count and signal strength

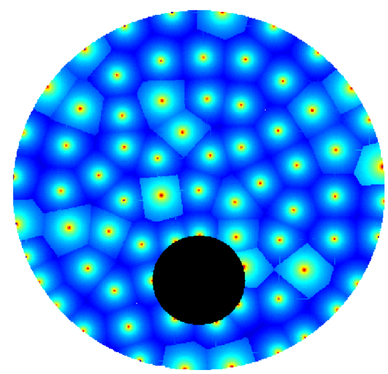


Fig. 8. The distribution of signal strength obtained by user in the environment with obstacle

deployment obtained by virtual force model. The blue points are the initial positions of nodes. The green points are the final positions obtained by the virtual force model. And the red lines indicate the relationship between them. In reality, we can directly deploy the wireless devices on the green points.

We compare the coverage rate between the initial deployment and the final deployment of nodes with different coverage radius in Fig. 5. The deployment calculated by the virtual force model converge to full coverage much faster than the initial deployment. It proves that the deployment calculated by the virtual force model has a more reasonable distribution.

Then, we need to choose k nodes among the nodes as access points. Fig. 4(b) illustrates the result obtained by the approximate algorithm of metric k-center problem. The green points represent the positions of nodes. Especially, the points with blue circle are selected as access points.

Based on the result in Fig. 4(b), we need to construct some broadcasting trees. Fig. 4(c) illustrates the broadcasting trees obtained by the distributed greedy strategy. The blue points stand for the position of nodes and the points with green circle are the access points. The red lines represent the path of broadcasting tree. The information would propagate along the broadcasting tree.

Fig. 7(a) illustrates the hierarchy hop number. Users in the blue area will connect to an access point directly. Users in the green area need one hop to connect to an access point while users in the brown area need two hops. From the result in Fig. 7(a), the usage of nodes mainly concentrates on the one-hop nodes which would cause the hot spot problem: the energy of one-hop nodes would be exhausted soon. The hot spot problem would affect all the children of the exhausted one-hop node. So that it is necessary to have some backup nodes, which are scheduled by our advanced virtual force model.

Based on the result in Fig. 4(c), we can calculate the transmit power of each node according to Eq. 16. Assuming that a mobile phone in the area would connect to its nearest device automatically, the distribution of the strength of signal which the mobile phone could get is illustrated in Fig. 7(b). The color of red represents the strongest strength while the color of purple represents the weakest strength.

We also provide a simulation of an area with obstacle. In this case, we assume that the obstacle absorb all the signal it confronts. We show the environment and the deployment of in Fig. 6(a). We show the selection of access points in Fig. 6(b). We show the broadcasting tree in Fig. 6(c). We show the distribution of signal strength obtained by user in Fig. 8.

V. Conclusion

In this paper, we propose a complete scheme, OR-Play, to maximize the coverage rate and to minimize the energy consumption of relay nodes in an arbitrary environment. The problem is decomposed into three phases. First, we provide an area coverage by determining the positions of wireless nodes, including access points and wireless relays. We solve it by introducing an advanced virtual force model in higher dimension, which not only provides an area coverage but also prolongs the network lifetime. Then, we select access points among the nodes. We consider the problem as a metric k-center problem and propose a 2-approximation algorithm. Finally, we define a new problem: k-minimum energy broadcasting problem to allocate the power. We construct the broadcasting trees by a distributed greedy strategy to establish the backbone of network. Power is allocated based on the broadcasting trees. The simulation result demonstrates that our scheme, OR-Play, provides the potential users stronger signal and lower hop count.

Acknowledgment

This work has been supported in part by the China 973 project (2014CB340303), the Opening Project of Key Lab of Information Network Security of Ministry of Public Security (The Third Research Institute of Ministry of Public Security) Grant number C15602, and the National Natural Science Foundation of China No. 61303202, No. 61672349, No. 61672353, No. 61472252, No. 61133006 and No. 61422208.

References

[1] R. Kershner, "The number of circles covering a set," *American Journal of mathematics*, vol. 61, no. 3, pp. 665–671, 1939.

[2] Z. Yun, X. Bai, D. Xuan, T. H. Lai, and W. Jia, "Optimal deployment patterns for full coverage and k-connectivity (k≤6) wireless sensor networks," *IEEE/ACM Transactions on Networking (TON)*, vol. 18, no. 3, pp. 934–947, 2010.

[3] X. Bai, C. Zhang, D. Xuan, and W. Jia, "Full-coverage and k-connectivity (k=14,6) three dimensional networks," in *IEEE International Conference on Computer Communications (INFOCOM)*. IEEE, 2009, pp. 388–396.

[4] I. Khoufi, P. Minet, A. Laouiti, and E. Livolant, "A simple method for the deployment of wireless sensors to ensure full coverage of an irregular area with obstacles," in *ACM International Conference on Modeling, Analysis and Simulation of Wireless and Mobile Systems (MSWiM)*, 2014, pp. 203–210.

[5] T. Amgoth and P. K. Jana, "Coverage hole detection and restoration algorithm for wireless sensor networks," *Springer Peer-to-Peer Networking and Applications (PPNA)*, pp. 1–13, 2015.

[6] H. A. Hashim, B. Ayinde, and M. Abido, "Optimal placement of relay nodes in wireless sensor network using artificial bee colony algorithm," *Journal of Network and Computer Applications (JNCA)*, vol. 64, pp. 239–248, 2016.

[7] Y. Zou and K. Chakrabarty, "Sensor deployment and target localization based on virtual forces," *IEEE International Conference on Computer Communications (INFOCOM)*, vol. 2, pp. 1293–1303, 2003.

[8] T.-Y. Lin, H. A. Santoso, and K.-R. Wu, "Global sensor deployment and local coverage-aware recovery schemes for smart environments," *IEEE Transactions on Mobile Computing (TMC)*, vol. 14, no. 7, pp. 1382–1396, 2015.

[9] J. E. Wieselthier, G. D. Nguyen, and A. Ephremides, "Energy-efficient broadcast and multicast trees in wireless networks," *Mobile Networks and Applications*, vol. 7, no. 6, pp. 481–492, 2002.

[10] A. Omri, M. O. Hasna, and K. B. Letaief, "Inter-relay interference management schemes for wireless multi-user decode-and-forward relay networks," *IEEE Transactions on Wireless Communications (TWC)*, vol. 14, no. 4, pp. 2072–2081, 2015.

[11] C. Ma, W. Liang, M. Zheng, and H. Sharif, "A novel local search approximation algorithm for relay node placement in wireless sensor networks," in *IEEE Wireless Communications and Networking Conference (WCNC)*. IEEE, 2015, pp. 1518–1523.

[12] J. Choi, D. To, Y. Wu, and S. Xu, "Energy-delay tradeoff for wireless relay systems using harq with incremental redundancy," *IEEE Transactions on Wireless Communications (TWC)*, vol. 12, no. 2, pp. 561–573, 2013.

[13] L. Erwu, W. Dongyao, L. Jimin, S. Gang, and J. Shan, "Performance evaluation of bandwidth allocation in 802.16 j mobile multi-hop relay networks," in *IEEE Vehicular Technology Conference*, 2007, pp. 939–943.

[14] S. Dasgupta, C. H. Papadimitriou, and U. Vazirani, *Algorithms*. McGraw-Hill, Inc., 2006.

[15] I. Gupta, D. Riordan, and S. Sampalli, "Cluster-head election using fuzzy logic for wireless sensor networks," in *IEEE Communication Networks and Services Research Conference*, 2005, pp. 255–260.

[16] J. A. Hartigan, "Clustering algorithms," 1975.

[17] D.-Z. Du, K.-I. Ko, and X. Hu, *Design and analysis of approximation algorithms*. Springer Science & Business Media, 2011, vol. 62.

[18] M. Čagalj, J.-P. Hubaux, and C. Enz, "Minimum-energy broadcast in all-wireless networks: Np-completeness and distribution issues," in *ACM International Conference on Mobile Computing and Networking*, 2002, pp. 172–182.

[19] A. Navarra, "Tighter bounds for the minimum energy broadcasting problem," in *IEEE Modeling and Optimization in Mobile, Ad-Hoc and Wireless Networks*, 2005, pp. 313–322.

[20] D. S. Michalopoulos, H. A. Suraweera, and R. Schober, "Relay selection for simultaneous information transmission and wireless energy transfer: A tradeoff perspective," *IEEE Journal on Selected Areas in Communications*, vol. 33, no. 8, pp. 1578–1594, 2015.

[21] Y. Luo, J. Zhang, and K. Letaief, "Transmit power minimization for wireless networks with energy harvesting relays," *IEEE Transactions on Communications (TOC)*, vol. 64, no. 3, pp. 987–1000, 2016.

[22] B.-C. Min, Y. Kim, S. Lee, J.-W. Jung, and E. T. Matson, "Finding the optimal location and allocation of relay robots for building a rapid end-to-end wireless communication," *Ad Hoc Networks*, 2015.

[23] R. Ruby, V. C. Leung, and D. G. Michelson, "Centralized and game theoretical solutions of joint source and relay power allocation for af relay based network," *IEEE Transactions on Communications (TOC)*, vol. 63, no. 8, pp. 2848–2863, 2015.

[24] H. Chen, P. Ren, L. Sun, and Q. Du, "A joint optimization of transmission mode selection and resource allocation for cognitive relay networks," in *IEEE International Conference on Communications (ICC)*, 2013, pp. 2852–2856.

[25] H. Chen, Y. Li, Y. Jiang, Y. Ma, and B. Vucetic, "Distributed power splitting for swipt in relay interference channels using game theory," *IEEE Transactions on Wireless Communications (TWC)*, vol. 14, no. 1, pp. 410–420, 2015.

[26] A. Bhattacharya and A. Kumar, "A shortest path tree based algorithm for relay placement in a wireless sensor network and its performance analysis," *Computer Networks*, vol. 71, pp. 48–62, 2014.

[27] M. Mezzavilla, K. Somasundaram, and M. Zorzi, "Joint user association and resource allocation in ue-relay assisted heterogeneous networks," in *IEEE International Conference on Communications Workshops (ICC)*, 2014, pp. 628–634.

[28] V. V. Vazirani, *Approximation algorithms*. Springer Science & Business Media, 2013.

[29] M. Luby, "A simple parallel algorithm for the maximal independent set problem," *SIAM Journal on Computing*, vol. 15, no. 4, pp. 1036–1053, 1986.

[30] V. L. Granatstein, *Physical Principles of Wireless Communications*. CRC Press, 2012.

[31] J. Edmonds, "Optimum branchings," *Journal of Research of the National Bureau of Standards B*, pp. 233–240, 1967.