Scheduling of Connected Autonomous Vehicles on Highway Lanes

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***Abstract*—With recent progress in vehicle autonomous driving and vehicular communication technologies, vehicle systems are developing towards fully connected and fully autonomous systems. This paper studies lane assignment strategies for connected autonomous vehicles in a highway scenario and their impact on the overall traffic efficiency and safety. We formulate a model of connected autonomous vehicles, which includes three features: traffic data available online, ultra-short reaction time, and cooperative driving. Based on this model, we propose a novel lane change maneuver *Politely Change Lane* (PCL), which achieves the tradeoff between traffic safety and efficiency. Its effectiveness is validated and evaluated by extensive simulations. The performance shows that PCL improves both safety and efficiency of the overall traffic, especially with heavy traffic.**

I. Introduction

Connected Car is a new term proposed in the automotive industry [1] [2] . Connected cars provide the ability to access vehicular networks and the Internet through wireless communication technologies, to provide infotainment for passengers and information for drivers with effects on driving efficiency and safety. For example, the driver can perform contactless payment for parking charges and fuel payments. Besides, connected cars, on the one hand, update their status to the networks in real-time, and on the other hand, retrieve traffic information online to assist drivers to make optimal driving decisions. Research shows that the quality of routes selected by individual drivers is below expectation and can be improved substantially using real-time traffic information [3]. It is reported by American National Highway Traffic Safety Administration that about 80% of accidents involving intoxicated drivers can be avoided if the cars are connected [2]. However, since the connected car is finally controlled by human drivers, there are still risks in safety and potential degrades in efficiency.

Autonomous driving is another valuable development direction in intelligent transportation systems [4]. Autonomous vehicles continuously sense the local environment in real-time and have accurate control. They can detect safety threats at an early stage and take action in time to avoid accidents. Therefore, high safety can be achieved by autonomous driving. However, since autonomous vehicles only obtain limited information through sensing, which is restricted to a small area or may even be inaccurate, they can hardly achieve globally optimal efficiency.

Considering the limitations in both connected and autonomous cars, we combine their characters together. We denote this kind of car by *Connected Autonomous Vehicle* (CAV). CAV has three unique features:

1) Accurate traffic data available online
2) Ultra-short reaction time
3) Cooperative driving

By virtue of these features, various aspects of traffic safety and efficiency can be improved at various granularities. In this work, we focus on lane assignment, i.e. making lane change decisions in a distributed and cooperative manner. Lane assignment is a fundamental driving task and a fine-grained decision. Intuitively, proper lane assignments can improve traffic efficiency. According to the US National Highway Traffic Safety Administration, about 9% (539,000) of crashes in 1999 were caused by unsafe lane changes [5] and more than 6% for the UK [6]. Therefore, proper lane assignments are also important to traffic safety.

In summary, our contributions include:

1) To the best of our knowledge, this is the first work on lane assignment for CAV. We formulate a basic model for CAV.
2) We propose a Politeness Index, which indicates the willingness to avoid affecting other vehicles during a lane change. We propose the Politely Change Lane (PCL) maneuver, which achieves a compromise between Never Change Lane and Aggressively Change Lane methods.
3) We carry out extensive simulations and the results validate that PCL improves both safety and efficiency of the overall traffic, especially with heavy traffic.

This paper is structured as follows. In Section II, some previous work related to traffic modeling and driver behavior is reviewed. In Section III, the basic traffic models for both manual and connected autonomous driving are presented. Based on the models, our lane change maneuvers are proposed in Section IV. The maneuvers are validated and evaluated by simulation in Section V. Finally, the paper is concluded and future work discussed in Section VI.

II. Related Work

A large number of papers have studied traffic modeling in recent decades. One basic model is the stochastic cellular automaton model for highway traffic proposed by Nagel and Schreckenberg [7]. It is a minimal model to produce basic features of real traffic [8].

This model possesses strong extensibility for more complex situations. Among all, the most important variation is the model for two lane traffic [9] [10]. Based on that, several works on the lane change behavior were carried out. Li et al. [11] studied the aggressive lane change behavior of fast vehicles and showed its positive impacts on traffic flow with moderate traffic density.

In this work, the cellular automaton model is aligned to the unique features of connected autonomous vehicles, including accurate traffic data available online, ultra-short reaction time, and cooperative driving. These features have not been much discussed. Treiber and Kesting [12] derived lane-changing rules with a "politeness factor" based on MOBIL (Minimizing Overall Braking Induced by Lane Changes) model. With this factor the lane-changing behavior can vary from egoistic to more cooperative. Hidas [13] modeled cooperative lane changing for a forced merging scenario. In their work, the behaviors are those of a human driver. In contrast, we study CAVs with such behaviors and do evaluations in terms of both traffic efficiency and safety.

III. TRAFFIC MODELS

A. Assumptions

We take an open straight highway with two symmetric lanes and homogeneous speed limitation as the scenario. Vehicles can enter or exit the highway at any speed below the speed limit. There are two sizes of vehicles: small vehicles (SV) and large vehicles (LV). Each vehicle has an independent pre-defined expected speed. Once a vehicle reaches its expected speed, it won't accelerate any more. For simplicity, we assume that the expected speed has a uniform distribution. Vehicles can be either manually driven vehicles or connected autonomous vehicles. These follow different driving models, which are presented in detail in following sections.

B. Manually Driving Model

The manually driving model extends the Nagel-Schreckenberg (N-S) cellular automaton model [7] [9] with following additional features: non-unified vehicle length, greedy lane selection, aggressive lane change, and independent expected speed.

The highway is modeled as a lattice of cells with two rows. Each row corresponds to a highway lane. The cells in a row are numbered as $1, 2, 3, ...$ in the direction of the highway, as shown in Fig. 1. The position of a vehicle i is notated as (l_i, x_i), where l_i is the lane ID and x_i is the cell ID. And it occupies the cells $x_i, x_i - 1, ..., x_i - L_i + 1$ on lane l_i, where L_i is the length of vehicle i in cells. For example, we set the length of SV to one cell and that of LV to three cells.

The model is discrete in time. The positions of vehicles are updated every iteration with the following processes.

1) Lateral Lane Change: All vehicles determine whether to change its lane or not in parallel. If the vehicle decided to change, it moves laterally to the target lane without longitudinal movement.

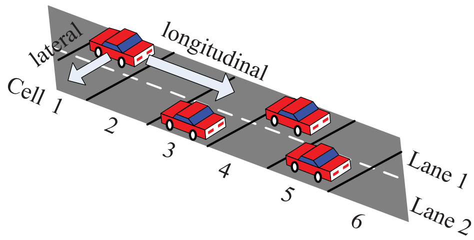

Fig. 1. An example of the highway model

The lane change is made only when the following conditions are met at the same time: there is a vehicle ahead so that the drive can't accelerate freely or is forced to brake; there is a lane better than the current one; there is no vehicle blocking the target lane. The conditions can be formalized as follows.

Condition 1: $gap_{head}(l_i, x_i) \leq v_i$.
Condition 2: $gap_{head}(tl_i, x_i) > gap_{head}(l_i, x_i)$.
Condition 3: $gap_{back}(tl_i, x_i) \geq L_i$.

If condition 1-3 are all met, update vehicle's position:

$$(l_i, x_i) \rightarrow (tl_i, x_i).$$

Where tl_i is the target lane ID of vehicle i; $gap_{head}(l, x)$ is the number of free cells before cell (l, x), exclusive; $gap_{back}(l, x)$ is the number of free cells after cell (l, x), inclusive; v_i is the speed of vehicle i, whose unit is number of cells per iteration.

2) Longitudinal Movement: After the lateral lane change process is performed for all vehicles, their positions are then updated longitudinally in parallel with the following steps:

Step 1: $tv_i = min(v_i + 1, vexp_i)$.
Step 2: $tv_i = min(tv_i, gap_{head}(l_i, x_i))$.
Step 3: If $tv_i < v_i$,
$tv_i = max(tv_i - 1, 0)$ with probability $p_{overbrake}$.
Step 4: $v_i \rightarrow tv_i$; $(l_i, x_i) \rightarrow (l_i, x_i + v_i)$.

Where v_i, tv_i, and $vexp_i$ are current speed, target speed, and expected speed of vehicle i in number of cells (per iteration), respectively; $p_{overbrake}$ is the probability used to model erratic driver behavior, in which the driver can possibly brake more than required.

C. Connected Autonomous Driving Model

The connected autonomous driving model is similar to the manual one, except for its car-following behavior.

As described in Section I, CAV has ultra-short reaction time. This requires only a short safe distance while following a car. Since the parallel update in the N-S model implies the reaction time of human drivers [7], a different update scheme should be used for CAVs. Specifically, the manually driven vehicles are updated in parallel in the first place as usual. CAVs are updated after that, sequentially from front (larger cell ID) to rear (smaller cell ID) along the highway. With this new workflow, the reaction time of CAVs is approximated to zero.

IV. CAV Lane Change Maneuvers

A. Baseline Maneuvers

We take two extreme cases of lane change maneuvers, i.e. safety-oriented and efficiency-oriented, as the baseline algorithms: *Never Change Lane* (NCL) and *Aggressively Change Lane* (ACL). In NCL, CAVs follow the car ahead and never change lane. To realize this maneuver, simply skip the first process Lateral Lane Change of the driving model. In ACL, CAVs make greedy and aggressive lane change as the human driver does, to which the current connected autonomous driving model already corresponds.

B. Politely Change Lane (PCL)

The PCL maneuver enables a tradeoff between traffic efficiency and safety. In PCL, the *Politeness Index* is introduced, notated as pol, $pol \in [0, 1]$. It indicates how much the vehicle takes account of the vehicles behind during a lane change.

Before a CAV changes lanes, if there is any other vehicle behind in the target lane, it estimates how much the speed of this vehicle will decrease in the case it changes lane. This deceleration can be calculated with the current speed and position of influenced vehicle retrieved online. The deceleration, together with politeness index, decide the probability of performing lane change p_{cl} according to this function:

$$f(\alpha_j, pol) = \begin{cases} 1 & \text{, if } \alpha_j = 0 \\ 0 & \text{, if } pol = 1 \\ max(-\dfrac{pol}{1-pol} \cdot \alpha_j + 1, 0) & \text{, otherwise} \end{cases} \tag{1}$$

Where $\alpha_j = \dfrac{\triangle v_j}{vexp_j} \in [0, 1]$, and j is the ID of the vehicle behind in the target lane.

According to the definition of p_{cl}, a CAV will be certain to change its lane only if the vehicle behind is not influenced by its action. Otherwise, the greater the politeness index is, the less likely the vehicle is to change lanes under the same α_j; the more the speed decrease of the influenced vehicle is, also, the less like the vehicle is to change lanes.

The maneuver with $pol = 0$ is exactly ACL. But NCL does not correspond to $pol = 1$ because if a politeness index is used, even if it is equal to one, the vehicle will change lanes in some cases. In contrast, NCL never does.

To implement PCL, we add a fourth condition in the Lateral Lane Change process.

Process 1: Lateral Lane Change

Condition 4: $rand() < p_{cl}$.

C. Cooperative Lane Change (Optional)

Cooperative lane change is an application of cooperative driving, one of the three CAV features mentioned in Section I. It is an optional function for the maneuvers proposed above.

Cooperative lane change is performed by two CAVs on two neighboring lanes. One wants to change its lane, called *initiator*, is blocked by the other, called *cooperative partner*. In this case, the initiator sends a lane change request to the partner. On receiving the request, the partner assesses the current traffic situation. If it can overtake the initiator in the next iteration with current speed or if it still blocks the initiator even when it stops immediately, the partner will simply ignore the request. Otherwise, the partner slows down to leave enough space for the initiator to perform the lane change. The initiator just keeps monitoring the target lane after the request, until there is enough space to change lanes.

The implementation of cooperative lane change is as follows. The initiator generates request when Condition 3 of the Lateral Lane Change process fails and the blocking vehicle is a CAV. The cooperative partner makes a decision on whether to accept or to ignore the request after Step 2 of the Longitudinal Movement process. If the request is accepted, the partner calculates the appropriate speed to leave enough space ahead and then applies this speed. Considering CAVs are updated sequentially from front (larger cell ID) to rear (smaller cell ID) along the highway, the initiator should be updated prior to the partner if the cell ID of their position is equal.

V. Simulation and Performance Evaluation

A. Metrics

To measure traffic efficiency, the *Actual Expected Speed Ratio* ($AESR$) is used:

$$AESR = \frac{\sum_{i=1}^{n} \frac{v_i}{V_i}}{n}. \tag{2}$$

Where V_i and v_i are the expected and actual average speed of vehicle i, respectively; n is the total number of vehicles.

In the traffic safety domain, three metrics are selected.

- Change Lane Count CL:

$$CL = \frac{N_{CL}}{n \cdot L}. \tag{3}$$

- Backward Distance BD at lane change:

$$BD = \frac{\sum_{i=1}^{N_{CL}} d_i}{N_{CL}}. \tag{4}$$

- OverTake OT on LV percent:

$$OT = \frac{N_{OT\_LV}}{N_{OT}}. \tag{5}$$

Where L is the length of the highway in number of cells; n is the total number of vehicles; N_{CL} is the number of lane changes of all vehicles; d_i is the backward distance in the i-th lane change in number of cells; N_{OT} is the number of overtakes; N_{OT_LV} is the number of overtakes on large vehicles.

Finally, the AOF (Aggregate Objective Function) for all the four metrics above is defined as:

$$AOF = \alpha_{AESR} \cdot w + \alpha_{CL} \cdot (1-x) + \alpha_{BD} \cdot y + \alpha_{OT} \cdot (1-z). \tag{6}$$

Where w, x, y, and z are normalized values of metrics $AESR$, CL, BD, and OT, respectively; α_i (i =

TABLE I
CONFIGURATION LIST OF SIMULATIONS

Item	Value	Item	Value
Lane Count	2	Speed Limit	10 cells/iteration
Cell Count	1000	Low Speed Limit	6 cells/iteration
Vehicle Count	1000	SV Length	1 cell
$p_{overbrake}$	0.5	LV Length	3 cells
Expected Speed	$[SPD_{low}, SPD_{max}]$ (uniform distribution)		
SV Count : LV Count	1 : 1 (uniform distribution)		

TABLE II
VARIABLE PARAMETER LIST OF SIMULATIONS

Item	Value
Lane Change Allowed	$true/false$
Cooperative LC Enabled	$true/false$
Politeness Index	pol
Penetration of CAV	θ
Departure Interval per Lane	λ (Poisson distribution)

$AESR, CL, BD, OT$) is the corresponding weight for each, $\sum \alpha_i = 1$. A larger AOF value indicates that the traffic flow is better in terms of both traffic efficiency and safety. Note that x and z are used in complement, because the greater value they have, the less safe the traffic is.

B. Simulations

We have implemented a simulator based on the driving models presented above. The full configurations are listed in Table I and Table II. The NCL maneuver is used when "Lane Change Allowed" is set to false. The ACL maneuver is used when *pol* is zero. Politeness Index with other values corresponds to PCL. All the results below are averaged over 5 simulation runs.

C. Performance Evaluation

First, we test the capacity of the simulated highway to figure out which departure intervals are proper for our following simulations. We induce a traffic flow full of small or large manually driven vehicles and measure the average vehicle-to-vehicle distance and **highway efficiency** η, which is defined as the ratio of average output rate to input rate of the highway, see Fig. 2 and Fig. 3. As can be seen from Fig. 2, the maximal input rate of the simulated highway appears at departure interval equal to one for large vehicles and at departure interval near 0.1 for small vehicles, because smaller departure intervals cannot further decrease the V2V distance. As can be seen from Fig. 3, the highway saturation point is not far from the departure interval equal to 0.1, because the highway efficiency drops dramatically if the flow is full of small vehicles. However, the large vehicle flow doesn't suffer such a bottleneck. The explanation is that large vehicles make far fewer lane changes than small vehicles, so the negative effect on traffic efficiency is decreased. Besides, the maximal highway input rate for large vehicle flow avoids saturation of

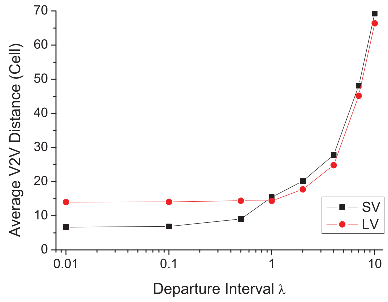

Fig. 2. Average Vehicle-to-Vehicle Distance with all manual small/large vehicles.

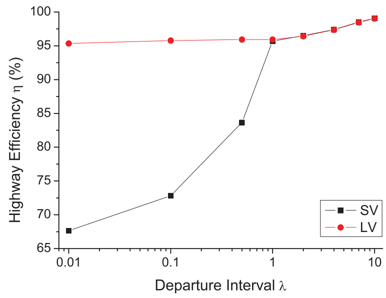

Fig. 3. Highway Efficiency η with all manual small/large vehicles.

the highway. Considering these factors, we choose a departure interval $\lambda = 2$ as an intermediate traffic condition, where neither the maximum input rate nor the highway saturation point are reached. We use $\lambda = 0.1$, 1, 4, and 10 for ultra-heavy, heavy, light, and ultra-light traffic, respectively.

We compare NCL and ACL under different penetrations of CAV using AESR. The simulations are conducted with average departure interval equal to two. According to Fig. 4, in the case that the CAVs never change lane, their AESR always stays rather low. Instead, the AESR of manually driven vehicles grows stably, with increasing penetrations of CAVs. Therefore, the curve for overall traffic in NCL bends down when there are more CAVs. In comparison, ACL gives CAVs much higher efficiency, but the manually driven ones suffer from it a lot, though they also use the ACL maneuver. For overall traffic, ACL outperforms NCL more with increasing penetration.

Fig. 5 proves the effectiveness of cooperative lane change. When cooperative lane change is enabled, CAVs can gain more under higher penetration, while the manually driven vehicles are hardly influenced. However, the improvement brought by cooperative lane change is only limited.

Fig. 6 and Fig. 7 show how the efficiency and safety metrics for both CAVs and manually driven vehicles change, when the politeness of CAVs increases. The simulations are conducted with $\lambda = 2$, $\theta = 50\%$, and pol from zero to one with 0.1 interval. Therefore, there are 11 groups of simulations. For each metric in each group, we evaluate it for manually driven vehicles, CAVs, and the overall traffic. There are a total of 33 values for each metric. The metrics are then normalized, in

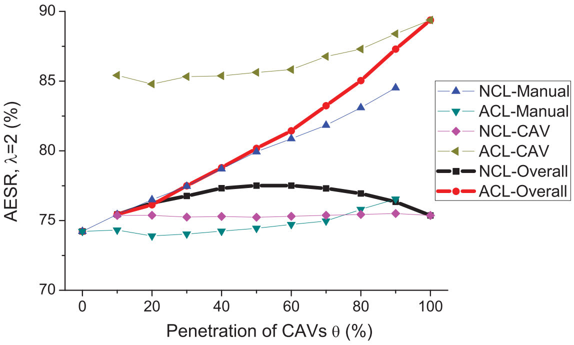

Fig. 4. Actual Expected Speed Ratio(AESR) of NCL and ACL with different penetrations.

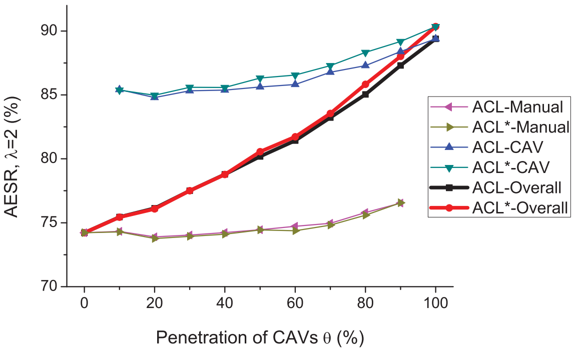

Fig. 5. Actual Expected Speed Ratio(AESR) of ACL and the ACL with cooperative lane change (ACL*) with different penetrations.

the following manner:

$$AESR_i \rightarrow \frac{AESR_i - AESR_{min}}{AESR_{max} - AESR_{min}} \cdot 100\%, i = 1, 2, ..., 33 \quad (7)$$

AOF is calculated with the weights $\alpha_{AESR} = 0.5$, $\alpha_{CL} = 0.1$, $\alpha_{BD} = 0.2$, and $\alpha_{OT} = 0.2$, so that the efficiency and safety domain have equal total weight.

According to the figures, as *pol* increases, the AOF for both CAVs and manually driven vehicles increases, but CAVs always perform better. For CAVs, although the AESR drops with higher *pol*, all three traffic safety metrics are improved significantly and this maintains the growth of AOF, see Fig. 6. On the other hand, the manually driven vehicles also benefit from the courtesy of CAVs in terms of AESR, CL, and OT, see Fig. 7. Note that the metrics change more significantly after $pol \geq 0.5$, which agrees with the definition of p_{CL} described in Section IV-B.

Fig. 8 and Fig. 9 show AOF and AESR improvement brought by PCL with various penetrations of CAVs when compared to ACL under same conditions. The improvement is most significant when the penetration rate is around 50%. For too low or too high penetrations, the PCL plays a secondary role in improving the performance. The phenomenon, that the AOF reaches maximum only after the AESR drops, can also been observed under other penetrations. And that is the point where traffic efficiency and safety are well balanced.

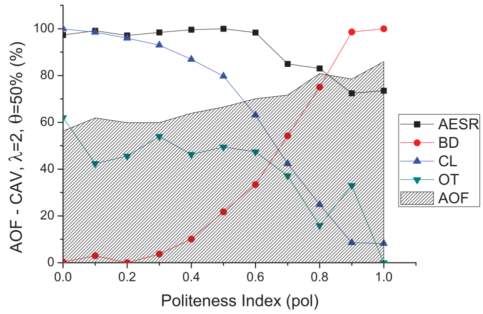

Fig. 6. Normalized metrics Actual Expected Speed Ratio (AESR), Backward Distance (BD) in lane change, Change Lane (CL) Count, Percent of OverTakes (OT) on large vehicles, and Aggregated Objective Function (AOF) for CAVs under $\lambda = 2$ and $\theta = 50\%$ with different PCL Politeness Indexes.

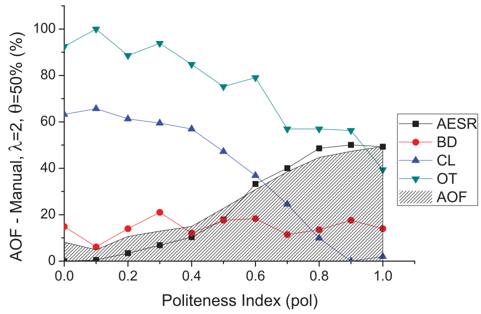

Fig. 7. Normalized metrics Actual Expected Speed Ratio (AESR), Backward Distance (BD) in lane change, Change Lane (CL) Count, Percent of OverTakes (OT) on large vehicles, and Aggregated Objective Function (AOF) for manually driven vehicles under $\lambda = 2$ and $\theta = 50\%$ with different PCL Politeness Indexes.

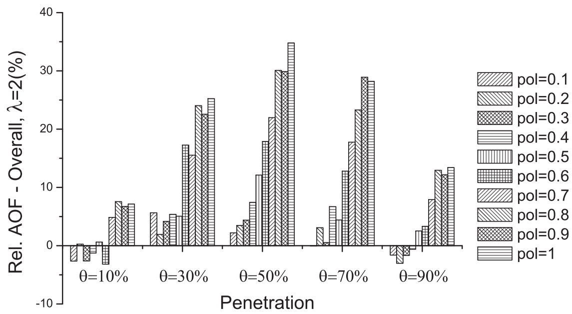

Fig. 8. Aggregated Objective Function (AOF) relative to ACL with $\lambda = 2$ and different PCL Politeness Indexes for θ=10%, 30%, 50%, 70%, and 90%.

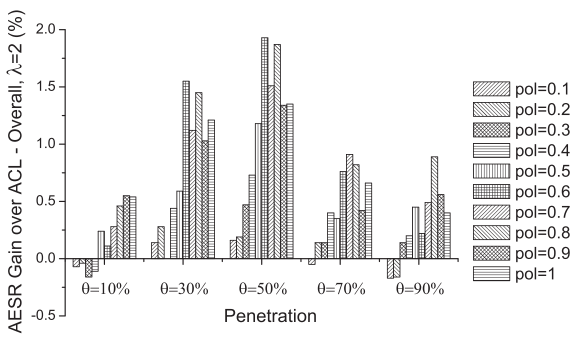

Fig. 9. Actual Expected Speed Ratio (AESR) Gain over ACL under $\lambda = 2$ with different PCL Politeness Indexes for θ=10%, 30%, 50%, 70%, and 90%.

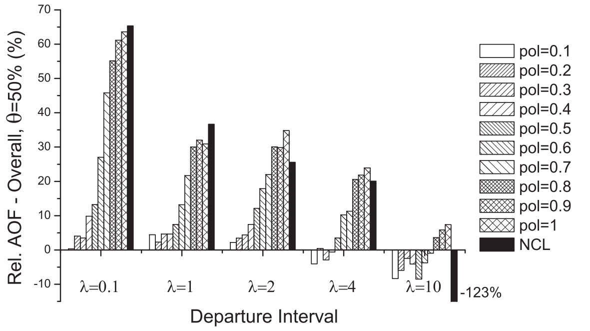

Fig. 10. Aggregated Objective Function (AOF) relative to ACL under $\theta = 50\%$ with different PCL Politeness Indexes for λ=0.1, 1, 2, 4, and 10.

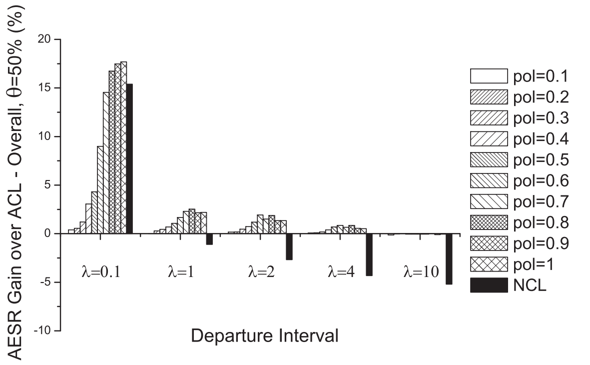

Fig. 11. Actual Expected Speed Ratio (AESR) Gain over ACL under $\theta = 50\%$ with different PCL Politeness Indexes for λ=0.1, 1, 2, 4, and 10.

Fig. 10 and Fig. 11 show the AOF and AESR improvement brought by PCL with various traffic densities when compared to ACL under the same conditions. As can be seen, the heavier the traffic is, the more significant improvement from PCL, in both traffic efficiency and safety. In contrast, with ultra-light traffic, the courtesy of CAVs is unnecessary. However, NCL can even outperform the best PCL in terms of AOF under heavy traffic ($\lambda = 1$ and $\lambda = 0.1$). Referring to Fig. 11, we can further infer that this performance improvement results from higher traffic safety. It agrees well with the real-life experience.

In conclusion, a summary of results is:

1) In light traffic, the more aggressive driving (ACL) of CAVs is adaptable, while the cooperative driving (PCL) does much better in heavy traffic, when considering both traffic efficiency and safety.
2) In non-light traffic, the CAVs should always behave as cooperatively as possible (high politeness index), which always leads to improvement in the overall traffic in terms of both efficiency and safety, even the CAVs themselves can benefit from it.
   a) The heavier the traffic is, the more significant this improvement becomes.
   b) This improvement never reaches the peak with too low or too high penetration of CAVs. Instead, it becomes most significant at moderate penetration.
   c) In terms of efficiency only, a politeness index around 0.6-0.8 delivers the best performance.
3) Cooperative lane change improves the efficiency of CAVs slightly, while manually driven vehicles are hardly influenced.

VI. Conclusion

In this work, highway traffic is introduced with connected autonomous vehicles (CAVs). The traffic model for CAVs is first defined. The safety-oriented and efficiency-oriented baseline lane change maneuvers, NCL and ACL, are proposed, which indicate two extreme cases. The Politeness Index is then introduced in the Politely Change Lane (PCL) maneuver to achieve a tradeoff between efficiency and safety. Extensive simulations are carried out and the results show that both traffic efficiency and safety can be improved by PCL, especially with heavy traffic and a moderate penetration rate.

Since in this work PCL only uses information on the traffic behind, some look-ahead maneuvers are an area for study in the future. Also, the limited deceleration of vehicles can be introduced to our model, in which case accidents may happen, and the effect of CAVs in such cases, e.g. the avoidance of multi-vehicle pile-ups, can then be analyzed.

References

[1] A.-M. Elliott. (2011, Feb.) The future of the connected car. [Online]. Available: http://mashable.com/2011/02/26/connected-car/

[2] K. Barry. (2012, Feb.) Ford: Connected cars can save us from gridlock. [Online]. Available: http://www.wired.com/autopia/2012/02/bill-ford-says-connected-cars-can-save-us-from-gridlock/

[3] Y. Yang, X. Li, W. Shu, and M. Wu, "Quality evaluation of vehicle navigation with cyber physical systems," in *GLOBECOM 2010, 2010 IEEE Global Telecommunications Conference*. IEEE, 2010, pp. 1–5.

[4] V. Wagner. (2012) Driving into the future: Autonomous cars. [Online]. Available: http://www.technewsworld.com/rsstory/74708.html

[5] N. H. T. S. Administration. (2003) Analysis of lane change crashes. [Online]. Available: http://www.nhtsa.gov/

[6] H. John. (2011) 6% of crashes caused by lane changing. [Online]. Available: http://www.honestjohn.co.uk/news/legal–motoring-advice/2011-09/blind-spots/

[7] K. Nagel and M. Schreckenberg, "A cellular automaton model for freeway traffic," *Journal De Physique I*, vol. 2, pp. 2221–2229, 1992.

[8] D. Chowdhury, L. Santen, and A. Schadschneider, "Statistical physics of vehicular traffic and some related systems," *Physics Reports-review Section of Physics Letters*, vol. 329, pp. 199–329, 2000.

[9] M. Rickert, K. Nagel, M. Schreckenberg, and A. Latour, "Two lane traffic simulations using cellular automata," *Physica A-statistical Mechanics and Its Applications*, vol. 231, pp. 534–550, 1996.

[10] W. Knospe, L. Santen, A. Schadschneider, and M. Schreckenberg, "A realistic two-lane traffic model for highway traffic," *Journal of Physics A: Mathematical and General*, vol. 35, 2002.

[11] X.-G. Li, B. Jia, Z.-Y. Gao, and R. Jiang, "A realistic two-lane cellular automata traffic model considering aggressive lane-changing behavior of fast vehicle," *Physica A-statistical Mechanics and Its Applications*, vol. 367, pp. 479–486, 2006.

[12] A. Kesting, M. Treiber, and D. Helbing, "General lane-changing model mobil for car-following models," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1999, pp. 86–94, 2007.

[13] P. Hidas, "Modelling lane changing and merging in microscopic traffic simulation," *Transportation Research Part C-emerging Technologies*, vol. 10, pp. 351–371, 2002.