Homework 9 - Inference

∗ If there is any problem, please contact TA.
Name:_____ Student ID:_____ Email: _____

Problem 1. (60 pts) Write down the principal solutions for the following sets of constraints:

(a) $\{X = Int,\ Y = X \to X\}$

(b) $\{Int \to Int = X \to Z\}$

(c) $\{X \to Y = Y \to Z,\ Z = U \to W\}$

(d) $\{Int = Int \to X\}$

(e) $\{X = Int \to X\}$

(f) $\{\}$

Solution. You just need to write down the principal solution to get points. Now you can try unification algorithm.
(a)

$$\begin{aligned}&(I, \{X = Int, Y = X \to X\})\\ \to&([x \to Int] \circ I, \{Y = Int \to Int\})\\ \to&([Y = Int \to Int] \circ [X \to Int] \circ I, \{\})\end{aligned}$$

Principal solution is: $S(X) = Int, S(Y) = Int \to Int$

(b)

$$\begin{aligned}&(I, \{Int \to Int = X \to Z\})\\ \to&(I, \{Int \to X, Int \to Z\})\\ \to&([X \to Int] \circ I, \{Int \to Z\})\\ \to&([Z \to Int] \circ [X \to Int] \circ I, \{\})\end{aligned}$$

Principal solution is: $S(X) = Int, S(Z) = Int$

(C)

$$\begin{aligned}&(I, \{X \to Y = Y \to Z, Z = U \to W\})\\ \to&(I, \{X \to Y, Y \to Z, Z = U \to W\})\\ \to&([X \to Y] \circ I, \{Y \to Z, Z = U \to W\})\\ \to&([Y \to Z] \circ [X \to Y] \circ I, \{Z = U \to W\})\\ \to&([Z = U \to W] \circ [Y \to Z] \circ [X \to Y] \circ I, \{\})\end{aligned}$$

Principal solution is: $S(X) = S(Y) = S(Z) = U \to W$

(d) No solution.
(e) No solution.
(f) the principal solution is I (or you can write [ ], they are the same). □

Problem 2. (40 pts)

Lemma 1. *If a set of constraints q has a solution, then it has a most general one.*

Prove this lemma.

Proof.

Lemma 2. *If a set of constraint q has a solution, the unification algorithm always return the principal solution*

The proof is in the reference book *Types and Programming Languages*, page 328, 22.4.5

Then lemma 1 is proved. □