Lab12-Turing Degree

CS363-Computability Theory, Xiaofeng Gao, Spring 2016

∗ Please upload your assignment to FTP or submit a paper version on the next class

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1. A *dominating set* for a graph $G = (V, E)$ is a subset D of V such that every vertex not in D is adjacent to at least one vertex in D. The domination number $\gamma(G)$ is the number of vertices in a smallest dominating set for G. The *Dominating Set* (DS) problem concerns finding a minimum $\gamma(G)$ for a given graph G.

   Prove that: SET-COVER$\equiv_p$ DOMINATING-SET.

2. Let A, B, C, be sets. Prove that

   (a) If A is B-recursive and B is C-recursive, then A is C-recursive.

   (b) If A is B-r.e. and B is C-recursive, then A is C-r.e.

   (c) If A is B-recursive and B is C-r.e., then A is not necessarily C-r.e.

3. Let A, B be any sets.

   (a) Show that $A \leq_T B$ iff $K^A \leq_m K^B$, and $A \equiv_T B$ iff $K^A \equiv_m K^B$.

   (b) Show that the previous question can be made effective in the following sense: there is a total computable function f such that if $c_A = \phi_e^B$, then $\phi_{f(e)} : K^A \leq_m K^B$. (*Hint.* Find total computable functions g, h such that (1) if $c_A = \phi_e^B$ then $K^A = W_{g(e)}^B$, (2) $\phi_{h(e)} : W_e^B \leq_m K^B$ for all e.)

4. Given an ascending sequence of Turing degrees:

$$\mathbf{b_0} < \mathbf{b_1} < \cdots < \mathbf{b_n} < \mathbf{b_{n+1}} < \dots$$

   Prove that no such ascending sequence of Turing degrees has a least upper bound.