Lab11-Reducibility

CS363-Computability Theory, Xiaofeng Gao, Spring 2016

1. Recall that $A \otimes B = \{\pi(a, b) \mid a \in A, b \in B\}$. Prove the following statements.

   (a) For any sets A, B, if $B \neq \varnothing$ then $A \leq_m A \otimes B$.

   (b) $A \equiv_m A \otimes \mathbb{N}$ for any set A,

   (c) $A \equiv_m A \otimes B$ if $A \neq \mathbb{N}$ and B is a non-empty recursive set.

2. Let $\mathbf{a}$, $\mathbf{b}$ be m-degrees.

   (a) Show that the least upper bound of $\mathbf{a}$, $\mathbf{b}$ is uniquely determined; denote this by $\mathbf{a} \cup \mathbf{b}$;

   (b) Show that if $\mathbf{a} \leq_m \mathbf{b}$ then $\mathbf{a} \cup \mathbf{b} = \mathbf{b}$;

   (c) Show that if $\mathbf{a}, \mathbf{b}$ are r.e., then so is $\mathbf{a} \cup \mathbf{b}$;

   (d) Let $A \in \mathbf{a}$ and let $\mathbf{a}^*$ denote $d_m(\overline{A})$. (Check that $\mathbf{a}^*$ is independent of the choice of $A \in \mathbf{a}$.) Show that $(\mathbf{a} \cup \mathbf{a}^*)^* = \mathbf{a} \cup \mathbf{a}^*$.

3. Show that the following sets all belong to the same m-degree:

   (a) $\{x \mid \phi_x = 0\}$,

   (b) $\{x \mid \phi_x \text{ is total and constant}\}$,

   (c) $\{x \mid W_x \text{ is infinite}\}$.