Lab06-Universal Program

CS363-Computability Theory, Xiaofeng Gao, Spring 2016

∗ Please upload your assignment to FTP or submit a paper version on the next class
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1. (a) Show that there is a decidable predicate $Q(x, y, z)$ such that
   i. $y \in E_x$ if and only if $\exists z.Q(x, y, z)$
   ii. if $y \in E_x$ and $Q(x, y, z)$, then $\phi_x((z)_1) = y$.

   (b) Deduce that there is a computable function $g(x, y)$ such that
   i. $g(x, y)$ is defined if and only if $y \in E_x$.
   ii. if $y \in E_x$, then $g(x, y) \in W_x$ and $\phi_x(g(x, y)) = y$; i.e. $g(x, y) \in \phi_x^{-1}(\{y\})$.

   (c) Deduce that if f is a computable injective function (not necessarily total or surjective) then f^{-1} is computable. (cf. exercise 2-5.4(1)).

2. (cf. example 3-7.1(b)) Suppose that f and g are unary computable functions; assuming that T_1 has been formally proved to be decidable, prove formally that the function $h(x)$ defined by $h(x) = \begin{cases} 1 & \text{if } x \in Dom(f) \text{ or } x \in Dom(g), \\ \uparrow & \text{otherwise,} \end{cases}$ is computable.

3. Show that there is a total computable function $k(e_1, e_2)$ such that $\phi_{k(e_1,e_2)}(x)$ is the characteristic function for predicate "$M_1(x)$ and $M_2(x)$", where M_1 and M_2 are both decidable predicate and $\phi_{e_1} = c_{M_1}$, $\phi_{e_2} = c_{M_2}$.

4. Show that there is a total computable function $s(x, y)$ such that for all x, y, $E_{s(x,y)} = W_x \cup E_y$.

5. Suppose that $f(x)$ is computable; show that there is a total computable function $k(x)$ such that for all x, $W_{k(x)} = f^{-1}(W_x)$.