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Review article

Coverage problem with uncertain properties in wireless sensor networks: A survey

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ABSTRACT

The coverage problem, as a fundamental problem for almost all types of applications in wireless sensor networks (WSNs), has been studied for over a decade. Because of its simplicity and ease of analysis, full coverage is widely adopted in many theoretical studies. However, a full coverage requirement can be relaxed into a partial one to avoid the overuse of sensors. Moreover, sometimes full coverage is not the best way to represent some real-world applications because of its strong restrictions and its deterministic characteristics. From this view, a better way is to introduce uncertainty into coverage problems. By analyzing the characteristics of partial or probabilistic coverage problems, and comparing them with full coverage problems, this survey is helpful to establish an overview of non-deterministic coverage problems, denoted as coverage problems with uncertain properties. This survey then sketches a general research framework to solve such problems. According to the framework, we first introduce a series of basic concepts of coverage problems with uncertain properties and then summarize the relevant models, such as detection models, network models, and deployment models. Based on these models, we discuss three main objectives, namely, to maximize coverage quality, to maximize network lifetime, and to minimize the number of sensors. Next, we illustrate several solution strategies for these three objectives, such as deployment, scheduling or selection, and movement or adjustment. Then, we classify the solutions (algorithms) from different aspects, i.e., traditional and heuristic, approximation, distributed and centralized, and random algorithms. In addition, the theoretical analysis for algorithms and the platforms for simulating the numerical experiments are also summarized. Finally, we discuss future challenges and directions for research of coverage problems with uncertain properties.

1. Introduction

Wireless Sensor Networks (WSNs) attract more and more attention from both academic and industrial communities in the recent decade. Sensors can detect, measure, and collect information from various environments. The information collected includes a variety of types, such as light, distance, vision, location, acceleration, sound, compass, rotation, magnetic, gravity, atmospheric pressure, temperature, and humidity, etc. With the rapid development of Micro-Electromechanical Systems, cheap and powerful sensors, which are viewed as lightweight "nodes", are massively produced. WSNs are composed of many such "nodes" dynamically without the help of any pre-defined infrastructure. The number of nodes can reach up to hundreds, even thousands, or millions.

Coverage problem is a fundamental problem in WSNs. When building WSNs, a set of sensors are deployed in a region of interest (ROI). Each active sensor can detect objects in its "detection range", which is a disk region centered at the location of the sensor. Therefore, we say an object in the ROI is "covered" if it can be detected by at least one sensor in WSNs. Consequently, the coverage problem is to determine whether the objects located at different places in the ROI can be covered by WSNs. Specially, we sometimes refer to "coverage quality" as the percentage of objects to be covered successfully by active sensors among all sensors deployed. Correspondingly, the coverage problem is also defined so as to study whether a coverage quality is achieved and how to achieve the required coverage quality for WSNs. On the one hand, we introduce many measurements, then, evaluate the WSNs from various aspects to determine whether the coverage quality is qualified. On the other hand, we adopt many methods to achieve a required coverage quality, for instance, to deploy sensors according to special patterns, to activate sensors in turn to extend network lifetime, to move sensor appropriately, or to adjust sensor parameters prop-

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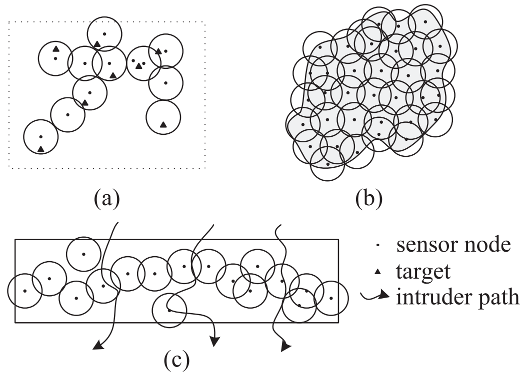


Fig. 1. Three kinds of typical coverage: (a) Target coverage. (b) Area coverage. (c) Barrier coverage.

erly. In all, researches on coverage problem benefit a lot of applications, such as battlefield surveillance, biological detection, smart space, industrial diagnostics, military facility, environment monitoring, and indoor guarding.

For different application scenarios, there are generally three basic types of coverage problems, namely, target coverage, area coverage, and barrier coverage. Typical examples are illustrated in Fig. 1, [1]. In these figures, the points represent the sensors and the cycles represent the detection range of sensors. Fig. 1(a) shows a scenario of target coverage, which focuses on the coverage of discrete target points (represented as small triangles). Fig. 1(b) shows a scenario of area coverage, which discusses a coverage on a continuous irregular region. Fig. 1(c) shows barrier coverage which studies an intruder detection problem along a long belt region by forming a barrier. All intrusion paths are detected by the sensor barrier. More details of these coverage problems are described in Section 2.

After identifying the basic coverage type for a coverage problem, the most important issue is its coverage quality requirements. The most common requirement is called *full coverage*, which in general means that all objects should be covered in the ROI of WSNs. Different coverage types may lead to different definitions of full coverage. For target or area coverage, it means that all points in the discrete target set or the continuous region should be covered. For barrier coverage, all intrusion paths should be detected by the sensor barrier(s). However, in some applications full coverage quality seems too strong, because the probability of event occurrence varies a lot geographically, which makes full coverage unnecessary in some places. For example, forest fire monitoring applications may require full coverage in dry regions, while they only require less than 50% of area to be covered in rainy regions. Therefore, the full coverage requirement could be relaxed to the so called *partial coverage*, which means that only a part of the target set, sub regions, and intrusion paths, are required to be covered.

Besides the coverage quality requirements for WSNs, another important issue for coverage problems is the sensor detection model. Researchers usually depict a sensor detection effect by deterministic models, e.g., the most popular 0/1 model. Using this model, a sensor's detection range is depicted as a disk centered at the location of the sensor, and its detection effect is deterministically equivalent for any point in the disk. That is, we use a binary variable to describe whether an object is covered by a sensor, 1 as "yes" and 0 as "no". The 0/1 model is also called the "disk" model in some literatures. Obviously, 0/1 model cannot precisely reflect the real-world situation without non-deterministic characteristics. Sometimes, we cannot easily assert that an object located around the boundary of a detection disk is definitely covered by a sensor. On the contrary, it may be detected with a probability according to the detection situation. Correspondingly, instead probabilistic models involve random variables to describe the sensor detection effect according to some factors, such as, Euclidean distance between a sensor and a target object, etc. The farther the distance is, the lower the detection probability is, and vice versa. Besides, probabilistic models could be used to describe many complicated environments where a sensor's detection effect could be influenced by noises, detection errors, objective delays, etc. Therefore, probabilistic detection models are adaptive to be used in new application scenarios of WSNs, such as robotic systems [2,3], people-centric networks [4], radar or transmit-receive (T-R) model systems [5], and other ad hoc networks.

Given the above two considerations, we discuss a special type of coverage problem in this survey, namely, **coverage problem with uncertain properties**, which mainly addresses coverage problems with partial or probabilistic properties. As a typical example for coverage with partial property, a p-percentage coverage problem is addressed by [6–10]. The p-percentage coverage means that p percent targets, area of ROI, or intrusion paths, are covered. Because a randomly selected target point or intrusion path is covered with a probability P (P is approximately equal to p in probability theory), it cannot be covered with a probability 1-P in a Monte-Carlo test. As a typical example for the coverage problem with probabilistic property, a P-probability coverage problem requires that objects or intruders should be covered or detected with a probability greater than or equal to P. Obviously, probabilistic coverage problems are usually with the usage of probabilistic detection models [11–21]. Besides uncertain properties in problem objectives, the uncertainty can also exist in detection models, deployment models, network models, environment simulations, and probabilistic estimations for network parameters, and so on.

As a research trend, the coverage problem with uncertain properties attracts more and more attentions. We consider two important reasons below.

First, the full coverage requirement is too restrict for many applications in WSNs. On the one hand, for those caring more about the network lifetime, we can actually sacrifice the coverage quality to extend the network lifetime by setting proper p-percentage or P-probability coverage threshold. On the other hand, for those caring more about coverage quality, to control the network lifetime can bring higher coverage quality level by tuning up p or P threshold for partial or probabilistic coverage. All these provide more flexibility for service providers to better design their WSN systems.

Second, probabilistic models are more practical. Coverage problems with uncertain properties usually use many kinds of probabilistic models, such as the probabilistic detection model [22–24], the probabilistic network model [13,16,25–29], the random noise model [30–32], and the random fault model [12,33,34], etc. Generally, these probabilistic models can better describe the physical world. As mentioned above, the 0/1 model for a sensor detection cannot describe the attenuation of sensing effect with target distance increasing as well as the random noise, such as the Gaussian noise model. Additionally, probabilistic models are also powerful to describe random node failures, node location inaccuracy, and imperfect time synchronization, etc.

Recently, many surveys summarized the coverage-related problems in WSNs. Majority of them focus on only one aspect of the coverage problem. The topic in [35] focuses on a special kind of deployment problem for WSNs, namely the planned deployment problem. It is a deterministic deployment problem aiming at coverage-related objectives such as to maximize coverage, maximize lifetime, maximize survivability, etc. In order to solve the special deployments, authors classify solutions into four types: Genetic Algorithms, Computational Geometry, Artificial Potential Fields, and Particle Swarm Optimization. Similarly, authors in [36] discuss deployment problems under a metaheuristics perspective. Authors survey several metaheuristics: single-solution based algorithm, evolutionary algorithm, swarm intelligence and their varieties. More nesting and hybrid methods based on basic metaheuristics are exploited. Instead of the deployment, authors in [37] summarize that evolutionary computation and swarm intelligence are comprehensively applied in wake-up scheduling which is another important energy management and optimization strategy. Another research direction for improving coverage quality is to use mobile sensors. Authors in [38,39] draw up a wide picture for a taxonomy of movement-assisted sensor deployment algorithms. Particularly, six classes of approaches are identified, and each of them is discussed in a special four-factor structure: assumptions, operating principals, advantages, and weaknesses in [39]. For a special kind of movement, authors in [40] discuss rotation strategy for the directional sensor networks.

The novelty of our survey is obvious compared with the above surveys. First, none of the above surveys takes uncertain properties as a key point for coverage problems although they involve some uncertain properties, such as probabilistic sensing model. Second, they merely consider one topic of the three main aspects for the coverage problem, respectively deployment, scheduling (wake-up), or movement. Instead, we exhaustively survey research work and bring all these aspects into our content as three main solution strategies. Besides, there are several surveys which overview the coverage problems generally, like [1,41–43], etc. However, none of them focus on coverage problems with uncertain properties. In all, there are three main contributions of our survey:

1. First, we summarize works on coverage problems with uncertain properties and give statistics on recent research literature over the last decade.
2. Second, we give a general research framework on the coverage problems with uncertain properties according to their special characteristics and potential research workflow. We discuss more details of the framework and its advantages at the end of Section 2.
3. Third, to the best of our knowledge, we initially summarized important optimization objectives and corresponding strategies for coverage problem with uncertain properties.

This survey is organized as follows: Section 2 presents the basic concepts of the coverage problem in WSNs, especially related to basic coverage types, detection objectives, and constraints for coverage problems with uncertain properties. Additionally, we provide a research framework on the coverage problem with uncertain properties (Fig. 4) and we tabulate statistics on all research literature involved in this survey. The later sections in this survey are organized according to the corresponding parts of the framework. Section 3 provides hypotheses, including detection, network, and deployment models. Next, Section 4 focuses on the problem objectives; there are main objectives, sub-objectives, and some restrictions on those objectives. Three important optimization objectives are given. Section 5 presents the solution strategies by discussing some typical researches. Section 5 also classifies the involved algorithms based on different problems and algorithmic categories. Section 6 summarizes important algorithmic theoretic analysis including model analysis, algorithm complexity, and approximation ratio results. Simulation platform collection will also be covered in this section. Section 7 introduces new directions in research of coverage problems with uncertain properties in the future. Finally, Section 8 concludes this survey.

2. Preliminaries and a research framework

In this section, we give the preliminaries for coverage problems in WSNs. We firstly introduce three basic coverage types, namely, target coverage, area coverage, and barrier coverage. Then, we give concepts of main coverage qualities and restrictions, such as partial

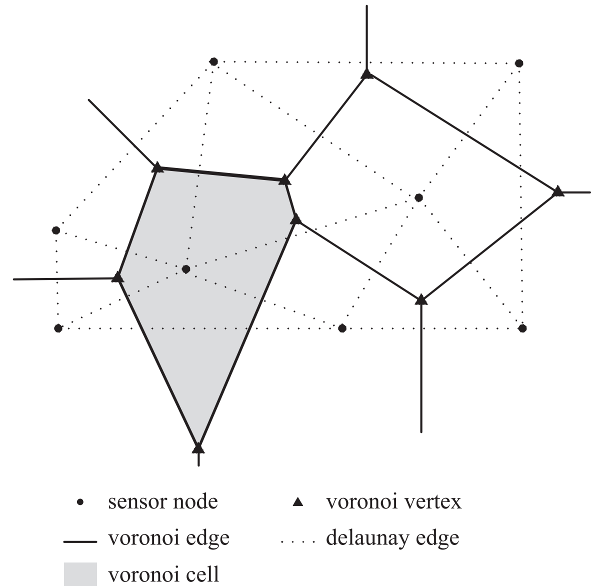


Fig. 2. Illustration of Voronoi diagram and Delaunay triangulation for 8 sensors [1].

coverage, probabilistic coverage, connectivity, and network overhead. Finally, we summarize a general research framework on coverage problems with uncertain properties. Meanwhile, we present statistics on research papers involved in this survey.

2.1. Basic coverage types

2.1.1. Target coverage

Target coverage will cover discrete targets. The locations of the targets (i.e., the spatial points) are usually given ahead. Fig. 1(a) illustrates the target coverage. Typically, the target coverage problem should select pre-deployed sensors to cover all discrete targets, which can be directly modeled as a classical *Set Cover* problem. However, a sensor might detect objects (events) with random mistakes. Many studies have considered False Alarm Probability as an important criterion [14,18]. In the (signal) detection theory, a false alarm occurs when a non-target event is identified as a target event. The False Alarm Probability addresses the false-positive probability among all event detections in WSNs.

2.1.2. Area coverage

The objective of area coverage is to cover the entire region of interest (ROI). Some papers also refer it as “field of interest”. In this survey, we use the former, especially when discussing area coverage. Fig. 1(b) illustrates an irregular ROI.

The area coverage is always discussed for a continuous space. Thus, an area coverage problem is usually hard to be transformed to a discrete space problem, which makes the area coverage problem more difficult to solve. The Voronoi division is a powerful tool for overcoming the infinite point property of area coverage. Fig. 2 illustrates a Voronoi graph and its corresponding Delaunay triangulation in 2*D* space where definite sensors are deployed as the Voronoi cell centers.

Definition 1 (Voronoi Cell and Voronoi Graph for WSN)**.** One Voronoi cell in 2*D* space for a certain sensor is a region composed of all points, which have the shortest distance to the sensor in comparison with to any other sensors. A Voronoi cell is usually a polygon and every edge of the polygon is called as a Voronoi edge. The intersection point of two or more Voronoi edges is called a Voronoi vertex. A Voronoi graph consists of all Voronoi cells. Two sensors are Voronoi neighbors when their Voronoi cells share one Voronoi edge.

A Delaunay graph is the dual graph of the corresponding Voronoi graph in graph theory. As illustrated in Fig. 2, the Delaunay edges (dashed) are orthogonal to their corresponding Voronoi edges (solid). However, Delaunay edges do not necessarily intersect Voronoi edges.

Assume that the sensors in set S are randomly deployed, a ROI $\mathcal{A}$ might or might not be completely covered under another random deployment. However, the Critical Sensor Density (CSD), λ_{csd}, helps to decide at least how many sensors are needed to completely cover the region under random deployment. The λ_{csd} for a tessellation can be computed as follows: Let A_p denote the area of a deployment pattern (a polygon), N_p denote the number of sensors that compose a pattern, and N_n denote the number of pattern blocks that share a node. Then λ_{csd} can be computed as Eq. (1) [44].

$$\lambda_{csd} = \frac{N_p}{A_p N_n} \tag{1}$$

It is worthwhile to point out that area coverage can be viewed as a generation of target coverage. Therefore, some studies focus on how to transform area coverage to target coverage by geometrical analysis [30,45–53]. The Voronoi and Delaunay graphs play a key role in the transformation. For some ROI with special shape, deployment of sensors has special results. Authors in [54] discuss the case for long belt and further give optimal solution for this case in [55].

2.1.3. Barrier coverage

Barrier coverage is to detect objects crossing a long belt shape region. Therefore, we must define a belt first [44].

Definition 2 (Ordinary Belt with Width w)**.** If l_1 and l_2 are two parallel curves with a separation width w, the region between l_1 and l_2 is referred to as a belt region $\mathcal{B}$ with width w. The two curves l_1 and l_2 are two parallel boundaries. We say $\mathcal{B}$ is ordinary (with respect to a sensor network deployed over $\mathcal{B}$) if it satisfies the following condition: for two sensors within $\mathcal{B}$, if their detection range D_1 and D_2 have an overlap, then $(D_1 \cup D_2) \cap \mathcal{B}$ is a connected sub region in $\mathcal{B}$.

Definition 3 (Barrier Coverage)**.** A path is said to be a *crossing path* if it crosses from one parallel boundary to the other. A sensor network deployed in a belt region is said to provide barrier coverage if and only if all crossing paths through the belt intersect the detection range of at least one sensor.

Aside from the traditional barrier coverage, the authors in [56] and [57] propose a variant of barrier coverage, namely *trap coverage*. A sensor network providing trap coverage guarantees that any moving object can move at most a known distance before it is detected, along any trajectory and by any speed. This means that coverage holes are allowed from the view of area coverage. Similarly, there is another variant, namely *sweeping coverage* [58–61], which considers mobile sensors to collect data detected by static sensors.

Additionally, for saving many sensors, mobile sensors, instead of static sensors, can be used to patrol along a line to dynamically form a barrier [62]. Fig. 3 shows two situations in this scenario. As shown in Fig. 3(a), many stable sensors are deployed along the barrier line, while in Fig. 3(b), if the defender knows the probability of intruder occurrence in the ROI as a prior knowledge, then mobile sensors can determine their own movement strategy. Otherwise, without the prior knowledge, keeping stable makes sensors to have the same efficiency as the moving ones in Fig. 3 (c).

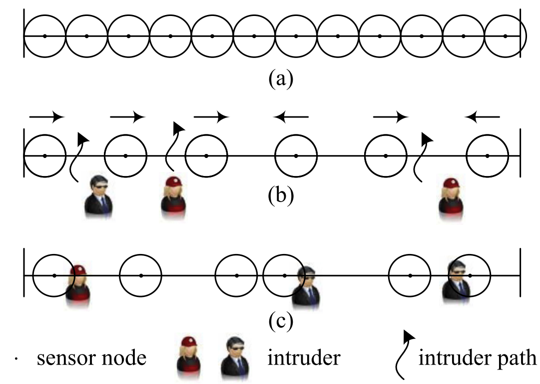


Fig. 3. (a) Full barrier coverage by static sensors; (b) and (c) Barrier coverage by mobile sensors [62].

Weibull distribution is used in [62] to estimate the intruder occurring probability as a prior knowledge.

2.2. Coverage quality and constraints

In this subsection, we first discuss the basic coverage quality, namely, **full coverage**. Then we give an overview and basic measurements for **partial coverage** and **probabilistic coverage**. Finally, we discuss connectivity and network overhead because they are usually important constraints in designing WSNs.

2.2.1. Full coverage

In fact, full coverage has been defined above in Section 2.1 for each basic coverage type: target, area, and barrier coverage. Full coverage is the basic coverage quality requirement, and other kinds of coverage quality stem from full coverage. In the early years, many studies focused on the full coverage with 0/1 model (disk model), such as [33,46]. Recently, many papers discuss full coverage problems while the sensing model has been changed to the probabilistic model, such as [63–66]. Moreover, when we use a probabilistic model as the detection model, a threshold will be predetermined. A target is detected if it is within some sensor's detection range and the detection probability on the target by the sensor is higher than the threshold. Some other complex criteria also justifies whether a target is covered. For instance, error rate can be set as a criterion because the detections by sensors may be disturbed by random noises in the environment [44,65].

2.2.2. Partial coverage

Partial Coverage studies how to relax coverage quality in order to lengthen the lifetime of the network. Adjusting coverage ratio is a simple and realistic way to achieve such purpose [47]. Coverage ratio measures the percentage of targets being covered for target coverage (the proportion of the covered region to the entire ROI for area coverage, and the percentage of paths detected to all paths for barrier coverage). For example, if 8 out of 10 targets are covered, then the coverage ratio is 80%. In a certain sense, full coverage can be viewed as a special partial coverage, which has a coverage ratio of 100%. In Section 4, we will formally define an important partial coverage problem, i.e., p-percentage coverage. Partial coverage merely adopts quantitative coverage ratio to control the coverage quality, which brings uncertain properties for coverage problems because which parts are detected or undetected is unknown and dynamic.

2.2.3. Probabilistic coverage

Probabilistic coverage describes coverage quality in probabilistic manner. The sensor detection model for probabilistic coverage is usually a probabilistic model. The coverage (detection) probability of a target can be expressed as Eq. (2)

$$P(t) = 1 - \prod_{i=1}^{n} [1 - p_i(t)] \tag{2}$$

where t denotes a target position in the ROI Ω (can be expressed by a coordinate in detail), $p_i(t)$ denotes the detection probability by the ith sensor node for t, and n is the number of sensors. We can calculate p_i according to different detection models [15,19,24]. $P(t)$ denotes the overall coverage probability at position t.

The objective of probabilistic coverage problem is usually to maximize the coverage probability for the entire ROI, and the value of maximum coverage probability can be expressed as

$$\max \int_{\Omega} R(t)P(t)dt \tag{3}$$

where $R(t)$ is an object (event) density function, for some $t \in \Omega$ [44,63,67]. If this constraint is more complex and has some other parameters, then it can be adjusted according to different requirements. For instance, in a wireless mobile sensor network (WMSN), if we need to minimize the distance of sensor movements, then we can change $P(t)$ to $P(t, \mathbf{s})$ and $\mathbf{s} = \{s_1, s_2, \cdots, s_N\}$ denotes the locations of sensors after the movements [68].

2.2.4. Connectivity

Apart from maintaining a certain coverage quality for WSNs, maintaining the connectivity of network is another important requirement. In the recent decade, considerable literature has been devoted to consider coverage and connectivity simultaneously. A classical theorem is proposed for a relation between sensor detection range and sensor communication range by [69] as seen below.

Theorem 1. *Supposing that a set of sensors S provides full 1-coverage for a convex region, then, if $R_c \geq 2R_s$, S forms a 1-connected network.*

In [69], the Voronoi graph is applied in proving the theorem. Some researches, such as [46,69], require a higher connectivity degree which is denoted as K-connected. If a network is K-connected, then the network will remain connected for any possible $K-1$ active nodes failing. A similar definition is given for K-coverage.

In [46], the optimal deployment is studied for achieving 4-connectivity and full 1-coverage under different ratios of communication range (denoted by r_c) to sensor detection range (denoted by r_s). Several deployment patterns are given, namely, diamond, triangle, square, and double-strip patterns. Several protocols, e.g., ASCENT [70], SPAN [71], AFECA [72], and GAF [73] aim to maintain network connectivity but do not guarantee coverage requirement. However, researchers, such as [69], extend them to a coverage-preserving version.

We introduce one metric for connectivity quality, namely, Packet Reception Rate (PRR), as a function taking the distance v between transmitter and receiver as a parameter, which can be expressed as

$$PRR(v) = \left(1 - \frac{1}{2}e^{-\frac{P_t - PL(v) - P_n}{2}}\right)^{8l} \tag{4}$$

where P_t is the output power of transmitter, $PL(v)$ is the path loss at distance v, P_n is the noise floor, and l is the frame length. The study in [74] provides a detailed derivation. As [75] mentioned, a probabilistic model for connectivity and coverage is more realistic. Thus, the PRR, as a network model, extends the sensor communication ability to a non-disk model. The PRR is usually required not to be larger than a certain threshold when such protocols autonomously construct connected networks.

Some researchers consider coverage and connectivity at the same time from the aspect of probabilistic analysis. For balancing the energy consumption of each sensor, authors in [76] propose a named Probability Density Function (PDF) for guiding sensor deployment and maximizing network lifetime. They mainly consider the energy consumption in data transmitting and data relaying to the sink node with respect to both coverage and connectivity. They propose that the regions closer to the sink should be with greater sensor density. Moreover, authors give both qualitative and quantitative analyses on the performance of the PDF based deployment scheme.

2.2.5. Network overhead

The overhead in WSNs has several meanings. The most common one is the power consumption for wireless transmissions. To ensure reliable data forwarding, a wireless link must preserve a basic channel quality, which is measured by its Signal to Interference and Noise Ratio (SINR). To preserve a given SINR, the power of the transmitter is a monotonically increasing function on the length of the current link between the sensor and the base station. A revised optimal coverage problem is given by combining coverage and communication cost with different weights in [68].

Another meaning of overhead refers to the metadata or the network routing information in a network application, which uses a portion of the available bandwidth of the communication. It is denoted as **protocol overhead**. These additional data, including protocol headers and application-specific information, are denoted as overhead because they do not contribute to the message content. Therefore, overhead should be controlled under a certain threshold in proposed algorithms and protocols. Simulations usually present the empirical overhead results to evaluate the efficiency of algorithms and protocols. Authors in [8] provide the message complexity of their distributed algorithm as $O(R)$, where R is the number of scheduling rounds. Authors in [77] also analyze the message complexity as $O((2+D)((1+m)/f+D))$ for each sensor, where f is the life cycle time, D is the number of other sensors in the current sensor's communication range, and m is the number of different sensor sets, each of which can provide local barrier coverage with a desired quality. Authors in [78] analyze algorithm complexity using overhead. The proposed Coverage Benefit Detection Algorithm (CBDA) has complexity $O(n)$, where n is the number of messages.

2.3. General framework for solving the coverage problem

In this subsection, we propose a framework for solving the coverage problem with uncertain properties, which is shown in Fig. 4. This framework depicts the basic research procedures for coverage problems in WSNs. We also give the statistics of the research papers involved in this survey. First, we summarize a research procedure into the following four steps:

1) Model construction;
2) Objective(s) formalization;
3) Solution design;
4) Result verification.

For each step, the framework gives main contents in the box below. For model construction, there are three main model types, namely, detection model, network model, and deployment model. The uncertain properties are obviously introduced for some of them, such as probabilistic detection model, random and pseudo-random deployment model, etc. For the objective(s) formalization step, the framework discusses coverage type, coverage quality, and main optimization objectives. We elaborate two kinds of coverage problem with uncertain properties, namely partial coverage, and probabilistic coverage. Moreover, three main optimization objectives also focus on uncertain properties because of their solv-

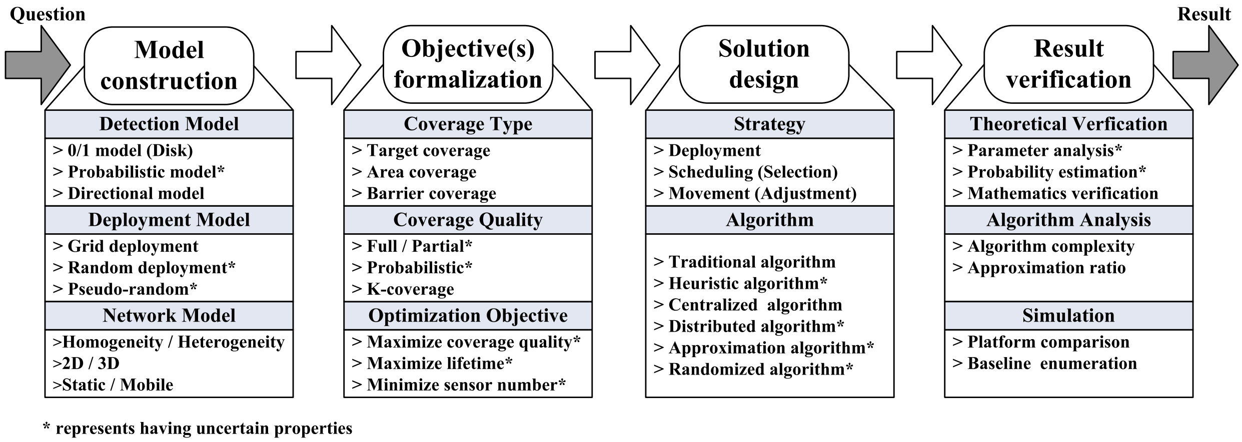


Fig. 4. A research framework for coverage problems in WSNs with uncertain properties.

ing a partial or a probabilistic coverage problem. As main objectives, three kinds of optimization problem will be clearly discussed in Section 4. For the solution design step, it involves strategies for sensor operations and algorithms. Among them, heuristic algorithms and random algorithms usually cannot give a deterministic result, which brings in uncertain properties. Finally, the result verification considers theoretical verification, algorithm analysis, and simulation. In the theoretical verification part, researchers take analyses on probabilistic parameter relationship, which brings a kind of uncertain properties. Moreover, probability estimation is also another main source of uncertain properties.

Several contributions of this survey are brought by the framework. First, we categorize three kinds of models from low level to high level. The detection model is about the detection feature of individual sensors. The deployment model is about the relationship between two or more sensors. The network model is about network features and the structure.

As the second contribution, we summarize three main optimization objectives for coverage problem with uncertain properties, namely, maximizing coverage quality, maximizing network lifetime, and minimizing the number of sensors. For example, maximizing coverage quality is to increase the level of coverage quality, which only exists for partial or probabilistic coverage and can never exist for full coverage. Additionally, maximizing network lifetime and minimizing the number of sensors are usually considered for saving energy under partial or probabilistic coverage problems.

The third contribution is to categorize strategies of sensor operations, including deployment, scheduling, and movement. Thus, we provide many discussions on strategies in Section 5 and closely analyze the characteristics of each strategy for three main optimization objectives.

Additionally, algorithms are important starting points for beginners to understand how to solve coverage problems with uncertain properties. Therefore, we also introduce and categorize algorithms from different aspects in Section 5. Moreover, we provide a list of tables to compare the complexity and approximation ratio for various algorithms in Section 6.

Although we summarize the framework of coverage problems with uncertain properties, some research is actually not strictly organized in accordance with the framework. For example, some authors want to elaborate their theory, or give a relation among network parameters. They do not pay much attention to algorithm design. However, the above research steps conserve a regular procedure to solve coverage problems with uncertain properties. In this survey, we organize our contents according to this framework. We focus on how to introduce uncertain properties into the coverage problem and how to solve the partial or probabilistic coverage problems correspondingly. Each of the following sections introduces one step of the framework, and items in each step are elaborated on by subsections. We focus on all the items marked with a star which involve uncertain properties.

Finally, we give two tables to summarize the literatures involved in this survey. In Table 1, we give statistics of research papers on coverage problems with uncertain properties. We classify them into journal papers or conference papers. From Table 1, we can see that coverage problems with uncertain properties attract more and more attentions, especially in the latest five years.

We give more details to see the distribution of papers in some representative journals and conferences in Table 2. We can see that coverage problems with uncertain properties receive more attentions than ten years ago by representative journals and conferences such as TIT (IEEE Transactions on Information Theory), TMC (IEEE Transactions on Mobile Computing), MobiCom (ACM International Conference on Mobile Computing and Networking) and so on. Later, it seems that research publications are transferred to other journals and conferences in the past five years such as TPDS (IEEE Transactions on Parallel and Distributed Systems), TON (ACM and IEEE Transactions on Networking), MobiHoc (ACM and IEEE International Symposium on Mobile Ad Hoc Networking and Computing), INFOCOM (IEEE International Conference on Computer Communications), SIGKDD (ACM International Conference on Knowledge Discovery and Data Mining) and so on. We see that the distribution of journals and conferences is broad. However, to the best of our knowledge, there is no previous survey paper summarizing the coverage problem with uncertain properties before.

3. Model construction

Coverage models are used to formulate the characteristics of coverage problems mathematically. An unique result will be decided under a deterministic model while an instance can lead to many different results under a non-deterministic models, especially representative probabilistic models. Compared to deterministic models, probabilistic models are more realistic but more complex. A coverage model consists of three types: detection model, network model, and deployment model and it is summarized in Table 3. For detection model, there are several subtypes whose detection functions are listed in “detail” column of the table. If a subtype is probabilistic, it is marked a “yes” in the “uncertainty” column while deterministic one is marked a “no”. For deployment model, the “detail” column provides comments on each subtype and the “uncertainty” column indicates if this model is involved in a probabilistic way. For network model, each subtype is discussed in two opposite sides and it is further divided from different as-

Table 1
Research paper statistics per year.

Category\ Year	2017	2016	2015	2014	2013	2012	2011	2010	2009	2008	2007	≤ 2006	Sum
Journals	1	17	8	14	9	10	8	9	5	5	4	7	97
Conferences	1	5	9	7	10	11	10	8	8	5	10	9	94
Total	2	22	17	21	19	21	18	17	13	10	14	16	191

Table 2
Papers in representative journals and conferences.

Journal		Conference	
TIT	[50]	MobiCom	[73,77,79,80]
TMC	[14,46,49,70,81–86]	MobiHoc	[87]
TPDS	[52,88–91]	INFOCOM	[8,57,62,67,92–96]
TON	[31,97,98]	SIGKDD	[99]
TOC	[100]	UbiComp	[101]
ComNet	[28,102–104]	SECON	[60,61,105]
TOSN	[25,69,106,107]	ICDCS	[5,12,108,109]
TWC	[26,63,110,111]	ICPP	[58]
IS	[112]	IWQoS	[113,114]
Performance Evaluation	[51]	CoNEXT	[115]
TITS	[116]	ICC	[33,74,117,118]
ComCom	[17,119]	ICCCN	[16,120,121]
Communications	[2]	IPCCC	[45]
JNCA	[122–127]	WCNC	[19,23,65,128–131]
Networks	[7,132]	GLOBECOM	[27,59,133–136]
PMC	[137–139]	WASA	[140–142]
The Journal of Supercomputing	[34,143]	WiCOM	[21,144–146]
WCMC	[6,147–149]	ISSNIP	[78,150]
Wireless Networks	[71,151–153]	COMSNETS	[154,155]
Ad Hoc Networks	[156–158]		
Communications Letters	[22,44,159]		
Sensors	[160–162]		
Wireless Sensor Systems	[66,163]		

Table 3
Illustration of coverage model.

Coverage model					
Type	Name	Detail			Uncertainty
Detection	0/1 model	$f(d(s_i,t)) = \begin{cases} 1, & \text{if } d(s_i,t) \le R \\ 0, & \text{if } d(s_i,t) > R \end{cases}$			No
	Attenuated model	$f(d(s_i,t)) = C/d^k(s_i,t)$			Yes
	Truncated attenuated model	$f(d(s_i,t)) = \begin{cases} 1, & \text{if } d(s_i,t) < R_s \\ e^{-kd(s_i,t)}, & \text{if } R_s \le d(s_i,t) \le R_u \\ 0, & \text{if } R_u < d(s_i,t) \end{cases}$			Yes
	Directional model	$f(d(s_i,t)) > 0, \beta \le \alpha/2;$			Yes/No
	Estimation model with loss	$Pr[PL(d) > \gamma] = Q[\frac{\gamma - PL(d)}{\theta}]$			Yes
	Transmitter-Receiver model	$P_r = P_r(\Delta P > \Delta P_h)$			Yes
Deployment	Grid deployment	Applied in the simple environment (Indoor) with best coverage			No
	Grid deployment with error	More realistic situation with considering wind, water, animals and other factors			Yes
	Random deployment	Applied in the tough environment (eattlefields)			Yes
	Pseudo-random deployment	According to some specific rules (Gaussian distribution)			Yes
Network	Homogeneous and heterogeneous models	Difference among sensors			N/A
		Detection range	Communication range	Computing power	
	2D and 3D models	Sensing region			
		2D plane	3D volume	3D surface	
	Static and mobile models	Sensor mobility			
		Mobile sensors	Static sensors	Hybrid sensors	

pects which are given in the “detail” column separately. e.g., when we compare the homogeneous models with heterogeneous models, the difference among sensors is focused and divided into three main types of difference: detection range, communication range, and computing power. Because the network model cannot be categorized into deterministic or probabilistic so an “N/A” is given for all network models. In the following subsections, we will introduce them separatively for more details.

3.1. Detection model

In this subsection, we first briefly review the traditional detection model, and then present several popular probabilistic detection models, which are widely used in WSNs. A detection model is presented as the geometric relation of a target point and a sensor, which measures the basic sensing capability and the detection quality of the sensor. In most cases, we formulate a detection function to depict the coverage performance from the Euclidean distances between the sensor and the target. Therefore, the detection quality of the sensor is commonly depicted as a disk centered at the position of the sensor.

A sensor detects a target by measuring the strength of the signal emitted from the target. The energy of the sensing signal attenuates with the distance increasing from the target. Thus, the detection function is modeled with Euclidean distance between sensor and target as parameter. For any point p, $d(s_i,t) = \sqrt{(x_i - x)^2 + (y_i - y)^2}$ denotes the Euclidean distance between the

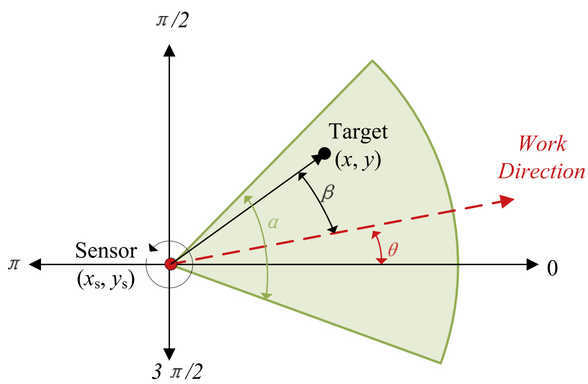


Fig. 5. Illustration for directional model.

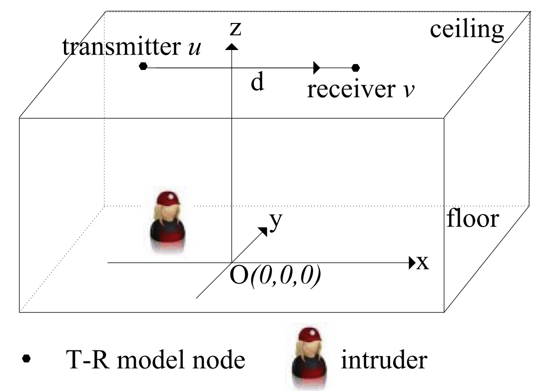


Fig. 6. The relation between the T-R model and the object [5].

sensor s_i and the target position t, where (x_i, y_i) and (x, y) are the Cartesian coordinates of the sensor s_i and the target point t. The function $f()$ taking the distance as a parameter indicates the detection effect and the R is the detection radius for 0/1 model.

3.1.1. Omnidirectional model

The 0/1 model [164] is the most widely used deterministic detection model in WSNs. It is the only model without uncertainty in Table 3 as a baseline in comparison with other probabilistic models.

Attenuated models consider the uncertainty in the sensing process and utilizes more information which the 0/1 model ignores. A basic attenuated model is given in Table 3, where k is a decay factor related to the physical characteristic of the sensor. Only when the event occurs at the location of the sensor, the signal received by the sensor is strongest. On the contrary, when the event occurs far from the sensor, the signal received by the sensor will become extremely weak.

The truncated attenuated model causes some neglecting and approximation of the detection probability when distance is too large or too small. An expression is given in Table 3, where (R_s, R_u) is the strip detection range. The R_s is the lower bound and the R_u is the upper bound. As a variety of this model, the lower bound R_s can be removed to follow the attenuating form as the middle equation of the full expression in Table 3 and ignore the lower truncation. The upper bound R_u can be removed as another variety of this model.

3.1.2. Directional model

In the above discussions, we consider the distance as an important factor for detection functions. Another factor which influences the detection function is the angle β between sensor working direction and the relative direction between sensor and the target point. It is the directional model which is mostly used for cameras [9] or other adjustable sensors [109,123]. Each sensor has only one working direction, $\theta \quad (-\pi \le \theta \le \pi)$, defined as its direction angle value relative to the positive x-axis. The effective detection field F_s (Field of View) of the sensor is in a sector shape, i.e., the pie-shaped zone formed of effective angle of view, denoted as angle α, and two sides in length of detection range r. A target point t is said to be covered by the sensor s_i if and only if the two conditions are satisfied. The $f(d(s_i, t))$ is a detection function of any type above, such as attenuated model proposed by [78]. Fig. 5 presents an illustration. For a 3D scenario, a directional 3D detection model is given for space volume coverage [136,139].

3.1.3. Estimation model with loss

The pass loss logarithmic normal model is an estimation model that predicts the path loss (event including the noise) that a signal encounters inside a building or a densely-populated region [21].

For the pass logarithmic normal model, the path loss PL (in db) at a distance is given in Table 3, where d_0 is the reference distance, β is the path loss component, indicating the rate at which the path loss increases with distance, x_σ is the zero-mean *Gaussian* distributed random variable with σ-variance (also in db), and $PL(d_0)$ is the mean path loss at reference distance d_0.

The pass logarithmic normal model captures various environmental factors with different received signal values at different locations while the distance between the target point and the sensor is the same. All the reference parameters are experimentally measured for the given event and the characteristics of the sensors are calculated using a free space path loss model. Each sensor has a receiving threshold value γ which describes the minimum signal strength correctly decoded at the sensor. The probability, with which the received signal level at a sensor will be above this receiving threshold γ, is given in Table 3 for an estimation model with loss. The function requires Q-function to compute probability according to the *Gaussian* process [20]. The Q-function is defined as follows:

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty \exp\left(-\frac{x^2}{2}\right) dx \tag{5}$$

For a given transmitting power and receiving threshold value, we can calculate the probability of receiving a signal above the receive threshold γ, at a given distance with the following probability function:

$$Pr[PL(d) > \gamma] = Q\left[\frac{\gamma - PL(d)}{\theta}\right] \tag{6}$$

where θ is the signal strength emitted by the target.

3.1.4. Transmitter-Receiver model

Transmitter-Receiver model (T-R model) is derived from the transceiver-free object detection. It uses the disturbance of the object to the radio environment to locate the target object. Compared with the traditional disk-like model, the T-R model is considered a more general model which has many new features with uncertain properties, such as the non-node centric (link centric), the interference, and the exclusivity between a different coverage unit (coverage unit may not be totally independent) [5,126]. The target objects are detected when the certain Transmitter-Receiver links (T-R links) have significant changes in the signal strength.

In a transceiver-free object detection system, T-R links are basic coverage units. The sensors continuously transmit packets and measure the signal strength of the corresponding links. We assume that a present object (or a person) is always on the ground (x, y, 0) and a T-R link $u(-d/2, 0, h)$, $v(d/2, 0, h)$ is deployed on the ceiling parallel to the ground, which is shown in Fig. 6. When targets

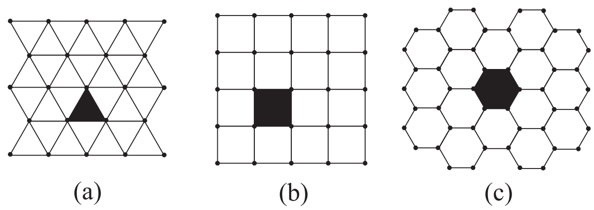


Fig. 7. Grid sensor deployment using (a) triangle, (b) square, and (c) hexagon pattern.

are present, additional wireless transmission paths will be added between transmitters and receivers. As a result, the signal strength is greatly influenced depending on the location of the object. By observing these T-R links, we can infer the positions of the target objects.

According to the pass loss log normal model, the difference (ΔP) of the received power at v is

$$\Delta P = c_1 + c_2 \log(4x^2 + d^2 + 4y^2 + 4h^2) + G \quad (7)$$

where c_1, c_1 are two constants. The constants d and h are separatively the distance between Transmitter and Receiver, and the height of ceiling. G is a random variable that can be modeled by the Rice distribution.

With the user threshold ΔP_h being appropriately set, the object detection probability P_r at a target point is the probability that ΔP is larger than a given threshold ΔP_h:

$$Pr = Pr(\Delta P > \Delta P_h) \quad (8)$$

3.2. Deployment model

A fundamental step in constructing a WSN is to deploy sensor nodes into an ROI. However, deployment models (or deployment strategies) are varied. Sensor deployment strategies can generally be classified into four categories. The first one is a deterministic model based on grid, and the other three are all deployment models with uncertain properties.

3.2.1. Grid deployment

Grid deployment means sensor nodes are deployed based on grids. Grid sensor deployments are of particular importance in many applications mainly because they provide a uniform and highly consistent partitioned space. According to the grid shape, grid deployment can be divided into more specific types. Triangle [33], square [114], and hexagon [88] are the most popular grid types that have been substantially discussed (Fig. 7). Such deployment can achieve the best coverage with the fewest sensor nodes. However, such deployment requires a simple environment and may be costly during the deployment process.

3.2.2. Grid deployment with error

In more realistic situations, considering wind, water, animals, and other factors, deployment errors may occur, and the location of the nodes may deviate from the original plan. In this case, the node location can be modeled by a probabilistic model. We can use two ways to deal with such problems. One is to decrease the length of the grid cells so that no region is left void even in the presence of worst-case errors. However, this approach may be too expensive when the worst-case magnitude of the errors is large. The other one is to only provide a probabilistic coverage guarantee [33].

3.2.3. Random deployment

Random deployment is the result of practical implementation for sensor deployment in battlefields or tough environments, for example, sensors may be air-dropped or launched via artillery. In these scenarios, the locations of sensors can be modeled by a static 2*D Poisson point* process. We can denote the sensor density to λ which measures the number of sensors per unit area. The total number of sensors located in a sensing region A, denoted to $N(A)$, follows a *Poisson* distribution of parameter $\lambda||A||$, where $||A||$ represents the area of region A. Thus, the probability can be described as

$$P(N(A) = k) = \frac{e^{-\lambda||A||}(\lambda||A||)^k}{k!} \quad (9)$$

The required number of sensors to cover a region using random deployment is in general approximately 3 to 10 times higher than that using grid deployment [150].

3.2.4. Pseudo-random deployment

In pseudo-random deployment, sensors are randomly but not uniformly deployed. This deployment type is usually used in two special cases. The first one is in a mobile network. Sensor nodes initially may be deployed randomly and then move around on the bases of some specific rules proposed by [165]. This strategy is also called a self-regulated strategy.

The second case is in a cluster network referenced below (in Section 3.3.1). We consider a WSN in a 2D plane with N sensors. These sensors are deployed according to the following 2*D Gaussian* distribution.

$$f(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} e^{-\left(\frac{(x-x_i)^2}{2\sigma_x^2} + \frac{(y-y_i)^2}{2\sigma_y^2}\right)} \quad (10)$$

where (x_i, y_i) is the deployment point, and x and y are the standard deviations for the orthogonal two dimensions, respectively. The distribution enables sensors to have a higher probability of being deployed near the sink node than in random deployment. Authors in [49] point out that the distribution relaxes the energy hole problem and increases the lifetime of the network.

3.3. Network model

In a coverage problem, many sensor nodes are deployed in the ROI to satisfy a specific coverage requirement. The region is also called the sensing region, and these sensor nodes form a WSN. Considering that the region and the sensor model differ significantly in different problems, the network can also be modeled in many different ways. This subsection introduces three kinds of WSN models.

3.3.1. Homogeneous and heterogeneous models

This model focuses on sensor types. A network where all sensor nodes have the same detection range, communication range, and computing power is called a homogeneous network, which is commonly used a simplification assumption [5,16,25,31,44,63,99,129,163,166].

By contrast, sensor nodes in a heterogeneous model may have different capabilities. In a heterogeneous model, two parameters are set for sensor nodes, namely, detection range and computing power. The detection range of a node is the region where any target can be detected by the node. This parameter is the most popular parameter considered in many papers [49,69,108,164]. Computing power is the processing capacity of a node. Adding some nodes with a high computing power to a network can, in general, effectively improve the total performance. However, the problem becomes complex if the computing power of nodes differs [49,82]. We call a network as a cluster network if it consists of some base stations (or called the sink nodes) with some advanced micro-controllers and many other common nodes.

Table 4
Illustration of problem objectives.

Objective type	Requirement		Comments
Achieving coverage quality	Achieving coverage ratio		Objective function: $\left\lvert \bigcup_{s_i \in C}(R_i) \bigcap \mathcal{A}_j \right\rvert > p_j \lvert \mathcal{A}_j \rvert$
	Achieving coverage probability		Objective function: $F_t = 1 - \prod_{s_i \in S}(1 - f(d(s_i, t)))$, $\forall t \in \mathcal{A}, F_t \geq P$
Main optimization with uncertainty	Maximizing coverage quality with uncertainty	Coverage ratio	Maximization on coverage ratio (Below 100%)
		Coverage probability	Objective function: $\max_s \int_\Omega R(t)P(t, s)dt$
	Maximizing the network lifetime		Sensor scheduling under partial or probabilistic coverage
	Minimizing the number of sensors		Selecting under partial or probabilistic coverage
Other objective	Probabilistic K-coverage		Combining probabilistic coverage and K-coverage
	Maximizing the weighted sum of targets		Partial coverage with weighted sum of targets
	Handling holes		Improving coverage ratio for partial coverage

Other parameters, such as communication range, are also considered. However, we usually do not consider power supply and mobile issues for a heterogeneous network.

3.3.2. 2D and 3D models

This model describes the sensing region, which can generally be categorized into three types, namely, 2*D* plane, 3*D* volume, and 3*D* surface. The problems in a 2*D* plane, such as crop sensing in regions or wildlife tracking on plains, have been discussed extensively in the past [12,13,19,50,51,77,110,167–169]. A 3*D* volume region is a 3*D* space whereas a 3*D* surface region is the surface of a 3*D* object. The 3*D* volume model is generally used for underwater sensor surveillance, air and water pollution monitoring, and forest monitoring. In most cases involving a 3*D* model, it actually does not substantially increase the difficulty of the problem. Researchers can extend most 2*D* plane algorithms to 3*D* volume directly [29,58,117,166,170]. The 3*D* surface model is discussed in some special cases, such as seismic monitoring on ocean floors or in mountainous regions [28,90,171]. This region is the most difficult one because we cannot use Euclidean distance to express distance. Thus, many existing 2*D* algorithms are not applicable.

3.3.3. Static and mobile models

This model depicts the sensor mobility. In a static model, the locations of all sensor nodes are fixed after they have been deployed [19,26,34,50,51,130,167–169]. In a mobile model, sensor nodes are equipped with a locomotive unit and can move around after deployment [52,62,68,96,165,172,173]. Mobile nodes can improve network performance by moving to the desired locations, but they also expend more energy for moving and need complex algorithms to calculate the moving paths. Apart from the above models, researchers consider a hybrid network that consists of both static nodes and mobile nodes [58,161,164,174].

4. Problem objectives

For coverage problems of WSNs, the foremost objective is to guarantee coverage quality. Coverage ratio and coverage probability are two common performance metrics used to estimate coverage quality. Meanwhile, they are the main topics in this survey. Coverage ratio reflects how much of the ROI is covered and is typically discussed as the partial coverage problem, which are elaborated in Section 2.2.2 by a series of researches [4,8,48,114,160,161,174–176]. Coverage probability reflects the degree of coverage for the entire ROI and is usually discussed as the probabilistic coverage problems, which are elaborated in Section 2.2.3 through many literatures [28,62,99,113,129,177]. As the second kind of objective, we present three kinds of optimization objectives considering efficiency of WSNs, namely, maximizing coverage quality, maximizing network lifetime, and minimizing the number of sensors. We respectively discuss their definitions and introduce their solutions in the next section. Finally, we present other related objectives of coverage problems with uncertain properties. We summarize all these objectives in Table 4.

4.1. Achieving coverage quality with uncertain properties

For coverage problems with uncertain properties, WSNs have different requirements for coverage quality, e.g., partial coverage, probabilistic coverage, and probabilistic *K*-coverage. In this subsection, we first introduce the definition of each coverage quality in detail, and then present relevant researches.

4.1.1. Achieving a certain coverage ratio

Experimental results in some papers show that the lifetime of WSNs will be extended greatly, if the coverage ratio of the network can decrease slightly, thereby providing partial coverage rather than full coverage [47,140,155]. Full coverage is actually unnecessary in some scenarios. In such a case, research about partial coverage is useful to prolong the lifetime of networks. Partial coverage can be defined when the coverage ratio $r < 1$.

Coverage ratio and lifetime are two important but conflictive requirements in WSNs. Improving both coverage ratio and lifetime at the same time is usually difficult. For partial coverage, authors in [49,144,169] focus on how to maximize the lifetime while the coverage ratio is guaranteed by a predetermined threshold. Authors in [8,48,161] have attempted to improve the coverage ratio while bound the network lifetime. A few studies have solved the partial target coverage problem [125]. Some researchers consider partial coverage under a camera or a directional detection model [9,123]. The partial coverage problem has also been investigated in the literature with many names, such as "*p*-percent coverage" [140], "α-coverage" [89], and "*Q*-coverage" [47]. The problem requires the network to cover at least "*p*-percent", "α-portion" or "*Q*-portion" of the ROI, target set, or crossing paths. In other words, if $p/100 = \alpha = Q$, such problems are actually the same. We denote this kind of problems as "*p*-percentage" coverage problems uniformly.

Here, we first introduce some important concepts and definitions about *p*-percentage coverage problem and provide some examples.

Definition 4 (*p*-percentage cover)**.** Give a real number p where $0 \leq p \leq 1$, a 2*D* region $\mathcal{A}$ and set S of N sensors s_i, for $i = 1, \ldots, N$. Sensor s_i has detection range R_i. $|\mathcal{A}|$ denotes the area of the ROI $\mathcal{A}$. A sub-set $C \in S$ *p*-percentage covers A (C is an *p*-percentage cover of $\mathcal{A}$) if [89]:

$$\left\lvert \bigcup_{s_i \in C}(R_i) \bigcap \mathcal{A} \right\rvert > p \mid \mathcal{A} \mid \tag{11}$$

The *p*-percentage coverage problem is considered as finding one or several *p*-percentage covers for ROI $\mathcal{A}$.

For large scale networks, separating the ROI into sub regions and assigning different coverage ratio to each sub region may be

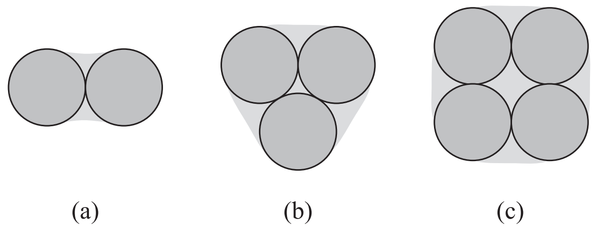


Fig. 8. Illustration of the space points covered by using the *P*-probability coverage model of: (a) 2 sensors; (b) 3 sensors; (c) 4 sensors [63].

a good strategy. In this way, users can deploy considerable sensors to the critical regions and deploy fewer sensors to trivial regions. Therefore, equipment costs can be reduced and the network lifetime can be prolonged using the same budget. We then introduce the extended *p*-percentage coverage problem as follows.

Definition 5 (extended *p*-percentage cover)**.** Give a 2*D* region $\mathcal{A}$ which is divided into *J* sub regions, a specific coverage ratio p_j for each sub regions $\mathcal{A}_j,\ j = 1, \ldots, J$, and set S of *N* sensors s_i, $i = 1, \ldots, N$. Sensor s_i has detection range R_i. $|\mathcal{A}|$ denotes the area of the ROI $\mathcal{A}$. A sub-set C extended *p*-percentage cover $\mathcal{A}$ (C is an extended *p*-percentage cover of $\mathcal{A}$) if [120]:

$$\Big|\bigcup_{s_i \in C}(R_i)\bigcap \mathcal{A}_j\Big| > p_j|\mathcal{A}_j|, \forall j = 1, 2, \ldots, J \tag{12}$$

The extended *p*-percentage coverage problem is considered as finding one or several extended *p*-percentage covers for ROI $\mathcal{A}$.

4.1.2. Achieving a certain coverage probability

In this subsection, we first briefly introduce the typical issue, namely data fusion, for the *P*-probability coverage problem. We then provide some concepts and definitions of achieving the *P*-probability coverage.

A probabilistic model is proposed to depict a sensor detection capability precisely and emphasize the information ignored by the classical disk model. Instead of assigning the detection probability as 0 easily if the target is out of the detection range as the classical 0/1 disk model does, we assign the detection probability of each sensor for a target and utilize the probability by combining the results of multiple sensors collaboratively to get a final decision.

Data fusion. As previously mentioned, data fusion is a significant issue of *P*-probability coverage. Detection performance can be improved by jointly considering the detections by multiple sensors. Fig. 8 indicates the regions that are considered as being covered when using the *P*-probability coverage model. The points within a disk are considered covered when only one sensor is used. They are also considered covered when more than one sensor is used. In addition, the points in light gray outside the circles are not covered by a single sensor, but these points are considered as covered by multiple sensors. These additionally covered target points or regions can be regarded as benefits gained by data fusion of using multiple sensors.

For any ROI *A*, the sensors near *A* that participates in the data fusion will detect whether an event occurs at *A*. Three classes of data fusion protocols are commonly used: value level fusion, feature level fusion, and decision level fusion [27]. Value level fusion and feature level fusion may be more precise when performing data fusion, but they require the member of fusion clusters to transmit raw data to the fusion center, which is extremely expensive, especially for a typical WSN with limited energy and bandwidth. Therefore, transmitting just the decision of each sensor to the fusion center in the study of the *P*-probability coverage is preferable, which is the decision level fusion. The WSN definitely detects the occurrence of an event if at least one sensor node does. Then, the decision level fusion rule can be expressed as

$$\lambda(S, t) = \bigcup_{s_i \in S} \lambda(s_i, t) \tag{13}$$

where $\lambda(S, p)$ denotes the decision result of data fusion of the WSN that detects the event that occurs at the target point *p*, and $\lambda(s_i, t)$ denotes the result of individual sensors s_i, and S is the fusion sensor set where s_i belongs.

Problem statement. Based on several aforementioned coverage models and the data fusion rule, the coverage of a target point *t*, thus is $F_p(S)$, which denotes the probability of an event occurring at *t* detected by active sensors in set S, can be described as follows [11,27,140,151]:

$$F_t(S) = 1 - \prod_{s_i \in S}(1 - f(d(s_i, t))) \tag{14}$$

where S is the set of active sensors, *f*() is a detection function which takes distance as its input, and *d*() is an Euclidean distance function of sensor s_i and target point *p*. Based on Eq. (14), we can obtain the precise definition of *P*-probability cover.

Definition 6 (*P*-probability cover)**.** Give a real number *P* where $0 < P < 1$, a 2*D* region *A* and set S of *N* sensors s_i, for $i = 1, \ldots, N$. Sensor s_i has detection range R_i. A sub-set $C \subset S$ is a *P*-probability cover of *A* if:

$$\forall t \in A, F_t(C) \geq P \tag{15}$$

The *P*-probability coverage problem is considered as finding one or several extended *P*-probability covers for ROI *A*. If we consider the weight of each sensor, we have the following weighted coverage definition.

Definition 7 (minimum weight *P*-probability coverage)**.** Give a real number *P* where $0 < P < 1$, a 2*D* region $\mathcal{A}$ and set S of *N* sensors, each $s_i \in S$ associates with the weight w_i, for $i = 1, \ldots, N$. Sensor s_i has detection range R_i. Find a *P*-probability cover C of *S* for region $\mathcal{A}$ with the minimum weights.

We set the parameter x_i as follows:

$$x_i = \begin{cases} 1, & \text{if sensor} s_i \text{is employed} \\ 0, & \text{otherwise} \end{cases} \tag{16}$$

and set the parameter δ_{ij} as:

$$\delta_{ij} = \begin{cases} 1, & \text{if target point} j \text{is covered by a sensor} s_i \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

Then we can expressed this problem as follows:

$$Minimize \quad \sum_{i=1}^{n} w_i x_i \tag{18}$$

subject to

$$\sum_{i=1}^{n} \delta_{ij} > 0, j = 1, 2, \ldots, m \tag{19}$$

$$F_t(C) \geq P \tag{20}$$

This kind of *P*-probability problem was proven to be a *NP*-hard problem [178]. Thus, we can hardly find an exact solution in polynomial time. Therefore, authors in [5] and [128] propose greedy algorithms and heuristic algorithms to approximately solve this problem.

Error estimation. Probabilistic model, different from the 0/1 disk model, is associated with uncertainty during the monitoring process. In fact, the detection of sensors always have noises, which disturb the decision when an event occurs at a target point [44]. Thus, the detection function of a single sensor can be expressed as the combination of actual signal and noise, as given by

$$y_i = \begin{cases} n_i, & \text{for non-detection} \\ n_i + sig_i, & \text{for detection} \end{cases} \tag{21}$$

where y_i is the received signal, n_i denotes noise, and sig_i denotes real signal. Thus, the probability of the false alarm and missed detection must also be considered in the coverage issue. We can measure the coverage quality by estimating the errors generated in the coverage process. Two basic errors, called False Positive (FP) and False Negative (FN), are proposed in the following definitions [5]:

Definition 8 (False Positive)**.** False Positive (FP) is the probability that there is no event occurring in the ROI, while the sensors detect the presence of one.

Definition 9 (False Negative)**.** False Negative (FN) is the probability that there is an event occurring in the ROI, while the sensors fail to detect it.

Definition 10 (*P*-error cover)**.** Given a real number P where $0 < P < 1$, a $2D$ region $\mathcal{A}$ and set S of N sensors s_i, for $i = 1 \ldots N$. A sub-set $C \subset S$ P-error cover $\mathcal{A}$(C is a P-error cover of $\mathcal{A}$) if:

$$\forall t \in \mathcal{A},\ FP(t) + FN(t) \le P \tag{22}$$

The *P*-error coverage problem is considered as finding one or several *P*-error covers for ROI $\mathcal{A}$.

Authors in [127] consider the 3*D* version of a false alarm under probabilistic detection models.

4.2. Optimization objectives w.r.t. efficiency

Besides achieving a certain coverage ratio or coverage probability in WSNs, several optimization objectives are proposed to improve the efficiency of WSNs, namely maximizing coverage quality, maximizing network lifetime, and minimizing the number of sensors, each of which is elaborated separately in the following subsections.

4.2.1. Maximizing coverage quality

In some situations, we want to guarantee the best coverage quality. As mentioned above, coverage ratio and coverage probability are two common performance metrics to estimate the coverage quality. Therefore, maximizing coverage ratio and maximizing coverage probability are the main objectives of coverage problems with uncertain properties. Besides, coverage ratio and coverage probability can be combined for more complex coverage objectives.

Maximizing coverage ratio with uncertainty. Authors in [133] give a mobile node navigation in a hybrid sensor network, which consists of both static and mobile sensors, to improve the dynamic coverage. The key feature of the proposed protocol is that mobile nodes are directed to move with a probability to maximize the coverage time of uncovered area.

Research like [161] considers an objective which is to determine the effectiveness of the coverage area, thus is, the desired coverage probability threshold, and to maximize the coverage ratio of the entire ROI with a heuristically Artificial Bee Colony (ABC) algorithm. Similarly, authors in [160,174,177] try to maximize the ratio of grid points whose detection probability is greater than probabilistic threshold C_{th} to the all grid points. The two heuristic algorithms implement the improved Virtual Forces (VF) algorithm, namely virtual force directed co-evolutionary particle swarm optimization (PSO), and particle swarm frog leaping hybrid optimization algorithm.

Some research considers the weight of importance over the entire region in the cost function for probabilistic coverage. Authors in [3] try to stabilize the robots under optimal configurations to minimize cost function with maximum coverage ratio. They employed a gradient descent method on this problem.

Maximizing coverage probability with uncertainty. Authors in [8] consider the different importance of each target. And authors a critical location coverage problem by using a point coverage model to maximize the event detection probability while meeting the network lifetime requirement. Then, they propose a sensor scheduling strategy.

The optimal coverage problem can be formulated as an optimization problem to maximize the expected event detection frequency by the sensors over the mission space Ω [68]:

$$\max_{s} \int_{\Omega} R(t)P(t,s)dx \tag{23}$$

where $P(t,\ s)$ is the joint probability that this event is detected (can be expressed by Eq. (14)), and $R(t)$, $t \in \Omega$, is an event density function which captures the frequency or density that a specific random event takes place. The metrics are similar to the ones in [66,179].

Authors in [172] evaluate a trade-off between improving target detection probability and reducing the total power consumption of WSNs by using a potential game theoretic approach.

Researches in [28,153] involve the intruder detection history recorded by the sensor nodes to determine which parts of the barrier is more vulnerable. Then, they relocate the available mobile sensor nodes to the vulnerable parts of the barrier in a timely manner. The heuristic algorithm helps to maximize the intrusion detection probability.

4.2.2. Maximizing network lifetime

Each sensor in WSNs is battery operated, and hence, energy efficiency is an important issue. In many application scenarios, replacing or recharging the batteries of the sensors is impossible when they are deployed in dangerous environments. To extend the lifetime of networks, the energy consumption of sensors must be balanced to avoid energy waste [22,25]. For different WSNs, energy consumption can be reduced by using the sleep-wakeup scheduling with a proper coverage quality.

Most of researches use a synchronous activity scheduling mechanism, where sensors periodically wake up synchronously, in order to determine their schedules for the next scheduling interval. For target coverage, these researches [9,65] schedule the sensor sleep-wakeup under different models such as camera fan-shape models and confident information models. Authors in [12] give a predefined failure probability requirements for each targets. Authors in [13] adopt the probabilistic coverage model (Communication Weighted Greedy Cover-Probabilistic Model) to extend the network lifetime with coverage and connectivity constraints. For area coverage, several protocols are proposed to guide sensor scheduling [144,145,155] and sensor movement [164] for probabilistic or partial coverage. Especially, research in [113] presents an analytical model to analyze object detection quality, even for high speed objects, and the model benefits performance evaluation of scheduling, network deployment, and sensing scheduling protocol design.

Instead of full coverage, partial or probabilistic coverage is a good choice for prolonging network lifetime. Some researchers analyze the improvement of lifetime and give relevant protocols [47,144,145,155,164,169]. Authors in [6] propose a Partial Target

Coverage (PTC) requirement to optimize the defined lifetime without dividing continuous time into discrete slots of different lengths. More specifically, for targets with different weights, they assign diversified coverage requirements. The most important target should be covered in the entire lifetime while others are covered in different pre-defined periods according to their importance. We can improve lifetime by moving sensors from a dense area to a sparse area [165] and balancing the static and mobile sensors [114]. Authors in [180] prolong the network life for active coverage with the cellular automata. Several researchers consider coverage and connectivity as the combined objective [34,51,82,130,140,151,181,182].

4.2.3. Minimizing the number of sensors

For some coverage problems, expensive sensors with strong properties must be used to meet some special requirements. In such cases, reducing the number of sensors may become a primary goal [58,93,117,121]. Unlike the cases referenced in Section 4.2.2, which aims to maximize the lifetime through minimizing the number of active nodes in each time slice, the objective in this subsection is to minimize the total number of sensor nodes.

In mobile social networks, authors in [4] aim to achieve a certain coverage for a given ROI over time with low cost in a People-Centric detection manner. They define an (α, T)-coverage which means that each point in the ROI is sensed by at least one mobile node with the probability of at least α during time period T. The proposed algorithms require a minimal number of mobile nodes inside the ROI and outside the ROI. A similar work is done in [101] for minimizing the incentive payments by selecting a near-minimal set of participants. They use smart phones to update air condition data through energy-efficient piggyback crowdsensing. Authors predicts the call and coverage probability of each mobile user based on historical records.

Researchers in [22,27] introduce the concept of probabilistic coverage based on a probabilistic detection model, i.e., every point in ROI has at least a probability δ or ϵ to be covered by a minimal number of sensors. Authors in [22] select a subset of sensor nodes with minimum weights to meet the coverage requirements, which is widely known as minimum weight sensor coverage problem (MWSCP). Particularly, authors in [27] explore the mathematical relationship between the probabilities of being covered by two adjacent points and transform minimum weight ϵ-full coverage problem into a point coverage problem. The two examples are special cases for minimizing the number of weighted sensors.

All examples above focus on area coverage. While, for probabilistic target coverage, some research considers the minimization of cost for sensor deployment [11,121]. Moreover, probabilistic set cover (PSC) problems are discussed in many literatures [15,16]. Sweep coverage, as a new kind of target coverage for data gathering, is introduced with uncertain properties [58] and it utilizes mobile nodes as supplementary in a sparse and probably disconnected sensor network. The authors analyze the minimal number of mobile sensors for a certain sweep delay.

A special geometrical analysis is required for a kind of coverage problems with fan-shape detection range on cameras. Authors in [183] address a plane coverage model, which is a simplified 3D scenario for multi-plane full or partial coverage. Research in [24] develops a novel probabilistic detection model for line-of-sight-based coverage (e.g., cameras) to tackle the sensor placement problem in a 3D scenario. Authors in [94] propose a novel local face-view barrier coverage, which achieves a statistical barrier coverage in camera sensor networks, leveraging intruder's trajectory lengths L along the barrier and head rotation angles δ. These coverage problems with detection probability bound and deployment pattern can guide practical camera sensor deployments and reduce the total budgets. Authors in [127] focus on detection issues by using a probabilistic detection model with five different collaborative detectors based on spatial correlation and signal detection theory. They aim at achieving the seamless 3D space coverage while the number of nodes required for a fixed space is minimized.

4.3. Other objectives

This subsection introduces several other objectives that are commonly discussed in the literature about coverage problems in WSNs, such as special K-coverage, maximizing the weighted sum of targets, and handling coverage holes.

4.3.1. Special K-coverage

In some scenarios, ensuring only coverage ratio $p=1$ or probability $P=1$ is deficient. K-coverage is thus proposed to maintain coverage ratio $p=1$ or probability $P=1$, even if K-1 sensors failed [50,52,77,184].

The K-coverage requirement is actually more sophisticated because it needs more sensors. Moreover, how to deploy less sensors to decrease the overlap area and prolong the lifetime is complicated. Also, K can be different for different targets (sub-regions), according to the weights of the targets (sub-regions) as mentioned by [52] and [69], which can be considered as a variant of the partial coverage problem.

On probabilistic K-coverage problems, authors in [108] and [102] extend the research on the K-coverage problem with a probabilistic detection model and heterogeneous network model. According to the studies, each slice must be covered by at least K sensors, and the total detection probability must be greater than a given threshold P_{th}. Researchers in [8] give another definition of probabilistic K-coverage, which addresses the condition where the total detection probability is greater than the predefined K.

4.3.2. Maximizing the weighted sum of targets

In real scenarios, the importance is different for different targets. For example, in a battlefield, we may want to pay more attention to a general than to a soldier. We hope that more important targets are more likely to be detected and even to be detected by more sensor nodes. Authors in [15] assign a weight to each target according to its importance. Then, they maximize the weighted sum of targets. Authors in [141] model intruding action with a Poisson distribution, which brings uncertain properties for the coverage problem. Then every barrier sensor is set with a weight of vulnerability according to which mobile sensors determine to protect them, then authors want to maximize the sum of weighted vulnerability for barrier sensors.

4.3.3. Handling coverage holes

Coverage holes are small regions in the ROI where a coverage quality requirement is not achieved. The holes may occur for many reasons, such as when the batteries of some sensor nodes run out, and when environmental elements destroy some sensor nodes, etc. Whatever the reason is, the uncertain properties are obviously introduced into the coverage problems. Therefore, researchers want to find the holes and improve coverage quality by repairing them. Two kinds of problems have been researched and discussed extensively. One problem is to monitor the network and to find these holes, such as [91,97,105,122,134,148]. The another is to repair these holes, such as [154,185–188]. Few researchers consider both of the problems together [103].

5. Solutions: strategies and algorithms

As mentioned in the previous section, main optimization objectives of coverage problems with uncertain properties can be generally classified into three categories: (1) maximizing coverage quality, (2) maximizing network lifetime, and (3) minimizing the number of sensors. In this section, we provide solutions for the three objectives. First, we discuss some main strategies. Afterwards, algorithms for coverage problems with uncertain properties are classified and summarized.

5.1. Basic strategies for different coverage objectives

In the process of finding solutions to achieve these objectives, several basic strategies are employed: deployment, scheduling (selection), and movement (adjustment). Deployment strategy means to allocate sensors specifically to achieve some certain coverage quality at the initial network building stage. Besides, we assume that sensors have two statements: activated and sleeping. Scheduling strategy means to choose a subset of sensors to be activated for a time interval and make other sensors sleeping, then iteratively activate another subset for the next time interval during the whole network lifetime. Selection strategy is a special scheduling because it is an one-time choice of the subset of sensors for whole life time. Movement strategy means to reallocate sensors for some reasons. The involved sensors are commonly mobile sensors. We also can adjust some parameters of sensors such as detection range for tunable sensors and working direction for directional sensors. We classify adjustment as a special movement strategy. One important thing we need to point out is that these strategies do not have essential conflicts. Thus, they may be combined to solve some problems. In the following three subsections, we just discuss these strategies respectively for some typical problems. We pay attentions to their representative applications for main objectives discussed above and clarify their characteristics of uncertain properties.

5.1.1. Deployment

Deployment is a common strategy discussed in many literatures. Researchers usually assume that they can determine the initial position for each sensor. Then they can construct the whole network efficiently with a small number of sensors. This assumption is too strong in practice. Additionally, the real position of the deployed sensor may deviate from its designed position, which brings uncertainty. In this subsection, we discuss some applications of deployment strategy for three main objectives: maximizing coverage quality, maximizing network lifetime, and minimizing the number of sensors.

Maximizing coverage quality. One of the essential responsibilities for WSNs is to maximize coverage quality. Authors in [66] propose a typical solution with a limited number of sensors by deployment.

For the objective of gathering as many features in the ROI as possible, authors deploy limited sensors considering the fact that the features do not distribute evenly. They define the Information Density and use it to model the Information Distribution which is shown in Fig. 9. Subfigure (a) shows the information distribution in 3D curved surface. The X-axis denotes the length of the region, the Y-axis denotes the width of the region, and the Z-axis denotes the quantity of information. Subfigure (b) is the projection of the information distribution in 2D space. The information distribution is illustrated with contour lines. N sensors are deployed to maximize the probabilistic coverage on the ROI under a probabilistic detection model. N-node coverage is obtained based on a probability multiplication principle as Equation (14). Here, we denote F_p as $P(x, y)$.

The double integral of the product expression indicates the amount of information captured from the entire region, which reflects the coverage capability of the N-node network. The expression is shown below:

$$F(x,y) = \iint_s \mathrm{D}(x,y) \cdot P(x,y) dxdy \tag{24}$$

where x and y are the coordinates in the region, $\mathrm{D}(x, y)$ represents the information distribution at (x, y).

Summarily, the objective is to optimize the $F(x, y)$ using sensor positions as parameters. The authors propose a heuristic algorithm to find the feasible positions to deploy sensors. The principal idea of the algorithm is to conduct an overall quick search in the whole solution space.

For bistatic radar (BR) network, authors in [98] study the optimal placement with a special kind of sensors, BRs, on a line segment (the formed barrier) to minimize its vulnerability which is calculated in a probabilistic form. Authors characterize the optimal placement problem by the placement order and placement spacing with definite M transmitters and N receivers.

Through discussing the local balanced structure under arbitrary placement orders and placement spacings, a binary search method is proposed to find an optimal placement. Universal theoretic analyses are given for the correctness of the method. It is the first work to explore the barrier coverage with BRs networks placement.

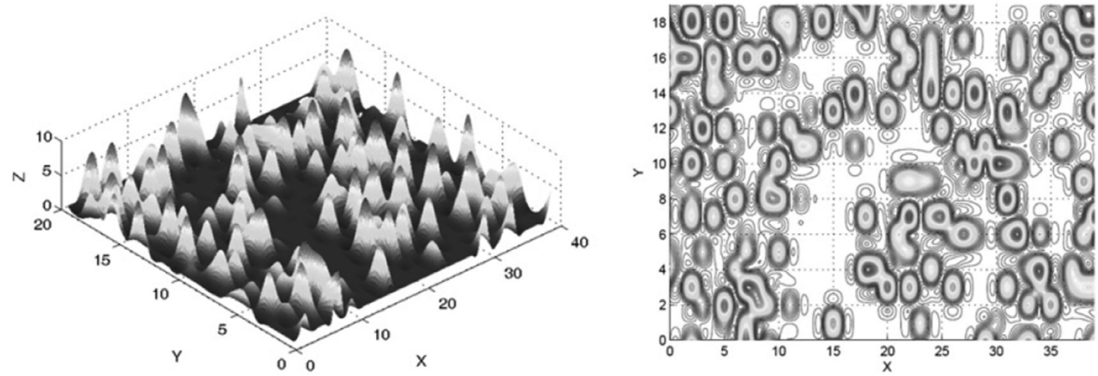
Maximizing network lifetime. Commonly, the lifetime of individual sensors depends on the battery duration, while the lifetime of WSNs can be defined as the duration within which the deployed sensors can steadily provide adequate coverage quality. Some papers [126,130] show that adjusting the deployment of sensor nodes can greatly extend the network lifetime by overcoming the unbalance of sensor distribution or inaccuracy of sensor position. Few work is published on this objective.

The coverage quality and lifetime are two conflict factors due to the constraints of battery power. Authors in [49] extensively discuss the relationship between the two factors and gave an analysis framework. By sacrificing coverage quality, such as using partial coverage, the lifetime will extend in the framework. Through theoretical analysis, the 2D Gaussian distribution of sensors is well-suited and the parameters σ_x and σ_y can be adjusted to deploy a certain number of sensors. Two deployment strategies are designed corresponding to $\sigma_x = \sigma_y$ (circular) and $\sigma_x \neq \sigma_y$ (elliptic) respectively. The basic idea is to partition the target area into squares and rectangles with the centers as the original points of Gaussian distribution. The proper length for the squares and rectangles are induced by their given analysis.

If the target field is accessible, then a deterministic sensor deployment strategy should be more effective. Authors in [130] propose a differential coverage problem, which means there is a difference of importance of different targets in ROI, under a probabilistic detection model. They gave a genetic algorithm to solve this problem with fewer deployed sensors. Therefore, they can balance the coverage quality and lifetime by localizing the genetic algorithm at more important subareas. Connectivity is also required in this paper.

Authors in [126] propose an optimal deployment pattern for a circle perimeter barrier coverage under a bistatic radar detection model. They focus on the structure property of a minimum cost deployment sequence. From a dynamic viewpoint for deployment strategy, they give two heuristically bigraphic algorithms to solve the movement optimization problems, respectively minimizing the total movement distance and minimizing maximal distance among single movements.

Minimizing number of sensors. Another important problem is to minimize the number of sensors [94,100,117,121]. The authors in [98] also consider a barrier coverage in bistatic radar networks alike [100] by deploying radar sensors. However, the latter one considers the cost difference between transmitter sensors and receiver sensors. The placement problem is to construct a belt barrier, which has the breadth no smaller than a predefined threshold, with minimum total placement cost.



(a). An example of information distribution in 3D curved surface. (b). The 2D concour lines of the example in (a).

Fig. 9. Information distribution model [66].

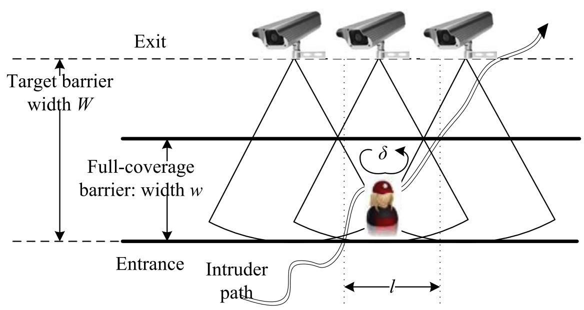


Fig. 10. Concept of local face-view camera barrier coverage [94].

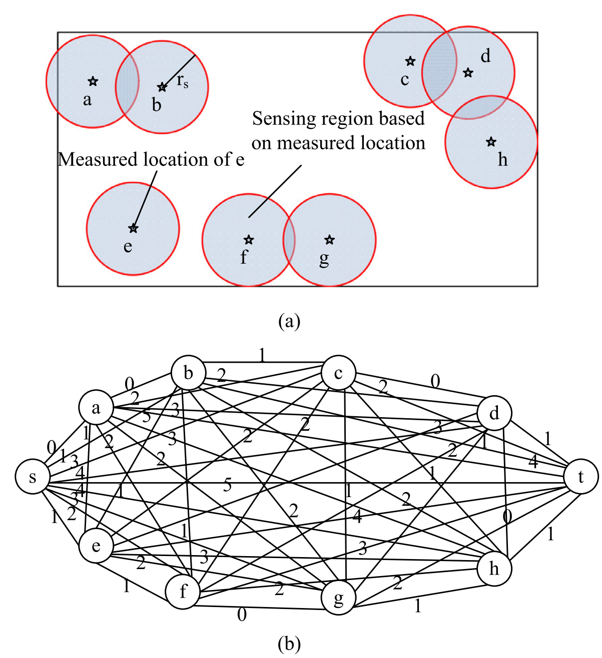


Fig. 11. Sensor network and its corresponding fault tolerant weighted barrier graph when only stationary nodes have location errors. (a) Deployed sensor network. (b) Fault tolerant weighted barrier graph [93].

In [94], authors propose a novel concept for achieving statistical barrier coverage in camera sensor networks, named Local Face-View Barrier Coverage (LFVBC). They denote the intruders' trajectory lengths along the barrier as l and head rotation angles as δ. Then they derive a rigorous probability bound for intruder detection based on a feasible deployment pattern. Fig. 10 shows an example of the LFVBC.

Besides, researchers in [117] study the partial coverage problem and the corresponding sensor saving rate in 3*D* lattice WSNs. Two popular 3*D* deployment patterns including cube and triangular prism are considered. The sensor saving rate in partial coverage with respect to full coverage is derived through mathematical models and theoretical analysis.

Other objectives. There are also some problems with other special objectives. For target probabilistic *K*-coverage, authors in [189] discuss an optimal deployment with the required sensing range to be optimal as well. They use an artificial bee colony algorithm to solve the deployment problem.

For barrier coverage, research in [93] discusses a barrier coverage when sensor nodes have location errors and mobile sensor nodes are involved to improve barrier coverage quality. Authors propose a fault tolerant weighted barrier graph which is shown in Fig. 11. They prove that the minimum number of mobile sensor nodes to form barrier coverage is the length of the shortest path on the graph.

In vehicular network applications, authors in [116] assume that the sensors are stochastically deployed outside the region. For such WSNs, they derive probabilistic expressions for *K*-coverage and connectivity using the exact geometrical information. They demonstrate an on-campus traffic monitoring system to count the number of vehicles, detect the direction of vehicles, and identify the vehicle (two-wheeler or four-wheeler) using sensors along both sides of the road, which is shown in Fig. 12.

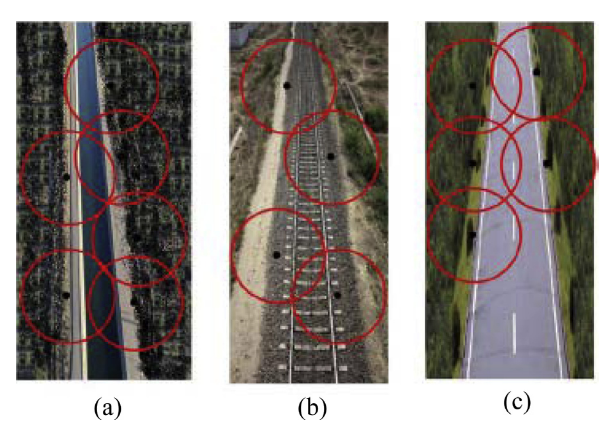


Fig. 12. Examples of the coverage of Field of Interest (FoI) with sensors deployed outside the edges. (a)–(c) Sensors monitoring canal water surface, railway track, and road surface, respectively [116].

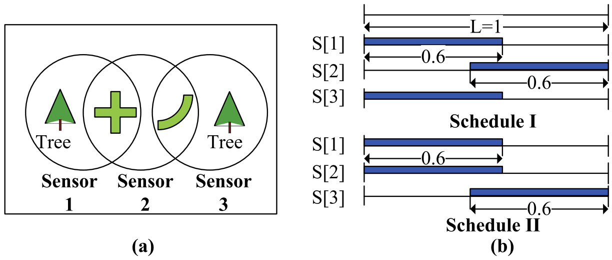


Fig. 13. A surveillance example, where three sensors monitor four critical locations [8].

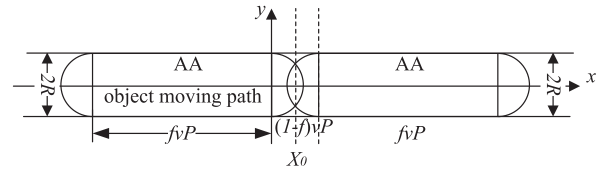


Fig. 14. The active area in the synchronized schedule when R is large [169].

5.1.2. Scheduling or selection

We employ the deployment strategy if we can determine the initial positions of sensors. However, it may be impossible in some situations, such as in a battlefield. Instead, network builders can only scatter a large number of sensors randomly to satisfy different coverage quality requirements. This leads to redundant sensors. To utilize these redundant sensors effectively, an intuitive idea is to schedule sensors to work alternatively. Scheduling strategy is commonly used to maximize the network lifetime and it is easy to be implemented by distributed algorithms. Selection is a special case of scheduling strategy because selection strategy has only one chance (or one time slot) to determine which sensors to be used. Additionally, sometimes we do not need a real-time coverage, instead, we only require that each target should be detected within an uncertain period. It is another type of uncertainty because we cannot know the starting point or duration of the coverage period. In this subsection, we discuss some examples of scheduling or selection strategy on optimization objectives as above.

Maximizing coverage quality. Scheduling strategy is considered to be used for some specially defined coverage quality. The problem MaxEct below is discussed in [8] where the authors propose a typical scheduling method. It divides many time slots and activates sensors for each slot.

Definition 11 (MaxEct Problem)**.** Given a unit-disk graph $G(N, E, P)$ with n sensors, a set of critical locations P whose importance factors are known, the battery life of each sensor B_i, $i = 1, \dots, n$, and a designated lifetime L, where $B_i < L$, MaxEct is to calculate an "on" schedule per cycle for each sensor such that the overall effective coverage time is maximized.

Fig. 13 uses a simplified example to illustrate how to calculate the total effective coverage time given the network and sensor schedule.

In Fig. 13(a), three sensors are required to monitor four targets, i.e., a curve, a crossing, and two trees, for 10 hours. The importance factor of the tree, curve, and crossing is assumed to be 1, 2, and 4, respectively. Since each sensor battery can only last for 6 hours, the existing sensors are not able to provide full coverage while meeting the network lifetime requirement. The authors trade the coverage for lifetime by dividing the mission duration of 10 h into ten cycles, and in each cycle, each sensor is active for, at most, $6/10 = 0.6$ h. Then the scheduling issue becomes how to place the "on" period (no greater than 0.6 hours) in each cycle so that the total effective coverage of the critical locations are maximized. Fig. 13(b) shows two possible solutions. In Schedule I, two trees are monitored for 0.6 h; the curve and the crossing are monitored for the full cycle of 1 h. Thus, the total effective coverage of Schedule I is $1^*0.6+1^*0.6+4^*1+2^*1 = 7.2$. Similarly, the total effective coverage of Schedule II is $1^*0.6+1^*0.6+4^*0.6+2^*1 = 5.6$. To simplify the presentation of solution, authors divide the algorithm into rounds. During each round, a subset of sensors, called local optimal sensors, are elected and their schedules are determined. Although the algorithm is only based on the computation of local optimal solution, it is a constant factor approximation to the global optimal solution. Their algorithm is equivalent to finding a maximal independent set in a distributed manner by labeling the local optimal sensors in each round.

Maximizing network lifetime. Currently sensor nodes are commonly battery powered. The available energy of each sensor for sensing and communication is limited because of the size and cost constraints, which greatly affect the lifetime of WSNs [9,14,15]. The lifetime of WSNs is often defined as the period from the network setup time to the time that the deployed network cannot provide adequate coverage [18,132,155]. Two sensor states exist, named active state and sleep state. The two states alternate during network operation. Sensors in active state perform monitoring tasks and communicate with central nodes, whereas sensors in sleep state deplete negligible energy in comparison with the active state.

In a randomly deployed sensor network, the scattered sensor nodes may be more than the necessary. If the region covered by one sensor can also be covered by other sensors, such sensors can be viewed as redundant and can be temporarily swaped into the sleep state. Sleep-wakeup scheduling sets the activation and deactivation of the sensors so that the WSNs always have a maximum available energy and lifetime [18,65,140,151,155,169].

Authors in [65], based on a new confident information coverage model, design a novel sensor scheduling algorithm to prolong the network lifetime. This algorithm organizes the sensors into a maximal number of set covers. Each cover is capable of providing the required coverage quality. The coverage will be achieved alternatively by the sensors in one set cover in active state, while all sensors in other set covers are in sleep state. The reconstruction random field is divided into evenly distributed grid cells, and the authors assume that each reconstruction point locates at the center of each grid cell.

For a partial coverage requirement, authors in [169] develop an analytical framework for object detection in sensor networks, and mathematically analyze average-case object detection quality by random and synchronized sensing scheduling protocols. The synchronized sensing schedule has a benefit that the object non-detected traveling distance, i.e., the distance by which an object travels before detection, is bounded. As shown in Fig. 14, the *active area AA* is the set of periodically repeated areas, except the last one when t_a (It is the time point t_a) is not a multiple of P (Here, P is one sensing period). Each repeated area is either a rectangle plus two overlapped half circles for a large R, or a rectangle plus two disjoint half circles for a small R. Denote $X_0 = (1-f)vP$ (f is the percentage of sensors in active state, v is the speed of moving target) as shown in Fig. 14, where it is assumed that $t_a > P$.

A refined scheduling strategy for a partial target coverage is proposed in [132] where the active sensors are classified into different roles: source and relay. Source nodes sense and transfer data while relay nodes only transfer data towards the unique sink node. Sensors with different roles consume different level of energy. The authors give a hybrid column generation, which is used to identify the optimal operation schedules. Moreover, a constraint programming strategy is provided to identify profitable network configura-

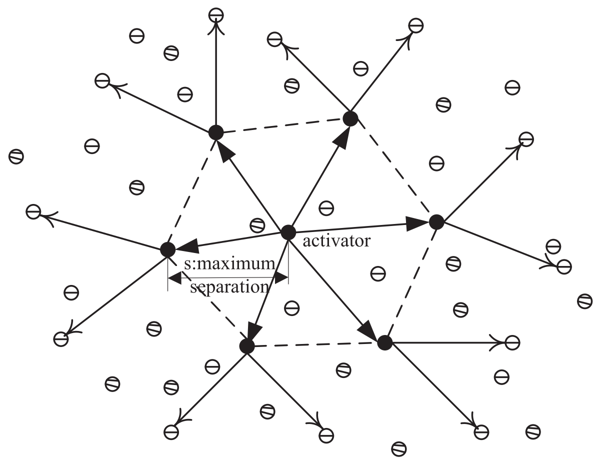


Fig. 15. A simplification of the node activation process in PCP. Activated nodes try to form a triangular lattice over the area [88].

tions leading to extend network lifetime, which is an optimal solution. Besides, an evolutionary algorithm is proposed to boost up the solution process and accelerate convergence.

Minimizing number of sensors. Designing and testing different coverage protocols for each sensing model is a costly task. To address this challenging task, authors in [88] propose a new probabilistic coverage protocol (denoted by PCP) that could employ different sensing models. They show that PCP works under 0/1 disk detection model or probabilistic detection models, with minimal changes. In Fig. 15, an optimization procedure is given. Optimality here means the minimum number of disks required. The idea of PCP is to activate a subset of deployed sensors to construct an *approximate* triangular lattice on the area to be covered. PCP starts by activating any sensor in the area, which is referred to as an activator. This sensor activates six other sensors located at vertices of the hexagon centered at its position. Each activated sensor, in turn, activates other sensors at vertices of its own hexagon.

Authors in [27] explore the mathematic relationship on sensing probability between two adjacent points and transform area ϵ-full coverage problem into point coverage problem. They then design an ϵ-full coverage optimization (FCO) to select a subset of sensors to provide ϵ-full coverage dynamically so that the lifetime of network is prolonged.

As a mobile social application, research in [4] has a goal to achieve (α, T)-coverage of a given Area of Interest (AoI) by a minimal set of mobile nodes. In this paper, the authors propose two algorithms: inter-location algorithm that selects a minimal number of mobile nodes from nodes inside the AoI considering the distance among them; and inter-meeting-time algorithm that selects nodes regarding to the expected meeting time among the nodes. To cope with the case that there is an insufficient number of sensors inside the AoI, they propose an extended algorithm which considers nodes outside the AoI. To further improve the accuracy, an updating mechanism which adapts the number of selected nodes based on their latest locations during the time period T is proposed. A similar work [190] is to select the minimal number of Roadside Access Points (RAPs) to disseminate information to mobile vehicles in the vehicle ad-hoc networks. They formulate the problem as a combining probabilistic set cover problem and feed it with the historic data. Finally, authors give a greedy algorithm to solve the special set cover problem and analyze its time complexity and approximation ratio. The two works represent a trend, named crowdsensing, which utilizes mobile smart devices to gather information in a certain area.

Other objectives. Research in [23] investigates the K-coverage problem under the probabilistic detection model, in which the detection probability by a sensor decays with the distance. Authors generalize the coverage problem and propose an algorithm to calculate the minimum degree of area coverage.

Similar to probabilistic K-coverage, authors in [112] propose a probabilistic-based dynamic non-deterministic α-K-coverage, which can guarantee that target moving area is covered by at least K sensors under at least α probability. They assume that target state (position and speed) estimation uncertainty of a moving target can be approximated by a Gaussian distribution. Then, the uncertainty at the next time can be illustrated by an uncertainty ellipsoid in the state space. Thus, authors propose an energy-efficient sensor scheduling scheme, Optimal Cooperation Scheduling Algorithm (OCSA).

In paper [78], authors address a probabilistic K-coverage problem on target set of interest with directional sensors with a probabilistic fan-shaped sensing model. Their objective is to set the subset of sensors so that each target probabilistic coverage sum is larger than K, and to maximize the lifetime of the network. Because the problem is NP-hard, two approximate algorithms, centralized Integral Linear Programming Algorithm (ILPA) and distributed Coverage Benefit Detection Algorithm (CBDA), are proposed. By modeling the problem as an ILP problem, ILPA loosens the integer constraint and converts the optimal LP solution to a feasible ILP solution with a proven approximation ratio guarantee. As a distributed algorithm, CBDA utilizes a back-off timer to decide active sensors and their direction with a large coverage benefit. It is proven that the solutions acquired by ILPA and CBDA are feasible.

The method addressed by [56] is to analyze the detection probability of a mobile target in the sensor network and define probabilistic trap coverage, which restricts the farthest displacement of a mobile target with a detection probability no less than the threshold. They develop the theory of circle graph, which can be generally applied in the area of intrusion detection, such as trap coverage and barrier coverage. They construct a circular graph to characterize the feature of probabilistic trap detection. In the circular graph, nodes represent sensors, and there is an edge between two sensors if they are neighbors in the sensor Voronoi graph. The capacity of the edge is the minimum detection gain of two corresponding sensors. The authors further study the practical issue how to schedule sensors to maximize the lifetime of a network while guaranteeing probabilistic trap coverage.

5.1.3. Movement or adjustment

To satisfy the requirements of modern applications, many new sensors are equipped with mobility components. Naturally, the movement strategy is exploited by these mobile sensors. Compared with the deployment strategy, there is a major difference since we can adjust the positions of sensors many times. Thus, in terms of time slots, movement is similar to the scheduling strategy, which exploits the movement strategy in each time slot. This strategy is particularly suitable for maximizing the coverage quality and some special objectives like fixing the coverage holes. Additionally, in relevant problems, the movement, instead of communicating or other actions, becomes the most energy-consuming action. Therefore, to save energy, minimizing movement distance became an important objective. Besides, the adjustment of sensor parameter setting, like working angle, detection range, etc., can be viewed as a special movement. In this subsection, we discuss movement or adjustment strategies for these main optimization objectives.

Maximize coverage quality. For area coverage, authors unify and extend several different existing strategies for multi-robot coverage

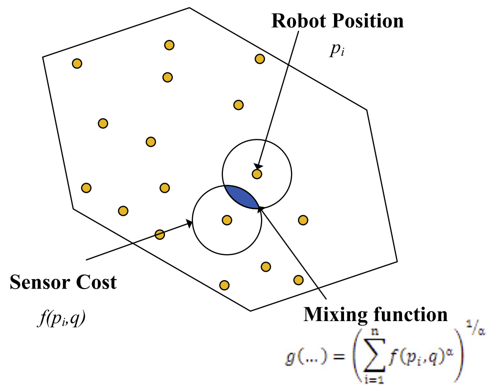


Fig. 16. The mixing function is illustrated in this figure. It determines how information from multiple robot sensors are combined, shown graphically as the intersection of the two circles in the figure [3].

control [3], including ones based on Voronoi partitions, probabilistic models, and artificial potential fields. The hybrid network is built with static and mobile sensors. Static ones are responsible for building Voronoi partitions while mobile ones are responsible for maximizing the probabilistic coverage objective in ROI by movement strategy. Fig. 16 shows the network scenario. The main idea is that robots should push away from one another to spread out over the ROI according to a mixing function requirement for coverage quality, meanwhile, they should not move too far from one another in order to avoid being disconnected.

Similar work is introduced in [191] where a Voronoi partition is used to guide mobile sensors to locate at the proper positions and adjust their sensing range for maximizing the coverage quality. Then, for trade-off between the coverage quality and the energy consumption, a scheduling strategy is used to minimize the energy consumption with partial or probabilistic coverage quality. The research discusses a novel multi-objective problem solved by a two-phase immune algorithm.

For barrier coverage, mobile sensors are driven to the positions near vulnerable static sensors frequently attacked for maximizing intrusion detection. Researchers assume that there is an intrusion probability based on the history. In [153], mobile sensors aid the static sensors for better coverage quality based on intrusion history. Fig. 17 illustrates how mobile nodes are assigned. Step 1: there is a line between a mobile node m_i to a static sensor node if their Euclidean distance is no greater than a limitation, say D. Step 2: two nodes u and v are added, and a bipartite graph is constructed. Then, a max-flow algorithm is applied to find the maximum flow from u to v. Step 3: the mobile nodes are relocated onto the static sensor nodes. This happens only if the max-flow value is equal to the number of mobile nodes. Otherwise, Steps 1 and 2 are repeated after D is adjusted properly. For calculating the value D, the authors propose a binary search algorithm and prove that the optimal solution of movement strategy can be found. Similarly, a coordinated sensor patrolling (CSP) algorithm is proposed in [62], which guides mobile sensors to form dynamic barrier for maximizing the intrusion detection in time slot. The heuristics is based on the past intruder arrival information. The total moving distance of the sensors during each time slot in CSP is minimum. The demonstration is in Fig. 17 (c). Similarly, intruder arrival time is modeled as random Poisson distribution, which is similar to the arrival model in [153]. It is an important uncertain property for this kind of barrier coverage.

For 3D case, a novel node sinking algorithm is given in [158]. Sensors are randomly scattered on the surface of water, and then each of them should be deployed under water by sinking vertically with a certain height to make a volumetrical coverage. Authors find

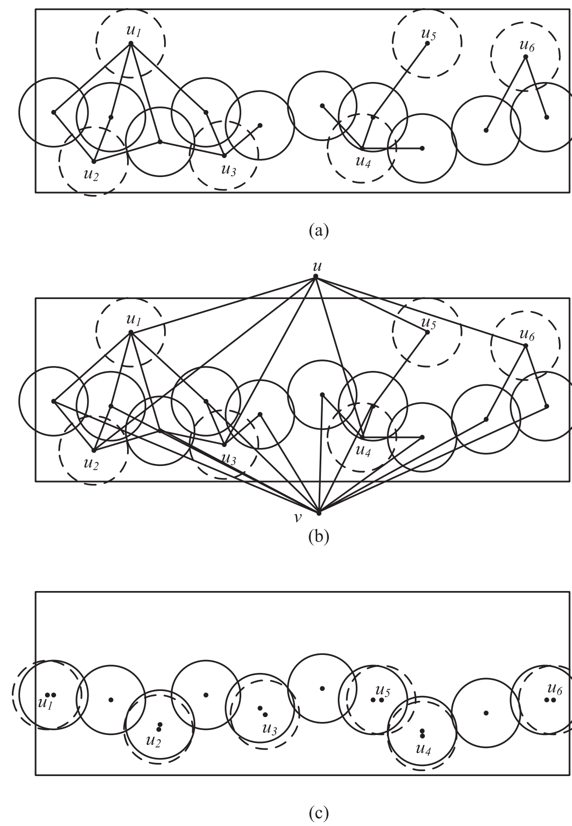


Fig. 17. Mobile sensors strengthen barrier [153].

an optimization solution with three stages, namely pattern node selection, network connectivity restoration, and coverage hole repair, by which these sensors are deployed for maximizing the covered volume ratio and keeping the connectivity.

For adjustment strategy, a distributed multi-round view coverage enhancing (VCE) algorithm is proposed in [138] by self-orientation of the camera sensors. In this algorithm, sensors are continuously rotated to reduce the overlapping view-coverage with their neighbors until reaching the stable state. The novelty is measuring the coverage quality with a finer granularity for face recognition. That means the view-coverage can be viewed as a special partial coverage. Thus, it replaces full-view coverage by adding a view-coverage ratio for each point of ROI.

Recently some researchers study how to adjust mixed parameters such as position and direction. In [123], authors address an area coverage problem with rotatable directional sensors. They aim to maximize the covered field ratio by selecting a suitable working direction for sensors in a distributed way. The Voronoi graph is applied to help decide a rotation angle of each sensor. Additionally, authors in [104] formulate a mixed objective function on the position and direction of sensors. The directional sensor in this research means each sensor has an attenuated elliptical detection model. They aim to maximize the objective function by adjusting the position and direction of sensors. They propose two algorithms: Concurrent Rotation and Motion Control (CRMC), and Staged Rotation and Motion Control (SRMC). The first one is a localized iterative algorithm derived from optimality conditions, so it aims at reaching a local maximum solution. The second algorithm unifies the orientations of the sensor nodes and then conducts the motions of the nodes, which decouples rotation and motion controls to reduce the computation complexity with a slight sacrifice in optimality.

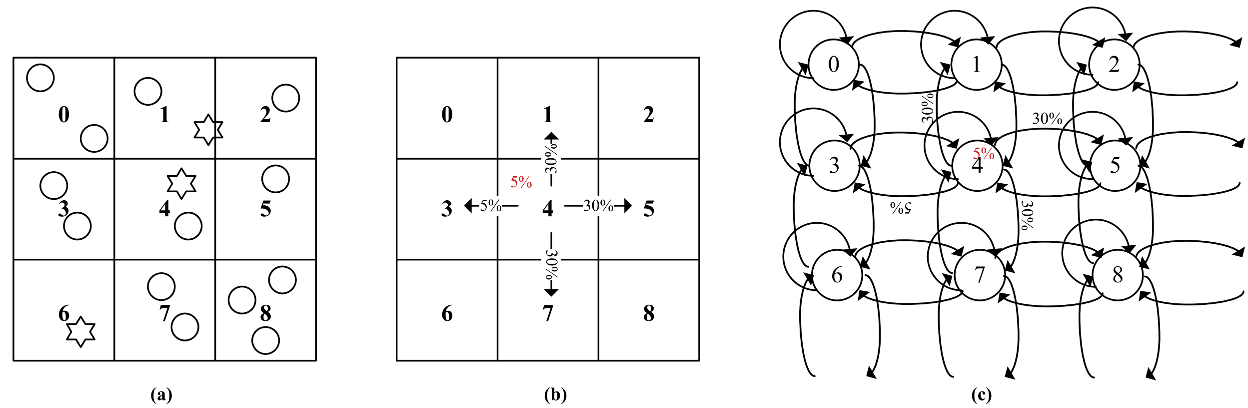


Fig. 18. Illustration for random walk for mobile sensors [114].

Maximizing network lifetime. Not much research explicitly takes on movement strategy for maximizing network lifetime. Instead, minimizing the movement distance, distance of individual sensor, or total distance of all sensors, becomes another objective for energy saving reasons because energy consumption on movement is huge in comparison with detection, communication, or other actions of sensors. As an example to maximizing network lifetime by movement strategy, authors in [114] discuss how to achieve a coverage with acceptable network lifetime with a hybrid sensor network. There are two main study issues. First, determining necessary coverage contributions from static sensors or mobile sensors. Second, scheduling the sensors to achieve the desired coverage contributions. A random walk model is presented for the mobile sensors. Fig. 18 shows the grid based solution. In Fig. 18(a), the stars represent mobile sensors and the circles represent static sensors. Fig. 18(b) shows the probabilities of mobile sensors for moving to or staying in a grid, determined according to the network configuration. Fig. 18(c) is a Markov chain for the random walk model, which is the uncertain properties for movement strategy. Additionally, the collaboration scheme just provides partial coverage quality.

For adjustment strategy, in [172], the authors apply a potential game theory to a power-aware mobile sensor area coverage problem combining the sensor movement strategy and detection voltage adjustment. The objective is to design an objective function to evaluate a trade-off between improving the detection probability and reducing the total energy consumption. A barycentric coordinator is given to represent the strategy space for sensor positions and detection voltage. In a ROI where each point has an importance, a distributed algorithm is running at each mobile sensor to find the local optimal position and voltage to update the barycentric coordinate based on equilibria of replicator dynamic.

Minimizing number of sensors. As a special target coverage problem, a sweep coverage is proposed by [58]. The authors logically categorize a wireless network into a stationary sensor network and a dynamic mobile network. When an event happens, the static node, which senses the event, becomes a point of interest (POI). Mobile nodes in the mobile network should move to the POI and collect information of the event with a high probability. A POI is swept if its data is retrieved by a mobile node. Each mobile node distributively guides the movement toward POIs by using local information only. Thus the basic idea is to build a virtual 3*D* map in the stationary sensor network, with POIs locating at the peaks, which is similar to the illustration in Fig. 9. Mobile sensors climb to these peaks with the help of local information, which is called the potential of the sensors. The objective is to minimize the number of mobile nodes under a certain setting of delay bound and information access probability to collect necessary information.

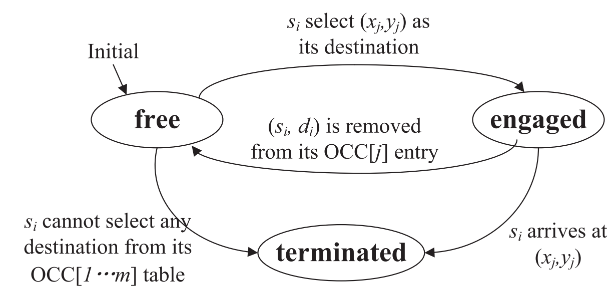


Fig. 19. The state transition diagram of each sensor s_i in the competition based dispatch scheme [52].

Other objectives. For area coverage, research in [52] considers minimizing both the number of sensors and their total movements. The authors solve the *K*-coverage sensor deployment problem to achieve *K*-coverage of the ROI Ω. They consider two sub-problems: a *K*-coverage placement problem, and a distributed dispatch problem. The placement problem asks how to determine a minimum number of required sensors and their locations in Ω to guarantee that Ω is *K*-covered and the network is connected. Particularly, the dispatch problem asks how to schedule mobile sensors to move to the designated locations so that the energy consumption is minimized. For the dispatch problem, they propose a pattern-based scheme and a competition-based scheme. While the pattern-based scheme allows sensors to derive the target locations on their own, the competition-based scheme allows mobile sensors to bid for their closest locations. The distributed competition based scheme is shown in Fig. 19. A probabilistic detection model is exploited and the arbitrary relation between communication distance and sensing distance of sensors brings uncertain properties into the *K*-coverage problem. Similarly, authors in [102] consider a *K*-coverage problem in mission oriented mobile WSNs, which is divided into two subproblems: namely sensor placement, and sensor selection. For the sensor placement problem, the sink of the network identifies locations in a ROI so that the region can be *K*-covered with the least sensors. First, the sink should decompose the ROI into Reuleaux triangle based on *Helly's Theorem* below (The geometric analysis of the Reuleaux triangle is shown in Fig. 20). For the sensor selection problem, they determine which sensors should move to the above-computed locations while minimizing the total energy consumption on sensor moving and communicating. Then, authors propose centralized and distributed approaches to solve the *K*-coverage problem in mission-oriented mobile WSNs. The uncer-

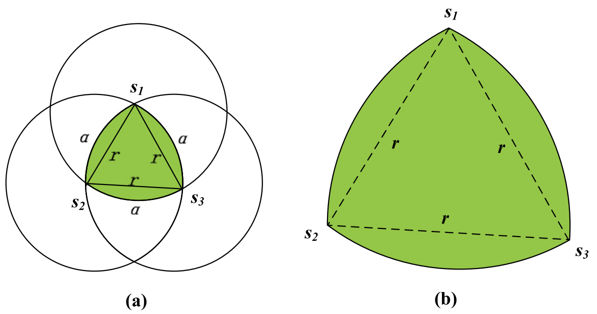


Fig. 20. (a) Intersection of three disks of radius r and (b) Reuleaux triangle of width r [102].

tainty is brought into by the probabilistic detection model and the randomly decomposition of ROI.

Theorem 2. *(Helly's Theorem) Let $K > 3$, the intersection of K sensing disks is not empty if and only if the intersection of any three of those K sensing disks is not empty.*

Another source of uncertain properties is from measurement error. The inaccuracy of sensor position causes coverage holes. Authors in [192] discuss the influence of measurement error on the Voronoi-based mobile sensor deployment for area coverage. A distributed robust Max-Area strategy, which uses information on error bounds, is proposed in order to obtain two polygons for each sensor, and it is shown that the exact Voronoi polygon by accurate measurement lies between them. A local spatial probability function is then derived for each sensor, which convertes the available information about the error bound into the likelihood of the points.

5.1.4. Communication related topics

In this subsection, we discuss more about communication related topics. For deployment strategy, authors in [102] apply the *Helly's Theorem* to determine the sensing range r_s for stochastic area K-coverage, then they investigate the relationship between r_s and the communication range r_t, that is $r_t \geq \sqrt{r_s}$. For selection strategy, authors in [162] use a two-approximation algorithm to find a Steiner tree T_s that spans the sink and all active sensors to provide connectivity. The Steiner points in T_s will be activated as relay sensors.

Many coverage problems with movement strategy is solved in a distributed way, which needs to consider the communication as well. Many researchers take sensors as multi-hop relay for communications with other sensors or the base station. In [149], authors address sensor deployment in underwater acoustic sensor networks to maximize coverage and guarantee connectivity. They assume that sensors can move vertically. Sensors are organized as a tree structure rooted at the surface station. Sensors deployed deeper transmit the sensed data by their parent sensors as relays. Researchers in [179] propose a distributed communication-aware coverage control by which the sensed information is routed to a fixed set of access points via a multi-hop network with probabilistic links. The scheme simultaneously optimizes coverage and routing of information. Specifically, optimization of the communication variables is performed periodically in the dual domain.

Opportunistic Sensor Network (OSN) is a novel network structure composed of mobile sensors. Its basic idea is that nodes (humans) equipped with sensors traverse the ROI, taking samples along their trajectory in periods. The nodes exchange their data to each other when they meet with a certain distance. The dynamic network with mobile sensors can populate the sensed data in an opportunistic way which provides monitoring on the ROI by pervasive communications. Therefore, the communications are necessary in OSNs and the cost of communications are so high that some research focuses on how to control the scale of communications. For example, authors in [152] propose the concept of opportunistic coverage, which has some unique characteristics based on the requirements of urban sensing applications and human mobility features such as spatiotemporal correlation, hotspot effects, and randomness. Additionally, authors in [80] study how the mobility in combination with the periodic sampling of nodes causes large differences in the sensor coverage. Their main insight is that areas, where over-sampling is prevalent, exhibit a high correlation with node contacts.

5.2. Algorithm design

In this section, the algorithms to solve coverage problems with uncertain properties are introduced. We provide an overview of each kind of algorithms. Also, some relevant papers are included for further explanation.

5.2.1. Traditional vs. heuristic algorithms

Traditional algorithms, such as greedy algorithm, dynamic programming, and backtracking, are popular solutions for the coverage problem. These classical algorithms are initial schemes and easy to analyze theoretically. We summarize researches on coverage problems with uncertain properties from three typical objectives: maximizing probability or ratio (in Table 5), maximizing network lifetime (in Table 6), and minimizing the number of sensors (in Table 7). In those tables, several typical solutions are used, such as greedy, programming, and optimization.

The greedy method is one of the most commonly used method in algorithm design. In our survey, traditional algorithms, particularly the greedy method, are the dominanting ones. For example, researches in [32,48,68,133,167,173,174] use greedy algorithm for maximizing coverage quality. Similarly, for maximizing network lifetime, there are greedy algorithms in [9,10,12,13,17,47,49,65,77,114,128,140,142,144,145,151,164,168,169,181,182,193]. And for minimizing the number of sensors, there are greedy algorithms in [112,121,183].

In our survey, we consider that programming consists of dynamic programming and (integer) linear programming. Authors in [151] use linear programming to maximize network lifetime while authors in [6] use dynamic programming. Authors in [15] use integer linear programming to minimize the number of sensors for a target coverage problem.

An optimization method is an algebraic or geographic deduction applied in algorithms [46,102,184]. An optimization method also contains Dijkstra algorithm [93,118], maximum flow algorithm [153] in a graph, and maximum matching algorithm in a bipartite graph [126]. Game theory based algorithm, such as [172], is classified into an optimization method.

Heuristic algorithms, such as swarm intelligence algorithms, genetic algorithms, neural network algorithms and clustering algorithms [29], always search a local optimal solution. Heuristic algorithms, especially swarm intelligence algorithms, attract more attentions due to their common nature with wireless sensor networks. Usually, they attract great attentions because of their similarity to WSNs. Authors adopt different swarm intelligence algorithms to solve probabilistic coverage problems. Swarm intelligence algorithm can also be called as particle swarm optimization. Given measure of coverage quality, a swarm intelligence algorithm can optimize the probabilistic coverage problem by iteratively improving a candidate solution. Authors in [7,125,130,194–196] apply genetic algorithms for probabilistic coverage problems. Genetic algorithm is an optimization method that mimics the process of natural selection. Generally, genetic algorithms use techniques inspired by natural evolution, including mutation and crossover, so

Table 5
Algorithm design for maximizing coverage quality.

Maximize	Algorithm Type						Sensor Model			Deployment			Type of Coverage			Strategy		
Probability or Ratio	Tra	Heu	Cen	Dis	App	Ran	0/1	Att	Tru	Grid	Rand	Pseu	Targ	Area	Barr	Dep	Sch	Mov
TAC [192]	✓			✓			✓				✓			✓				✓
GLOBECOM [133]	✓			✓			✓			✓				✓				✓
WN [153]	✓						✓				✓				✓		✓	
CDCECC [68]	✓			✓			✓			✓			✓				✓	
WPC [32]	✓		✓					✓			✓			✓		✓		
TII [48]	✓			✓					✓	✓				✓		✓		
CIAC [174]	✓			✓					✓		✓			✓				
JNCA [123]	✓			✓			✓				✓			✓				✓(T)
WPMC [184]	✓		✓						✓		✓				✓		✓	
Automatica [179]	✓			✓				✓			✓			✓				✓
IJRR [3]	✓		✓				✓				✓			✓				✓
ComNet [103]		✓		✓				✓			✓			✓				✓
WCMC [149]		✓	✓				✓				✓			✓				✓
IF [191]		✓	✓				✓		✓		✓			✓				✓
AHN [158]		✓	✓				✓				✓			✓				✓
TON [98]		✓	✓					✓(T-R)		✓					✓	✓		
PMC [138]		✓		✓			✓				✓			✓				✓(T)
JNCA [124]		✓		✓			✓				✓			✓				✓
IDEAL [175]		✓	✓					✓				✓		✓		✓		
JICS [177]		✓	✓						✓		✓			✓			✓	
WSS [66]		✓	✓						✓			✓		✓		✓		
Sensor [161]		✓	✓						✓		✓			✓				✓
Sensor [160]		✓	✓						✓		✓			✓				✓
INFOCOM [62]	✓		✓	✓		✓			✓		✓			✓				✓
INFOCOM [8]				✓	✓		✓		✓			✓					✓	
TFECCS [172]	✓			✓					✓		✓			✓				✓

Abbreviations in this table: Tra - Traditional algorithm. Heu - Heuristical algorithm. Cen - Centrialized algorithm. Dis - Distributed algorithm. App - Approximation algorithm. Ran - Randomized algorithm. 0/1 - 0/1 model. Att - Attenuated model. Tru - Truncated model. Grid - Grid deployment. Rand - Random deployment. Pseudo - Pseudo-random deployment. Targ - Target coverage. Area - Area coverage. Barr - Barrier coverage. Dep - Deployment. Sch - Scheduling. Mov - Movement. T - Tuning. T-R - Transmitter-Receiver.

it is more complicated to implement. Some researches, such as [66,135,136,197], exploit clustering algorithms. To guarantee the performance of WSNs, clustering algorithms usually cluster the sensors according to some heuristic rules. Then the sensors in the same cluster work in the same wake-up cycle. In our survey, we do not include any papers applying neural network algorithm to probabilistic coverage problem. However, we think neural network algorithms and other heuristic algorithms can be applied for some appropriate scenarios.

Applications of heuristical algorithms cover three main objectives, including maximizing coverage quality in Table 5, maximizing network lifetime in Table 6, and minimizing the number of sensors in Table 7. Particularly, nearly all of them introduce uncertain properties by exploiting attenuated and truncated detection model and they are only required to achieve a partial or probabilistic coverage.

The solution that we obtain through a heuristic algorithm is usually not the best one, and the computational complexity of heuristic algorithms is higher than traditional ones. Moreover, we can hardly prove the effectiveness of these solutions theoretically. Nevertheless, heuristic algorithms are still useful in solving coverage problems because many coverage problems are NP-hard and a precise solution is difficult to acquire.

5.2.2. Centralized vs. distributed algorithm

Algorithms for coverage problems can also be categorized into centralized and distributed algorithms. A centralized algorithm is an algorithm that is executed based on global information. In a centralized algorithm, at least one node, usually called a sink node, should gather global information and send commands to others. The entire network primarily depends on the sink node. Therefore, the sink node should be the dominant node in the network. Compared with the distributed algorithms, the communication cost is much higher for the sink node which runs centralized algorithms. However, based on global information, centralized algorithms usually obtain a more accurate result. In addition, centralized algorithms are easier to design and analyze.

Some papers transform the original problems into some basic problems. Authors in [6,184] use a classical linear programming method to solve the problems while some researchers in [161,189] transform it into a problem solved by an artificial bee colony method. Set cover problem is also another feasible form for this problem [65]. All of these problems are solved on the assumption that the sink node has global information. In some papers [93,118,141,153], graphic algorithms are applied to solve the problems. Authors in [93,118] depict the problems into shortest path problems and use a Dijkstra algorithm to solve them. Authors in [141,153] exploit a maximum flow algorithm of the graph to solve the problem.

On the other hand, some papers use divide-and-conquer methods to solve the problems [17,23,32,66,142,175,198]. The general idea of those papers is similar as following steps.

1) Divide the ROI into many sub regions based on geometric information.
2) In each sub region, find a solution consisting of a special node as a cluster head, which gathers all the information within this region, and other common sensors.
3) Merge all the solutions and make modifications or optimizations.

By contrast, each node makes decisions based on its local information in distributed algorithms. The node collects only a part of the information so the communication costs can be reduced for the sink node. However, the entire network communication cost is increased because of numerous behaviors of the individual nodes. A network designer should design complicated protocols to control the behaviors of individual nodes. Using distributed algorithms is necessary because collecting the global information is difficult or

Table 6
Algorithm design for maximizing network lifetime.

Maximize	Algorithm Type						Sensor Model			Deployment			Type of Coverage			Strategy		
Lifetime	Tra	Heu	Cen	Dis	App	Ran	0/1	Att	Tru	Grid	Rand	Pseu	Targ	Area	Barr	Dep	Sch	Mov
ComCom [17]	✓			✓				✓			✓		✓					✓
IWQoS [114]	✓		✓				✓			✓				✓				✓
MSN [164]	✓			✓				✓		✓				✓				✓
WN [151]	✓		✓	✓			✓				✓						✓	
EJWCN [181]	✓			✓			✓				✓						✓	
WiOPT [9]	✓		✓				✓				✓		✓				✓	
PDPA [47]	✓			✓			✓				✓			✓			✓	
WCNC [128]	✓		✓				✓		✓		✓			✓			✓	
WASA [140]	✓		✓	✓			✓				✓			✓			✓	
TMC [49]	✓		✓				✓					✓		✓		✓		
MOBICOM [77]	✓			✓			✓				✓				✓		✓	
ISCIT [182]	✓		✓				✓				✓			✓			✓	
WiCom [144]	✓		✓				✓				✓			✓			✓	
WD [10]	✓			✓			✓				✓		✓				✓	
ICDCS [12]	✓		✓				✓				✓		✓				✓	
WCNC [65]	✓		✓					✓			✓		✓				✓	
WiCom [145]	✓			✓				✓			✓			✓			✓	
EUC [168]	✓		✓					✓			✓			✓			✓	
IJDSN [193]	✓			✓					✓		✓			✓			✓	
TPDS [169]	✓			✓					✓		✓				✓		✓	
ICOIN [13]	✓		✓						✓		✓		✓				✓	
WASA [142]	✓		✓				✓				✓				✓			✓
ICC [118]	✓		✓					✓(T-R)				✓			✓		✓	
JNCA [126]	✓		✓					✓(T-R)				✓			✓	✓		✓
WINET [151]	✓		✓	✓			✓				✓			✓			✓	
WICOM [6]	✓		✓				✓				✓		✓				✓	
JNCA [125]		✓	✓				✓				✓		✓				✓	
Networks [132]		✓	✓				✓				✓		✓				✓	
Networks [7]		✓	✓				✓				✓		✓				✓	
TOSN [107]		✓		✓			✓				✓			✓				✓
TMC [82]		✓		✓			✓				✓			✓			✓	
SEMCCO [189]		✓	✓				✓		✓		✓		✓			✓		
WCNC [130]		✓	✓						✓		✓			✓		✓		
CMA [196]		✓	✓						✓		✓			✓			✓	
IJPEDS [18]		✓	✓						✓		✓		✓				✓	
Networks [7]		✓	✓				✓				✓		✓				✓	
GLOBECOM [136]		✓		✓			✓					✓			✓		✓	
INFOCOM [67]		✓	✓					✓			✓				✓		✓	
ICDCS [108]			✓		✓							✓		✓			✓	
TWC [26]				✓	✓				✓		✓				✓		✓	
PE [51]				✓		✓	✓					✓		✓			✓	
IJPCC [64]				✓		✓			✓			✓		✓			✓	
COMSNETS [155]	✓			✓			✓					✓		✓			✓	
MPE [20]	✓			✓				✓			✓		✓				✓	
TOSN [25]			✓			✓	✓				✓			✓			✓	
ISSNIP [78]			✓			✓		✓			✓		✓				✓	
TOSN [69]				✓		✓		✓			✓			✓			✓	
TMC [14]	✓			✓				✓			✓		✓				✓	
WSC [180]	✓			✓			✓			✓				✓			✓	
TJS [34]	✓			✓				✓			✓			✓			✓	
VTC [165]		✓	✓				✓					✓		✓				✓

Abbreviations in the table are the same as Table 5.

even impossible in many real situations. Thus, although the distributed algorithms are hard to design, many studies on distributed algorithms have been conducted.

Among these papers, there are two major kinds of idea in opposite directions: adding and removing methods. For the adding method, all sensors are asleep initially. Then each sensor will decide whether it should awaken according to its local information [8,20,48,193]. In these methods, the core is to find a suitable local requirement that can meet global demands. For example, some papers design distributed algorithms based on patterns [56], and sensors decide their movements by themselves [107,124,179]. On the other hand, for the removing method, the authors assume that all the sensors have been awakened and each sensor should decide whether it is redundant [20,155,181]. For these methods, the criterion of the redundancy plays a decisive role.

Readers may be confused about several algorithms to distributed algorithms, like parallel or localized algorithms. The parallel algorithm can be executed a piece at a time on many different nodes and then recombined in the end to obtain a correct result. It is a kind of distributed algorithm because the node may wait for the results of its neighbor nodes, whereas the contrary is incorrect. The localized algorithm produces a localized solution rather than a global solution. For example, the localized algorithm is usually used to solve barrier coverage problems. Designing a global barrier is difficult, which forces many researchers to use a localized algorithm to gain a local barrier [26,56,143].

Moreover, in some cases, centralized and distributed algorithms can be both achieved or even transformed to each other. Many papers have proposed algorithms of both kinds and compare their performance through experiments. The algorithm in [62] is used

Table 7
Algorithm design for minimizing the number of sensors.

Minimize	Algorithm Type						Sensor Model			Deployment			Type of Coverage			Strategy		
Sensor Num	Tra	Heu	Cen	Dis	App	Ran	0/1	Att	Tru	Grid	Rand	Pseu	Targ	Area	Barr	Dep	Sch	Mov
Sensors [162]	✓		✓					✓		✓				✓			✓	
PMC [139]	✓		✓						✓		✓			✓			✓	
IS [112]	✓		✓				✓				✓				✓			✓
ICON [15]	✓		✓						✓		✓		✓				✓	
TOC [100]	✓		✓					✓(T-R)		✓					✓	✓		
DASFAA [190]	✓		✓					✓			✓		✓				✓	
UbiComp [101]	✓		✓					✓			✓		✓				✓	
ICCCN [121]	✓		✓					✓		✓			✓			✓		
ICEE [183]	✓		✓					✓			✓			✓			✓	
INFOCOM [94]	✓		✓				✓			✓					✓	✓		
INFOCOM [93]	✓		✓				✓				✓				✓			✓
ComNet [102]	✓		✓	✓			✓		✓			✓		✓		✓		
TMC [46]	✓		✓				✓			✓				✓		✓		✓
MobiCom [80]		✓		✓			✓				✓			✓			✓	
ICL [22]		✓	✓						✓		✓			✓			✓	
TPDS [52]		✓	✓	✓			✓		✓	✓				✓		✓		✓
ICCCN [16]		✓	✓					✓			✓		✓			✓		
WiCOM [21]		✓	✓					✓			✓		✓			✓		
SAS [11]		✓	✓						✓		✓		✓			✓		
JIPS [4]			✓			✓	✓				✓			✓			✓	
WCNC [23]			✓		✓				✓		✓			✓			✓	

Abbreviations in the table are the same as Table 5.

to decentralize the traditional centralized coordinated sensor patrolling (CSP) algorithm for the barrier coverage problem and its designers propose two improved algorithm called simple distributed CSP (S-DCSP) and general distributed CSP (G-DCSP). Authors in [102] discuss the multi-coverage for a network with some mobile sensors and propose both centralized and distributed algorithms to solve it. Authors in [140] study the partial coverage problem and design both two kinds of scheduling algorithms.

Authors in [88] bridge deterministic and probabilistic detection models for area coverage with minimal changes. It is a framework protocol with advantages including a minimum number of activated sensors, fewer total energy consumed, and longer network lifetime in comparison with other protocols.

5.2.3. Approximation algorithm

Many coverage optimization problems are *NP*-hard. Aside from heuristic algorithms, approximation algorithms are used to find approximate solutions instead of the global optimal solutions. With the use of approximation algorithms, certain properties of the approximate solution are proven. For example, a ρ-approximation algorithm $\mathscr{A}$ has an approximate solution will not be greater (or less, depending on the objective) than a factor ρ times the optimal solution which is denoted as *OPT*. The performance of algorithm $\mathscr{A}$ is defined as a function $f(X)$ to any instance x of the problem. The factor ρ is called the relative performance guarantee. For minimization and maximization, authors in [199] propose the following expressions:

$$\begin{cases} OPT \leq f(X) \leq \rho OPT, & \rho \gneq 1 \quad \text{for Min} \\ \rho OPT \leq f(X) \leq OPT, & \rho \lneq 1 \quad \text{for Max} \end{cases} \tag{25}$$

A number of techniques have been developed to design approximation algorithms for different types of problems [199,200]. These techniques include greedy strategy, restriction, partition, guillotine cut, relaxation, integer linear programming, primal-dual schema and local ratio, and semi-definite programming, etc. For the probabilistic coverage problem, the most popular technique is linear programming. Many combinatorial optimization problems can be transformed into integer linear programs. For all integer linear programming methods, the general approach to finding an approximation solution has two steps. In the first step, it searches for the optimal solutions in the corresponding relaxed linear programming by methods such as the simplex method, the ellipsoid method, or the interior-point method. In the second step, rounding the optimal solution of the linear programming to a feasible solution of the original integer linear programming produces an approximation. The rounding methods include combinatorial rounding, pipage rounding, iterated rounding, and random rounding.

However, an approximation ratio for approximation algorithm is difficult to analyze. Among more than a hundred papers, only five papers provide a proposal and an analysis on approximate ratios [8,23,26,56,78]. Work in [8] addresses a weighted location (target) coverage problem with time constraint, which maximizes efficient coverage time. Authors model it as an integer programming proven as NP-hard. Then, distributed scheduling is designed to approximately solve the problem in each time slice, during which each sensor is on duty starting at some time point and lasting for a corresponding slot according to its evenly partitioned battery. The battery partitions are the same for each time slice, and on-or-off scheduling on sensors is the same in each time. The scheduling tries to minimize the time slot overlap of the sensors that cover the same locations (targets). The strategy is to choose the sensor with the highest coverage gain greedily.

The work in [26] is to build a barrier coverage to detect an intruder crossing the barrier from one side to another side with at least a detection probability ϵ. The maximum speed of the intruder and the sensor detection cycle are pre-specified. The study proved that when an intruder goes along the middle perpendicular path, it can be detected with a minimal probability. The authors transform the problem into a minimum cut problem by constructing the selected sensor neighbor graph with the perpendicular-crossing coverage probability as the edge weight. The authors in [56] propose a similar intrusion detection problem with the same basic idea for computing coverage probability. They call it a trapping problem that considers any point as a starting point for intruders and lets the detection probability be no less than a given threshold after the intruder moves for a given distance. In this paper, the authors divide the entire coverage area into small sub regions using the cycles of the sensors. In computing the coverage probability, the algorithm only considers the sensors that are close to the sub region, or at most at D distance, which can be the radius of the detection range. Moreover, the algorithm views the sub region as a virtual starting point and view the area outside of the scope of sensor detection as a virtual terminal point. The problem can be deduced as a barrier coverage problem [26]. All the sub regions are satisfied for a probabilistic coverage, and the entire area meets the trap coverage requirement.

For the proposed sensor selection problem, the authors in [78] give two approximation algorithms that are both designed similar to the greedy algorithm for the set cover problem. The authors in [23] define a similar *K*-coverage problem to the work of [78], but the former is focused on area coverage. This paper uses decretized transformation from an algorithm, which is first proposed by [106].

5.2.4. Randomized algorithm

A randomized algorithm employs a degree of randomness as part of its logic. It is a natural inspiration to use randomized algorithms for probabilistic coverage problems, which is strongly related to randomness. Several works give randomized solutions to cope with different aspects of the probabilistic coverage problem, including model analysis, algorithm design, and simulation verification. For network models, the authors in [49] determine the parameters, such as node density and detection range, in terms of desirable detection probability in a random deployment scenario. Authors in [33] exploit a triangular grid sensor deployment with random displacement error to deduce the coverage percentage function by using grid tile length and error radius as two inputs.

The randomized algorithm is also called a probabilistic algorithm that decides on some logic steps under a uniform or a precomputed probabilistic factor. In [25], an energy efficient, constant-time, and randomized scheme is designed for a partial area to reach the desired coverage quality. Each sensor can be active only during one round of the entire network cycle. Therefore, it is randomly choosen at which round it will be active. Thus, the probability $1/\xi$ is chosen for each round where ξ is the number of rounds. The same authors in [51] improve the random scheme for sensor selection, with the constraint of minimum the distance between any pair of selected sensors, by using a *Poisson* sampling technique. All these works focus on scheduling the sensors to maximize the network lifetime.

The randomized algorithm is also used to address some new problems. Research in [62] uses a random distribution model where the sensor is randomly moved to build the barrier based on prior knowledge of an approaching intruder. In addition, authors in [4] use a Markov chain to describe the pattern of people movement and to randomly choose a group of people to cover objectives in a man-made map. Many authors consider randomized simulations to verify analysis results. A typical Monte-Carlo simulation is adopted by many researchers such as [33,49]. The simulations are randomly repeated a hundred and even a thousand times to obtain the average results.

5.3. Algorithm taxonomy

In this subsection, we add some remarks on the algorithm taxonomy for coverage problems with uncertain properties from three typical objectives: maximizing coverage quality (in Table 5), maximizing network lifetime (in Table 6), and minimizing the number of sensors (in Table 7). We list involved representative papers in these tables by denoting each paper as “its abbreviated name of journal or conference + its reference number”. The main categorization in these tables is “Algorithm Type”. Each paper proposes at least one algorithm. The algorithms can be Traditional, Heuristic, Centralized, Distributed, Approximation, or Randomized. One algorithm is exclusively Traditional, Heuristic, Approximation, and Randomized. There are several papers proposing more one algorithms. Particularly, these papers describe centralized and distributed versions of the proposed algorithm at the same time.

We also classify the algorithms according to several important features that have been introduced in the previous sections: detection model, deployment, coverage type, and strategy. Through these classifications, we find some relationships between them. For example, nearly all papers for maximizing coverage quality use a probabilistic sensor model: attenuation (Transmitter-Receiver model is a special attenuation also denoted as Att with T-R for indicating) and truncation. Particularly, the papers for maximizing coverage ratio use the disk model to address area coverage. In addition, papers on maximizing network lifetime consider a random deployment and use sensor selection (wake-up scheduling is a special selection also denoted as Sel). Moreover, papers on minimizing the number of sensors need to optimize the positions of sensors by deploying static sensors and moving mobile sensors (Tuning is a special movement also denoted as Mov with *T* for indicating). we use abbreviations for these labels. The abbreviations are listed on the bottom of the Table 5 and it is used the same for Tables 6 and 7.

6. Theoretical verification and algorithm analysis

This section first gives an overview of theoretical verifications on coverage problems with uncertain properties. The main objective is to get a relationship between network parameters, and the main method is to apply probabilistic estimation on theoretical parameter relationship instead of proposing any explicit algorithms. Another analyses are to verify the correctness and the accuracy of algorithms in mathematics verification ways. For algorithm analyses, most of the researchers mainly discuss time complexity, space complexity, and approximation ratio. For simulation experiments, analyses can be classified into two kinds. On the one hand, authors verify theoretical results by simulations. On the other hand, authors prove that the proposed algorithms have better performance and more accurate results by comparing them with previous literatures. We separately summarize theoretic analyses in the following subsection. Related simulation platforms and tools are also introduced at the end of this section.

6.1. Relationship between model parameters

Some researchers focus on analyzing the relationship of coverage model parameters, including detection models, density of sensors, and network performance (lifetime and coverage quality). In this subsection, analyses of the relationship of coverage model parameters are classified into model analysis, analysis on the density of sensors, and other analyses. We list some papers about relationship analysis and provide an overview of their contributions.

6.1.1. Model analysis

Detection and network models have an important influence on the solutions of coverage and the network performance. We expect that a model is as simple as possible, meanwhile, we expect that the model can depict the real world.

Some papers, such as [31,163], investigate the influence of detection models for coverage quality. By comparing 0/1 disk models with probabilistic models, previous researchers prove that the probabilistic model is more accurate. For example, the authors in [163] prove that the probabilistic model improved coverage quality even by 12%. The authors in [57] propose a new probabilistic network model, called trap coverage, which is used to ensure that any moving objects could only move for a maximum distance before they are detected. Using this model, some coverage holes are permitted only if the radius of the hole is short enough. Apparently, fewer sensors are required to achieve this kind of coverage quality. Authors of [31] propose the new sensor cooperation model. Thus, the data fusion improves coverage quality.

Other researchers propose a new network model to describe a specific environment. For instance, a novel K-coverage model is proposed in [115]. It calculates the probability of K-coverage and the minimum number of sensors required. The authors in [44] propose a new model called information coverage, which considers another cooperation of sensors. According to the simulation comparison of information coverage and other models, there is a significant reduction in the number of required sensors. Authors in [201] also use information coverage models to analyze the density difference between the 0/1 detection model and the probabilistic model. They verify assumption in a greenhouse testbed for precise agriculture, which shows that the number of sensors decreases in both grid deployment and random deployment. The authors in [5] propose the T-R model, which describe many new coverage features, including probabilistic coverage and link-centric coverage units. Experimental findings indicate that the average error is only 8%. The authors in [159] propose a new traffic model to analyze the intrusion detection applications. The traffic model is based on the probabilistic detection model.

6.1.2. Analysis on the density of sensors

Determining the density or the number of sensors is usually studied because a sufficient number of sensors is necessary, and the required number of sensors must be determined. Most similar studies analyze the sensor density under a certain coverage quality in a specific environment. The work in [110] is to analyze the critical sensor density for a partial coverage under the border effects and to first apply the density to an arbitrary convex area. In their further work [116], they analyze another scenario where the sensors are scattered nearby outside of the border of ROI and calculate the density for area coverage with a certain coverage ratio. The authors in [81] characterize the sensor density for intrusion detection problems. They propose an approach to estimate the critical sensor density. The authors in [147] analyze the sensors density required for a given coverage quality, such as full K-coverage and partial K-coverage, in an one-dimensional scenario. The authors make a sensor *Poisson* distribution assumption and consider the border effect. The authors in [202] investigate the tight upper bound of the number of sensors for the path coverage problem. The authors in [50] study the relationship between the changing on probability of the K-coverage and the sensor density. Authors in [111] consider the sensor density for full confident information coverage under probabilistic detection model. The authors in [139] derive probabilistic expressions for K-coverage and M-connectivity for 3D heterogeneous directional WSNs.

6.1.3. Other analysis

Other analyses include analyses on the deployment patterns, random errors in the grid based deployment, and the analytical frameworks for performance evaluation, and so on. The analysis on deployment patterns and random errors of deployment can be used to deploy sensors efficiently. The analysis on frameworks for performance evaluation can be used to balance coverage quality and network lifetime, and to optimally configure network parameters. Some papers propose analytical frameworks to evaluate performance in different environments. The authors in [203] propose an analytical model for an object-tracking coverage problem. The authors in [137] address the problem on mobile target detection, and derive a framework to evaluate network performance. The authors in [204] present a new performance function for a probabilistic track coverage. The work in [205] is to provide an analytic method to evaluate performance for the stochastic event coverage. The authors in [157] use local information to analyze the probabilistic coverage in a distributed way by individual sensors without GPS. The authors in [86] study the optimal placement pattern based on the confident information coverage model. They first analyze situations when sensors are deployed at n vertices of a regular n-sided polygon and then extend it to acute cyclic polygons. Finally, the authors conclude that regular triangular lattice is the optimal placement pattern.

Other researchers analyze random errors in the grid based deployment and the deployment patterns. The authors in [33] study efficient deployment planning with random errors and figure out the expressions for the relationship between average coverage percentage and radius of the random errors. The work in [89] is to research a specific and useful problem and the authors propose a framework which transforms almost any full coverage algorithm to a corresponding partial coverage algorithm.

6.2. Complexity and approximation ratio analysis

A few number of studies provide theoretical analysis on the complexity of algorithms for solving coverage problems with uncertain properties. We summarize them in Table 8. All the algorithms in the table are proven as polynomial. For space limitation, we do not elaborate parameters in each time complexity expression. Readers can find them in the corresponding references. The most of involved papers with complexity analysis have a common characteristic, that is, sensors are randomly deployed in the initial stage. Therefore, it is a trend to focus on sensor selection and scheduling optimization for probabilistic coverage problem instead of a grid-based deployment. In addition, work in [193] addresses a space complexity $O(N)$.

A few papers propose approximation algorithms and prove the approximation ratio. Authors in [8] prove that the maximizing efficient coverage time problem (MaxEct) is an NP-hard problem and provide an algorithm with approximation ratio 0.5. Researchers in [26] propose an algorithm called minimum weight barrier with ε-ratio $w_c/w_{OPT} = \frac{N}{N+F(G)/\varphi(\epsilon)-1}\rho\theta\psi^{-1}[\varphi(\epsilon)]$. The study in [23] is the only one about minimizing the number of sensors with theoretical analysis. The authors define a variable ℓ to discretize sensing function and let ℓ as the approximation level, which is another name of approximation ratio. The authors in [56] apply Edmonds-Karp algorithm to solve the probabilistic trap coverage and use a distributed approximation algorithm to schedule sensors for maximizing network lifetime. The approximation ratio for lifetime is $L/L^* \geq (1/2)[1-(O(1)/E)]$. The authors in [78] propose both centralized and distributed algorithms to solve probabilistic K-coverage problem. The integral linear programming (ILP) is used to approximately find the optimal K-coverage set with a $b\rho$-approximation ratio. Recent work proposed in [162] implements a Steiner tree algorithm to solve target coverage problem with selection strategy. Authors prove that the algorithm has approximation ratio $N/N_{opt} \leq m(\ln(1-\lambda(dis_{min}))/(1-p_{min}))$. Authors in [190] give a greedy algorithm to solve the Roadside Access Points (RAP) selection problem like an extension of the classical Set Cover problem with approximation ratio $1+\ln(\frac{\theta*\tau*|V|}{opt})$.

6.3. Simulation and experiment platform

After solving the coverage problem, researchers need to verify the performance of the algorithms theoretically by performance bound, approximation ratio, and complexity analysis. Meanwhile, they also need to compare their design to some other related works practically. Due to the difficulties to build testbed or large scale WSNs, researchers usually provide simulations and numerical experiments to verify the efficiency and correctness of their designs. Simulation also provides a good opportunity to study large scale networks and convenient comparisons between different algorithms and solutions. This subsection provides an overview on simulation platforms.

Table 8
Algorithm time complexity summarization.

–		Strategy		
		Select	Schedule*	Movement
Coverage Problem Objective	Maximize Coverage Probability or Maximize Coverage Ratio	ILB and IMTB [4]: $O(n(M+L)^2T)$ EWOT [4]: $O((n+n')(M+L)^2T)$ OBCA [184]: $O(V_c^*E_c^{*3}+V_t^*E_t^{*2}log(V_t^*))$ DPCCP+SPAN [129]: $O(M^2)$	MaxEct [8]: $O(dR)$	EX-VFA [174]: $O(nmk)$ Alg [68]: $O(NV^2)$ CIVA [191]: $O(Max_{gen}.n_g.N_{clon}.p_s)$ Alg [149]: $O(nd^2)$
	Maximize Lifetime	MWBA [26]: $O(\|V\|\|E\|^2)$ CPCA [167]: $O(N(N+Y))$ Improved Edmonds-Karp [56]: $O(k\|V\|\|E_d\|^2)$ and $O(F\|V\|\|E_d\|^3) for\, k \leq \|E_d\|$	ISCPk [193]: $O(N^2)$ OAS [6]: $O(N^3 * M^3)$ ECSS [34]: $O(n)$ SLICE [9]: $O((nq)log(nq))$ MEASURE-SLICE [9]: $O((nq)^3log(nq))$ PCH [10]: $O(\nu\|S_s\|^2\|T_0\|+\nu(\|E\|+(S_s\cup S_r)log(S_s\cup S_r)))$ GR-COV [182]: $O(N^2)$ ILPA [78]: $O(n^3)$ CBDA [78]: $O(n)$	Alg.1 [49]: $O(((\sigma_{max}(m_1+c_1))+s_1)N_{max})$ Alg.2 [49]: $O(N_{max}[\sigma_{x_{max}}\sigma_{y_{max}}(N_{points}m_2+N_{sector}c_2)+s])$ Coverage Optimal Alg [165]: $O(\sum_{i=0}^{K_{repeat}} N \prod_{l=1}^{K=\llcorner R/r_i \lrcorner} N_{N_l})$
	Minimize Number of Sensors	Alg [48]: $O(\ell nNlogN)$ Alg [139]: $O(n^{t_{max}})$ CrowdRecruiter [101]: $O(N^2)$ Alg [190]: $O(VR^3)$ PSCA [162]: $O(m^2nk)$	Greedy-OCSA Heuristic [112]: $O((k-N_L)^2$	GPBGA [121]: $O(I^2 \times J \times Q)$ CLBGA [121]: $O(I^2 \times J \times Q)$

Deployment of sensors in all above papers is random style.

MATLAB. MATLAB is one of the most popular mathematic tools in scientific and research communities. MATLAB is viewed as a multi-paradigm numerical computing environment and a fourth-generation programming language. As an additional package, Simulink adds graphical multi-domain simulation and model-based design for dynamic and embedded systems. Most researches on the probabilistic coverage problem exploit MATLAB in data analysis, calculation, algorithm evaluation, and simulation [2,3,62,77,79,128,137,147,180,189,206]. And some directly use probabilistic detection models [14,16,18,19,29,48,67,160].

Networks Simulator (NS). The authors in [155] uses the Networks Simulator (NS) as a simulator of discrete event networks. NS creates an open simulation environment for networking in the research community. NS-2 is developed by C++ and NS-3 is written in C++ and Python. Users can configure the parameters of networks and have a deeper look into the entire network operation process by the animation demonstration as well as the performance analysis graphics.

A WSN protocol LEACH is an NS-2 simulation tool with the source code and documentation. LEACH protocol in WSNs is an entry-level agreement, that is, a lot of new agreements are based on it. Here, a lot of researchers use LEACH as a framework to integrate new protocols [64] or create a new match of LEACH [145,167].

Tossim. Tinyos Tossim is a free, open source, and software component-based operating system targeting WSNs. Because TOSSIM runs the same code as sensor hardware, the simulation compiler directly compiles simulation program from component table applied by TinyOS. Therefore, it is useful for applications which need hardware expanding. The authors in [165] use it to simulate a network which considers a joint-optimization of coverage, routing, energy efficiency, and delay.

QualNet. QualNet is a commercial version of GloMoSim (Global Mobile Information System Simulator) used by Scalable Network Technologies (SNT) for their defense projects. GloMoSim is a network protocol simulation software that simulates wireless and wired network systems. GloMoSim is designed using the parallel discrete event simulation capability provided by a parallel programming language, called Parsec. Authors in [4] simulate the people-centric mobile sensor network and random movements on a man-made map.

Simlib. Simlib used by [25] is a simple simulation language for discrete event complex system model. Simlib provides a convenient resource management function. Thus, users can conveniently add new module types and new modules to the resource pool. The resource module library contains thermal equipment, network elements, control algorithms, and the commonly used algorithm modules. To some extent, Simlib is not only for network simulation, but for any industrial system simulation.

ILOG CPLEX and LINGO. Finally, we introduce two mathematic tools rather than platforms. The most famous IBM ILOG CPLEX is an optimizer which solves very large linear programming problems. The authors in [82,182] exploit CPLEX to solve two linear programming problems for modeling a partial area coverage. Another linear, interactive, and general optimizer is LINGO launched by LINDO systems company. LINGO is used to solve nonlinear programming and used for some linear and nonlinear equations, etc. LINGO 13.0 adds a group of powerful probabilistic problem solving functions. A maximizing network lifetime problem for probabilistic target coverage is transformed into a dynamic programming problem which is solved in LINGO by [6].

A comparison of these platforms is given in Table 9.

7. Open challenges and new problems

In this section, we attempt to discuss future research of coverage problems with uncertain properties, including: challenging problems and new problems.

7.1. Challenging problems

This subsection discusses some difficult problems for which no perfect solution has been developed so far. The partial and probabilistic versions of these coverage problems are easily induced.

Table 9
Simulation platform statistics.

Characteristic	MATLAB	Network Simulator (NS)	Tossim	QuelNet	Simlib	CPLEX	LINGO
Platform support	Universal	Unix,Windows	TinyOS	Universal	Universal	Universal	Universal
Programming language	MATLAB script	C++,C,Tcl,Otcl,Python	C++,C,nesC	Parsec	C++,C,nesC	C++,C	LINGO script
Code exportable	No	Limited	Yes	No	Yes	Yes	No
Schalability to large networks	Good	Fair (Some cases)	Excellent	Excellent	Excellent	–	–
Protocol design/optimization	None	Possible	Possible	Possible	Possible	–	–
Mobile support	No	Yes	No	Yes	No	–	–
Dynamic network topolopy support	No	Yes	No	Yes	Yes	–	–
3*D* radiomodel	Yes	No	No	No	No	–	–
Probabilistic sensing model	Yes	Yes	Yes	Yes	Yes	–	–

7.1.1. Special K-coverage problem

Although researchers have developed many solutions in the past, many difficult problems for *K*-coverage still exist. One such problem is the 3*D* *K*-coverage problem. The 2*D* *K*-coverage problem is usually solved according to Helly's theorem in [102]. The theorem states that a regular pentagon is *K*-covered if the central area of the regular pentagon contains at least *K* active sensors. Then, the original problem is transformed into a puzzle problem, which is easy to handle. However, in a 3*D* case, because the transformed polyhedron cannot fill the space of interest easily, a similar method is not feasible. The 3*D* *K*-coverage can have uncertain versions, partial or probabilistic 3*D* *K*-coverage, which are more complicated.

Another remaining unresolved problem is the optimal *K*-coverage pattern problem. The objective is to find an optimal sensor deployment pattern that ensures that every point in the area is *K*-covered while the number of sensor nodes is minimized. The optimal pattern when *K* equals to 1 is proposed by [207], and the optimal pattern when *K* equals to 2 is discussed by [92]. At present, no optimal pattern has been proposed for the case when *K* is larger than 2.

7.1.2. Full-view coverage problem

Visual sensors, such as cameras, can collect visual information from a ROI and provide more information to identify an intruder. The most famous problem of this type is the famous gallery camera problem. Under a sector shape detection range, a new coverage quality is defined as full view coverage. To provide a full view of the intruder, the camera must face the direction of the intruder from a certain required angle. Compared with an area coverage problem, such as the gallery camera problem, a full view barrier coverage proposed by [87] needs few cameras for intruder detection. Camera rotation are involved for the full view coverage problem. How to select the fewest cameras and rotate them to form a full view barrier is an NP-hard problem, and no perfect solution for this problem has been developed until now, especially, when we talk about probabilistic full-view coverage problems.

7.1.3. Sweep coverage problem

The sweep coverage problem can be viewed as a special target coverage. The difference between sweep coverage and the common target coverage is that sensors keep moving to patrol targets of interest in sweep coverage problem. The requirement of sweep coverage is that every target is scanned once, and mobile sensors report the detection record to a sink node during a certain period. The objective could be to minimize the average distance of sensors movements, which is an NP-hard problem. Authors in [131] propose an algorithm based on the traveling salesman problem (TSP) with only one mobile sensor. Further research can consider exploiting multi-mobile sensors and probabilistic detection models to optimize the sweep plan.

7.2. New problems

New problems contain state-of-the-art innovations, such as new network models, new methods, and new scenarios for coverage problems.

7.2.1. Particular sensing model

The authors in [5] present a novel link-centric probabilistic detection model called T-R model, which fills the gap between the traditional 0/1 model and real-world sensing behavior. This transceiver-free model changes the existing coverage problems, such as barrier coverage, trap coverage, and sweep coverage. The people-centric model [4] describes the movement of people with a smart device that can randomly walk and cover the entire map. A camera fan-shape model [9] and sensing adaptable model are also developed. Based on these new models, some fundamental coverage problems should be changed and their new definitions must be given. Researchers can find some promising research directions from these newly appearing problems. For example, authors in [4] define a new (α, T) coverage where each target in a ROI can be covered by at least one mobile node with a probability α during a period *T*.

7.2.2. Heterogeneous network

A heterogeneous network contains several types of sensors, where sensors have different sensing ranges and different communication ranges. This kind of heterogeneity affects network modeling. For heterogeneous networks, researchers should perform a different probabilistic analysis and consider more complicated relations between sensors in comparison with homogeneous networks [49,108,155]. In the future, different kinds of sensors, with uncertain properties, should be used to build heterogeneous WSNs.

Networks with mobile sensors are another type of heterogeneity network. Initial deployment is usually accomplished by randomly scattering static sensors. Mobile sensors recover the coverage hole and improve probabilistic coverage quality [17,114,133,164,173,174]. For future work, mobile sensors should be used to satisfy different coverage requirements per target or sub-area, such as dynamic coverage [133] and mission-oriented [102].

7.2.3. Coverage with crowdsensing

Centralized control on mobile sensor networks provides a static coverage quality, which correspondingly needs a high cost for network buildings. However, nowadays pervasive mobile smart devices (smart phones or smart tablets) and powerful cloud infrastructure provide the opportunities for providing coverage, such as monitoring the environment, through namely *crowdsourcing*. In crowdsourcing, coverage tasks are delivered to device-carrying participants. Therefore, a lot of services are provided by the imbedded sensors in smart devices, such as, waste disposal, indoor Wi-Fi mapping, Bluetooth or RFID systems integrated with sensing, place and route characterization, transportation analysis, and noise pollution monitoring and so on. These services, namely *crowdsensing*, underlie the smart city, smart building, and Internet of things (IoT) [208,209]. However, the monitoring under the crowdsensing is dynamic and uncertain because the device carrier trajectory pattern cannot be explicitly depicted. Several researchers try to study how to softly control the movements of participants. Authors in [95] discuss how to use the game incentive to inspire the participants to explore more regions where the population is sparse. Authors reuse the incentives mechanisms in location-based gaming and social networking apps, instead of money incentive, to exert limited control over people's actions. They call this approach Crowd Soft Control (CSC).

In vehicular crowdsensing application, authors in [210] address the problem of how to select a limited number of vehicles to provide maximum road coverage for a certain monitoring task such as traffic estimation and environment monitoring. The trajectory of each vehicle is unpredictable. To evaluate the performance of the proposed algorithms, authors introduce a realistic scenario, Urban Heat Island (UHI). Offline and Online algorithms are given separately.

7.2.4. Coverage with budget constraint

In commercial applications, the budget is an important factor which causes a tradeoff between budget constraint and coverage quality. In previous sections, we have discussed the objective: minimizing the number of sensors. Naturally, the objective reflects consideration on budget constraint because less sensors deployed for coverage means explicitly lower cost.

For crowdsensing applications, more research focuses on a new topic: coverage with a budget constraint. In [211], a task assignment framework for mobile crowdsensing applications is given, namely CrowdTasker, which addresses the problem of how to select participants for sensing and arrange them to sense in different task cycles under a budget constraint. Different from uniform payments for participants, the CrowdTasker divides the incentives into base incentive and bonus incentive. The base incentive is given to each selected participant and it is static. The bonus incentive is dynamic according to the number of cycles where some participant contributes the sensing readings. The objective is to maximize the coverage quality which can be viewed as a probabilistic coverage quality. The authors of [211] study the related work mentioned above which ensures a predefined percentage of subareas being covered by the selected participants, in order to minimize the overall incentive payment under the probabilistic coverage constraint [101]. However, the incentive payment for each individual participant in previous work is prefixed so the objective is transformed to minimize the number of participants. Additionally, an energy-efficient solution for sensed information updating is given, namely piggyback [85]. Piggyback reduces energy consumption by leveraging smartphone opportunities, such as 3G call, to perform sensing tasks and return sensor readings. Therefore, 3G call history and trajectory of participants are used to estimate the coverage quality with uncertainty.

7.2.5. Nature inspired approaches and new testbeds

As a hot trend, nature inspired approaches would be exploited more and more in coverage problem with uncertain properties. This is because that acquiring fast and direct algorithms is more significant than chasing for an optimal solution, especially in cases of results with uncertain properties for coverage problems. As discussed in previous sections, heuristic algorithms contains many nature inspired approaches such as genetic algorithm, swarm algorithm, and etc. Another name for this kind of approaches is metaheuristic. Much work has been done and is summarized in some review papers [36,37,39].

Uncertain properties in coverage problem are studied to better model the real environment. Therefore, relying only on a simulation platform is not enough. Researchers should implement these coverages by using a sensor testbed, such as MICA2 motes [14], GreenOrbs [28], and even a real network scenario, to study the applicability of the proposed approaches and to assess their performance in real-world scenarios and large scale WSNs. How to use the testbeds efficiently should be considered in future work.

8. Conclusion

Compared with ideal and strict full coverage problems, partial or probabilistic coverage problems are more natural and helpful for describing the real-world applications. Coverage problems with uncertain properties are introduced in this survey. We first introduce a series of basic concepts, then discuss relevant models, such as detection, network, and deployment models. In particular, this survey studies three main optimization objectives, namely, maximizing coverage quality, maximizing network lifetime, and minimizing the number of sensors in coverage problem with uncertain properties. For solving these optimization objectives, we discuss corresponding basic solution strategies: deployment, scheduling or selection, and movement or adjustment of sensor parameters and giverepresentative problems and their solutions. Besides, we emphasize on the taxonomy of the involved algorithms as traditional and heuristic, approximate, distributed and centralized, and random viewpoints. We also give theoretic model, algorithm complexity, approximation ratio analyses and a simulation platform summary. It is worthwhile to emphasize that the key ideas of uncertain properties are included into our proposed research framework of coverage problems which is a useful guide for readers to study related topics about coverage problems with uncertain properties. Finally, challenging and new problems are also discussed for future research.

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References

[1] B. Wang, Coverage Control in Sensor Networks, Springer, 2010, doi:10.1007/978-1-84996-059-5.
[2] F. Santoso, Sub-optimal decentralised control algorithms for blanket and k-barrier coverage in autonomous robotic wireless sensor networks, IET Commun. 4 (17) (2010) 2041–2057.
[3] M. Schwager, D. Rus, J.J. Slotine, Unifying geometric, probabilistic, and potential field approaches to multi-robot coverage control, Int. J. Rob. Res. 30 (3) (2011) 371–383.
[4] A. Ahmed, K. Yasumoto, Y. Yamauchi, M. Ito, Probabilistic coverage methods in people-centric sensing, J. Inform. Process. Syst. 19 (2011) 473–490.
[5] D. Zhang, Y. Liu, L.M. Ni, Link-centric probabilistic coverage model for transceiver-free object detection in wireless networks, in: IEEE International Conference on Distributed Comput. Systems (ICDCS), 2010, pp. 116–125.
[6] Y. Gu, Y. Ji, J. Li, B. Zhao, An optimal algorithm for solving partial target coverage problem in wireless sensor networks, Wireless Commun. Mobile Comput. (WCMC) 13 (13) (2013) 1205–1219.
[7] A. Rossi, A. Singh, M. Sevaux, Column generation algorithm for sensor coverage scheduling under bandwidth constraints, Networks 60(3) (2012) 141–154.
[8] C. Liu, G. Cao, Distributed critical location coverage in wireless sensor networks with lifetime constraint, in: IEEE International Conference on Computer Communications (INFOCOM), 2012, pp. 1314–1322.

[9] Y. Hong, D. Kim, D. Li, B. Xu, W. Chen, A.O. Tokuta, Maximum lifetime effective-sensing partial target-coverage in camera sensor networks, in: IEEE International Symposium on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt), 2013, pp. 619–626.
[10] D. Zorbas, D. Glynos, C. Douliger, Connected partial target coverage and network lifetime in wireless sensor networks, in: IFIP conference on Wireless Days (WD), 2009, pp. 1–5.
[11] B. Carter, R. Ragade, A probabilistic model for the deployment of sensors, in: IEEE Sensors Applications Symposium (SAS), 2009, pp. 7–12.
[12] J. He, N. Xiong, Y. Xiao, Y. Pan, A reliable energy efficient algorithm for target coverage in wireless sensor networks, in: IEEE International Conference on Distributed Computing Systems (ICDCS), 2010, pp. 180–188.
[13] C.-M. Kim, Y. Kim, I.-S. Kang, K.-W. Lee, Y.-H. Han, A scheduling algorithm for connected target coverage under probabilistic coverage model, in: IEEE International Conference on Information Networking (ICOIN), 2012, pp. 86–91.
[14] G. Xing, X. Chang, C. Lu, J. Wang, K. Shen, R. Pless, J.A. O'Sullivan, Efficient coverage maintenance based on probabilistic distributed detection, IEEE Trans. Mobile Comput. (TMC) 9(9) (2010) 1346–1360.
[15] J.-W. Huang, C.-M. Hung, K.-C. Yang, J.-S. Wang, Energy-efficient probabilistic target coverage in wireless sensor networks, in: IEEE International Conference on Networks, 2011, pp. 53–58.
[16] P. Wu, X. Cao, X. Wu, G. Chen, Energy-efficient stochastic target coverage in sensor surveillance systems, in: IEEE International Conference on Computer Communications and Networks (ICCCN), 2012, pp. 1–6.
[17] D. Zorbas, T. Razafindralambo, Prolonging network lifetime under probabilistic target coverage in wireless mobile sensor networks, Comput. Commun. (CC) 36(9) (2013) 1039–1053.
[18] P. Chen, W. Hu, Sleep-wake up scheduling with probabilistic coverage model in sensor networks, Int. J. Parallel Emergent Distrib. Syst. 29 (1) (2014) 1–16.
[19] S. Adlakha, M. Srivastava, Critical density thresholds for coverage in wireless sensor networks, in: IEEE Wireless Communications and Networking Conference (WCNC), 2003, pp. 1615–1620.
[20] Y. Tian, Y. Ou, H.R. Karimi, Y. Liu, J. Han, Distributed multitarget probabilistic coverage control algorithm for wireless sensor networks, Math. Probl. Eng. (5) (2014) 1–8.
[21] D. Zhang, M. Xu, Y. Chen, S. Wang, Probabilistic coverage configuration for wireless sensor networks, in: International Conference on Wireless Communications, Networking and Mobile Computing (WiCOM), 2006, pp. 1–4.
[22] J. Chen, J. Li, S. He, Y. Sun, H.-H. Chen, Energy-efficient coverage based on probabilistic sensing model in wireless sensor networks, IEEE Commun. Lett. 14(9) (2010) 833–835.
[23] H.-L. Wang, W.-H. Chung, The generalized k-coverage under probabilistic sensing model in sensor networks, in: IEEE Wireless Communications and Networking Conference (WCNC), 2012, pp. 1737–1742.
[24] V. Akbarzadeh, C. Gagne, M. Parizeau, M. Argany, M.A. Mostafavi, Probabilistic sensing model for sensor placement optimization based on line-of-sight coverage, IEEE Trans. Instrum. Meas. 62(2) (2013) 293–303.
[25] W. Choi, G. Ghidini, S.K. Das, A novel framework for energy-efficient data gathering with random coverage in wireless sensor networks, ACM Trans. Sensor Netw. (TOSN) 8(4) (2012) 36.
[26] J. Chen, J. Li, T.H. Lai, Energy-efficient intrusion detection with a barrier of probabilistic sensors: global and local, IEEE Trans. Wireless Commun. (TWC) 12(9) (2013) 4742–4755.
[27] Q. Yang, S. He, J. Li, J. Chen, Y. Sun, Energy-efficient probabilistic full coverage in wireless sensor networks, in: IEEE Global Communications Conference (GLOBECOM), 2012, pp. 591–596.
[28] X. Liu, K. Wu, Y. Zhu, L. Kong, M. Wu, Mobility increases the surface coverage of distributed sensor networks, Comput. Netw. (ComNet) 57(11) (2013) 2348–2363.
[29] K. Shaila, M. Sajitha, V. Tejaswi, Probabilistic model for single and multi--sensing intrusion detection in wireless sensor networks, Iosrjournals Org 16 (2014) 51–66.
[30] W. Wang, V. Srinivasan, K.-C. Chua, B. Wang, Energy-efficient coverage for target detection in wireless sensor networks, in: ACM International Symposium on Information Processing in Sensor Networks, 2007, pp. 313–322.
[31] R. Tan, G. Xing, B. Liu, J. Wang, X. Jia, Exploiting data fusion to improve the coverage of wireless sensor networks, ACM/IEEE Trans. Netw. (TON) 20(2) (2012) 450–462.
[32] S. Kumar, Impact of interference on coverage in wireless sensor networks, Wireless Pers. Commun. 74 (2) (2014) 683–701.
[33] G. Takahara, K. Xu, H.S. Hassanein, Efficient coverage planning for grid-based wireless sensor networks, in: IEEE International Conference on Communications (ICC), 2007, pp. 3522–3526.
[34] Y. Zeng, C.J. Sreenan, N. Xiong, L.T. Yang, J.H. Park, Connectivity and coverage maintenance in wireless sensor networks, J. Supercomput. 52(1) (2010) 23–46.
[35] D.S. Deif, Y. Gadallah, Classification of wireless sensor networks deployment techniques, IEEE Commun. Surv. Tutorials 16 (2) (2014) 834–855.
[36] C. Tsai, P. Tsai, J. Pan, H. Chao, Metaheuristics for the deployment problem of WSN: a review, Microprocess. Microsyst. - Embedded Hardware Des. 39 (8) (2015) 1305–1317.
[37] P. Musílek, P. Krömer, T. Barton, Review of nature-inspired methods for wake-up scheduling in wireless sensor networks, Swarm Evol. Comput. 25 (2015) 100–118.
[38] B. Wang, H. Lim, D. Ma, A survey of movement strategies for improving network coverage in wireless sensor networks, Comput. Commun. (CC) 32 (13–14) (2009) 1427–1436.
[39] M.R. Senouci, A. Mellouk, K. Asnoune, F.Y. Bouhidel, Movement-assisted sensor deployment algorithms: a survey and taxonomy, IEEE Commun. Surv. Tutorials 17 (4) (2015) 2493–2510.
[40] M.A. Güvensan, A.G. Yavuz, On coverage issues in directional sensor networks: a survey, Ad Hoc Netw. (AHN) 9 (7) (2011) 1238–1255.
[41] C. Zhu, C. Zheng, L. Shu, G. Han, A survey on coverage and connectivity issues in wireless sensor networks, J. Netw. Comput. Appl. (JNCA) 35 (2) (2012) 619–632.
[42] F. Wu, Y. Gui, Z. Wang, X. Gao, G. Chen, A survey on barrier coverage with sensors, Front. Comput. Sci. 10 (6) (2016) 968–984.
[43] S. He, J. Chen, J. Li, Y. Sun, Energy-Efficient Area Coverage for Intruder Detection in Sensor Networks, Springer Briefs in Computer Science, Springer, 2014, doi:10.1007/978-3-319-04648-8.
[44] B. Wang, W. Wang, V. Srinivasan, K.C. Chua, Information coverage for wireless sensor networks, IEEE Commun. Lett. 9(11) (2005) 967–969.
[45] F. Li, R. Raskar, A. Lippman, Analysis on probabilistic view coverage for image sensing - a geometric approach, in: IEEE International Performance Computing and Communications Conference (IPCCC), 2008, pp. 271–280.
[46] X. Bai, Z. Yun, D. Xuan, T.-H. Lai, W. Jia, Optimal patterns for four-connectivity and full coverage in wireless sensor networks, IEEE Trans. Mobile Comput. (TMC) 9(3) (2010) 435–448.
[47] H. Bai, X. Chen, Y.-C. Ho, X. Guan, Percentage coverage configuration in wireless sensor networks, in: International Symposium on Parallel and Distributed Processing and Applications (ISPA), 3758, 2005, pp. 780–791.
[48] K.W. Derr, M. Manic, Wireless sensor network configuration part ii:adaptive coverage for decentralized algorithms, IEEE Trans. Industr. Inf. (TII) 9(3) (2013) 1728–1738.
[49] D. Wang, B. Xie, D.P. Agrawal, Coverage and lifetime optimization of wireless sensor networks with gaussian distribution, IEEE Trans. Mobile Comput. (TMC) 7 (12) (2008) 1444–1458.
[50] P.-J. Wan, C.-W. Yi, Coverage by randomly deployed wireless sensor networks, IEEE Trans. Inf. Theory (TIT) 52 (6) (2006) 2658–2669.
[51] W. Choi, S. K.Das, Cross: a probabilistic constrained random sensor selection scheme in wireless sensor networks, Performance Evaluation 66 (12) (2009) 754–772.
[52] Y.-C. Wang, Y.-C. Tseng, Distributed deployment schemes for mobile wireless sensor networks to ensure multi-level coverage, IEEE Trans. Parallel Distrib. Syst. (TPDS) 19(9) (2008) 1280–1294.
[53] M.O. Rahman, M.A. Razzaque, C.S. Hong, Probabilistic sensor deployment in wireless sensor network: A new approach, in: IEEE International Conference on Advanced Communication Technology, 2007, pp. 1419–1422.
[54] B. Wang, H. Xu, W. Liu, H. Liang, A novel node placement for long belt coverage in wireless networks, IEEE Transactions of Computers (TOC) 62 (12) (2013) 2341–2353.
[55] B. Wang, H. Xu, W. Liu, L.T. Yang, The optimal node placement for long belt coverage in wireless networks, IEEE Transactions of Computers (TOC) 64 (2) (2015) 587–592.
[56] J. Chen, J. Li, T.H. Lai, Trapping mobile targets in wireless sensor networks: an energy-efficient perspective, IEEE Trans. Veh. Technol. 62(7) (2013) 3287–3300.
[57] P. Balister, Z. Zheng, S. Kumar, P. Sinha, Trap coverage: allowing coverage holes of bounded diameter in wireless sensor networks, in: IEEE International Conference on Computer Communications (INFOCOM), 2009, pp. 136–144.
[58] M. Xi, K. Wu, Y. Qi, J. Zhao, Y. Liu, M. Li, Run to potential: sweep coverage in wireless sensor networks, in: IEEE International Conference on Parallel Processing (ICPP), 2009, pp. 50–57.
[59] Y. Feng, X. Gao, F. Wu, G. Chen, Shorten the trajectory of mobile sensors in sweep coverage problem, in: IEEE Global Communications Conference (GLOBECOM), 2015, pp. 1–6.
[60] X. Gao, X. Zhu, Y. Feng, F. Wu, G. Chen, Data ferry trajectory planning for sweep coverage problem with multiple mobile sensors, in: IEEE International Conference on Sensing, Communication, and Networking (SECON), 2016, pp. 1–9.
[61] Z. Chen, X. Zhu, X. Gao, F. Wu, J. Gu, G. Chen, Efficient scheduling strategies for mobile sensors in sweep coverage problem, in: IEEE International Conference on Sensing, Communication, and Networking (SECON), 2016, pp. 1–4.
[62] S. He, J. Chen, X. Li, X. Shen, Y. Sun, Cost-effective barrier coverage by mobile sensor networks, in: IEEE International Conference on Computer Communications (INFOCOM), 2012, pp. 819–827.
[63] B. Wang, K.C. Chua, V. Srinivasan, W. Wang, Information coverage in randomly deployed wireless sensor networks, IEEE Trans. Wireless Commun. (TWC) 6(8) (2007) 2994–3004.
[64] J. Lu, L. Bao, T. Suda, Probabilistic self-scheduling for coverage configuration in wireless ad-hoc sensor networks, Int. J. Pervasive Comput. Commun. 4 (1) (2008) 26–39.
[65] X. Deng, B. Wang, N. Wang, W. Liu, Y. Mo, Sensor scheduling for confident information coverage in wireless sensor networks, in: IEEE Wireless Communications and Networking Conference (WCNC), 2013, pp. 1027–1031.
[66] L. Zhang, D. Li, H. Zhu, L. Cui, Open: an optimisation scheme of n-node coverage in wireless sensor networks, IET Wireless Sens. Syst. 2(1) (2012) 40–51.
[67] G. Yang, D. Qiao, Barrier information coverage with wireless sensors, in: IEEE International Conference on Computer Communications. (INFOCOM), 2009, pp. 918–926.

[68] W. Li, C.G. Cassandras, Distributed cooperative coverage control of sensor networks, in: IEEE Conference on Decision and Control and European Control Conference, 2005, pp. 2542–2547.
[69] G. Xing, X. Wang, Y. Zhang, C. Lu, R. Pless, C.D. Gill, Integrated coverage and connectivity configuration for energy conservation in sensor networks, ACM Trans. Sensor Netw. (TOSN) 1 (1) (2005) 36–72.
[70] A. Cerpa, D. Estrin, Ascent: adaptive self-configuring sensor networks topologies, IEEE Trans. Mobile Comput. (TMC) 3 (3) (2004) 272–285.
[71] B. Chen, K. Jamieson, H. Balakrishnan, R. Morris, Span: an energy efficient coordination algorithm for topology maintenance in ad hoc wireless networks, Wireless Netw. 8 (5) (2002) 481–494.
[72] J. Heidemann, Adaptive energy-conserving routing for multihop ad hoc networks, Research Report 527, USC/Information Sciences Institute, 2000.
[73] Y. Xu, J.S. Heidemann, D. Estrin, Geography-informed energy conservation for ad hoc routing, in: ACM International Conference on Mobile Computing and Networking (MobiCom), 2001, pp. 70–84.
[74] S. Slijepcevic, M. Potkonjak, Power efficient organization of wireless sensor networks, in: IEEE International Conference on Communications (ICC), 2001, pp. 472–476.
[75] M. Hefeeda, H. Ahmadi, An integrated protocol for maintaining connectivity and coverage under probabilistic models forwireless sensor networks, Ad Hoc Sensor Wireless Netw. 7 (3-4) (2009) 295–323.
[76] S. Halder, S.D. Bit, Design of a probability density function targeting energy-efficient node deployment in wireless sensor networks, IEEE Trans. Netw. Serv. Manag. (TNSM) 11 (2) (2014) 204–219.
[77] A. Chen, S. Kumar, T.-H. Lai, Designing localized algorithms for barrier coverage, in: ACM International Conference on Mobile computing and networking (MobiCom), 2007, pp. 63–74.
[78] Y. Wu, J. Yin, M. Li, Z. En, Z. Xie, Efficient algorithms for probabilistic k-coverage in directional sensor networks, in: IEEE International Conference on Intelligent Sensors, Sensor Networks and Information Processing (ISSNIP), 2008, pp. 587–592.
[79] S. Kumar, T.-H. Lai, A. Arora, Barrier coverage with wireless sensors, in: ACM International Conference on Mobile Computing and Networking (MobiCom), 2005, pp. 284–298.
[80] H. Wennerström, C. Rohner, Towards even coverage monitoring with opportunistic sensor networks, in: ACM Workshop on Challenged Networks, CHANTS@MobiCom, 2016, pp. 81–86.
[81] Y. Wang, X. Wang, B. Xie, D. Wang, D.P. Agrawal, Intrusion detection in homogeneous and heterogeneous wireless sensor networks, IEEE Trans. Mobile Comput. (TMC) 7 (6) (2008) 698–711.
[82] A. Chamam, S. Pierre, On the planning of wireless sensor networks: energy-efficient clustering under the joint routing and coverage constraint, IEEE Trans. Mobile Comput. (TMC) 8 (8) (2009) 1077–1086.
[83] S. He, J. Chen, X. Li, X.S. Shen, Y. Sun, Mobility and intruder prior information improving the barrier coverage of sparse sensor networks, IEEE Trans. Mobile Comput. (TMC) 13 (6) (2014) 1268–1282.
[84] P.K. Sahoo, W.-C. Liao, Hora: a distributed coverage hole repair algorithm for wireless sensor networks, IEEE Trans. Mobile Comput. (TMC) 14 (7) (2015) 1397–1410.
[85] H. Xiong, D. Zhang, L. Wang, H. Chaouchi, Emc^3: energy-efficient data transfer in mobile crowdsensing under full coverage constraint, IEEE Trans. Mobile Comput. (TMC) 14 (7) (2015) 1355–1368.
[86] J. Zhu, B. Wang, The optimal placement pattern for confident information coverage in wireless sensor networks, IEEE Trans. Mobile Comput. (TMC) 15 (4) (2016) 1022–1032.
[87] Y. Wang, G. Cao, Barrier coverage in camera sensor networks, in: ACM/IEEE International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc), 2011, pp. 3967–3974.
[88] M. Hefeeda, H. Ahmadi, Energy-efficient protocol for deterministic and probabilistic coverage in sensor networks, IEEE Trans. Parallel Distrib. Syst. (TPDS) 21(5) (2010) 579–593.
[89] Y. Li, C.T. Vu, C. Ai, G. Chen, Y. Zhao, Transforming complete coverage algorithms to partial coverage algorithms for wireless sensor networks, IEEE Trans. Parallel Distrib. Syst. (TPDS) 22(4) (2011) 695–703.
[90] L. Liu, H. Ma, On coverage of wireless sensor networks for rolling terrains, IEEE Trans. Parallel Distrib. Syst. (TPDS) 23(1) (2012) 118–125.
[91] B. Liu, O. Dousse, P. Nain, D. Towsley, Dynamic coverage of mobile sensor networks, IEEE Trans. Parallel Distrib. Syst. (TPDS) 24(2) (2013) 301–311.
[92] X. Bai, Z. Yun, D. Xuan, B. Chen, W. Zhao, Optimal multiple-coverage of sensor networks, in: IEEE International Conference on Computer Communications (INFOCOM), 2011, pp. 2498–2506.
[93] Z. Wang, H. Chen, Q. Cao, H. Qi, Z. Wang, Fault tolerant barrier coverage for wireless sensor networks, in: IEEE International Conference on Computer Communications (INFOCOM), 2014, pp. 1869–1877.
[94] Z. Yu, F. Yang, J. Teng, A.C. Champion, D. Xuan, Local face-view barrier coverage in camera sensor networks, in: IEEE International Conference on Computer Communications (INFOCOM), 2015, pp. 684–692.
[95] J.P. Rula, F.E. Bustamante, Crowdsensing under (soft) control, in: IEEE Conference on Computer Communications (INFOCOM), 2015, pp. 2236–2244.
[96] Z. Chen, X. Gao, F. Wu, G. Chen, A ptas to minimize mobile sensor movement for target coverage problem, in: IEEE International Conference on Computer Communications (INFOCOM), 2016, pp. 1–9.
[97] G. S.Kasbekar, Y. Bejerano, S. Sarkar, Generic coverage verification without location information using dimension reduction, ACM/IEEE Trans. Netw. (TON) 20(6) (2012) 1991–2004.
[98] X. Gong, J. Zhang, D. Cochran, K. Xing, Optimal placement for barrier coverage in bistatic radar sensor networks, ACM/IEEE Trans. Netw. (TON) 24 (1) (2016) 259–271.
[99] P. Zhang, W. Chen, X. Sun, Y. Wang, J. Zhang, Minimizing seed set selection with probabilistic coverage guarantee in a social network, in: ACM International Conference on Knowledge Discovery and Data Mining (SIGKDD), 2014, pp. 1306–1315.
[100] B. Wang, J. Chen, W. Liu, L.T. Yang, Minimum cost placement of bistatic radar sensors for belt barrier coverage, IEEE Trans. Comput. (TOC) 65 (2) (2016) 577–588.
[101] D. Zhang, H. Xiong, L. Wang, G. Chen, Crowdrecruiter: selecting participants for piggyback crowdsensing under probabilistic coverage constraint, in: ACM Conference on Ubiquitous Computing (UbiComp), 2014, pp. 703–714.
[102] H.M. Ammari, On the problem of k-coverage in mission-oriented mobile wireless sensor networks, Comput. Netw. (ComNet) 56(7) (2012) 1935–1950.
[103] W. Li, Y. Wu, Tree-based coverage hole detection and healing method in wireless sensor networks, Comput. Netw. (ComNet) 103 (2016) 33–43.
[104] F. Li, J. Luo, S. Xin, Y. He, Autonomous deployment of wireless sensor networks for optimal coverage with directional sensing model, Comput. Netw. (ComNet) 108 (2016) 120–132.
[105] H. Chintakunta, H. Krim, Divide and conquer: localizing coverage holes in sensor networks, in: IEEE Communications Society Conference on Sensor, Mesh and Ad Hoc Communications and Networks (SECON), 2010, pp. 359–366.
[106] C.-F. Huang, Y.-C. Tseng, H.-L. Wu, Distributed protocols for ensuring both coverage and connectivity of a wireless sensor network, ACM Trans. Sensor Netw. (TOSN) 3 (1) (2007) 319–321.
[107] T.P. Lambrou, Optimized cooperative dynamic coverage in mixed sensor networks, ACM Trans. Sensor Netw. (TOSN) 11 (3) (2015) 46:1–46:35.
[108] H.M. Ammari, J. Giudici, On the connected k-coverage problem in heterogeneous sensor nets the curse of randomness and heterogeneity, in: IEEE International Conference on Distributed Computing Systems (ICDCS), 2009, pp. 265–272.
[109] X. Gao, R. Yang, F. Wu, G. Chen, J. Zhou, Optimization of full-view barrier coverage with rotatable camera sensors, in: IEEE International Conference on Distributed Computing (ICDCS), 2017.
[110] H.P. Gupta, S.V. Rao, T. Venkates, Critical sensor density for partial coverage under border effects in wireless sensor networks, IEEE Trans. Wireless Commun. (TWC) 13(5) (2014) 2374–2382.
[111] B. Wang, J. Zhu, L.T. Yang, Y. Mo, Sensor density for confident information coverage in randomly deployed sensor networks, IEEE Trans. Wireless Commun. (TWC) 15 (5) (2016) 3238–3250.
[112] K. Shi, H. Chen, Y. Lin, Probabilistic coverage based sensor scheduling for target tracking sensor networks, Inf. Sci. (IS) 292 (2015) 95–110.
[113] S. Ren, Q. Li, H. Wang, X. Chen, X. Zhang, Analyzing object detection quality under probabilistic coverage in sensor networks, in: IEEE International Workshop on Quality of Service (IWQoS), 2005, pp. 107–122.
[114] D. Wang, J. Liu, Q. Zhang, Probabilistic field coverage using a hybrid network of static and mobile sensors, in: IEEE International Workshop on Quality of Service (IWQoS), 2007, pp. 56–64.
[115] S.A.R. Zaidi, M. Hafeez, D.C. McLernon, M. Ghogho, A probabilistic model of k-coverage in minimum cost wireless sensor networks, in: ACM Conference on Emerging Network Experiment and Technology (CoNEXT), 2008, p. 33.
[116] H.P. Gupta, S.V. Rao, V. Tamarapalli, Analysis of stochastic k-coverage and connectivity in sensor networks with boundary deployment, IEEE Trans. Intell. Transp. Syst. (TITS) 16 (4) (2015) 1861–1871.
[117] Y. Wang, W. Chu, Z. Xiao, Y. Zhang, Partial sensing coverage in 3d wireless lattice sensor networks, in: IEEE International Conference on Communications (ICC), 2014, pp. 24–29.
[118] R. Wang, S. He, J. Chen, Z. Shi, F. Hou, Energy-efficient barrier coverage in bistatic radar sensor networks, in: IEEE International Conference on Communications (ICC), 2015, pp. 6743–6748.
[119] H. Mostafaei, Stochastic barrier coverage in wireless sensor networks based on distributed learning automata, Comput. Commun. (CC) 55 (2015) 51–61.
[120] S. Gao, X. Wang, Y. Li, p-percent coverage schedule in wireless sensor networks, in: IEEE International Conference on Computer Communications and Networks(ICCCN), 2008, pp. 548–553.
[121] J. Zhu, B. Wang, Sensor placement algorithms for confident information coverage in wireless sensor networks, in: IEEE International Conference on Computer Communication and Networks (ICCCN), 2014, pp. 1–7.
[122] H.-C. Ma, P.K. Sahoo, Y.-W. Chen, Computational geometry based distributed coverage hole detection protocol for the wireless sensor networks, J. Netw. Comput. Appl. (JNCA) 34(5) (2011) 1743–1756.
[123] T.-W. Sung, C.-S. Yang, Voronoi-based coverage improvement approach for wireless directional sensor networks, J. Netw. Comput. Appl. (JNCA) 39 (2014) 202–213.
[124] M. Vecchio, R. Lopez-Valcarce, Improving area coverage of wireless sensor networks via controllable mobile nodes: a greedy approach, J. Netw. Comput. Appl. (JNCA) 48 (2015) 1–13.
[125] F. Carrabs, R. Cerulli, C. D'Ambrosio, A. Raiconi, A hybrid exact approach for maximizing lifetime in sensor networks with complete and partial coverage constraints, J. Netw. Comput. Appl. (JNCA) 58 (2015) 12–22.
[126] J. Chen, B. Wang, W. Liu, Constructing perimeter barrier coverage with bistatic radar sensors, J. Netw. Comput. Appl. (JNCA) 57 (2015) 129–141.

[127] Y. Xiang, Z. Xuan, M. Tang, J. Zhang, M. Sun, 3D space detection and coverage of wireless sensor network based on spatial correlation, J. Netw. Comput. Appl. (JNCA) 61 (2016) 93–101.
[128] J.-P. Sheu, H.-F. Lin, Probabilistic coverage preserving protocol with energy efficiency in wireless sensor networks, in: IEEE Wireless Communications and Networking Conference (WCNC), 2007, pp. 2631–2636.
[129] Y. Tian, S. Zhang, Y. Wang, A distributed protocol for ensuring both probabilistic coverage and connectivity of high density wireless sensor networks, in: IEEE Wireless Communications and Networking Conference (WCNC), 2008, pp. 2069–2074.
[130] Y. Li, Y. Song, Y.-H. Zhu, R. Schott, Deploying wireless sensors for differentiated coverage and probabilistic connectivity, in: IEEE Wireless Communications and Networking Conference (WCNC), 2010, pp. 1–6.
[131] D. Zhao, H. Ma, L. Liu, Mobile sensor scheduling for timely sweep coverage, in: IEEE Wireless Communications and Networking Conference (WCNC), 2012, pp. 1771–1776.
[132] F. Castaño, E. Bourreau, A. Rossi, M. Sevaux, N. Velasco, Partial target coverage to extend the lifetime in wireless multi-role sensor networks, Networks 68 (1) (2016) 34–53.
[133] T. Wimalajeewa, S.K. Jayaweera, A novel distributed mobility protocol for dynamic coverage in sensor networks, in: IEEE Global Telecommunications Conference (GLOBECOM), 2010, pp. 1–5.
[134] Y. Feng, P. Martins, L. Decreusefond, Connectivity-based distributed coverage hole detection in wireless sensor networks, in: IEEE Global Telecommunications Conference (GLOBECOM), 2011, pp. 1–6.
[135] X. Fan, V.O.K. Li, The probabilistic maximum coverage problem in social networks, in: IEEE Global Telecommunications Conference (GLOBECOM), 2011, pp. 1–5.
[136] J. Wang, X. Zhang, 3d percolation theory-based exposure-path prevention for optimal power-coverage tradeoff in clustered wireless camera sensor networks, in: IEEE Global Communications Conference (GLOBECOM), 2014, pp. 305–310.
[137] P. Medagliani, J. Leguay, G. Ferrari, V. Gay, M. Lopez-Ramo, Energy-efficient mobile target detection in wireless sensor networks with random node deployment and partial coverage, Pervasive Mobile Comput. (PMC) 8 (3) (2012) 429–447.
[138] C. Yang, W. Zhu, J. Liu, L. Chen, D. Chen, J. Cao, Self-orienting the cameras for maximizing the view-coverage ratio in camera sensor networks, Pervasive Mobile Comput. (PMC) 17 (2015) 102–121.
[139] H.P. Gupta, S.V. Rao, T. Venkatesh, Analysis of stochastic coverage and connectivity in three-dimensional heterogeneous directional wireless sensor networks, Pervasive Mobile Comput. (PMC) 29 (2016) 38–56.
[140] Y. Li, C. Ai, Z. Cai, R. Beyah, Sensor scheduling for p-percent coverage in wireless sensor networks, in: International Conference on Wireless Algorithms, Systems, and Applications (WASA), 2008, pp. 27–40.
[141] B. Xu, D. Kim, D. Li, J. Lee, H. Jiang, A.O. Tokuta, Fortifying barrier-coverage of wireless sensor network with mobile sensor nodes, in: International Conference on Wireless Algorithms, Systems, and Applications (WASA), 2014, pp. 368–377.
[142] Y. Wang, S. Wu, X. Gao, F. Wu, G. Chen, Efficient line k-coverage algorithms in mobile sensor network, in: International Conference on Wireless Algorithms, Systems, and Applications (WASA), 2015, pp. 581–591.
[143] A. Ahmadi, M. Shojafar, S.F. Hajeforosh, M. Dehghan, M. Singhal, An efficient routing algorithm to preserve k-coverage in wireless sensor networks, J. Supercomput. 68 (2) (2014) 599–623.
[144] Y. Mao, Z. Wang, Y. Liang, Energy aware partial coverage protocol in wireless sensor networks, in: International Conference on Wireless Communications, Networking and Mobile Computing (WiCOM), 2007, pp. 382–394.
[145] Y. Tian, Y. Wang, S.-F. Zhang, A distributed probabilistic-coverage node scheduling scheme for large wireless sensor networks, in: International Conference on Wireless Communications, Networking and Mobile Computing (WiCOM), 2007, pp. 2464–2468.
[146] D. Wang, Q. Qin, J. Yu, The algorithm of coverage base on probability sensing model in wireless sensor network, in: International Conference on Wireless Communications, Networking and Mobile Computing (WiCOM), 2012, pp. 1–4.
[147] L. Li, B. Zhang, J. Zheng, A study on one-dimensional k-coverage problem in wireless sensor networks, Wireless Commun. Mobile Comput. (WCMC) 13 (1) (2013) 1–11.
[148] W. An, N. Qu, F.-M. Shao, X. Xiong, S. Ci, Coverage hole problem under sensing topology in flat wireless sensor networks, Wireless Commun. Mobile Comput. (WCMC) 16 (5) (2016) 578–589.
[149] F. Senel, Coverage-aware connectivity-constrained unattended sensor deployment in underwater acoustic sensor networks, Wireless Commun. Mobile Comput. (WCMC) 16 (14) (2016) 2052–2064.
[150] P.A. Birjandi, L. Kulik, E. Tanin, K-coverage in regular deterministic sensor deployments, in: IEEE International Conference on Intelligent Sensors, Sensor Networks and Information Processing (ISSNIP), 2013, pp. 521–526.
[151] T. Yardibi, E. Karasan, A distributed activity scheduling algorithm for wireless sensor networks with partial coverage, Wireless Netw. 16(1) (2010) 213–225.
[152] D. Zhao, H. Ma, L. Liu, Energy-efficient opportunistic coverage for people-centric urban sensing, Wireless Netw. 20 (6) (2014) 1461–1476.
[153] B. Xu, Y. Zhu, D. Kim, D. Li, H. Jiang, A.O. Tokuta, Strengthening barrier-coverage of static sensor network with mobile sensor nodes, Wireless Networks 22 (1) (2016) 1–10.
[154] P.K. Sahoo, J.-Z. Tsai, H.-L. Ke, Vector method based coverage hole recovery in wireless sensor networks, in: IEEE International Conference on Communication Systems and Networks (COMSNETS), 2010, pp. 1–9.
[155] G.H. Prabhat, S.V. Rao, T. Venkatesh, Sleep scheduling for partial coverage in heterogeneous wireless sensor networks, in: IEEE International Conference on Communication Systems and Networks (COMSNETS), 2013, pp. 1–10.
[156] C. Lee, D. Shin, S.W. Bae, S. Choi, Best and worst-case coverage problems for arbitrary paths in wireless sensor networks, Ad Hoc Netw. 11 (6) (2013) 1699–1714.
[157] H. Chizari, T. Poston, S.A. Razak, A.H. Abdullah, S. Salleh, Local coverage measurement algorithm in gps-free wireless sensor networks, Ad Hoc Netw. 23 (2014) 1–17.
[158] Z. Wang, B. Wang, A novel node sinking algorithm for 3d coverage and connectivity in underwater sensor networks, Ad Hoc Netw. (AHN) 56 (2017) 43–55.
[159] I. Demirkol, F. Alagoz, H. Delic, C. Ersoy, Wireless sensor networks for intrusion detection: packet traffic modeling, IEEE Commun. Lett. 10(1) (2006) 22–24.
[160] X. Wang, S. Wang, J. Ma, An improved co-evolutionary particle swarm optimization for wireless sensor networks with dynamic deployment, Sensors 7(3) (2007) 354–370.
[161] O. Celal, D. Karaboga, B. Gorkemli, Probabilistic dynamic deployment of wireless sensor networks by artificial bee colony algorithm, Sensors 11(6) (2011) 6056–6065.
[162] A. Shan, X. Xu, Z. Cheng, Target coverage in wireless sensor networks with probabilistic sensors, Sensors 16 (9) (2016) 1372.
[163] A. Hossain, S. Chakrabarti, P.K. Biswas, Impact of sensing model on wireless sensor network coverage, IET Wireless Sens. Syst. 2 (3) (2012) 272–281.
[164] N. Ahmed, S.S. Kanhere, S. Jha, Ensuring area coverage in hybrid wireless sensor networks, in: IEEE International Conference on Mobile Ad-Hoc and Sensor Networks (MSN), 2007, pp. 548–560.
[165] G. Zhang, K. Sivan, F.P.S. Chin, C.C. Ko, Quality of service and coverage optimization in wireless sensor networks, in: IEEE Vehicular Technology Conference (VTC), 2008, pp. 2824–2828.
[166] J. Zhang, R. Wang, Y. Qian, Q. Wang, A coverage control algorithm based on probability model for three-dimensional wireless sensor networks, in: IEEE International Symposium on Distributed Computing and Applications to Business, Engineering & Science (DCABES), 2012, pp. 169–173.
[167] J. Lu, T. Suda, Coverage-aware self-scheduling in sensor networks, in: IEEE Annual Workshop on Computer Communications, 2003, pp. 117–123.
[168] J. Lu, L. Bao, T. Suda, Coverage-aware sensor engagement in dense sensor networks, J. Embedded Comput. 3 (1) (2009) 3–18.
[169] S. Ren, Q. Li, H. Wang, X. Chen, X. Zhang, Design and analysis of sensing scheduling algorithms under partial coverage for object detection in sensor networks, IEEE Trans. Parallel Distrib. Syst. (TPDS) 18(3) (2007) 334–350.
[170] Z. Xiao, M. Huang, J. Shi, J. Yang, J. Peng, Full connectivity and probabilistic coverage in random deployed 3d wsns, in: IEEE International Conference on Wireless Communications & Signal Processing (WCSP), 2009, pp. 1–4.
[171] S. Temel, N. Unaldi, O. Kaynak, On deployment of wireless sensors on 3-d terrains to maximize sensing coverage by utilizing cat swarm optimization with wavelet transform, IEEE Trans. Syst. Man Cybern. Syst. 44(1) (2014) 111–120.
[172] N. Hayashi, T. Ushio, T. Kanazawa, Potential game theoretic approach to power-aware mobile sensor coverage problem, IEICE Trans. Fundam. Electron. Commun. Comput. Sci. 94-A (3) (2011) 929–936.
[173] J. Zhang, S. Li, Q. Li, Y. Liu, N. Wu, Coverage hole recovery algorithm based on perceived probability in heterogeneous wireless sensor network, J. Comput. Inf. Syst. 10 (7) (2014) 2983–2990.
[174] J. Chen, X. Wang, The coverage problem in heterogeneous wireless sensor network: an improved algorithm of virtual forces, in: Chinese Intelligent Automation Conference, 255, 2013, pp. 619–630.
[175] Y.-N. Guo, M. Chen, C. Wang, Multi-objective quantum cultural algorithm and its application in the wireless sensor networks' energy-efficient coverage optimization, in: International Conference on Intelligent Data Engineering and Automated Learning (IDEAL), 2013, pp. 161–167.
[176] F. Shen, C. Liu, J. Zhang, A distributed coverage-aware sleep scheduling algorithm for wireless sensor networks, in: International Conference on Information Technology: New Generations (ITNG), 2009, pp. 524–527.
[177] H. Sun, J. Zhao, Application of particle sharing based particle swarm frog leaping hybrid optimization algorithm in wireless sensor network coverage optimization, J. Inf. Comput. Sci. 8(14) (2011) 3181–3188.
[178] I.K. Altinel, N. Aras, E. Güney, C. Ersoy, Binary integer programming formulation and heuristics for differentiated coverage in heterogeneous sensor networks, Comput. Netw. (ComNet) 52 (12) (2008) 2419–2431.
[179] Y. Kantaros, M.M. Zavlanos, Distributed communication-aware coverage control by mobile sensor networks, Automatica 63 (2016) 209–220.
[180] X. Xu, X. Zhang, L. Wang, Simulating energy efficient wireless sensor networks using cellular automata, in: IEEE Winter Simulation Conference (WSC), 2011, pp. 3207–3216.
[181] H.O. Sanli, H. Cam, Collaborative event-driven coverage and rate allocation for event miss-ratio assurances in wireless sensor networks, EURASIP J. Wireless Commun. Netw. 2010 (2) (2010). 50–50
[182] A. Chamam, S. Pierre, Dynamic sensor activation for maximizing network lifetime under coverage constraint, in: IEEE International Symposium on Communications and Information Technologies (ISCIT), 2007, pp. 971–976.

[183] Z.M. Radani, S. Samavi, F. Fooladgar, Multi plane volumetric coverage in wireless visual sensor network, in: IEEE Iranian Conference on Electrical Engineering (ICEE), 2012, pp. 758–762.

[184] Y.-L. Lai, J.-R. Jiang, Barrier coverage with optimized quality for wireless sensor networks, in: IEEE International Symposium on Wireless Personal Multimedia Communications, 2012, pp. 192–196.

[185] X.-H. Deng, C.-G. Xu, F.-Y. Zhao, Y. Liu, Repair policies of coverage holes based dynamic node activation in wireless sensor networks, in: IEEE/IFIP International Conference on Embedded and Ubiquitous Computing (EUC), 2010, pp. 368–371.

[186] A. Ou, T. Yang, J. Yu, J. Wu, Y. Wu, Modeling wireless sensor network coverage based on random radius, in: International Congress on Image and Signal Processing (CISP), 5, 2011, pp. 2603–2606.

[187] F. Li, S. Xiong, L. Wang, Recovering coverage holes by using mobile sensors in wireless sensor networks, in: IEEE International Conference on Computational Intelligence and Security (CIS), 2011, pp. 746–749.

[188] A.K. Sagar, D.K. Lobiyal, Mobility based energy efficient coverage hole maintenance for wireless sensor network, in: International Conference on Quality, Reliability, Security and Robustness in Heterogeneous Networks, 2013, pp. 115–127.

[189] S. Mini, S.K. Udgata, S.L. Sabat, Sensor deployment for probabilistic target k–coverage using artificial bee colony algorithm, in: International Conference on Swarm, Evolutionary, and Memetic Computing (SEMCCO), Springer Berlin Heidelberg, 2011, pp. 654–661.

[190] Y. Hu, M. Xiao, A. Liu, R. Cheng, H. Mao, Nearly optimal probabilistic coverage for roadside data dissemination in urban vanets, in: International Workshops on Database Systems for Advanced Applications (DASFAA), 2016, pp. 238–253.

[191] M. Abo-Zahhad, N. Sabor, S. Sasaki, S.M. Ahmed, A centralized immune-voronoi deployment algorithm for coverage maximization and energy conservation in mobile wireless sensor networks, Inf. Fusion 30 (2016) 36–51.

[192] J. Habibi, H. Mahboubi, A.G. Aghdam, Distributed coverage control of mobile sensor networks subject to measurement error, IEEE Trans. Autom. Control (TAC) 61 (11) (2016) 3330–3343.

[193] J. Yu, S. Ren, S. Wan, D. Yu, G. Wang, A stochastic k-coverage scheduling algorithm in wireless sensor networks, Int. J. Distrib. Sens. Netw. 2012 (2012) 178–193.

[194] F. Castano, A. Rossi, M. Sevaux, N. Velasco, A column generation approach to extend lifetime in wireless sensor networks with coverage and connectivity constraints, Comput. Oper. Res. 52 (2014) 220–230.

[195] D.T.H. Ly, N.T. Hanh, H.T.T. Binh, N.D. Nghia, An improved genetic algorithm for maximizing area coverage in wireless sensor networks, in: ACM International Symposium on Information and Communication Technology, 2015, pp. 61–66.

[196] J. Jia, J. Chen, G. Chang, Y. Wen, J. Song, Multi-objective optimization for coverage control in wireless sensor network with adjustable sensing radius, Comput. Math. Appl. 57 (11-12) (2009) 1767–1775.

[197] L. Wang, J.-Y. Yang, Y.-Y. Lin, W.-J. Lin, Keeping desired qos by a partial coverage algorithm for cluster-based wireless sensor networks, J. Netw. 9 (12) (2014) 3221–3229.

[198] Y. Wang, S. Wu, X. Gao, F. Wu, G. Chen, Minimizing mobile sensor movements to form a line k-coverage, Peer-to-Peer Netw. Appl. (PPNA) (2016) 1–6.

[199] V.V. Vazirani, Approximation algorithms, Springer, 2001.

[200] D.-Z. Du, K.-I. Ko, X. Hu, Design and analysis of approximation algorithms, 62, Springer-Verlag GmbH, 2011.

[201] B. Wang, X. Deng, W. Liu, L.T. Yang, Confident information coverage in sensor networks for field reconstruction, IEEE Wireless Commun. 20 (6) (2013) 74–81.

[202] M. Noori, S. Movaghati, M. Ardakani, Characterizing the path coverage of random wireless sensor networks, EURASIP J. Wireless Commun. Netw. 4 (2) (2010). 50–50

[203] S. Ren, Q. Li, H. Wang, X. Chen, X. Zhang, A study on object tracking quality under probabilistic coverage in sensor networks, ACM Sigmobile Mobile Comput. Commun. Rev. 9 (1) (2005) 73–76.

[204] S. Ferrari, G. Zhang, T.A. Wettergren, Probabilistic track coverage in cooperative sensor networks, IEEE Trans. Syst., Man, Cybern.–Part B 40 (6) (2011) 1492–1504.

[205] H. She, Z. Lu, A. Jantsch, D. Zhou, L.-R. Zheng, Stochastic coverage in event–driven sensor networks, in: IEEE International Symposium on Personal Indoor and Mobile Radio Communications (PIMRC), 2011, pp. 915–919.

[206] S.E. Nezhad, H.J. Kamali, M.E. Moghaddam, Solving k-coverage problem in wireless sensor networks using improved harmony search, in: International Confrence on Broadband and Wireless Computing,Communication and Applications (BWCCA), 2010, pp. 49–55.

[207] R. Kershner, The number of circles covering a set, Am. J. Math. 61(1) (1938) 665–671.

[208] L. Atzori, A. Iera, G. Morabito, The internet of things: a survey, Comput. Netw. (ComNet) 54 (15) (2010) 2787–2805.

[209] J. Gubbi, R. Buyya, S. Marusic, M. Palaniswami, Internet of things (iot): a vision, architectural elements, and future directions, Future Gen. Comput. Syst. 29 (7) (2013) 1645–1660.

[210] K. Han, C. Chen, Q. Zhao, X. Guan, Trajectory-based node selection scheme in vehicular crowdsensing, in: IEEE/CIC International Conference on Communications (ICCC), 2015, pp. 1–6.

[211] H. Xiong, D. Zhang, G. Chen, L. Wang, V. Gauthier, Crowdtasker: Maximizing coverage quality in piggyback crowdsensing under budget constraint, in: IEEE International Conference on Pervasive Computing and Communications (PerCom), 2015, pp. 55–62.

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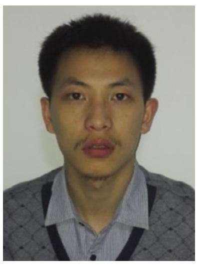

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