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Wireless networking, dominating and packing

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Abstract Recently, there are many papers published in the literature on study of dominating sets, connected dominating sets and their variations motivated from various applications in wireless networks and social networks. In this article, we survey those developments for wireless networking, dominating, and packing problems.

Keywords Wireless network · Dominating set · Packing

1 Introduction

Recently, there is a news at **AP NewsBreak** "Airspeed systems failed on US planes" which contains the following statements:

> On at least a dozen recent flights by U.S. jetliners, **malfunctioning equipment made it impossible for pilots to know how fast they were flying, federal investigators have discovered**. A similar breakdown is believed to have played a role in the **Air France crash into the Atlantic that killed all 228 people aboard in June**. (June, 2009)

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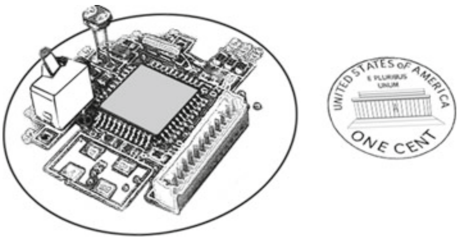

Fig. 1 An example of a sensor (compared with a cent)

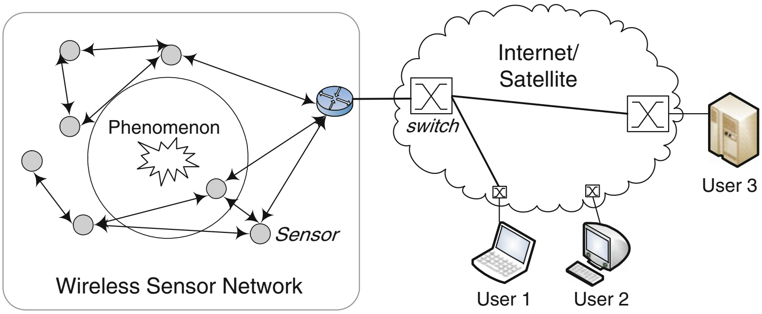

Fig. 2 A sample wireless sensor network

While a car's speedometer uses tire rotation **to calculate speed, an airplane relies on sensors** known as Pitot tubes to measure changing air pressure.

Like the fatal Air France flight, the newly discovered Northwest incidents and the two other malfunctions under investigation all involved planes with sensors made by the European electronics giant Thales Corp. **The Air France crash called into question the reliability of the sensors and touched off a rush to replace them.**

So, 228 people were killed by failure of sensors which measure the speed of airplane. This surprising news made sensor an important issue in our life. Nowadays, sensors are existing everywhere in our dairy life. It is usually a small but not very expensive device with three functions: sensing, computation and communication (see Fig. 1 as an example).

In this article, we mainly study sensor communication. There are two types of communication: wired and wireless. Obviously, the communication between sensors in an airplane is wired communication. However, what we are interested in this article is wireless communication, that is, all sensors form a *wireless sensor network* (see Fig. 2 as an example).

The wireless sensor network is widely applied in battle field (Fig. 3), environmental monitoring, agriculture, healthcare industry, biological system (Fig. 4), and traffic control, etc.

The sensors in the network are incorporated with integrated circuits to provide networking capability. The wireless sensor network is essentially an ad hoc wireless

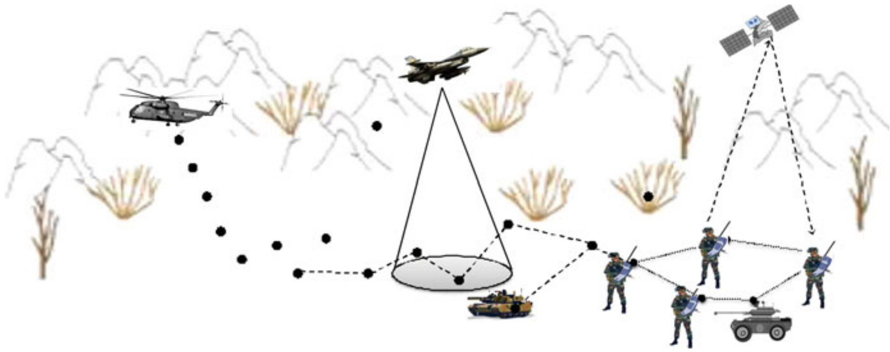

Fig. 3 Sensor application in a battle field control

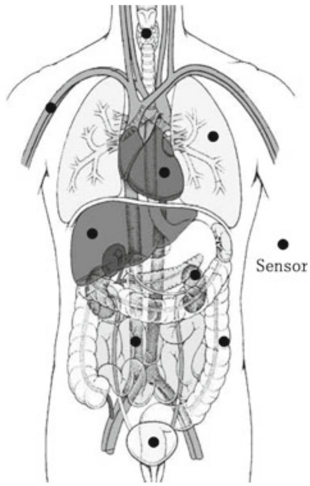

Fig. 4 Sensor application in biological system

network which is composed of many sensors, possibly of many types. Usually, there is no central administration in the network, and the network is hardware-infrastructureless. Therefore, in the wireless sensor network, each sensor is not only a mobile host but also a router. In other words, the sensors are able to forward the received data packages according to routing protocols.

Although the wireless network is quite different from wired network, and the routing protocols in the wired network can not be applied directly in the wireless network, some technologies of wired networks are still simulated by wireless networks in their developments [35]. One of such technologies is *backbone* [15], which can be used for reducing the communication overhead and increasing network reliability. In the wireless sensor network, the corresponding simulation is called a *virtual backbone*, which plays a more important role in wireless communication [3,32,34,36,42]. A virtual backbone is a *connected dominating set* in the network. That is, it is a subset of sensors such that they form a connected sub-network and every sensor is either in the subset or adjacent to a sensor in this subset. An additional advantage of virtual

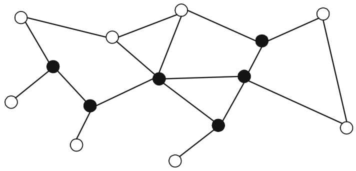

Fig. 5 A virtual backbone in wireless network

backbone in wireless network is the saving of storage. Without virtual backbone, routing table has to be stored in every sensor node while with virtual backbone, only sensor nodes in the virtual backbone needs to store routing tables. An example is shown in Fig. 5, in which black nodes form a virtual backbone.

Dominating sets and connected dominating sets are traditional research subjects in graph theory [33]. Say, given a graph $G = (V, E)$, a subset $D \subseteq V$ of nodes is called a *dominating set* if every node not in D has a neighbor in D. Furthermore, D is called a *connected dominating set* if the subgraph induced by D is connected. In graph theory, the minimum size of (connected) dominating set in G is called the *(connected) domination number* of G. Determination of the domination number and the connected domination number in various graphs have attracted many research efforts [6,11], since computing the dominating number and the connected dominating number in general graph is NP-hard [21].

When the connected dominating set plays a role of virtual backbone in wireless networks, it is also naive to ask for the minimum cardinality. Computing the minimum connected dominating set becomes a hot research topic in both theoretical computer science and computer network due to its impact in wireless communication.

Guha and Khuller [22] gave a two-stage greedy approximation within a factor of $3 + \ln \delta$ from optimal for the minimum connected dominating set where δ is the maximum node degree of input graph. They also showed that this is almost the best possible result, i.e., there is no polynomial-time approximation for the minimum connected dominating set within a factor of $\rho \ln \delta$ from optimal for any $0 < \rho < 1$ unless $NP \subseteq DTIME(n^{O(\log \log n)})$. This lower bound result is obtained by a transformation from the result of Feige [16] on the set-cover problem. Ruan et al. [31] improved upper bound from $3 + \ln \delta$ to $2 + \ln \delta$. Du et al. [13] showed a tight upper bound that the minimum connected dominating set in general graph can be polynomial-time approximated within a factor of $\alpha(1 + \ln \delta)$ from optimal for any $0 < \alpha < 1$.

While the research on the determination of the dominating number and the connected dominating number is still going on in combinatorics group, the computer science people move their attention more and more to compute approximation solution for those mathematical models of wireless networks. In this article, we are going to survey those developments in the direction to approximation solution, especially in graphs which are mathematical models of wireless sensor networks.

2 Unit disk graphs

The simplest mathematical model for wireless sensor networks is the unit disk graph. A unit disk is a disk with diameter one. A *unit disk graph* is associated with a set of unit disks in the Euclidean plane. Each node is the center of a unit disk. An edge exists between two nodes u and v if and only if disks associated with u and v intersects each other. That is, if and only if the distance between u and v is at most one.

When all sensors are identical, they have the same communication range and two sensor nodes are able to communicate each other if and only if their distance is within the communication range, which is a disk with certain radius. Therefore, it is isomorphic to a unit disk graph.

Computing the minimum connected dominating set in unit disk graphs is still NP-hard [10]. However, computing approximation solutions is not as hard as in general graphs. The first polynomial-time constant-approximation is obtained by Wan et al. [37]. The most important part of this work is the frame of their approximation solution. Indeed, it is followed later on by almost all approximation solutions [5,17,18,26,28,38] for the minimum connected dominating set in unit disk graphs with good performance which is provable in theory and comparable in computational experiments.

This frame consists of two stages. In the first stage, construct a dominating set and in the second stage, connect the dominating set into a connected dominating set.

For construction of a dominating set, the best-known easy way is to build a maximal independent set. This is based on the fact that every maximal independent set is a dominating set and can be computed as follows: In each iteration, choose an uncolored node, color it in black and its uncolored neighbors in grey until no uncolored node is left.

Since the maximal independent set is a part of approximation solution, estimating the size of a maximal independent set would be a very important part of analysis of this type of approximation. Consider a minimum connected dominating set C^*. For each node $u \in C^*$, let us construct a disk with center u with radius one, called *dominating disk* of u. Then all nodes should lie in the union of dominating disks of nodes in C^* since C^* is a dominating set. All nodes in a maximal independent set are away from each other with distance larger than one.

Then order nodes in $C^* = \{x_1, x_2, \ldots, x_{opt}\}$ in such way that every $C_j^* = \{x_1, \ldots, x_j\}$ for $1 \leq j \leq opt$ induces a connected subgraph.

First, consider the dominating disk of x_1. How many independent nodes can it contain? Suppose $y_1, \ldots, y_k$ are independent nodes in the dominating disk of x_1 an shown in Fig. 6. Then $\angle y_1 x_1 y_2 > 60^o$, $\angle y_2 x_1 y_3 > 60^o, \ldots, \angle y_k x_1 y_1 > 60^o$. Therefore, $k \leq 5$.

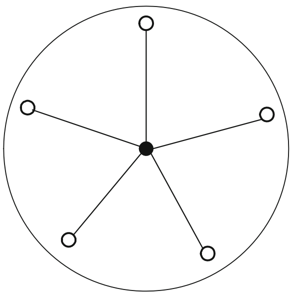

Fig. 6 Independent nodes in a dominating disk of x_1

Next, we consider the dominating disk of x_j for $j \geq 2$. How many independent nodes can be contained in the dominating disk of x_j, but not in the dominating disk of x_i for any $i = 1, 2, \ldots, j-1$? Suppose $y_1, y_2, \ldots, y_k$ are those independent nodes. Since C_j^* induces a connected subgraph, there must exist i, $1 \leq i \leq j-1$, such that x_i lies in the dominating set of x_j. Since $y_1, y_2, \ldots, y_k$ do not lie in the dominating disk of x_i, $x_i, y_1, y_2, \ldots, y_k$ are independent nodes lying in the dominating disk of x_j. Therefore, $k \leq 4$. This concludes that the total number of independent nodes lying in the union of dominating disk of nodes in C^* is at most $1 + 4opt$ where $opt = |C^*|$. This is a key lemma for Wan et al. [37] to see the constant performance ratio of their approximation.

Lemma 1 (Wan et al. [37]) *Any maximal independent set has size at most* $1 + 4opt$ *where opt is the size of the minimum connected dominating set.*

What is the possible best upper bound for the size of a maximal independent set? One has the following conjecture.

Open Problem 1 (*Conjecture*) In a unit disk graph, any maximal independent set has size $|mis| \leq 2 + 3 \cdot opt$ where opt is the size of the minimum connected dominating set.

Several methods have been established to attack this conjecture. The first method is based on new results on disk packing. Wu et al. [43] showed that two dominating disks of u and v with distance at most one can contain at most eight independent nodes and using this fact, showed that $|mis| \leq 3.8 \cdot opt + 1.2$. Wan et al. [38] showed that if the dominating disk of u_0 contains u_1, u_2, u_3, then the union of dominating disks of u_0, u_1, u_2, u_3 can contain at most 15 independent nodes and with this fact, further showed that $|mis| \leq 1\frac{1}{3} + 3\frac{2}{3} \cdot opt$. Here $|mis|$ is the size of maximal independent set, while $|opt|$ is the size of optimal connected dominating set.

Funke et al. [17] use a simple area argument. Define the *adjacent area* of a node x to be a disk with center x and radius 1.5. Then two adjacent nodes have at least $\frac{9}{2}\arccos\frac{1}{3} - \sqrt{2}$ area in common. Thus, a minimum connected dominating set can have at most

$$(opt - 1)\left(\pi 1.5^2 - \frac{9}{2}\arccos\frac{1}{3} + \sqrt{2}\right) + \pi 1.5^2$$

area. For every node y in a maximal independent set, draw a disk with center y and radius 0.5. All such disks are disjoint and lie in the adjacent areas of nodes in the minimum connected dominating set. Therefore,

$$|mis| \leq \frac{(opt - 1)(\pi 1.5^2 - \frac{9}{2}\arccos\frac{1}{3} + \sqrt{2}) + \pi 1.5^2}{0.25\pi} \leq 3.748 \cdot opt + 9.$$

Funke et al. [17] claimed a much better bound $|mis| \leq 3.453 \cdot opt + 8.291$. However, the proof involves a unproved fact about disk packing. Therefore, this bound is not recognized as proved. With a combinatorial method, Gao et al. [19] Showed that

$|mis| \leq 3.453 \cdot opt + 4.839$. Very recently, Li et al. [25] employed a quite complicated geometry argument and push this bound further to $|mis| \leq 3.4306 \cdot opt + 4.8185$. In another direction, Wan et al. [38] presented an example to show that $|mis| = 3 \cdot opt + 2$ is reachable.

While research on Open Problem 1 is still quite active, progress on the second stage is also going quite fast. A naive way to connect a dominating set into a connected one is to construct a minimum spanning tree. This construction can be combined with the construction of maximal independent set. Starting from the second iteration, choose an uncolored node adjacent to a grey node instead of making an arbitrary choice. Then a minimum spanning tree for black nodes would have each "edge" containing only one grey node. Thus, we need only $|mis| - 1$ grey nodes to connect all black nodes into connected one. This gives us a connected dominating set with size $2|mis| - 1 \leq 6.8612 \cdot opt + 8.637$.

Certainly, employing the Steiner tree to do the job is a better idea. A *Steiner tree* for a given subset of nodes, called *terminals*, in a graph is a tree interconnecting all terminals such that every leaf is a terminal. Every node other than terminals in the Steiner tree is called a *Steiner node*. Clearly, for interconnecting dominating set, we prefer to minimize the number of Steiner nodes instead of the total edge length, the objective in classic Steiner tree problems. This means that we need to study the following problem.

Steiner Tree with Minimum Number of Steiner Nodes (ST-MSN) Given a unit disk graph G and a dominating set D of nodes, compute a Steiner tree for D with the minimum number of Steiner nodes.

Although the ST-MSN problem in unit disk graph has not been studied very much, its geometric version in the Euclidean plane has been studied extensively [7,14,27]. Some results in the Euclidean plane can be extended to unit disk graphs. By performing such an extension, Min et al. [28] obtained a polynomial-time 3-approximation for the ST-MSN problem and hence their approximation solution for the minimum connected dominating set has size at most $6.4306 \cdot opt + 4.8185$ based on the number bound for $|mis|$.

For the classical network Steiner tree problem, current best approximation is constructed by greedy strategy [30]. The same thing happens here. The best performance for interconnecting dominating set is also achieved by a greedy algorithm. Actually, there are several greedy algorithms designed in the literature [18,26,38]. It is reported in [25] that the greedy approximation in [38] reaches performance ratio 6.075.

The upper bound on $|mis|$ is a big burden in establishing the performance ratio of approximation designed in above two stage frame.

Open Problem 2 Can we find an efficient construction for maximal independent set such that obtained set has size significantly smaller than $3 \cdot opt + 2$?

Finding such an efficient construction may greatly improve all approximations that we mentioned above.

From another side to look, we may have the following question: Is there a lower bound for the performance ratio of polynomial-time approximation for the minimum connected dominating set in unit disk graphs? The answer is No. Indeed, Cheng et al. [9] already showed the existence of a PTAS (polynomial-time approximation scheme).

That is, for any $\varepsilon > 0$, there exists a polynomial-time approximation within a factor of $1+\varepsilon$ from optimal. However, the running time of such an approximation is $O(n^{1/\varepsilon^2})$. It cannot be implemented to use in the real world. Therefore, one is still making efforts to look for approximations with a smaller performance ratio and a low-degree polynomial for running time.

Constructing connected dominating set is a real-world problem. With some specific application background, additional requirement may apply, such as message-optimal [1], distributed construction [37], weakly connected [8], bounded diameter [23], fault tolerant [44], r-hop dominating [24] and shortest path condition [41,42], etc. Those requirements give more potential research topics about connected dominating sets in unit disk graphs in the future.

3 Node-weighted unit disk graph

When wireless sensor networks contain more types of sensors with the same communication range, but different costs, the mathematical model would be the unit disk graph with node weight. For virtual backbone, instead of connected dominating set with minimum cardinality, we would prefer to have connected dominating set with minimum total weight.

Suppose we would like still to use the two stage construction. In the first stage, we want to construct the minimum weight dominating set in input node-weighted unit disk graph. The first observation is that we cannot use the maximal independent set any longer. Indeed, we do not have efficient way to construct a maximal independent set with minimum total weight unless $NP = P$ [21]. For arbitrarily chosen one, we have no upper bound for its total weight. Actually, it was a long-standing open problem whether there exists a polynomial-time constant approximation for minimum weight dominating set in unit disk graph.

Ambühl et al. [2] solved this open problem by constructing a 72-approximation with partition technique. Gao et al. [20] introduced a new technique, called *double partition* and using it, they constrict a $6+\varepsilon$. Dai and Yu [12] and Zou et al. [47] further improved the performance ratio to $5+\varepsilon$ and $(4+\varepsilon)$, respectively.

For minimum weight connected dominating set in unit disk graph, Ambühl et al. [2] spent $17opt$ to connect the 72-approximation of weighted dominating set into a 89-approximation of weighted connected dominating set where opt is the object function value of an optimal solution, i.e., the minimum total weight of connected dominating set. Gao et al. [20] reduce the spending for interconnection from $17opt$ to $4opt$. Zou et al. [48] further reduced this spending to $3.85 \cdot opt$.

Open Problem 3 Does the minimum weight dominating set have a PTAS in unit disk graphs?

Nieberg et al. [29] developed a technique, called the *local neighborhood-based scheme technique*, which can construct a PTAS for minimum dominating set in polynomial growth bounded graphs. The unit disk graph is also a polynomial growth bounded graph. Therefore, Wang et al. [40] tried to expend the local neighborhood-based scheme technique to the weighted case. They are successful in some special case, however, it is still open in general.

Since all approximations constructed with partition techniques have very high running time, it is hard to implement for application in the real world. Therefore, the following open problem receives more attention.

Open Problem 4 Can we construct a constant approximation for the minimum weight (connected) dominating set in unit disk graphs without using partition?

4 Unit ball graph

When identical sensors are deployed into a field which is not flat, the mathematical model would be the unit ball graph. A graph is called a *unit ball* graph if all its nodes lie in the 3-dimensional Euclidean space and an edge (u, v) exists if and only if the distance between u and v is at most one. In other word, if let each node u associate a ball with center u and diameter one, called a *unit ball*, then edge (u, v) exists if and only if ball u and ball v intersect each other.

Constructing a connected dominating set in unit ball graphs can also be done in two stages. In the first stage, we construct a maximal independent set. Now, to establish an upper bound for the size of a maximal independent set, we have to study problems on sphere packing.

Call the ball at center u with radius one the *dominating ball* of node u. The following is the first packing problem that we would like to study: How many independent nodes can lie in the dominating ball of a node?

This problem has a close relationship to Gregory-Newton Problem, a well-known packing problem as follows: How many unit balls can touch a unit ball such that they do not touch each other?

Gregory-Newton Problem was proposed in 1694 and was solved by Hoppe in 1874 (see [46]). The answer is 12. Actually, an icosahedron has 12 vertices and the distance from the center to a vertex is smaller than the edge length. Zong [46] is an excellent reference book on Gregory-Newton Problem, which also study the problem in high Euclidean spaces.

If we allow surrounding balls touch each other, what is the answer to Gregory-Newton Problem, is it different from the case that not allow them to touch each other? In spaces with different dimensions, the answer is different. For example, in the Euclidean plane, there exist only five unit disks which touch a unit disk, but themselves not touch each other. However, if we allow themselves to touch, then the answer is six (Fig. 7).

However, in the 3-dimensional Euclidean space, does not matter, we allow surrounding balls touch each other or not, there are at most 12 unit balls touch a unit ball.

From the answer of Gregory-Newton Problem, could we conclude that the dominating ball of a node can contain at most 12 independent nodes? The implication was made in [4] without any additional explanation. However, we feel that it needs a mathematical proof.

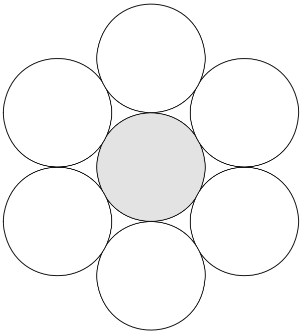

Fig. 7 Six unit disks touch a unit disk

Now, we can immediately obtain $|mis| \leq 11 \cdot opt + 1$ in unit ball graphs by the argument similar to the proof for $|mis| \leq 4 \cdot opt + 1$ in unit disk graphs. We can also improve this bound by studying dominating balls of two nodes.

Open Problem 5 How many independent nodes can lie in the union of dominating balls of two nodes with distance at most one?

A conjectured answer is 20. A progress made in Zhang (private communication) is that at most 22 independent nodes can lie in the union of dominating balls of two nodes with distance at most one. This can improve a little the upper bound for the size of maximal independent set.

For connecting a maximal independent set into a connected dominating set, we can also employ the minimum spanning tree [4] and greedy algorithms [49]. They are all simple extensions from the Euclidean plane to the 3-dimensional Euclidean space.

Not every technique used in the Euclidean plane can be used in the 3-dimensional Euclidean space. For example, the technique for establishing the existence of PTAS for the minimum connected dominating set in unit disk graphs [9] does not work in 3-dimensional Euclidean space. Zhang et al. [45] discovered a new technique to overcome this difficulty and showed that in any Euclidean space, the minimum connected dominating set in unit ball graphs has PTAS.

5 Conclusion

In this survey, we discuss recent developments for wireless networking, dominating, and packing problems. Especially, we categorize researches on dominating set by topology models and problem characteristics. We first introduce the commonly used technologies to select a connected dominating set with minimum cardinality, and then illustrate the idea to evaluate the approximation performance for designs in several literatures. Next, we summarize latest results to select a minimum connected dominating set under unit disk graph, node-weighted graph, and unit ball graph.

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