Minimum Power Strongly Connected Dominating Sets in Wireless Networks

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Abstract

Consider a digraph $G = (V, E)$. A subset of vertices $D \subseteq V$ is called a strongly connected dominating set if the subgraph induced by D is strongly connected and every vertex not in D is incident to an in-edge coming from D and also to an out-edge going to D. Strongly connected dominating set is a virtual backbone in ad-hoc wireless networks. It should be kept active all the time. Therefore, constructing an energy-efficient strongly connected dominating set is significant. Since this problem is $\mathcal{NP}$-hard, in this paper we give an $O(\ln n)$-approximation for the minimum power strongly connected dominating set, which is the weighted version where each node has been assigned a power function.

Keywords: Wireless Network, Connected Dominating Set, Energy Efficient

1 Introduction

In wired networks, we usually select a group of nodes to construct a network backbone for efficient routing, where only selected nodes can forward data but the entire network will receive them. However, in many application of wireless networks, such as wireless ad-hoc or sensor networks, there is no fixed or predefined infrastructure, resulting no physical backbones. Nodes in such kind of networks always communicate with each other via a shared medium, by either single-hop or multi-hop routing strategies. Therefore, a virtual backbone can be formed by constructing a *Connected Dominating Set* (CDS).

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The formal definition of CDS is: Given an undirected graph $G = (V, E)$, a subset $C \subseteq V$ is a CDS of G, if (1) for each node $u \in V$, u is either in C or there exist a node $v \in C$ such that $(u, v) \in E$, and (2) the subgraph $G[C]$ induced by C is connected. $G[C] = (C, E')$, where $E' = \{(u, v)|(u, v) \in E \text{ and } u, v \in C\}$. CDS (as wireless virtual network backbone) has been widely used for efficient routing in mobile ad-hoc networks to help routing and adapting network topology changes. Thus it is desirable to construct a *minimum connected dominating set* (MCDS) to reduce the traffic during communication and simplify the connectivity management.

The MCDS problem has been studied intensively in *Unit Disk Graph* (UDG), in which all nodes have the same transmission range. However, in practice, based on the power limitation, geographical difference etc., the transmission ranges of all nodes are not necessarily equal. In such cases, a wireless ad-hoc network can be modeled as a directed graph $G = (V, A)$, where each node $v \in V$ is located on the two-dimensional Euclidean plane and has a transmission range $r(v)$. We say that there exists a directed $(v, u) \in A$ if and only if $d(v, u) \leq r(v)$. Such a graph is called a *Disk Graph* (DG).

In a DG, the set of nodes forming virtual network backbone is called the *Strongly Connected Dominating Set* (SCDS). Its definition is shown as follows: Given a digraph $G = (V, A)$, a subset $C \subseteq V$ is a SCDS of G, if (1) for each node $u \in V$, u is either in C or there exist two nodes $v, w \in C$ such that $(u, v) \in A$ and $(w, u) \in A$; (2) the subgraph $G[C]$ induced by C is strongly connected. To minimize the traffic during communication (minimize $|C|$), it is desirable to construct the *Minimum Strongly Connected Dominating Set* (MSCDS).

Example 1. *In Fig.1(a), set $\{u, v, y\}$ of black nodes forms a SCDS, and also a MSCDS. Set $\{v, w\}$ is not a SCDS since node x can only be reached from y, which is not in $\{v, w\}$. Set $\{v, x, y\}$ is not a SCDS since the subgraph it induced is not strongly connected. In Fig.1(b), set $\{u, v, x, y\}$ of gray nodes forms a SCDS, but not a MSCDS.*

While a lot of researches have focused on the study of CDS in UDG as homogeneous networks, the study of SCDS in DG as heterogeneous networks attracts more and more

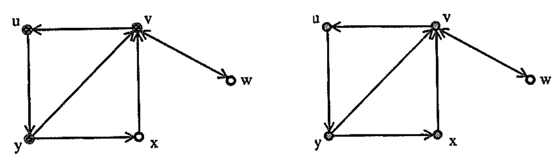

Figure 1: (a) MSCDS; (b) a SCDS but not MSCDS

attention. However, compared with MCDS, MSCDS is harder to achieve. In this paper we study a more general problem — the *Minimum Power Strongly Connected Dominating Set* (MPSCDS): Given a set of n nodes, each of which has a given level of transmission power, we want to find a SCDS such that the total power is minimized. Simply speaking, MPSCDS is the weighted version of MSCDS. If we assign $p(v) = 1$ for each v, then MPSCDS becomes MSCDS. Our contribution is presenting a heuristic algorithm to compute MPSCDS with approximation ratio $2(1 + \ln(n-1))$, n is the number of nodes in this graph. This is done by transforming MPSCDS to a related problem: *Minimum Energy Broadcast Routing Problem* (MEB).

The remainder of this paper is structured as follows: Section 2 describes the related works on CDS, mainly focused on MSCDS. Section 3 describes three heuristic algorithms solving MEB which were given in our previous work [10]. The heuristic algorithm for MPSCDS and its theoretical analysis are exhibited in Section 4. Finally, we give a conclusion in Section 5.

2 The Related Works

CDS has been studied extensively in the literature [2, 22, 12, 14, 15, 16, 21]. For MCDS in general graphs, Guha and Khuller [9] showed a greedy $(\ln\Delta + 3)$-approximation and Ruan *et al.* [13] showed a greedy $(\ln\Delta + 2)$-approximation where Δ is the maximum degree in the graph.

For MCDS in UDG, Cheng *et al* [5] showed the existence of a polynomial-time approximation scheme (PTAS), i.e., for any $\varepsilon > 0$, there exists a polynomial-time $(1+\varepsilon)$-approximation. However, its running time grows rapidly as ε goes to 0 and hence is not worth implementing in practice. Therefor, several approximations have been constructed with distributed implementations [1, 3, 11, 20]. For SCDS in digraph, Thai *et al.* [17, 18, 7] gave several constructions. Those constructions mainly try to reduce the size of SCDS.

Clark *et al.* [6] has proved that an MCDS in UDG is $\mathcal{NP}$-hard, and Thai [19] showed that the MCDS problem in Disk Graphs with Bidirectional links is $\mathcal{NP}$-hard. The basic idea is to use polynomial time reduction from *Set Cover Problem* to CDS. Since minimum set cover problem is NP-complete, the MSCDS is $\mathcal{NP}$-hard. Because MSCDS is a special case of MPSCDS, MPSCDS is $\mathcal{NP}$-hard. Since the lower bound of the approximation ratio for set cover problem is $O(\ln n)$, $O(\ln n)$ is also a lower bound for MPSCDS.

3 Preliminaries

In this section, we will introduce another problem –*Energy Efficient Broadcast Routing Problem*, which will be the footstone to solve MPSCDS problem. MEB has been studied in our previous paper [10], and three approximation heuristics have been discussed to solve this problem. In our next section, we will solve MPSCDS problem based on these algorithms for MEB.

Firstly, Let's discuss the definition of MEB. The network is modeled by a digraph $G = (V, A)$, while each node v has its power level $p(v)$. Suppose T is a broadcast directed tree with source s. Let $NL(T)$ denote the set of non-leaf nodes of T. Then, the total power cost $PC(T)$ can be represented as $PC(T) = \sum_{v \in NL(T)} p(v)$. The *Minimum Energy Efficient Broadcast Routing Problem*(MEB) is: Given a broadcast request with source s and $p(v)$ for each node v, find a broadcast tree rooted at s such that total power cost of tree is minimized.

The MEB problem has been proved to be $\mathcal{NP}$-hard [8], and three heuristic algorithms were proposed in [10]. The approximation ratio of one algorithm is proven to be $(1 + 2\ln(n-1))$, which is based on Node-weighted Steiner Tree. The detailed algorithms will be described in the appendix. For the readers who are interested in the numerical simulation results, please refer to [10].

4 Algorithm for MPSCDS

We will firstly propose a general algorithm based on any algorithm of the MEB. Construct an auxiliary directed graph $\overleftarrow{G} = (V, \overleftarrow{A})$ corresponding to $G = (V, A)$, while $\overleftarrow{A} = \{(v,u) | \forall (u,v) \in A\}$. That is, changing the direction of all arcs in A, and keep the power of each node in $\overleftarrow{G}$.

The main idea of the general algorithm includes three steps: firstly, randomly take a node v as source and get a broadcast tree T rooted from v in G using the algorithm of the MEB problem. Secondly, get a broadcast tree $\overleftarrow{T}$ sourced from v in $\overleftarrow{G}$ using the same algorithm for the MEB problem. Finally, union set $NL(T)$ and $NL(\overleftarrow{T})$ to get a strongly connected dominating set of G. This algorithm is represented as following:

Alg.1 A general algorithm for MPSCDS	
1	Randomly take a node s in G as source. Use the algorithm of MEB to get a broadcast tree T of G sourced from s.
2	Use the same algorithm of MEB to get a broadcast tree $\overleftarrow{T}$ of $\overleftarrow{G}$ sourced from s.
3	Return $NL(T) \cup NL(\overleftarrow{T})$ as the solution.

Theorem 1. *Alg.1 returns a right solution for MPSCDS. That is,* $NL(T) \cup NL(\overleftarrow{T})$ *is a strongly connected dominating set of* G.

Proof. Firstly, we prove that $G[NL(T) \cup NL(\overleftarrow{T})]$ is strongly connected. For any two nodes u and v in $NL(T) \cup NL(\overleftarrow{T})$, since $\overleftarrow{T}$ is a broadcast tree of $\overleftarrow{G}$ rooted at s, then there is a directed path from s to u in $\overleftarrow{G}$. Therefore there is a directed path $P(u,s)$ from u to s in G. Since T is a broadcast tree of G sourced from s, there is a directed path $P'(s,v)$ in G. So $P(u,s) + P'(s,v)$ is a directed path from u to v. Since $NL(T) \cup NL(\overleftarrow{T})$ is a set with non-leaf of T and $\overleftarrow{T}$, $P(u,s) + P'(s,v)$ must be a directed path in $G[NL(T) \cup NL(\overleftarrow{T})]$. For the same reason, there is a directed path in $G[NL(T) \cup NL(\overleftarrow{T})]$ from v to u.

Secondly, we prove that $NL(T) \cup NL(\overleftarrow{T})$ is a dominating set in G. For any node $v \in V \backslash (NL(T) \cup NL(\overleftarrow{T}))$, v is a leaf of both T and $\overleftarrow{T}$. Then there is a node $u \in NL(T)$ such that $(u,v) \in T$, that is, $(u,v) \in A$. Besides, there is a node $w \in NL(\overleftarrow{T})$ such that $(w,v) \in \overleftarrow{T}$, that is $(v,w) \in A$. Therefore, $NL(T) \cup NL(\overleftarrow{T})$ is a dominating set in G. From the above, $NL(T) \cup NL(\overleftarrow{T})$ is a strongly connected dominating set of G. □

In the following, we call the algorithm of MEB as AL-MEB.

Theorem 2. *If the AL-MEB has approximation ratio* α, *then the general algorithm based on AL-MEB satisfies:*

$$P(C) \leq 2\alpha(P(C^*) + \max p(v)),$$

where C *is a solution produced by the general algorithm based on AL-MEB for MPSCDS problem, and* C^* *is an optimal solution for MPSCDS problem.*

Proof. Suppose T and $\overleftarrow{T}$ are the broadcast tree of G and $\overleftarrow{G}$ produced by the AL-MEB, respectively, T^* and $\overleftarrow{T^*}$ are optimal broadcast tree of G and $\overleftarrow{G}$ respectively. Then we have

$$\begin{aligned} P(C) &= \sum_{v \in NL(T) \cup NL(\overleftarrow{T})} p(v) \\ &\leq \sum_{v \in NL(T)} p(v) + \sum_{u \in NL(\overleftarrow{T})} p(u) \\ &\leq \alpha\Big(\sum_{v \in NL(T^*)} p(v) + \sum_{u \in NL(\overleftarrow{T^*})} p(u)\Big) \\ &= \alpha P(T^*) + \alpha P(\overleftarrow{T^*}) \end{aligned} \quad (1)$$

Recall that C^* is a strongly connected dominating set. We consider the following two cases:

Case 1: $s \in C^*$. Since C^* is a strongly connected dominating set, we can get a broadcast tree T_1 of G sourced from s and also get a broadcast tree $\overleftarrow{T_1}$ of $\overleftarrow{G}$ sourced from s such that any node in V/C^* must be a leaf of T_1 and $\overleftarrow{T_1}$. Note that some nodes in C^* may be leaves of T_1 and $\overleftarrow{T_1}$. Because T^* and $\overleftarrow{T^*}$ are the optimal broadcast tree of G and $\overleftarrow{G}$, we have:

$$\begin{aligned} \sum_{v \in NL(T^*)} p(v) &\leq \sum_{v \in NL(T_1)} p(v) \\ &\leq \sum_{v \in C^*} p(v) \\ &= P(C^*), \end{aligned} \quad (2)$$

$$\begin{aligned} \sum_{u \in NL(\overleftarrow{T^*})} p(u) &\leq \sum_{u \in NL(\overleftarrow{T_1})} p(u) \\ &\leq \sum_{u \in C^*} p(u) \\ &= P(C^*), \end{aligned} \quad (3)$$

Case 2: $s \notin C^*$. Since C^* is a strongly connected dominating set, there must be two nodes $w, r \in C^*$, such that $(s,w) \in A$ and $(r,s) \in A$. By the same reason, we can get a broadcast tree T_1 of $G \backslash \{s\}$ sourced from w and also get a broadcast tree $\overleftarrow{T_1}$ of $\overleftarrow{G} \backslash \{s\}$ sourced from r such that any node in $V \backslash (C^* \cup \{s\})$ must be a leaf of T_1 and $\overleftarrow{T_1}$. Then $T_1 \cup (s,w)$ is a broadcast tree of G sourced from s, and $\overleftarrow{T_1} \cup (s,r)$ is a broadcast tree of $\overleftarrow{G}$ sourced from s. Then we have:

$$\begin{aligned} \sum_{v \in NL(T^*)} p(v) &\leq p(s) + \sum_{v \in NL(T^*) \backslash \{s\}} p(v) \\ &\leq p(s) + \sum_{v \in NL(T_1)} p(v) \\ &\leq p(s) + \sum_{v \in C^*} p(v) \\ &= p(s) + P(C^*) \end{aligned} \quad (4)$$

$$\begin{aligned}\sum_{u\in NL(\overleftarrow{T^*})} p(u) &\le p(s) + \sum_{u\in NL(\overleftarrow{T^*})/\{s\}} p(u)\\ &\le p(s) + \sum_{u\in NL(\overleftarrow{T_1})} p(u)\\ &\le p(s) + \sum_{u\in C^*} p(u)\\ &= p(s) + P(C^*) \qquad (5)\end{aligned}$$

From (1)-(5), we have

$$\begin{aligned}P(C) &\le \alpha(P(T^*) + P(\overleftarrow{T^*}))\\ &\le \alpha(p(s) + P(C^*) + p(s) + P(C^*))\\ &= 2\alpha(p(s) + P(C^*))\\ &\le 2\alpha(\max_{v\in V} p(v) + P(C^*))\end{aligned}$$

Then we finished our proof. □

Since the Node-Weighted Steiner Tree Based Heuristic (see Alg.3 in the appredix) produces a $(1+2\ln(n-1))$ approximation for MEB, using it in Alg.1 will produce an approximation algorithm to solve MPSCDS with approximation ratio $2(1+2\ln(n-1))$.

Theorem 3. *The algorithm using the Node-Weighted Steiner Tree based Heuristic of MEB satisfies*

$$P(C) \le 2(1+2\ln(n-1))(P(C^*) + \max p(v)).$$

5 Conclusion

Computing energy-efficient strongly connected dominating set in digraphs is NP-hard. In this paper, we introduced a general algorithm to solve MPSCDS problem and gave a heuristic with performance ratio $O(\ln n)$.

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6 Appendix: Three Heuristics for MEB

6.1 Transforming the MEB to Directed Steiner Tree Problem

For the network graph G, we split each node $v \in V$ into two nodes v' and v'' and connect them by a new arc from v' to v'' with weight $p(v)$. For each arc $(v_1, v_2) \in A$, (v_1'', v_2') is a new arc with weight 0. Then the MEB problem in G can be transformed to the following problem in G': finding a directed tree T in G' which is rooted from s' and covers all the nodes in V' such that the sum of the weights of all arcs in T is minimized. This is a typical directed Steiner Tree Problem, and we can used any heuristics such as [4] listed.

6.2 Greedy Heuristic

We can use greedy heuristic to compute MEB. We introduce two sets, one is cover-set, containing the nodes which transmit/relay messages, the other is covered-set, containing the nodes that are outgoing neighbors of the nodes in the cover-set. Then the algorithm can be shown as follows:

Alg. Greedy Heuristic

Input: $G = (V, A)$ and s
Output: T: a broadcast tree rooted from S
$C = \{s\}$; (the cover-set)
$D = V_s$; (the covered-set)
While $(D \neq V - \{s\})$ do
    choose $v_i \in D - C$ such that
    $\max(\lvert V_i \cap (V - (\{s\} \cup D))\rvert / p(v_i))$;
    $C = C \cup \{v_i\}$;
    $D = D \cup V_i$;
Construct the broadcast tree T from C

Here $V_i = \{v_j \mid (v_i, v_j) \in A\}$ is a set of outgoing neighbors of node v_i. This algorithm can output a broadcast tree in time $O(n^2)$.

6.3 A Node-Weighted Steiner Tree-Based Heuristic

For any set U of nodes which contains source s, Let H_U be the subgraph of G in which an arc e of G is present if and only if the initial node of e belongs to U. A strongly connected component of H_U is said to be an *orphan* if it does not contain the source s and has no incoming arc. Fix a set U of nodes which contains s. An arborescence is said to be legal (with respect to U) if 1) all the sinks are heads and 2) it contains at least two heads if the root is not s (a root is also counted as a head if it is a head). The quotient cost of a legal arborescence is defined as the ratio of its cost to the number of heads in this arborescence. A *min-quotient* legal arborescence rooted at v is a legal arborescence rooted at v with the smallest quotient cost. The quotient cost of a node v is defined as the quotient cost of a min-quotient legal arborescence rooted at v.

The algorithm uses a greedy strategy iteration. Initially, U contains only S. In each iteration, it selects a node v of the smallest quotient cost; next, adds the nonsink nodes $U(v)$ of the min-quotient legal arborescence rooted at v to U, and updates H_U and the set of orphans. This operation is repeated until there is no orphan component. The algorithm is formally presented as the following:

Alg. NWST for MEB

Input: $G = (V, A)$ and s
Output: T: a broadcast tree rooted from S
$U = \{s\}$; (transmitting node set)
$O = \{\{i\} \mid i \in V \backslash \{s\}\}$; (set of orphans)
While $(\lvert O\rvert \neq 0)$ do
    choose v with smallest quotient cost;
    $U = U \cup U(v)$;
    Update H_U and O;
    Recalculate quotient cost for each node
Construct the broadcast tree T from C

D.Li [10] proved that this algorithm has approximation ratio at most $(1 + 2\ln(n-1))$.