We have the following lemma for buyers in AEGIS-MP.

**Lemma 6.** *For AEGIS-MP, it holds that* $\text{OPT}(\mathbb{L}^J, \vec{\mathcal{S}}^J) < etJ \times \text{OPT}(\mathbb{W}^J, \vec{\mathcal{S}}^J)$*, where* $t = (1+\kappa)$.

Due to the limitation of space, we leave the detailed proof to our technical report [30].

We now present the approximation ratio of AEGIS-MP.

**Theorem 3.** *AEGIS-MP achieves* $(\delta_{max} + e\delta_{max}tJ^2 + etJ)$*-approximation.*

*Proof.* Let $\mathbb{O}$ be the set of winners in the optimal allocation, and winners are allocated bundles from $\vec{\mathcal{S}}^O = (\mathbf{S}_1^O, \mathbf{S}_2^O, \cdots, \mathbf{S}_n^O)$. We partition the winners into three categories, and bound the value of them separately.

▷ We denote the winners that also stay in $\mathbb{W}^J$ by $\mathcal{N}_1$. In this case, winners might win other interested channel bundles in the optimal allocation, so we get

$$\text{OPT}(\mathcal{N}_1, \vec{\mathcal{S}}^O) \leq \delta_{max} \times \text{OPT}(\mathbb{W}^J, \vec{\mathcal{S}}^J). \tag{6}$$

▷ We turn to another set of winners $\mathcal{N}_2$, which is the subset of losers in AEGIS-MP, *i.e.*, $\mathcal{N}_2 \subseteq \mathbb{L}^J$. Buyer $i$ belongs to $\mathcal{N}_2$ if and only if she is a winner in the optimal allocation $\mathbb{O}$, and her allocated bundle $\mathbf{S}_i^O$ is not included in $\mathbf{S}_i^J$, which is the bundle that the buyer $i$ declares in AEGIS-MP when she drops out. We have the following claim for winners $\mathcal{N}_2$.

Claim 1: $\text{OPT}(\mathcal{N}_2, \vec{\mathcal{S}}^O) < e\delta_{max}tJ^2 \times \text{OPT}(\mathbb{W}^J, \vec{\mathcal{S}}^J)$.

*Proof.* Let $\mathbb{F}^j = \left\{ i \in \mathcal{N}_2 \;\middle|\; |\mathbf{S}_i^j| = |H_i|, |\mathbf{S}_i^{j+1}| < |H_i| \right\}$ be the set of buyers from $\mathcal{N}_2$ that first shrink their bundles in the $j$th iteration, and we have $\mathcal{N}_2 = \bigcup_{j=0}^{J-1} \mathbb{F}^j$. According to the first and third properties of undominated strategy in Definition 9, we can conclude that $\mathbf{S}_i^j$ contains some interested channel bundles for all $i \in \mathbb{N}$ and $0 \leq j \leq J-1$. Therefore, for any $i \in \mathbb{F}^j$, we have

$$V_i(\mathbf{S}_i^O) \leq \delta_{max} \times V_i(\mathbf{S}_i^J), \quad \forall i \in \mathbb{F}^j. \tag{7}$$

According to the **First Time Shrink** property, all bundles $\vec{\mathcal{S}}^{j+1}$ of buyers in $\mathbb{F}^j$ are disjoint. Additionally, by the **Shrinking Sets** property, we have $\mathcal{S}_i^J \subseteq \mathcal{S}_i^{j+1}$, implying bundles $\vec{\mathcal{S}}^J$ of buyers in $\mathbb{F}^j$ are also disjoint. Therefore, $(\mathbb{F}^j, \vec{\mathcal{S}}^J)$ is a valid allocation. Since $\mathbb{F}^j \subseteq \mathcal{N}_2 \subseteq \mathbb{L}^J$, we get

$$\text{OPT}(\mathbb{F}^j, \vec{\mathcal{S}}^J) \leq \text{OPT}(\mathbb{L}^J, \vec{\mathcal{S}}^J). \tag{8}$$

Combining with Inequalities (7)(8), we conclude that

$$\begin{aligned} & \text{OPT}(\mathbb{F}^j, \vec{\mathcal{S}}^O) = \sum_{i \in \mathbb{F}^j} V_i(\mathbf{S}_i^O) \leq \delta_{max} \sum_{i \in \mathbb{F}^j} V_i(\mathbf{S}_i^J) \\ = \;& \delta_{max} \text{OPT}(\mathbb{F}^j, \vec{\mathcal{S}}^J) \leq \delta_{max} \text{OPT}(\mathbb{L}^J, \vec{\mathcal{S}}^J). \end{aligned} \tag{9}$$

Using Inequality (9) and Lemma 6, we get

$$\text{OPT}(\mathbb{F}^j, \vec{\mathcal{S}}^O) < e\delta_{max}tJ \times \text{OPT}(\mathbb{W}^J, \vec{\mathcal{S}}^J).$$

Finally, we conclude that

$$\text{OPT}(\mathcal{N}_2, \vec{\mathcal{S}}^O) \leq \textstyle\sum_{j=0}^{J-1} \text{OPT}(\mathbb{F}^j, \vec{\mathcal{S}}^O) < e\delta_{max}tJ^2 \text{OPT}(\mathbb{W}^J, \vec{\mathcal{S}}^J) \tag{10}$$

□

▷ We denote the winners in $\mathbb{L}^J \backslash \mathcal{N}_2$ by $\mathcal{N}_3$. According to the definition of $\mathcal{N}_2$, the allocated bundles of winners in $\mathcal{N}_3$ are contained in bundles $\vec{\mathcal{S}}^J$, together with Lemma 6, we get

$$\begin{aligned} & \text{OPT}(\mathcal{N}_3, \vec{\mathcal{S}}^O) \leq \text{OPT}(\mathbb{L}^J \backslash \mathcal{N}_2, \vec{\mathcal{S}}^J) \\ \leq \;& \text{OPT}(\mathbb{L}^J, \vec{\mathcal{S}}^J) < (etJ) \text{OPT}(\mathbb{W}^J, \vec{\mathcal{S}}^J). \end{aligned} \tag{11}$$

We now combine these three types of winners together (Inequalities (6)(10)(11)), and conclude that $\text{OPT}(\mathbb{O}, \vec{\mathcal{S}}^O) \leq \text{OPT}(\mathcal{N}_1, \vec{\mathcal{S}}^O) + \text{OPT}(\mathcal{N}_2, \vec{\mathcal{S}}^O) + \text{OPT}(\mathcal{N}_3, \vec{\mathcal{S}}^O) < (\delta_{max} + e\delta_{max}tJ^2 + etJ)\text{OPT}(\mathbb{W}^J, \vec{\mathcal{S}}^J)$. □

From the above analysis, we now can get our main result for AEGIS-MP according to Definition 4.

**Theorem 4.** *AEGIS-MP is an implementation of an* $\mathcal{O}(\delta_{max}tJ^2)$*-approximation in undominated strategies.*

# 6. EVALUATION RESULTS

In this section, we show our evaluation results. We implement AEGIS using network simulation, and compare its performance with CRWDP [5] and NSR-MP. CRWDP is an unknown single-minded combinatorial spectrum auction, and NSR-MP is a variant of AEGIS-MP. Neither CRWDP nor NSR-MP considers channel spatial reusability.

## 6.1 Methodology

We use two complementary datasets, namely Google Spectrum Database [9] and GoogleWiFi [32], to evaluate the performance of our mechanisms. We take Google Spectrum Database as our first dataset. We first extract WiFi nodes in an area (Latitude range: $[40°25'18'', 39°38'29'']$, Longitude range: $[-76°34'40'', -74°52'20'']$) from WiGLE.net [27], and we then query Google Spectrum Database the available TV white spaces and corresponding maximum permissible power for each WiFi node, which is considered as a portable device in the database. Portable devices can work on unused TV channel 21 through 51, except channel 37, 38, 39. To generate conflict graphs, we apply a simple Free Space propagation model [26] to predict the interference range between nodes, and consequently create the conflict graphs.[3] We also evaluate our mechanisms in a practical conflict graph, built from exhaustive signal measurements, in the second data set. The second dataset, GoogleWiFi, records 78 APs in a $7km^2$ residential area of the Google WiFi network in Mountain View, California. It was collected by a research group from UC Santa Barbara in April 2010 [32].

We build a set of auction configurations by sampling WiFi nodes in the first data set, and the number of WiFi nodes varies from 200 to 2000 with increment of 200. For the second data set, we assume the number of leasing channels can be one of three values: 6, 12 and 24. We consider the case of single-minded buyers and the case of multi-minded buyers, who can have up to 10 interested bundles (*i.e.*, $l_i \leq 10$). For each buyer $i$, her $l_i$ interested channel bundles are randomly generated from her available channel set, and the valuations on bundles are uniformly distributed over $(0, 1]$. The maximum closeness parameter of valuation is set as 5, *i.e.*, $\delta_{max} = 5$. The minimum monetary unit in the auction systems is set as $\epsilon = 10^{-5}$. In AEGIS-MP, since buyers may have multiple undominated strategies at their decision points, we assume that buyers randomly select one of them. All the results of performance are averaged over 200 runs.

**Metrics:** We evaluate the following five metrics:

► *Social Welfare:* The sum of winning buyers' valuations on their allocated bundles of channels.

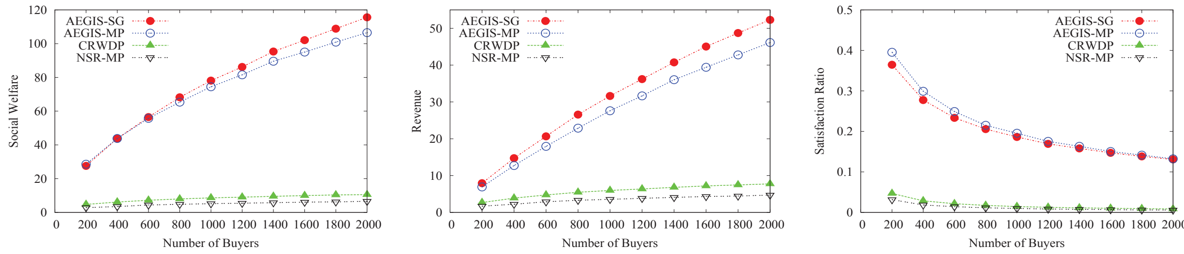
► *Revenue:* The sum of payments received from buyers.

► *Satisfaction Ratio:* The fraction of winners over buyers.

► *Channel Utilization:* The number of radios worked on each channel.

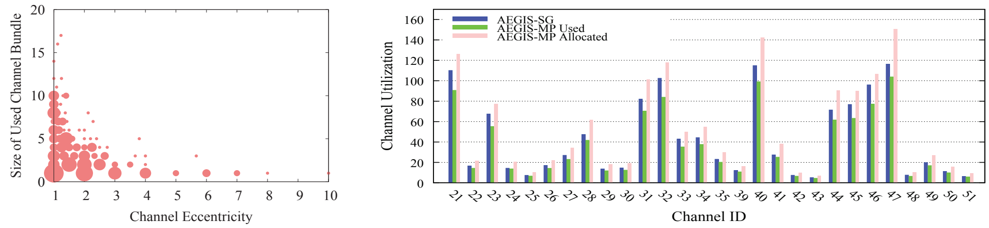
► *Channel Eccentricity:* The ratio of allocated channels over actually used channels for one buyer. In AEGIS-MP, for each winner, the final allocated bundle may contain multiple interested bundles and uninterested channels, but the winner only use one interested bundle. Therefore, we use channel eccentricity to measure this channel over-allocation.

[3]Other propagation models, *e.g.*, Egli and Longley-Rice [26], could be used to generate more accurate conflict graphs.




(a) Social Welfare (b) Revenue (c) Satisfaction Ratio

**Figure 3: Performance of AEGIS, CRWDP and NSR-MP on Google Spectrum Dataset.**




(a) Channel Eccentricity (b) Channel Utilization

**Figure 4: Channel eccentricity and channel utilization of AEGIS.**

## 6.2 Performance on Google Spectrum Dataset

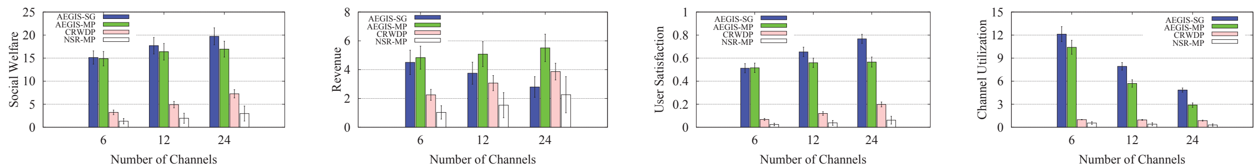
By varying the number of buyers, we collect a set of performance data, as illustrated in Figure 3. We can see that AEGIS always outperforms the other two mechanisms, CRWDP and NSR-MP. This result demonstrates that exploiting channel spatial reusability can significantly improve the performance of spectrum auction systems. Figure 3 also shows that when the number of buyers increases, the social welfare and revenue increase, while the satisfaction ratio decreases. On one hand, AEGIS allocates channels more efficiently among more buyers, hence the social welfare and revenue increase. On the other hand, larger number of buyers leads to more intense competition on limited channels, thus decreases the satisfaction ratio. We also observe from Figure 3 that revenue is much lower than social welfare. Similar to previous work [31], we can institute reserve prices for channels to increase revenue, and make a trade-off between revenue and social welfare. How to determine an optimal reserve price is out of the scope of this paper. Intuitively, multi-minded auction mechanisms should perform better than single-minded ones because of the more feasible bundle choices for buyers. However, as shown in Figure 3, AEGIS-MP is slightly worse than AEGIS-SG in terms of social welfare and revenue. As we will discuss later, the channel eccentricity of winners in AEGIS-MP is the main reason for this degradation of system performance.

We now present the evaluation results of channel eccentricity and channel utilization. The channel eccentricity for AEGIS-SG is always equal to 1, because the allocated bundle is exactly buyer's interested bundle. Figure 4(a) shows the channel eccentricity of AEGIS-MP. We randomly select one instance from the 200 simulation instances when the number of buyers is fixed at 2000, and calculate the channel eccentricity for each winner. The placement of a circle in Figure 4(a) indicates one set of winners with the same channel eccentricity and the same size of used channel bundle. The size of a circle is logarithmic to the number of winners. Though some of winners' channel eccentricities are equal to 1, there exist about 67% winners, whose channel eccentricities are larger than 1. On one hand, AEGIS-MP just stimulates buyers to take undominated strategies, such that buyers can still maintain multiple bundles or uninterested channels in their active bundles during the auction. On the other hand, buyers only use the most valuable channel subset among their allocated bundles. Therefore, the size of allocated bundle can be larger than that of actually used bundle in some cases, leading to channel over-allocation.

The channel eccentricity affects the channel utilization of AEGIS-MP. By fixing the number of buyers at 2000 and running 200 simulation instances, we record the average channel utilization for each channel, and plot the results in Figure 4(b). We do not include CRWDP and NSR-MP in this analysis, because they do not consider channel spatial reusability. As shown in the figure, different TV channels have different channel utilization. The reason is that TV white spaces are spatially heterogeneous, *e.g.*, channel 47 can be accessed to almost all buyers, while channel 33 are only available to around 36% buyers. In AEGIS-MP, we distinguish between allocated channels and used channels. We can observe from Figure 4(b) that the allocated number is always larger than the used number for each channel. This is because some winners have channel eccentricity higher than 1. We can also see from Figure 4(b) that the channel utilization of AEGIS-MP is always lower than that of AEGIS-SG. The reason is that the winners with high channel eccentricity in AEGIS-MP disables some possible allocations of their interfering neighbors. From the above analysis, we can get that buyers' manipulated strategies on channel demands indeed impact the performance of spectrum auction systems.

## 6.3 Performance on GoogleWiFi Dataset

Figure 5 shows the system performance of AEGIS, CRWDP, and NSR-MP on GoogleWiFi dataset when there are 6, 12, 24 channels. Since channels are accessible to all buyers in this setting, we average the channel utilization on all channels on this dataset. Generally, the evaluation re-




(a) Social Welfare (b) Revenue (c) Satisfaction Ratio (d) Channel Utilization

**Figure 5: Performance of AEGIS, CRWDP and NSR-MP on GoogleWiFi Dataset.**

sults are similar with those on Google Spectrum Dataset. Again, AEGIS achieves better performance than CRWDP and NSR-MP. Figure 5 also shows that when the number of channels increases, the social welfare and satisfaction ratio increase, and channel utilization decreases. The reason is that fixing the number of buyers, larger supply of channels results in more trades in the auction, and thus increases social welfare and satisfaction ratio. The channel utilization decreases because buyers can be allocated to more channels when the number of channels increases. For revenue, AEGIS-SG decreases with the number of channels, while AEGIS-MP, CRWDP and NSR-MP increase. The clearing price calculation in AEGIS-SG is based on critical bid. When larger number of channels are accessible in the auction, more buyers are allocated channels, reducing the critical bids for winners. Hence, the revenue of AEGIS-SG decreases. Though the clearing price of CRWDP is also calculated based on critical bid, there still exist considerable losers when the number of channels becomes large. Therefore, the critical bids for winners still stay high, so that the revenue continue to grow with the increase of channels. The clearing prices of AEGIS-MP and NSR-MP are the bids of winners at the end of the auctions. Larger supply of channels leads to more winners, and thus revenues in AEGIS-MP and NSR-MP become higher.

# 7. CONCLUSION

Considering the five challenges for designing a practical spectrum auction mechanism, we have proposed AEGIS, which is the first framework of unknown combinatorial auction mechanisms for heterogeneous spectrum redistribution. For the case with unknown single-minded buyers, we have designed a direct revelation combinatorial auction mechanism, call AEGIS-SG. AEGIS-SG achieves strategy-proofness and approximately efficient social welfare. We have further considered the case with unknown multi-minded buyers, and designed an iterative ascending combinatorial auction, namely AEGIS-MP. AEGIS-MP is implemented in undominated strategies, and has a good approximation ratio. We have implemented AEGIS and evaluated its performance on two practical datasets. Compared with the existing work, AEGIS achieves superior performance, in terms of social welfare, revenue, satisfaction ratio, and channel utilization.

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