

# STAMP: A Strategy-Proof Approximation Auction Mechanism for Spatially Reusable Items in Wireless Networks

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**Abstract**—The advent of participatory sensing markets and spectrum markets based on the wireless networks have led to a new kind of auction dealing with spatially reusable items, which can be shared by multiple parties that are geographically far apart enough from each other. Simply applying traditional auctions to spatially reusable items is vulnerable to bid manipulation, and may lead to low allocation efficiency. In this paper, we study the problem of auctioning spatially reusable items. We propose STAMP, which is a Strategy-proof Approximation auction Mechanism for sPatially reusable items in wireless networks. STAMP can be implemented with any existing maximum independent set algorithm, and can guarantee the allocation efficiency as high as the algorithm based on. Evaluation results show that STAMP achieves much better performance than existing mechanisms, in terms of allocation efficiency.

## I. INTRODUCTION

Auction has been regarded as an efficient way to reallocate resources for more than two thousand years. Recently, with the rapid development of the Internet and wireless technology, many practical applications have given rise to a new trend of research, which focuses on auction mechanism design for spatially reusable items. Participatory sensing market (*e.g.*, Sensorly and Ear-Phone) [18], [16] and spectrum auction market (*e.g.*, FCC spectrum auctions) [13], [17] are good examples. In the participatory sensing market, the information sensed by multiple participants who are far away enough from each other is valid to the service aggregator; while in the spectrum auction market, multiple users who are not within the interference range of each other can use the same frequency band simultaneously. Unfortunately, simply applying traditional auctions to spatially reusable items is vulnerable to bid manipulation, and may lead to low allocation efficiency [22]. Therefore, it is highly needed to design novel auction mechanisms that can deal with spatially reusable items.

However, there are two major challenges when designing auction mechanisms for spatially reusable items. One challenge, which is inherited from traditional auction design, is

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strategy-proofness (please see Section Problem Formulation for the definition), which intuitively means that a buyer cannot get larger utility by submitting a bid other than her true valuation. A strategy-proof auction mechanism is attractive, because it can prevent the buyers from manipulating their bids, and such protects truthful buyers benefit. The other challenge is maximization of allocation efficiency (please see Section Problem Formulation for the definition). Multiple buyers who are far away enough from each other can share a spatially reusable item without splitting the value of it. A critical goal of the service aggregator who posts information sensing tasks in the participatory sensing market is to maximize the proportion of task completed in order to guarantee high quality of serve to its customers; and a goal of the auctioneer in the spectrum market is to maximize channel utilization due to the scarcity of spectrum resource. However, the optimization problem normally becomes computationally intractable when considering spatially reusable items.

In this paper, we model the problem of spatially reusable item allocation/assignment as a sealed-bid auction, and propose STAMP, which is a Strategy-proof Approximation auction Mechanism for sPatially reusable items in wireless networks. STAMP may achieve an allocation efficiency as high as the best ever known maximum independent set algorithm. The detailed contributions of this paper are as follows.

- We design a sealed-bid auction mechanism for spatially reusable items, namely STAMP, and theoretically prove that STAMP is a strategy-proof auction mechanism. STAMP can be implemented with any known maximum independent set algorithm, and therefore can achieve an allocation efficiency as high as the best ever known such algorithm.
- We implement STAMP and compare its performance with related auction mechanisms. Our evaluation results show that STAMP achieves much higher allocation efficiency than existing auction mechanisms.

The remaining parts of this paper are organized as follows. In section II, we briefly review related works in participatory sensing markets, spectrum markets and incentive mechanisms design. In section III, we present the model and review some related solution concepts. In Section IV, we present

the detailed design of STAMP, and prove its properties. In section V, we show the evaluation results of STAMP. Finally, we conclude our work in Section VI.

## II. RELATED WORKS

To our best knowledge, there are only a few research studies on the strategy-proof auction mechanism design for participatory sensing markets. [5] designed an auction mechanism to motivate user participation. [21] designed a platform-centric model and a user-centric model for participatory sensing. Both of them did not consider the spatial reusability of the tasks. There are also a few mechanism design works for spectrum markets. [22] proposed VERITAS to ensure truthfulness in spectrum auction. However, the mechanism may suffer from low allocation efficiency due to the greedy allocation algorithm. Later, [20] proposed SMALL to achieve strategy-proofness by sacrificing a bounded number of buyers. Although SMALL shows better allocation efficiency than VERITAS, it still cannot exploit the power of the best existing maximum independent set algorithm. In contrast, our mechanism STAMP not only guarantees strategy-proofness, but also can achieve an allocation efficiency as high as the best ever known such algorithm.

There also exist many works on auction mechanism design [14], [9]. In [1], Archor and Tardos characterized the relationship between monotonicity and truthfulness for single-parameter agents. [11] shown how to deliver an approximation mechanism given an approximation algorithm under certain cases. [8] presented the design of two polynomially tractable, universally incentive-compatible randomized mechanisms for combinatorial auctions with general bidder preferences.[7] proposed the first monotone randomized PTAS for minimizing the cost of parallel related machines.

## III. PROBLEM FORMULATION

We consider a spatially reusable items market (*e.g.*, participatory sensing market and secondary spectrum market), and model it as a sealed-bid auction. There is a “seller” (*e.g.*, service aggregator and spectrum owner), who has a spatially reusable item (*e.g.*, information sensing task or idle channel) for sale. The item can be allocated to more than one buyer, if they are out of the conflict distance (*e.g.*, information validation range and interference range). There is also a set of “buyers” (*e.g.*, mobile phone users and local wireless service applications), who want to bid for the item and get profit (*e.g.*, compensation from the service aggregator and revenue from serving her own customers). The buyers submit their sealed bids simultaneously at the beginning of the auction, such that the buyers cannot know each other’s bid. Throughout the paper, we consider a collusion free environment.

In the auction, the seller has a spatially reusable item with conflict distance  $d$ . Denote the set of buyers by  $\mathcal{N} = \{1, 2, \dots, n\}$ . Each buyer  $i \in \mathcal{N}$  has a private valuation  $v_i$  for the item. This is commonly known as *type* in the literatures. We denote the profile of buyer valuations by:

$$\vec{v} = (v_1, v_2, \dots, v_n).$$

Each buyer  $i$  chooses her bid  $b_i$  based on her type. Then, we denote the profile of bids by:

$$\vec{b} = (b_1, b_2, \dots, b_n).$$

For convenience, we let  $\vec{v}_{-i}$  denote the valuation profile of buyers other than buyer  $i$ . Similarly, we can define  $\vec{b}_{-i}$ . We also use  $\vec{v}_{-\mathcal{M}}$  ( $\vec{b}_{-\mathcal{M}}$ ) to represent the valuation (bid) profile of buyers other than the set  $\mathcal{M} \subseteq \mathcal{N}$  of buyers.

We use a graph  $G = (V, E)$  to represent the conflict among buyers. Here,  $V$  is the set of vertices, and  $E$  is the set of edges. Each buyer is represented by a vertex in the conflict graph  $G$ , and there is an edge between a pair of buyers  $i$  and  $j$ , if their geographic distance is no larger than  $d$ . Any pair of buyers who are connected in  $G$  cannot both be the item. Here, we denote the set of neighbors of  $i$  in  $G$  by  $N_i$ . We can use a vector to represent the outcome of the item allocation, which is a function of  $\vec{b}$  and  $G$ :

$$\begin{aligned} \vec{x}(\vec{b}, G) &= (x_1, x_2, \dots, x_n), \\ \text{s.t.}, x_i &= \begin{cases} 1, & \text{the item is allocated to } i; \\ 0, & \text{otherwise.} \end{cases} \end{aligned}$$

The auctioneer (or the seller, if the seller is trustworthy) not only determines the item allocation, but also calculates the payment profile:

$$\vec{p}(\vec{b}) = (p_1, p_2, \dots, p_n).$$

We note that in the participatory sensing markets, the mobile phone users consume their own resource to sense information and receive compensations from the service aggregator, so the valuations (*i.e.*, the cost induced in the process of sensing), the bids, and the payments are all negative; while in the spectrum auction markets, the buyers can get profit through providing service via the channel bought and need to pay the spectrum seller, so the valuations, the bids and the payments are positive.

We can now define the utility of buyer  $i \in \mathcal{N}$  as the difference between her valuation and payment:

$$u_i(\vec{b}) = x_i v_i - p_i(\vec{b}).$$

We consider that the buyers are rational and each buyer’s goal is to maximize her own utility.

We now recall the definition of *Dominant Strategy* [15], [10], *Strategy-Proofness* [12] and *Allocation Efficiency*, which will be used in the following parts of this paper.

**Definition 1** (Dominant Strategy). *Strategy  $a_i$  is a player  $i$ ’s dominant strategy, if for any  $a_i' \neq a_i$  and any strategy profile of the other players  $a_{-i}$ ,*

$$u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i}).$$

The concept of *incentive-compatibility* is based on dominant strategy. It means that revealing truthful information is the dominant strategy for every player. A company concept is *individual-rationality*, which intuitively means that for every player who truthfully participates the auction is expected to gain non-negative utility. We now can introduce the definition of Strategy-Proof Mechanism.

**Definition 2** (Strategy-Proof Mechanism). *A mechanism is strategy-proof when it satisfies both incentive-compatibility and individual-rationality.*

**Definition 3** (Allocation Efficiency). *The allocation efficiency of an auction mechanism is the total number of items the buyers win.*

The design goal of our auction mechanism is to achieve strategy-proofness and allocation efficiency maximization. Although the problem of allocation efficiency maximization can be formulated as the following binary program:

*Objective:*

$$\text{Maximize } \sum_{k=1}^n x_k$$

*Subject to:*

$$\begin{aligned} x_i + x_j &\leq 1, & \forall (i, j) \in E; \\ x_i &\in \{0, 1\}, & \forall i \in \mathcal{N}, \end{aligned}$$

it is known that the problem is computationally intractable [19]. Therefore, we should seek for an approximation approach.

#### IV. DESIGN OF STAMP

In this section, we present the design of STAMP, and prove its strategy-proofness and approximation ratio of allocation efficiency. STAMP consists of an item allocation algorithm and a payment scheme. The item allocation algorithm comprises initial item allocation and item reallocation algorithms, STAMP can achieve an allocation efficiency as good as the best known maximum independent set algorithm by allowing to use any such algorithm. By performing the item reallocation algorithm, the mechanism allocates items to buyers with higher valuations. In the payment scheme, by charging the buyers the minimum value by bidding which she can still win the item, STAMP guarantees strategy-proofness.

##### A. Item Allocation

STAMP starts with the initial item allocation algorithm. During the initial item allocation, STAMP allocates the item using the best known maximum independent set approximation algorithm for general bounded degree graphs [6], [3], so that the buyers who are connected in  $G$  are not allocated the item concurrently, *i.e.*:

$$x_i + x_j \leq 1, \quad \forall (i, j) \in E. \quad (1)$$

STAMP then performs item reallocation algorithm. It visits all the buyers iteratively, from the smallest index to the highest index (*i.e.*, from 1 to  $n$ ).

Assume that STAMP is attempting to perform the item reallocation for  $i$  (if  $i$  is allocated an item). STAMP finds

a set of buyers  $w_i$  such that:

$$\forall l \in w_i :$$

$$1. l \in N_i. \quad (2)$$

$$2. x_l = 0. \quad (3)$$

$$3. x_k = 0, \forall k \in N_l. \quad (4)$$

$$4. l \notin w_j, \forall j \in \{1, 2, \dots, i-1\}. \quad (5)$$

$$5. l \geq i. \quad (6)$$

Here, constraint (2) guarantees that the buyers who are connected are not allocated the item concurrently. Constraint (3) and (4) jointly guarantee that the buyers selected in  $w_i$  are all able to be allocated the item. Constraint (5) and (6) jointly guarantee that STAMP's allocation is monotone [1].

If  $w_i \neq \emptyset$ , for the buyers who bid higher than  $i$  in  $w_i$ , STAMP allocates the item to them instead of  $i$ . The allocation outcome of the other buyers remains the same. STAMP then visit the next buyer, *i.e.*:

$$x_i = 0, x_j = 1, \forall j \in w_i, b_j > b_i.$$

If  $w_i = \emptyset$ , STAMP visits the next buyer.

The item reallocation algorithm runs until all the buyers are visited exactly once.

The algorithm for item allocation is formally stated in Algorithm 1. Function  $MIS(G)$  can be any existing maximum independent set algorithm and it allocates the item to the corresponding buyers. Algorithm 1 returns the allocation result and  $\vec{w}$ , which is used in the payment determination.

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#### Algorithm 1 Item Allocation of STAMP

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**Input:** A set of buyers  $N$ , a profile of bids  $\vec{b}$ , a conflict graph  $G$ .

**Output:** A vector of item allocation  $\vec{x}$  and a vector of group assignment  $\vec{w}$ .

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1:  $\vec{x}, \vec{w} \leftarrow 0$ ;
2:  $\vec{x} \leftarrow MIS(G)$ ;  $i \leftarrow 1$ .
3: repeat
4:    $w_i = \{l | l \in N_i \wedge x_l = 0 \wedge x_k = 0 \wedge l \notin w_j \wedge l \geq i, \forall k \in N_l \forall j \in \{1, 2, \dots, i-1\}\}$ .
5:   if  $w_i \neq \emptyset$  then
6:     for all  $k \in w_i$  do
7:       if  $\exists b_k > b_i$  then
8:          $x_i \leftarrow 0$ .
9:       end if
10:      if  $b_k > b_i$  then
11:         $x_k \leftarrow 1$ .
12:      end if
13:    end for
14:  end if
15:   $i \rightarrow i + 1$ 
16: until  $i = n$ .
17: return  $\vec{x}, \vec{w}$ .
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The computational complexity of the maximum independent set algorithm is  $O(n^c)$  [6]. Notice that there is only one round

of reallocation, so there are at most  $n$  times of reallocations. The computational complexity for reallocation is  $O(n^2)$ . The mechanism takes a  $O(n^{2+\kappa})$  time to determine which buyers are allocated the item.

### B. Payment Determination

In this subsection, we present the payment scheme of STAMP. For a winning buyer, STAMP charges her the minimum price that she can still win by bidding this value (this is commonly known as the critical value [22]) and the buyers who are not allocated the price 0:

$$p_i(\vec{b}) = \begin{cases} \underset{b_i}{\operatorname{argmin}} (x_i = 1), & \text{the item is allocated to } i; \\ 0, & \text{otherwise.} \end{cases}$$

Combining the result of  $\vec{w}$  of Algorithm 1, the value of  $\underset{b_i}{\operatorname{argmin}} (x_i = 1)$  is computed as following:

$$\underset{b_i}{\operatorname{argmin}} (x_i = 1) = \max(A, B),$$

$$\text{Where } A = \begin{cases} b_j, & \text{if } i \in w_j \text{ for some } j; \\ 0, & \text{otherwise.} \end{cases}$$

$$B = \max_{k \in w_i} b_k.$$

We can now draw the following lemmas and theorems:

**Lemma 1.** *STAMP's allocation algorithm satisfies monotonicity.*

*Proof:* For an arbitrary buyer  $i$ , we distinguish two cases:

- If  $i$  is allocated in the initial item allocation, she is not contained in any  $w_j$ , then if she bids higher than all the buyers in  $w_i$ , she wins the item; otherwise, she does not win the item.
- If  $i$  is not allocated in the initial item allocation, she is contained in exactly one  $w_j$ . In order to win the item, she first needs to bid higher than buyer  $j$ . What's more, she also needs to bid higher than all the buyers in  $w_i$ , or she still does not win the item. Therefore, if she bids higher than the larger one of  $b_j$  and the highest bid in  $w_i$ , she wins the item; otherwise, she does not win the item.

According to the above analysis, we can draw the conclusion that the allocation algorithm of STAMP is monotone. ■

The monotonicity of STAMP's allocation algorithm and the critical payment scheme guarantee STAMP's incentive compatibility [1].

**Lemma 2.** *STAMP achieves incentive-compatibility.*

*Proof:* For a buyer  $i$ , we prove that she cannot increase her utility by misreporting. We divide the proof into two cases:

- 1) If  $i$ 's valuation satisfies  $v_i \geq \underset{b_i}{\operatorname{argmin}} (x_i^f = 1)$ , then  $i$  is allocated the item when bidding truthfully. We can further distinguish two cases of misreporting:

- $i$  misreports  $b'_i (> v_i)$ , then  $i$  is still allocated the item, her utility remains unchanged:

$$\begin{aligned} u_i(b'_i, \vec{b}_{-i}) &= v_i x_i - \underset{b_i}{\operatorname{argmin}} (x_i = 1) \\ &= u_i(\vec{b}). \end{aligned}$$

- $i$  bids  $b'_i (< v_i)$ , then her utility remains unchanged or becomes 0:

$$u_i(b'_i, \vec{b}_{-i}) = \begin{cases} v_i x_i - \underset{b_i}{\operatorname{argmin}} (x_i = 1), & i \text{ wins;} \\ 0, & \text{otherwise.} \end{cases}$$

- 2) If  $i$ 's valuation for winning the item is  $v_i < \underset{b_i}{\operatorname{argmin}} (x_i = 1)$ , then  $i$  is not allocated the item when bidding truthfully. We can further distinguish two cases of misreporting:

- $i$  misreports  $b'_i (< v_i)$ , then  $i$  is still not allocated the item, her utility remains unchanged:

$$u_i(b'_i, \vec{b}_{-i}) = 0 = u_i(\vec{b}).$$

- $i$  bids  $b'_i (> v_i)$ , then her utility remains unchanged or even negative:

$$u_i(b'_i, \vec{b}_{-i}) = \begin{cases} v_i x_i - \underset{b_i}{\operatorname{argmin}} (x_i = 1), & i \text{ wins;} \\ 0, & \text{otherwise.} \end{cases}$$

Thus we can conclude that the buyers' dominant strategy is bidding truthfully and STAMP achieves incentive compatibility. ■

**Lemma 3.** *STAMP achieves individual-rationality.*

*Proof:* For a buyer  $i$ , assume that she bids truthfully by  $b_i = v_i$ . We divide the proof into two cases:

- 1) If  $b_i = v_i \geq \underset{b_i}{\operatorname{argmin}} (x_i = 1)$ , then  $i$  is allocated the item and her utility is non-negative:

$$\begin{aligned} u_i(\vec{b}) &= v_i x_i - p_i(\vec{b}) \\ &= v_i - \underset{b_i}{\operatorname{argmin}} (x_i = 1) \\ &\geq 0. \end{aligned}$$

- 2) If  $b_i = v_i < \underset{b_i}{\operatorname{argmin}} (x_i = 1)$ , then  $i$  is not allocated the item and her utility is zero:

$$u_i(\vec{b}) = 0.$$

Since the utility of the buyers are non-negative when they bid truthfully, we can conclude that STAMP achieves individual rationality. ■

**Theorem 1.** *STAMP achieves strategy-proofness.*

*Proof:* This is clear from Lemma 2, Lemma 3 and the definition of strategy-proof auction mechanism. ■

**Theorem 2.** *STAMP achieves an asymptotic approximation ratio of:*

$$\min\{\kappa/\mu, [\kappa' \log(\log \Delta)]/\Delta\}$$

(where  $\kappa$  is a positive constant,  $\kappa'$  is a constant depends on  $\kappa$ , and  $\Delta$ ,  $\mu$  are the maximum and the average degrees of the  $G$ , respectively) in terms of allocation efficiency [6].

*Proof:* During the item reallocation procedure, there is no decrease in the allocation efficiency, so the approximation ratio of the allocation efficiency is solely determined by the initial item allocation algorithm. Since the initial item allocation is equivalent to the maximum independent set algorithm in  $G$  and the asymptotic approximation ratio of the best ever known algorithm [6] is:

$$\min\{\kappa/\mu, [\kappa' \log(\log \Delta)]/\Delta\}$$

(where  $\kappa$  is a positive constant,  $\kappa'$  is a constant depends on  $\kappa$ , and  $\Delta$ ,  $\mu$  are the maximum and the average degrees of the  $G$ , respectively), we can conclude this statement. ■

### C. A Toy Example

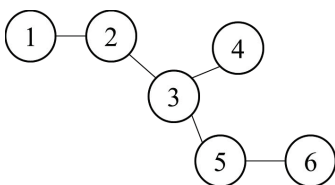


Fig. 1. A Toy Example

We now give a toy example to illustrate how STAMP works. Suppose that we have a set of  $n = 6$  buyers and the profile of the valuations and bids is:

$$\vec{v} = \vec{b} = (3, 1, 4, 5, 3, 1).$$

The graph  $G$  is shown in Figure 1. Suppose that after the initial item allocation, buyer 1, 3 and 6 is allocated the item:

$$\vec{x} = (1, 0, 1, 0, 0, 1).$$

Then STAMP performs the item reallocation:

- 1) For buyer 1,  $w_1 = \{2\}$  and  $b_2 \leq b_1$ , so she wins the item with payment 1.
  - 2) For buyer 2, she is not allocated the item so her payment is 0.
  - 3) For buyer 3,  $w_3 = \{4, 5\}$ . The only reallocation is between buyer 3 and buyer 4:
- $$\vec{x} = (1, 0, 0, 1, 0, 1).$$
- 4) For buyer 4,  $w_4 = \emptyset$ . She wins the item with payment 4.
  - 5) For buyer 5, she is not allocated the item so her payment is 0.
  - 6) For buyer 6,  $w_6 = \emptyset$ . She wins the item with payment 0.

The allocation and payment vectors are:

$$\vec{x} = (1, 0, 0, 1, 0, 1), \vec{p} = (1, 0, 0, 4, 0, 0).$$

## V. EVALUATION

In this section, we implement STAMP and evaluate its performance in terms of allocation efficiency.

### A. Methodology

We run the mechanism for over 1000 times to evaluate its performance. There are four different kinds of settings described later. In all these four settings, there is a single spatially reusable item for sale and the buyers' valuations lie in the range of  $(0, 1]$ .<sup>1</sup> Here, we use a heuristic maximum independent set algorithm [2] in the initial allocation.

We investigate the allocation efficiency to measure the mechanism's performance. We compare the performances of STAMP with VERITAS and SMALL which are mechanisms for homogeneous spatially reusable items.

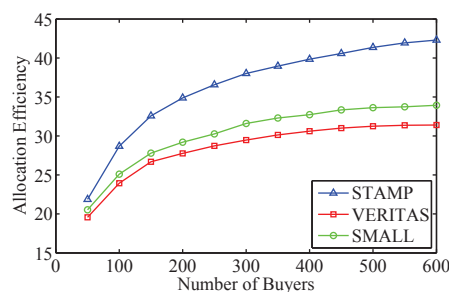


Fig. 2. Fixed Area and Varying Number of Buyers

First, we fix the terrain area to be 2000 meters  $\times$  2000 meters, and the buyers are randomly distributed in this area. The number of buyers varies from 50 to 600 with step of 50. We also fix  $d = 300$  meters [4]. Figure 2 shows the comparison results on allocation efficiency of STAMP, VERITAS and SMALL. The results show that STAMP outperforms VERITAS and SMALL in terms of allocation efficiency in any situation. When the number of buyers increases, all mechanisms' allocation efficiency increase and the advantage of STAMP becomes more obvious, but the rate of increase drops since the average degree of  $G$  also increases.

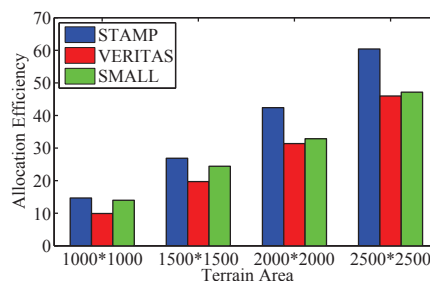


Fig. 3. Varying Area and Fixed Number of Buyers

<sup>1</sup>The ranges of buyers' valuations can be chosen differently from the ones used here. However, the evaluation results of using different ranges are similar to each other. As a result, we only show the results for the above ranges in this paper.

Second, we fix the number of buyers to be 500 and the terrain area varies from 1000 meters times 1000 meters to 2500 meters times 2500 meters with step of 1000 in side length and  $d = 300$  meters. Figure 3 shows the comparison results on allocation efficiency of STAMP, VERITAS and SMALL. The results show that STAMP outperforms VERITAS and SMALL in terms of allocation efficiency in the four terrain areas. When the area becomes larger, all mechanisms' allocation efficiency also increase and the differences between STAMP and VERITAS, STAMP and SMALL all increase.

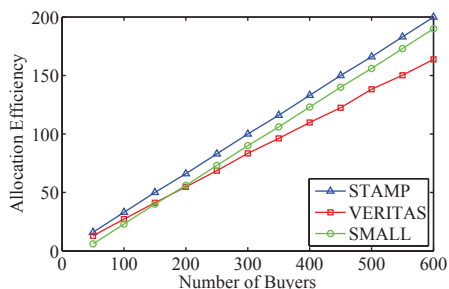


Fig. 4. Fixed Average Degree and Varying Number of Buyers

Third, we fix the average degree of the  $G$  to be 4, and the number of buyers varies from 50 to 600 with step of 50. Figure 4 shows the comparison results on allocation efficiency of STAMP, VERITAS and SMALL for this setting. The results show that STAMP outperforms VERITAS and SMALL in terms of allocation efficiency. The difference of STAMP and VERITAS increases along with the number of buyers while the difference of STAMP and SMALL remains the same all the time.

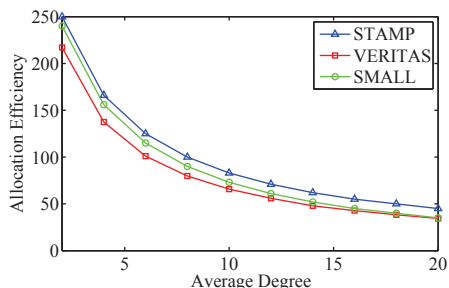


Fig. 5. Varying Average Degree and Fixed Number of Buyers

Finally, we fix the number of buyers to be 500 and the average degree of the  $G$  varies from 2 to 20 with step of 2. Figure 5 shows the comparison results on allocation efficiency of STAMP, VERITAS and SMALL for the setting. The results show that STAMP outperforms VERITAS and SMALL in terms of allocation efficiency when the average degree of  $G$  varies. The allocation efficiency of all mechanisms decrease when the average degree of  $G$  increases. This is because when the average degree of  $G$  increases, the size of the maximum independent set of  $G$  becomes smaller and so does the number of winners.

From the above results, we can draw the conclusion that STAMP achieves superior performance in terms of allocation efficiency.

## VI. CONCLUSION

In this paper, we have introduced the problem of spatially reusable items allocation in which there was a single seller who wanted to sell spatially reusable items and performed an auction to allocate the item to a group of buyers. We have designed a strategy-proof approximation auction mechanism, namely STAMP, to solve the problem. STAMP achieves an allocation efficiency as high as the best ever known maximum independent set algorithm. For future work, we are going to consider auction mechanisms for some more challenging settings like online auction markets.

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