A Strategy-Proof Auction Mechanism for Adaptive-Width Channel Allocation in Wireless Networks

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***Abstract*—Efficient wireless channel allocation is becoming a more and more important topic in wireless networking. Dynamic channel allocation is believed to be an effective way to cope with the shortage of wireless channel resource. Up to now, a number of auction mechanisms have been designed to solve the problem of dynamic channel redistribution. Such designs deal with either the problem of single channel allocation or the problem of multiple channels allocation with an assumption of the same per-channel valuation. However, considering the recent outcomes of researches on throughputs of adaptive-width channels and the needs of wireless users in practice, we need to provide buyers with a way to submit various combinatorial bids for channels. This motivates our work on designing a more practical auction mechanism to solve the problem of channel redistribution. In this paper, we propose SPECIAL, which is a Strategy-Proof and EffiCIent multi-channel Auction mechanism for wireLess networks. SPECIAL guarantees the strategy proofness of the channel auction, exploits wireless channels' spatial reusability, and achieves high channel allocation efficiency. Numerical results demonstrate that SPECIAL prevents buyers from manipulating the auction, and achieves high performance.**

***Index Terms*—Wireless networks, channel allocation, combinatorial auction.**

I. Introduction

With the surging deployment of wireless communication devices and the emergence of software-defined radios, the shortage of radio spectrum is becoming a more and more serious problem. On the one hand, each protocol in the IEEE 802.11 [1] standard specifies a number of overlapping wireless channels, among which only limited number of orthogonal channels are available (*e.g.*, 3 orthogonal channels in IEEE 802.11b/g and 12 in IEEE 802.11a). On the other hand, traditional static allocation of the wireless channels hinders the already limited spectrum resource from being used efficiently [2]–[4]. Hence, considering the insufficient usage of current available wireless channels, it is highly important to carefully allocate the wireless channels, in order to promote the performance of wireless networks.

Among the best-known market-based allocation mechanisms, auctions are outstanding on both perceived fairness and allocation efficiency [5]. Thus, auction is a natural way to distribute goods, including wireless channels. For example, since 1994, the Federal Communications Commission (FCC) and its counterpart across the world have been using auctions to assign channels. However, designing a feasible channel auction mechanism has its own challenges. The first challenge, which is not only limited to channel auctions but applies to auctions in general, is strategy-proofness meaning that by reporting true valuation of the good as the bid, each buyer can maximize her payoff. Since the buyers always want to maximize their own payoff, they may manipulate the auction to seek for more benefit if the auction mechanism is not properly designed. Such misbehavior may hurt the benefit of truthful buyers, and thus discourage truthful buyers from participating in the auction. The second challenge is the efficiency of the channel allocation. Different from conventional goods, wireless channels have a property of spatial reusability, which means that wireless users that are well geographically separated can use the same channel simultaneously. With this property, the well-known Vickrey-Clarke-Groves auction becomes not appropriate to solve the problem of channel allocation in general cases, because even if a powerful central authority exists, computing the optimal channel allocation is NP-complete in a multi-hop wireless network [6], [7].

In recent years, a number of elegant channel auction mechanisms in wireless network (*e.g.*, [8]–[11]) have been proposed to solve the problem of dynamic channel allocation. In these papers, it is commonly assumed that every buyer either bids for only one channel, or bids for multiple channels with the same per-channel price. However, doubling the number of channels, especially contiguous channels, a buyer's valuation does not necessarily double. It has been shown that the

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saturated throughput is a concave non-decreasing function on channel width [12]. For example, the saturated throughput on a 40MHz channel is less than twice of that on a 20MHz channel. Consequently, according to the saturated throughput on a channel, a buyer's valuation is reasonably expected to be a concave non-decreasing function on the width of the channel she gets. Different buyers may have different valuation functions. Therefore, considering the need for various numbers of channels due to various valuations, it is more reasonable to give the buyers the flexibility to submit various combinatorial bids for contiguous channels.

In this paper, we present SPECIAL, which is a Strategy-Proof and EffiCIent multi-channel Auction mechanism for wireLess networks. As far as we know, we are the first to combine flexible bids with combinatorial auction to study the problem of dynamic channel allocation. Combinatorial auction, in which a large number of items are auctioned concurrently and bidders are allowed to express preferences on bundles of items [13]–[16], has the capability of providing the proper expression of the problem of combinatorial wireless channel allocation. Furthermore, SPECIAL is fundamentally different from traditional combinatorial auction, as it allows multiple users that are geographically separated to use the same channel due to spatial reusability. In SPECIAL, all the buyers simultaneously submit their sealed bids for available channels. A bid specifies the maximal price the buyer would like to pay for each combination of contiguous channels. Then, SPECIAL decides the auction winners, channel allocation, and charges based on the bids. Our analysis shows that SPECIAL achieves strategy-proofness, and exploits wireless channels' spatial reusability.

We make the following contributions in this paper:

- We present a combinatorial auction mechanism, namely SPECIAL, for the problem of channel allocation in multi-hop wireless networks. To the best of our knowledge, we are the first to introduce flexible bids for different numbers of contiguous channels.
- Our analysis shows that SPECIAL is a strategy-proof channel auction mechanism.
- Our simulation results verify that SPECIAL does prevent buyers form manipulating their bids. Furthermore, SPECIAL achieves good performance on channel utilization. SPECIAL performs at least as well as TRUST, VERITAS, and SMALL with any number of buyers involved.

The rest of the paper is organized as follows. In Section II, we briefly review the related works. In Section III, we present technical preliminaries. In Section IV, we describe SPECIAL in detail, and prove its strategy-proofness. We present evaluation results in Section V. Finally, we conclude the paper and point out potential future works in Section VI.

II. Related Work

In this section, we review related works on channel allocation involved with selfish participants, as well as some recent related works on auction design.

Earlier, Félegyházi *et al.* [17] studied Nash Equilibria in a static multi-radio multi-channel allocation game. Later, Wu *et al.* [18] designed a mechanism for the multi-radio multi-channel allocation game, converging to a much stronger equilibrium state, called strongly dominant strategy equilibrium (SDSE). These works considered the problem in a single collision domain. For multiple collision domains, a number of strategy-proof auction-based spectrum allocation mechanisms (*e.g.*, VERITAS [8], TRUST [9], and SMALL [11]) have recently been proposed to solve the channel allocation problem. VERITAS uses a greedy spectrum allocation algorithm to distribute channels and a critical value based pricing mechanism to charge winning bidders. TRUST integrates double auction and radio spectrum allocation, and considers both buyers and sellers valuations on the channels. Improving TRUST, SMALL allocates channels to groups without having to sacrifice a good transaction, which includes a channel and a group of buyers, enabling all channels to be sold and limiting the number of sacrificed buyers almost linearly with the number of buyers. A min-max coalition-proof Nash equilibrium channel allocation scheme has been proposed in [19] to study the multi-radio channel allocation problem in multi-hop wireless networks. In [20], Wu *et al.* have studied the problem of adaptive-width channel allocation. Yang *et al.* [21] proposed a framework for truthful and profit maximizing double auctions for wireless spectrum. Dong *et al.* [22] designed truthful double auction using separate designs of the buyer and seller side auctions. Chen *et al.* [23] and Wang *et al.* [24] are also recent works on double auction design for channel allocation. Peng *et al.* [25] proposed faithful auction mechanisms for distributed wireless spectrum allocation. Chen *et al.* [26] designed a truthful spectrum auction framework that achieves information theoretic security and greatly reduces both computation and communication overhead. Wang *et al.* [27] proposed a strategy-proof and false-name-proof auction framework for large scale dynamic spectrum access. Chen *et al.* [28] proposed a distributed game based channel allocation algorithm for wireless sensor and actuator networks. In [29], an auction system for recall-based cognitive radio networks was studied. There are also some works on online spectrum auction design (*e.g.*, [10], [30], [31]) and on privacy-preserving spectrum auction design (*e.g.*, [32], [33]).

Another important work on channel allocation game is [34], where the authors proposed a graph coloring game model and discussed the price of anarchy under various topology conditions such as different channel numbers and bargaining strategies. However, none of the above work considers the saturated throughput of contiguous channels.

In addition, besides spectrum allocation, auction mechanisms are also applied to other research problems. *e.g.*, Xu *et al.* [35] studied the problem of resource allocation for device-to-device underlaying networks using combinatorial auction. Deng *et al.* [36] designed truthful mechanism to study secure communication in wireless cooperative systems. Aggarwal *et al.* [37] designed truthful auctions for pricing keywords in search engines. Machine learning techniques provide potential enhancement to auction mechanism design [38], and channel allocation within TV white space [39] is also worth investigation.

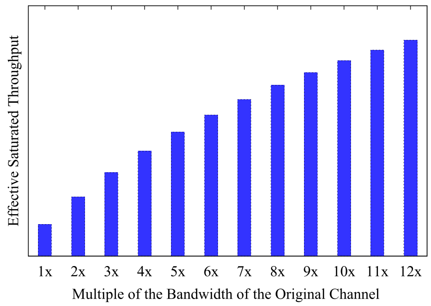


Fig. 1. Effective saturated throughput as a function on the multiple of the bandwidth of the original channel.

III. Technical Preliminaries

In this section, we present our auction model for the problem of combinatorial channel allocation, and review several solution concepts from game theory and mechanism design.

A. Auction Model

We model the problem of wireless channel allocation as a *combinatorial channel auction*. In this auction, there is a wireless service provider, called "seller", who possesses the license of a number of wireless channels and wants to lease out regionally idle channels; and there is a set of static nodes, called "buyers", such as WiFi access points and WiMAX base stations, who want to lease channels in order to provide services to their customers. A channel can be leased to multiple buyers, if these buyers can transmit simultaneously and receive signals with an adequate Signal to Interference and Noise Ratio (SINR). Different from existing channel auction mechanisms, our combinatorial channel auction allows buyers to bid for various numbers of contiguous channels.[1] The auction is sealed-bid and private, meaning that the buyers simultaneously submit their bids privately to the "auctioneer" without any knowledge of others, and do not collude. The objective of the auction is to efficiently allocate the channels to the buyers based on their bids, without violating interference constraints between the buyers.

We assume that the seller has a set of contiguous, orthogonal, and homogenous channels $K = \{1, 2, \ldots, k\}$ to lease out (*e.g.*, 3 orthogonal channels in IEEE 802.11b/g and 12 in IEEE 802.11a. For the other protocols working on different spectrum bands, there can be even more orthogonal channels.). The available channels are numbered from 1 to k. As is shown in paper [12], contiguous original channels can be combined to get a wider channel. Such a combined channel can normally provide higher throughput than a single original channel. In Figure 1, we present the function of effective saturated throughput of a channel on the multiple of the bandwidth of the original channel, which is derived from the experiment results of [12] and mathematical analysis of [40]. In our auction, a (combined) channel can be leased to one or a group of non-conflicting buyers. (We will define buyer group in Section IV-A.)

[1]Our model of combinatorial channel auction is a variant of traditional combinatorial auctions, which allow buyers to place bids on any combinations of discrete items. In our model, the buyers bid for contiguous channels, which can be accessed with a single radio.

We denote the set of buyers by $N = \{1, 2, \ldots, n\}$, where each buyer has a unique identification number. In this paper, we denote the buyers' identification number by 1 through n. We assume that each of the buyers only has a single radio,[2] and can tune its radio to work on an original channel or a wider channel combined by several contiguous original channels. Let v_i^q be buyer i's valuation of a wider channel combined by q $(1 \le q \le k)$ contiguous original channels. Then the valuation vector of a buyer i can be denoted as:

$$\vec{v}_i = (v_i^1, v_i^2, \ldots, v_i^k).$$

A buyer's valuation function is private information to the buyer herself and is commonly named *type*. We assume that the valuation function is also a concave non-decreasing function. On one hand, this is directly the result coming from Figure 1; on the other hand, it is widely assumed that network users may hold a decreasing marginal valuation for data transmission rate, which also leads to a concave non-decreasing valuation function. Therefore, this implies

$$\frac{v_i^x}{x} \ge \frac{v_i^y}{y}, \forall i \in N, \forall x, y, \quad s.t.\ x < y \wedge 1 \le x, y \le k, \tag{1}$$

In practice, it is more reasonable to give the buyers the flexibility to submit various combinatorial bids for channels. In our combinatorial channel auction, we allow each buyer to submit an independent bid b_i^q for each number q $(1 \le q \le k)$ of contiguous channels. Similarly, we denote a buyer i's bid vector by:

$$\vec{b}_i = (b_i^1, b_i^2, \ldots, b_i^k).$$

According to inequality (1), we have

$$\frac{b_i^x}{x} \ge \frac{b_i^y}{y}, \forall i \in N, \forall x, y, \quad s.t.\ x < y \wedge 1 \le x, y \le k, \tag{2}$$

when buyers truthfully submit their bids.

In our combinatorial channel auction, the strategy s_i of a buyer $i \in N$ is to report a bid vector, in which $b_i^q = s_i(v_i^q, q)$, based on her channel valuation v_i^q, for each $q (1 \le q \le k)$. The strategy profile $\vec{s}$ of all the buyers is represented by the following vector:

$$\vec{s} = (s_1, s_2, \ldots, s_n).$$

According to the notation convention, let $\vec{s}_{-i}$ represent the strategy profile of all the buyers except buyer i.

We assume that all the buyers are rational, and their objectives are to maximize their own utilities. Here, we define the utility of a buyer $i \in N$ as

$$u_i(\vec{s}) = v_i(\vec{s}) - p_i(\vec{s}), \tag{3}$$

where $v_i(\vec{s})$ is player i's valuation on the outcome of the strategy profile $\vec{s}$, and $p_i(\vec{s})$ is a charge for using the allocated channel(s). We assume that a buyer has no preference over different outcomes, if the utility is the same to the buyer herself.

[2]We note that our channel auction mechanism can be extended to the case of multiple radios by modeling each radio as a virtual buyer [11].

B. Solution Concepts

We recall several important solution concepts from game theory and mechanism design. First, we recall the definition of *Dominant Strategy*:

Definition 1 (Dominant Strategy [41], [42]): A dominant strategy of a player is one that maximizes her utility regardless of what strategies the other players choose. Specifically, $s_i^\star$ is player i's dominant strategy, if for any $s_i' \neq s_i^\star$ and any strategy profile of the other players $\vec{s}_{-i}$, we have

$$u_i(s_i^\star, \vec{s}_{-i}) \geq u_i(s_i', \vec{s}_{-i}). \quad (4)$$

Before recalling the definition of Strategy-proof Mechanism, we define direct-revelation mechanisms first. A direct-revelation mechanism is a mechanism, in which the only strategy available to players is to make claims about their preferences to the mechanism. In our combinatorial channel auction, the strategy of a buyer is to report a bid based on her channel valuation. A direct-revelation mechanism is strategy-proof if it satisfies two conditions, *incentive-compatibility* and *individual-rationality*. Incentive-compatibility means reporting truthful information is a dominant strategy for each player. Individual-rationality means each player can always achieve at least as much expected utility from faithful participation as without participation. The formal definition of *Strategy-Proof Mechanism* is as follows.

Definition 2 (Strategy-Proof Mechanism [43], [44]): A direct-revelation mechanism is strategy-proof if reporting truthful information is a dominant strategy for each player, and each player can always achieve at least as much expected utility from faithful participation as without participation.

In our combinatorial channel auction, the strategy-proofness means that no buyer $i \in N$ can increase her utility by reporting a bid $b_i^q \neq v_i^q$ for any q $(1 \leq q \leq k)$. In other words, it is every buyer's best strategy to simply submit her valuation as the bid in our combinatorial channel auction.

IV. Design of SPECIAL and Strategy-Proofness

In this section, we present our design of SPECIAL, and prove its strategy-proofness.

A. Auction Design

The design of SPECIAL is composed of three main components: *buyer grouping and bid integration*, *group-channel allocation*, and *winner selection and charging*. First, we divide the buyers into multiple groups with a bid-independent method, and define the integrated group bid, which is used instead of buyers' individual bids in the channel allocation process. Then, we present an algorithm to determine which channels are assigned to every buyer group. Finally, we show our winner selection criteria and charging scheme that guarantee the strategy-proofness of our combinatorial channel auction. As shown in inequality (1) and (2), we claim that each buyer has a concave non-decreasing valuation function on the number of channels, and require that the bids submitted by each buyer should also be a concave non-decreasing function on the number of channels.

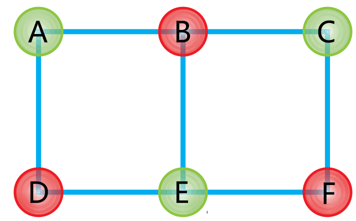


Fig. 2. A toy example with 6 buyers (A-F).

1) Buyer Grouping and Bid Integration: Considering the spatial reusability of the channels, SPECIAL divides all the buyers into multiple non-conflicting groups. Each group can be assigned with a distinct channel. The assigned channel is either an original channel or a wider channel that is composed of several original contiguous channel. To prevent the buyers from manipulating the auction, here we group the buyers using a bid-independent method. As in [9] and [11], SPECIAL uses a conflict graph to capture the radio transmission interference among the buyers. Any pair of buyers, who are in the radio transmission interference range of each other, have a line connecting them in the conflict graph. Then the calculation of bid-independent groups can be implemented by a certain existing graph coloring algorithm (*e.g.*, [45]), such that no two buyers have interference between each other in the same group. We note that the buyers have no control on which group they are in, when the above grouping strategy is used.

We denote the set of buyer groups by

$$G = \{g_1, g_2, \ldots, g_m\},$$

where m is the number of the buyer groups. The buyer groups in G should satisfy the following two requirements:

$$\bigcup_{1 \leq j \leq m} g_j = N,$$

meaning that all the buyers are involved, and

$$g_j \cap g_f = \emptyset, \forall g_j, g_f \in G \wedge j \neq f,$$

meaning that no buyer can be in multiple groups.

Figure 2 shows a toy example with 6 buyers (A-F). There exists several feasible grouping results, *e.g.*, $g_1 = \{A, C, E\}$ and $g_2 = \{B, D, F\}$.

From now on, we consider the buyer groups as competitors in the combinatorial channel auction. We now define the integrated group bid for each of the buyer groups. Although a natural way to define the group bid is to simply sum up all the bids for each number of contiguous channels from the group members, this way may allow some of the buyers to manipulate the group bid by reporting untruthful bids [9]. Therefore, to guarantee the strategy-proofness of the auction, we let the group bid be proportional to the smallest bid for each number of contiguous channels in the group, and sacrifice the buyers who may benefit from manipulating the group bid. The sacrificed will not be granted any channel. Two types of buyers have to be sacrificed when computing a group's bid for q contiguous channels:

(1) The buyer who submitted the smallest bid for q contiguous channels in the group. We will present the detailed

analysis in Section IV-A3a and IV-A3b. In the case of ties, *i.e.*, more than one buyer submits the smallest bid in the group, the tied buyer with smallest identification number will be selected as the sacrificed buyer.

(2) The buyer who can benefit by manipulating her bid for other numbers of contiguous channels than q in order to make herself win q contiguous channels. In Section IV-A3c, we will present our scheme to identify such cheating buyers in order to achieve strategy-proofness.

Here, we claim that the number of sacrificed buyers is always no more than two in a buyer group.[3] So we define the integrated group bid (IGB) φ_j^q for each group $g_j \in G$ on q contiguous channels as

$$\varphi_j^q = \max((|g_j| - 2) \cdot \theta_j^q, 0), \tag{5}$$

where

$$\theta_j^q = \min_{l \in g_j}(b_l^q). \tag{6}$$

We denote the IGB vector of group g_j as

$$\vec{\varphi}_j = (\varphi_j^1, \varphi_j^2, \ldots, \varphi_j^k).$$

According to inequality (2), we can get that φ_j^q is also a concave non-decreasing function on q, for every $g_j \in G$. Note that in our design of SPECIAL, we consider the allocation of contiguous channels. However, if the aggregate throughput of non-contiguous channels is still a concave non-decreasing function on the number of channels, we can obtain the same result on φ_j^q, meaning SPECIAL can be applied to that case as well.

We note that even in the special case that the group has only two buyers, *i.e.*, $\varphi_j^q = 0$, the valid winning buyers in group g_j are still be charged according to our charging scheme, indicating that the integrated group bid (IGB) is not linked to the winning buyers charge.

2) Group-Channel Allocation: After forming the buyer groups, we present our algorithm that allocates contiguous channels to the buyer groups based on their IGBs.

For ease of comparison between IGBs, we define per-channel integrated group bid (PIGB) ξ_j^q for each buyer group g_j on q contiguous channels:

$$\xi_j^q = \frac{\varphi_j^q}{q}. \tag{7}$$

Similarly, we denote the PIGB vector of group g_j as

$$\vec{\xi}_j = (\xi_j^1, \xi_j^2, \ldots, \xi_j^k).$$

Since φ_j^q is a concave non-decreasing function on q, we can get that ξ_j^q is a non-increasing function on q, such that

$$\xi_j^x \geq \xi_j^y, \ \forall x < y \wedge 1 \leq x, y \leq k, \ \forall g_j \in G. \tag{8}$$

[3] When most of the groups have only one buyer, this problem regresses to the channel allocation in a single collision domain. Existing works like [18], [20] have provided possible solutions.

Algorithm 1 Algorithm for Group-Channel Allocation GCA()

Input: The set of buyer groups G, the number of available channels k, and a set $\xi = \{\xi_j^q | g_j \in G, 1 \leq q \leq k\}$ of PIGBs.

Output: A vector $\vec{r}$ of numbers of channels allocated to every group, and a channel allocation vector $\vec{ca}$.

$\vec{r} \leftarrow 0^m$, $\vec{ca} \leftarrow (0,0)^m$, $k' \leftarrow k$
while $k' > 0$ **do**
  $\xi_j^q \leftarrow \max(\xi)$
  $r_j \leftarrow q$, $k' \leftarrow k' - 1$
  $\xi \leftarrow \xi \setminus \{\xi_j^q\}$
end while
$k' \leftarrow 1$
for $j = 1$ to m **do**
  if $r_j > 0$ **then**
    $ca_j \leftarrow (k', k' + r_j - 1)$
    $k' \leftarrow k' + r_j$
  end if
end for
return $(\vec{r}, \vec{ca})$.

For the ease of comparison between PIGBs, we define the preference relation as

$$(a, h) \prec (b, j) \Leftrightarrow a < b \vee (a = b \wedge h < j),$$

where a and b are values of PIGBs, and h and j are the identification numbers of buyer groups. In the case of ties in the process of channel allocation, we determine that the group with higher group number has higher priority to be allocated a channel.

Algorithm 1 shows the pseudo-code of group-channel allocation algorithm GCA() used in SPECIAL. The algorithm takes in the set of buyer groups G, the number of available channels k, and a set $\xi = \{\xi_j^q | g_j \in G, 1 \leq q \leq k\}$ of PIGBs, and then outputs a vector $\vec{r}$ of the numbers of original channels allocated to each group, and a vector $\vec{ca}$ which determines the channels allocated to every group. Generally, GCA() is a greedy algorithm, and according to ineuqation (8), we can get that, for every group, the number of its allocated channels increases one by one, if any. Therefore, after the execution of the GCA(), there will be no available channel left. Each element r_j in $\vec{r}$ means that r_j contiguous channels are allocated to buyer group g_j; each element $ca_j(x, y)$ in $\vec{ca}$ means that the original channels $\{z | x \leq z \leq y\} \subseteq K$ are allocated to g_j.

We note that although Algorithm 1 can efficiently allocate the channels to the buyer groups according to their PIGBs, it cannot guarantee strategy-proofness. In the next sub section, we will present a method to strengthen Algorithm 1 in order to achieve strategy-proofness.

3) Winner Selection and Charging: In this section, we consider how to determine winners in each winning buyer group who has been assigned channel(s) and their charges for using the assigned channel(s). The design of this part directly determines the auction mechanism's properties. A carefully designed winner selection and charging scheme can guarantee the strategy-proofness of the auction. In this

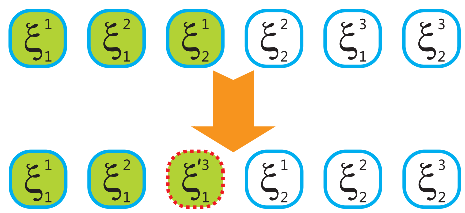


Fig. 3. An illustration of preemptive bidding. PIGBs are sorted in non-increasing order. A buyer in group g_1, who can only get 2 channels by bidding truthfully, may get 3 channels by submitting an untruthful bid for 3 channels.

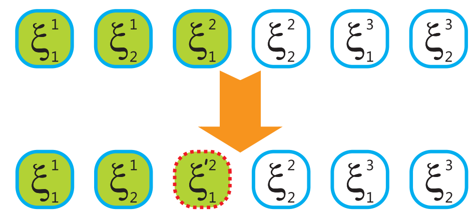


Fig. 4. An illustration of depreciated bidding. PIGBs are sorted in non-increasing order. A buyer in group g_1 can increase her utility by submitting an untruthful bid that is lower than any others' bids in the group and her own valuation for 2 channels.

section, we analyze possible cheating actions of the buyers, and then strengthen our winner selection and charging scheme step by step to achieve strategy-proofness.

Our analysis shows that there are three cheating actions, say, *preemptive bidding*, *depreciated bidding*, and *retreat for advancing*, through which a buyer may improve her utility. We provide a method to prevent each of the cheating actions, respectively. In the following, we continue to use the toy example shown in Figure 2 with 3 channels for auctioning to illustrate the effect of buyers' cheating actions.

a) Preemptive bidding: The cheating action of preemptive bidding means that a buyer $i \in g_j$ submits a cheating bid vector to make PIGB $\xi_j'^q$ be selected as a winning bid, which would never be selected as a winning group bid if the buyer i bids truthfully. Thus, group g_j wins q channels, and so does i.

Figure 3 shows the effect of preemptive bidding. The PIGBs are sorted in non-increasing order. Suppose the sorted PIGBs shown in solid-border round corner squares represent the case, when buyer $A \in g_1$ bids truthfully. In this case, 2 channels are allocated to g_1 and 1 channel to g_2. Then, we assume that buyer $A \in g_1$ submits an untruthful bid $b_A'^3 \neq v_A^3$, such that PIGB ξ_1^1, ξ_1^2, and $\xi_1'^3$ (indicated by the dashed-border round corner square) are sequentially selected as winning bids. Consequently, group g_1 wins 3 channels, and the buyer A may also get 3 channels.

From this example, we observe that buyer A's truthful bid $b_A^{\star 3}$ must be the minimum bid in $\{b_A^{\star 3}, b_C^3, b_E^3\}$; otherwise, buyer A's cheating bid on 3 channels cannot increase the PIGB ξ_1^3 of group g_1. So $v_A^3 = b_A^{\star 3} = \theta_1^3$. If we charge every winner in group g_1 the price θ_1^q, where q channel(s) will be allocated to group g_1, then even if buyer A successfully get 3 channels, her utility will be negative or zero, because θ_1^3 will be at least as large as v_A^3.

Formally, we define the charging scheme as follows. If a group g_j wins q contiguous channels, each potential winning buyer $i \in g_j$ is charged a uniform price, which is equivalent to the smallest bid for q contiguous channel in the group. Here, we define the charge of every buyer for using the allocated channels as

$$p_i = \theta_j^q \cdot \eta_i, \tag{9}$$

where η_i decides whether buyer i is selected as a winner or not.

Lemma 1: For any winning group bid ξ_j^q, if we charge each winner $i \in g_j$ with θ_j^q, preemptive bidding can be prevented.

b) Depreciated bidding: The cheating action of depreciated bidding means that a buyer $i \in g_j$ may submit a lower cheating bid $b_i'^q$ than the truthful one $b_i^{\star q} = v_i^q$, with no influence on the channel allocation. Such a cheating action may decrease the charge to the winners in g_j, if $\xi_j'^q$ is a winning PIGB and $b_i'^q$ appears to be the smallest bid for q channels in the group g_j. As a result, if i is selected as an auction winner, her utility can be increased through depreciated bidding.

Figure 4 shows the cheating action of depreciated bidding. Suppose buyer group g_1 wins 2 channels when buyer A bid truthfully (*i.e.*, $b_A^2 = v_A^2$). We assume that buyer A submits a lower bid $b_A'^2 < b_A^{\star 2}$, such that g_1 still wins 2 channels, but $\xi_1'^2 < \xi_1^2$. So $\min(b_A'^2, b_C^2, b_E^2) \leq \min(b_A^{\star 2}, b_C^2, b_E^2)$. Then buyer A's utility becomes

$$\begin{aligned} u'_A &= v_A^2 - \min(b_A'^2, b_C^2, b_E^2) \\ &\geq v_A^2 - \min(b_A^{\star 2}, b_C^2, b_E^2) \\ &= u_A. \end{aligned}$$

Hence, buyer A may get her utility increased by depreciated bidding.

From this example, we observe that if a buyer $i \in g_j$ can benefit from depreciated bidding, she must appear to be the one who has the smallest bid for q channels when ξ_j^q is a winning PIGB. Therefore, after allocating q channels to buyer group g_j, the buyer, who has the smallest bid for q channels in the group g_j, should be excluded from the set of winners.

Lemma 2: If ξ_j^q is a winning PIGB, we can prevent depreciated bidding by excluding the buyer $i = \underset{i \in g_j}{argmin}(b_i^q)$ from the winner set. *i.e.*, let $\eta_i = 0$.

c) Retreat for advancing: The cheating action of retreat for advancing means that if a buyer $i \in g_j$ bids truthfully, PIGB ξ_j^q will be selected as a winning PIGB for group g_j; but if buyer i submits several cheating bids, another PIGB $\xi_j^{q'}$ ($q' < q$) is selected as the final winning PIGB for group g_j. Consequently, buyer i's utility u_i may be increased.

Figure 5 shows the effect of retreat for advancing. If buyer A bids truthfully for 2 channels $b_A^{\star 2} = v_A^2$, buyer group g_1 will be allocated 2 channels. When buyer A submits a cheating bid $b_A'^2$, only 1 channel is allocated to buyer group g_1. Suppose buyer A is selected as a winner with/without cheating. If $v_A^1 - \min\{b_A^1, b_C^1, b_E^1\} > v_A^2 - \min\{b_A'^2, b_C^2, b_E^2\}$, then the buyer A can get her utility increased by this method of cheating.

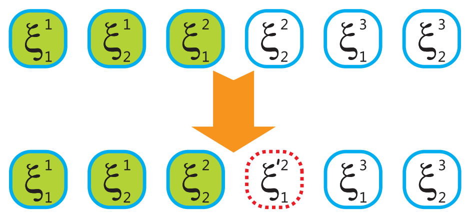


Fig. 5. An illustration of retreat for advancing. PIGBs are sorted in non-increasing order. A buyer in group g_1 may get her utility increased, by submitting an untruthful bid for 2 channels to make group g_1 win 1 channel instead of 2 channels won when bidding truthfully.

From this example, we can observe that if a buyer $i \in g_j$ benefit from winning q' ($q' + 1 \leq q$) channels instead of q through retreat for advancing, then the auction must exhibit the following two properties:

$$i = \underset{l \in g_j}{argmin}(b_l^{q'+1}), \tag{10}$$

and

$$(\min_{h \neq j}(\xi_h^{r_h}), \underset{h \neq j}{argmin}(\xi_h^{r_h})) \prec \left(\frac{\max\left((|g_j| - 2) \cdot \min_{l \neq i \wedge l \in g_j}(b_l^{q'+1}), 0\right)}{q' + 1}, j \right), \tag{11}$$

where r_h is the number of channels group h wins by $GCA()$.

To guarantee strategy-proofness, we have to exclude each such buyer who satisfies the above two criteria from the set of winners in each group.

Lemma 3: For every buyer group $g_j \in G$, if there exists buyer $i \in g_j$ satisfying the condition (10) and (11), we can exclude such buyers from winners to prevent the cheating action of retreating for advancing. *i.e.*, let $\eta_i = 0$.

We use Algorithm 2 to summarize our method to determine winners. We note that the time complexity of GCA() is $O(kn)$ and can be further improved to $O(klogn)$ by using heap sort. In WIN() winner selection and charging (line 3 $\sim$ 13) can be completed in $O(n + k)$, so the time complexity of WIN() is $O(n + klogn)$. With a well-known greedy graph coloring algorithm [46] adopted for buyer grouping, the overall time complexity of SPECIAL is $O(n^2 + klogn)$. In practice, the number of users is normally much larger than the number of channels, i.e., $n \gg k$. Thus, the time complexity of SPECIAL becomes $O(n^2)$.

Finally, we use an example to illustrate the process of winner selection and charge determination by SPECIAL. We still use the grouping result shown in Figure 2 with 3 channels for auctioning. Suppose buyers A-F submit their bid vectors as shown in Table I, thus we can obtain that $\vec{\varphi}_1 = (2, 2.6, 3.6)$ and $\vec{\varphi}_2 = (1.8, 2.8, 3)$. Consequently, we can obtain that $\vec{\xi}_1 = (2, 1.3, 1.2)$ and $\vec{\xi}_2 = (1.8, 1.4, 1)$, and after sorting, we have $\xi_1^1 > \xi_2^1 > \xi_2^2 > \xi_1^2 > \xi_1^3 > \xi_2^3$. According to GCA(), each PIGB is selected in descending order until all channels are allocated. Therefore, winning PIGBs are ξ_1^1 and ξ_2^2, i.e., g_1 wins 1 channel, and g_2 wins 2 channels. Next, SPECIAL selects winners in each winning groups and determines their corresponding charges.

Algorithm 2 Algorithm for Winner Selection WIN()

Input: The set of buyer groups G, the number of available channels k, and a set $\xi = \{\xi_j^q | g_j \in G, 1 \leq q \leq k\}$.

Output: A set W of winners in the combinatorial channel auction.

1: $W \leftarrow \emptyset$, $pm \leftarrow 0$, $\xi' \leftarrow \xi$
2: $(\vec{r}, \vec{ca}) \leftarrow GCA(G, k, \xi')$
3: **for all** $r_j > 0$ **do**
4: $\quad T \leftarrow g_j \setminus \{\underset{i \in g_j}{argmin}(b_i^{r_j})\}$
5: $\quad$ **if** $r_j < k$ **then**
6: $\quad\quad pm \leftarrow \underset{i \in g_j}{argmin}(b_i^{r_j+1})$
7: $\quad\quad d \leftarrow \underset{h \neq j}{argmin}(\xi_h^{r_h})$
8: $\quad\quad$ **if** $(\xi_d^{r_d}, d) \prec \left(\frac{\max((|g_j|-2) \cdot \min_{l \neq pm \wedge l \in g_j}(b_l^{r_j+1}), 0)}{r_j+1}, j \right)$ **then**
9: $\quad\quad\quad T \leftarrow T \setminus \{pm\}$
10: $\quad\quad$ **end if**
11: $\quad$ **end if**
12: $\quad W \leftarrow W \cup T$
13: **end for**
14: return W.

TABLE I

BID VECTORS OF BUYERS, AN ILLUSTRATIVE EXAMPLE

Group j	g_1			g_2		
Buyer i	A	C	E	B	D	F
b_i^1	2	2.5	2.5	1.8	2	2
b_i^2	3	2.6	3	2.8	2.9	3
b_i^3	3.6	3.75	4.2	3.3	3	3.3

Since $A = \underset{i \in g_1}{argmin}(b_i^1)$ and $B = \underset{i \in g_2}{argmin}(b_i^2)$, A and B will be excluded from the winners. Then in g_1, we have $pm = C$ and $d = 2$. Following line 8 in WIN(), we can obtain $(1.4, 2) \prec (1.5, 1)$, so C has to be excluded from the winners as well. Note that in g_2, this situation does not hold for buyer D. Therefore, E in g_1 wins 1 channel and will be charged $\theta_1^1 = 2$, while D and F in g_2 both win 2 channels and will be charged $\theta_2^2 = 2.8$.

B. Strategy-Proofness

Theorem 1: SPECIAL is a strategy-proof combinatorial channel auction mechanism.

Proof: We prove that SPECIAL satisfies both *individual rationality* and *incentive compatibility* as follows.

1) Individual Rationality: We can see that each truthful buyer's utility is always no less than 0. By not taking part in the auction, a buyer cannot get a channel, and her utility is 0. So participating is not worse than staying outside the auction. Therefore, SPECIAL satisfies the individual rationality.

2) Incentive Compatibility: We will prove that no buyer can increase her utility by submitting a cheating bid, which is not equal to her valuation, no matter what the other buyers do. That is to say, truthful bidding is every buyer's dominant strategy. In our previous analysis in Section IV-A3, we have pointed out three representative kinds of cheating actions capturing the essential of possible cheatings, and thus designed the corresponding schemes to prevent them happening. We now prove that considering all these three cheating actions and exploiting all our schemes in Section IV-A3, our design is sufficient to make SPECIAL achieve strategy-proofness.

Suppose a buyer i belongs to a buyer group g_j that wins r_j and r'_j channels, when buyer i bids truthfully and not, respectively. Here, we note that group g_j wins r_j or r'_j channels does not guarantee that the buyer i also get r_j or r'_j channels, because buyer i can be out of the set of winners. We consider the possible change of buyer i's utility in three cases:

a) $r'_j > r_j$*:* Group g_j gets more channels when buyer i bids untruthfully. In this case, we can get that

$$b_i'^{q} > b_i^{\star q} = \theta_j^q, \forall q \in \{x | r_j + 1 \le x \le r'_j\}.$$

So we can get that

$$b_i'^{r'_j} \ge \theta_j'^{r'_j} > b_i^{\star r'_j} = v_i^{r'_j}.$$

Thus, if buyer i wins a certain number of channel(s) finally (*i.e.*, $\eta_i = 1$), we can get that

$$\begin{aligned} u'_i &= v_i^{r'_j} - \theta_j'^{r'_j} \\ &< v_i^{r'_j} - b_i^{r'_j} \\ &= 0 \end{aligned}$$

Otherwise, $u'_i = 0$. Therefore, it shows that if buyer i manipulates her bid to win $r'_j > r_j$ channel(s), she will have $u'_i \le u_i$.

b) $r'_j = r_j$*:* Group g_j still gets the same number of channels when the buyer i bids untruthfully. If $r'_j = r_j = 0$, then buyer i's utility is still 0. So we focus on the cases, in which $r'_j = r_j > 0$. We now distinguish two cases as follows:

- Buyer i wins r_j channels when she submits a truthful bid. In this case, with the fact that $i \neq \underset{l \in g_j}{argmin}(b_l^{r_j})$, her utility is

  $$u_i = v_i^{r_j} - \min_{l \in g_j \wedge l \neq i}(b_l^{r_j}).$$

  To improve her utility, she has to decrease the charging price for herself. However, she will not manage to reach it unless she decreases her bid $b_i^{r_j}$ to $b_i'^{r_j}$ such that

  $$b_i'^{r_j} < \min_{l \in g_j \wedge l \neq i}(b_l^{r_j}).$$

  However, if she does so, she will win no channel because of lemma 2, leading to $u'_i \le u_i$.
- The buyer i wins no channel when she submits a truthful bid, meaning $u_i = 0$ (*i.e.*, $\eta_i = 0$). Due to lemma 2 and 3, which can lead to $\eta_i = 0$, we further distinguish two cases:
  - Using depreciated bidding, *i.e.*, $i = \underset{l \in g_j}{argmin}(b_l^{r_j})$.

    For this case, if she wants to improve her utility, the only possible method is to submit an untruthful bid $b_i'^{r_j} \ge \min_{l \in g_j \wedge l \neq i}(b_l^{r_j})$. But if she does, we will get that

    $$\begin{aligned} u'_i &= v_i^{r'_j} - \min_{l \in g_j \wedge l \neq i}(b_l^{r_j}) \\ &\le v_i^{r'_j} - b_i'^{r_j} \\ &= 0 \end{aligned}$$
  - Using retreat for advancing, *i.e.*,

    $$\begin{cases} (\min_{h \neq j}(\xi_h^{r_h}), \underset{h \neq j}{argmin}(\xi_h^{r_h})) \\ \prec \left(\frac{\max\left((|g_j|-2) \cdot \min_{l \neq i \wedge l \in g_j}(b_l^{r_j+1}), 0 \right)}{r_j+1}, j \right) \\ i = \underset{l \in g_j}{argmin}(b_l^{r_j+1}) \end{cases} \tag{12}$$

    For this case, if she wants to improve her utility, the only possible method is to submit an untruthful bid $b_i'^{r_j+1} > b_i^{\star r_j+1}$, such that $i \neq \underset{l \in g_j}{argmin}(b_l'^{r_j+1})$. But if she does so, we will get that group j will win $r_j + 1$ channels, because

    $$\begin{aligned} \xi_j'^{r_j+1} &= \frac{\max\left((|g_j| - 2) \cdot \min_{l \neq i \wedge l \in g_j}(b_l^{r_j+1}), 0 \right)}{r_j + 1} \\ &\Rightarrow (\min_{h \neq j}(\xi_h^{r_h}), \underset{h \neq j}{argmin}(\xi_h^{r_h})) \prec (\xi_j'^{r_j+1}, j) \end{aligned}$$

    However, this contradicts with the condition $r'_j = r_j$.

    So this cheating method cannot happen in this case.

Therefore, it appears that if buyer i manipulates the auction, achieving that $r'_j = r_j$, we can always have $u'_i \le u_i$.

c) $r'_j < r_j$*:* Group g_j gets less channels when buyer i bids untruthfully. If $r'_j = 0$, then buyer i's utility becomes 0. Consequently, we focus on the case of $r'_j > 0$. In this case, we can get that

$$i = \underset{l \in g_j}{argmin}(b_l'^{r'_j+1}),$$

because otherwise, $\xi_j'^{r'_j+1} = \xi_j^{r'_j+1} \ge \xi_j^{r_j}$, leading to the result that group j wins at least $r'_j + 1$ channel(s). We can also get that

$$\begin{aligned} &\frac{\max((|g_j| - 2) \cdot \min_{l \in g_j \wedge l \neq i}(b_l^{r'_j+1}), 0)}{r'_j + 1} \\ &\ge \xi_j^{r'_j+1} \\ &\ge \xi_j^{r_j} \end{aligned}$$

and

$$(\min_{h \neq j}(\xi_h^{r'_h}), \underset{h \neq j}{argmin}(\xi_h^{r'_h})) \prec (\xi_j^{r_j}, j)$$

Thus, we can conclude that

$$(\min_{h\neq j}(\xi_h^{r'_h}), \underset{h\neq j}{argmin}(\xi_h^{r'_h})) \prec \left(\frac{\max((|g_j|-2)\cdot \min_{l\in g_j \wedge l\neq i}(b_l^{r'_j+1}), 0)}{r'_j+1}, j\right)$$

Hence, according to lemma 3, buyer i will be excluded from the winner set, which results in that $u'_i = 0$.

Therefore, if buyer i manipulates her bid to win $r'_j < r_j$ channel(s), her utility $u'_i \leq u_i$.

All in all, we have proved that truthful bidding is every buyer's dominant strategy. Therefore, SPECIAL satisfies incentive compatibility. Since SPECIAL satisfies both incentive compatibility and individual rationality, we conclude that SPECIAL is a strategy-proof combinatorial channel auction mechanism. ■

V. Numerical Results

We implement SPECIAL and evaluate its performance using network simulations. The objective of our simulations is twofold. One is to demonstrate that our mechanisms prevent buyers from misreporting channel valuations. The other is to measure the influence of our mechanisms on the system performance.

A. Methodology

We implement SPECIAL based on a greedy graph coloring algorithm [46] and present its performance. In the simulation, buyers are randomly distributed in various terrain areas (including 1000 × 1000, 1500 × 1500, 2000 × 2000 and 2500 × 2500 meters). The number of buyers varies from 20 to 600. Same as [47], we set interference range to be 1.7 times transmission range. The outdoor transmission range of IEEE 802.11n is about 250 meters. Therefore, the radio interference range is set to 425 meters. The numbers of channels for leasing can be 6, 12 or 24.

We assume that the vector of any buyer's channel valuations is concave non-decreasing and randomly distributed over $[0, k]$, where k denotes the number of channels.[4] In our simulations, we define the function of the valuation of each buyer as

$$\begin{cases} y_1 = \Delta_1 = rand[0,1]; \\ \Delta_t = \Delta_{t-1} * rand[0,1], \quad \forall t,\ 2\leq t\leq k; \\ y_t = y_{t-1} + \Delta_t, \quad \forall t,\ 2\leq t\leq k \end{cases} \tag{13}$$

All the results on performance are averaged over 4000 runs.

Node Behavior: In our simulations, we compare two kinds of node behavior:

- *Honest Reporting:* Reporting one's true channel valuation vector as her bid.
- *Misreporting:* Reporting an arbitrary bid vector other than one's true channel valuation vector. The misreported bid vector is also non-decreasing in the range $[0, k]$.

[4]The ranges of buyers' channel valuations can be different from the ones used here. However,the evaluation results of using different ranges are identical. Therefore, we only show the results for the above ranges in this paper.

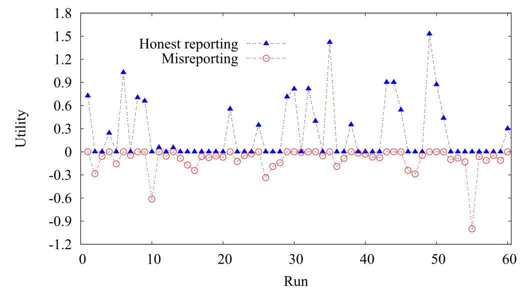


Fig. 6. Utilities obtained by a randomly selected buyer 121 when it honestly reports vs. misreports channel valuation. The figure demonstrates that the buyer can never benefit from misreporting.

Metrics: We evaluate two metrics:

- *Utility:* Utility is the difference between buyer's channel valuation and payment. This metric reflects the impacts of a buyer's behavior on her own. The target of our evaluation is to verify that, with SPECIAL, a buyer cannot get more utility by misreporting.
- *Channel Utilization:* Average number of users working on each channel. According to FCC's mission to make spectrum available so far as possible and improve all wireless users' utilities, we choose channel utilization as our primary metric. Because that channel utilization reflects how many users are working on each channel on average indicates how well our auction facilitates the redistribution of available channels.

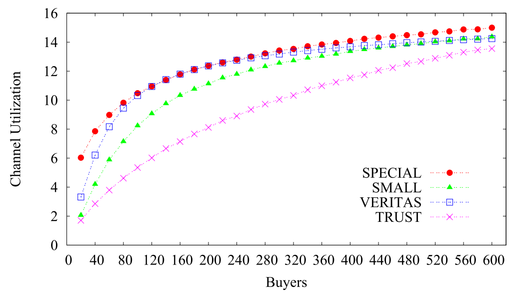
B. Impacts of Buyer's Behavior on Her Utility

In our first simulation, we demonstrate that, a buyer cannot increase her own utility by misreporting channel valuation. For this purpose, we randomly sample buyers and record the utilities they obtain by honest reporting and misreporting, respectively. Since the two utilities are same in most of runs of the simulation, we only show the first 60 records in which the two utilities are different. However, the simulation is repeated more than 4000 times. The number of buyers in this simulation is set to 200, the number of channels for auctioning is 12, and the terrain area is 2000 × 2000 meters.

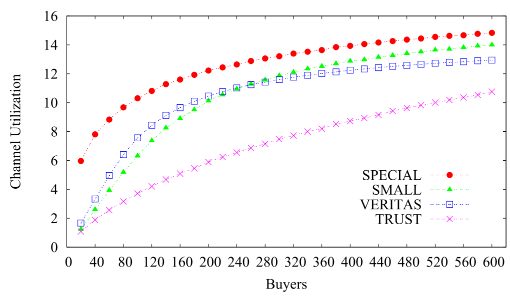
Figure 6 shows the utilities of a randomly selected buyer (buyer 121). The results for the other nodes are similar to that of buyer 121. In each run, the other buyers randomly choose to be honest or not, and their bid vectors do not change when evaluating buyer 121's utilities of honest reporting and misreporting. From the figure, we can see that the utility of misreporting is always lower than that of honest reporting, when misreporting results in a different utility. We can also see that the utility obtained by honest reporting is always non-negative, while misreporting leads to about 70% negative utility in the records presented.

C. Impacts on Channel Auction Performance

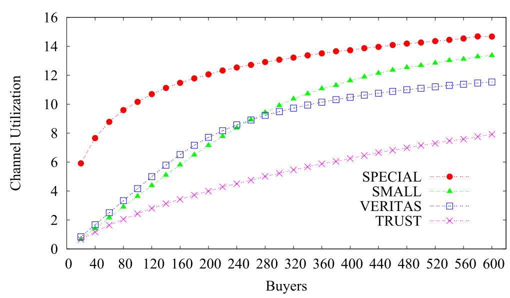
In this set of our simulations, we compare the performance of SPECIAL, in terms of channel utilization, with three existing strategy-proof channel auction mechanism: TRUST [9], VERITAS [8], and SMALL [11]. In TRUST, VERITAS, and SMALL, the per-channel valuation of a player is equal to that of a single channel in SPECIAL.



(a) 6 channels



(b) 12 channels



(c) 24 channels

Fig. 7. Channel Utilization of SPECIAL, TRUST, VERITAS and SMALL for auctioning 6, 12, and 24 channels.

Figure 7 shows the channel utilization of SPECIAL, TRUST, VERITAS and SMALL. The number of channels for auctioning are 6, 12 and 24, respectively. The number of buyers varies from 20 to 600. The terrain area is 2000 $\times$ 2000 meters. These three figures show that SPECIAL performs at least as well as any other one with any number of buyers involved. Since SPECIAL enables all buyers to flexibly submit their bids for adaptive-width channels, large groups with high bids can win more channels than others, leading to the result that compared with TRUST, VERITAS and SMALL, SPECIAL enables more users to operate on each channel on average. The only chance that VERITAS's channel utilization approaches that of SPECIAL happens when there are 6 channels and 120-240 buyers. This is because SPECIAL needs to sacrifice some buyers to guarantee strategy-proofness. Furthermore, the advantage of SPECIAL dramatically increases with the increasing number of channels for auctioning.

Figure 8 shows an unique property of SPECIAL, *i.e.*, SPECIAL can achieve almost the same channel utilization with different numbers of channels for auctioning. However, this property does not hold for TRUST, VERITAS, or SMALL as shown in Figure 7.

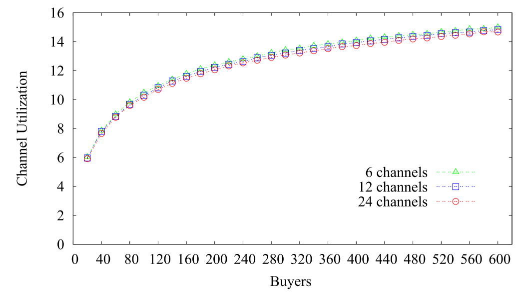


Fig. 8. Channel Utilization of SPECIAL for auctioning 6, 12 and 24 channels.

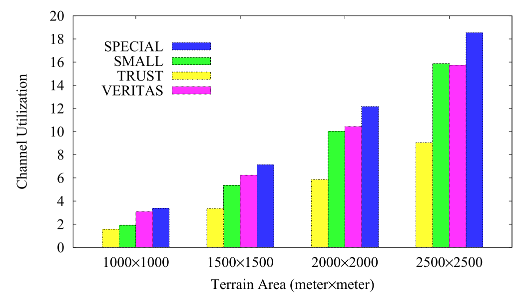


Fig. 9. Channel utilization of SPECIAL, TRUST, VERITAS and SMALL with the constant density of buyers over terrain area. The density is 1/20000 m^2, and the number of channels for auctioning is 12.

Figure 9 shows the channel utilization of SPECIAL, TRUST, VERITAS and SMALL with the constant density of buyers over terrain area. We fix the density on 1/20000 m^2. The number of channels for auctioning is 12, while the terrain area varies as 1000×1000, 1500×1500, 2000×2000 and 2500×2500 meters. Again, the results show that SPECIAL outperforms all the other auction mechanisms.

VI. Conclusion and Future Work

In this paper, we have modeled the spectrum allocation problem as a combinatorial auction, and proposed a strategy-proof and efficient spectrum allocation mechanism, called SPECIAL. We have implemented SPECIAL and performed extensive evaluations on it. Our evaluation results verify that SPECIAL is a strategy-proof auction mechanism. Compared with existing works, SPECIAL achieves better performance in terms of channel utilization. Furthermore, our evaluation results show that SPECIAL can achieve steady channel utilization with different numbers of channels for auctioning.

There are several interesting directions for future work. One direction is to extend SPECIAL to be resistant to collusion among users, since users are normally selfish and may form collusive groups in practice to further manipulate bids for their own interests. Yet, another direction is to consider the manipulation on the number of radios, *i.e.*, a user can buy a wider channel but actually split it into multiple narrow channels and tune each of her radios on a subchannel. Although this may be detected by communication techniques, it is still interesting to investigate the resistance to the cheating behavior on the number of radios in the scope of game theory. Furthermore, another important direction is to incorporate the demand for various qualities of service and experience into the valuation function for adaptive-width channels.

In addition, designing auction mechanisms for channel allocation in multi-hop wireless networks without using a graph coloring algorithm is worth further investigation.

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