Strong-Incentive, High-Throughput Channel Assignment for Noncooperative Wireless Networks

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Abstract—Channel assignment is a very important topic in wireless networks. In this paper, we study FDMA channel assignment in a noncooperative wireless network, where devices are *selfish*. Existing work on this problem has considered Nash Equilibrium (NE), which is not a very strong solution concept and may not guarantee a good system performance. In contrast, in this work, we introduce a payment formula to ensure the existence of a Strongly Dominant Strategy Equilibrium (SDSE), a different solution concept that gives participants much stronger incentives. We show that, when the system converges to an SDSE, it also achieves global optimality in terms of system throughput. Furthermore, we extend our work to the case in which some radios have a limited tunability. We show that in such a case, nevertheless, it is generally impossible to have a similar SDSE solution; with additional assumptions on the numbers of radios and the types of channels, etc., we can again achieve an SDSE solution that guarantees optimal system throughput. Besides this extension, we also consider other extensions of our strategic game to achieve throughput fairness and to deal with possibly inconsistent information caused by players joining and leaving. Finally, we evaluate our design with simulated experiments. Numerical results verify that the system does converge to the globally optimal channel assignment with the proposed payment formula, and that the system throughput is significantly higher than that achievable with the random-based and NE-based channel assignment schemes.

Index Terms—Communication/networking, algorithm design, economics, security.

1 Introduction

The radio spectrum is a scarce resource in this age of fast growing wireless communications. To better utilize the radio spectrum, Frequency Division Multiplexing Access (FDMA) is introduced to divide the carrier bandwidth into channels of different frequencies, each carrying a signal at the same time. Some wireless systems also use Code Division Multiple Access (CDMA), Time Division Multiple Access (TDMA), or Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) to allow multiple radio transceivers to access the same frequency channel. With the emergence of software-defined radios, the problem of channel assignment, which assigns radio transceivers to available channels, has gained increasing importance. Due to the limitation on the number of available channels, careful channel assignment is needed to mitigate the performance degradation of wireless networks because of interference among different users.

In recent years, a large number of channel assignment schemes for wireless networks (e.g., [1], [2], [3], [4], [5], [6], [7], [8], [9]) have been proposed. In general, they assumed that all the nodes are "well behaved" or "cooperative." However, this assumption may not be valid in general ad hoc networks [10]. In practice, a node can easily deviate from the protocol to seek for more benefit for itself. So it is crucial to study how to provide incentives for the selfish nodes to behave cooperatively. In a recent work, Félegyházi et al. [11], [12] studied Nash Equilibria (NEs), which is a standard solution concept from game theory, in a noncooperative multiradio multichannel allocation game. While their work is elegant and intriguing, NE does not provide an ideal solution to the problem of channel assignment. There are two reasons: 1) NE is not a very strong solution concept. More specifically, when in an NE, a player of the game has incentives to keep its equilibrium strategy only under the assumption that all other players are also keeping their equilibrium strategies. When this assumption is not valid, NE does not provide incentives for the game player. 2) More importantly, NE is usually not social efficient, which means that the system performance is not maximized. Therefore, when the system converges to one of the NEs, it could be the case that some of the selfish nodes benefit at the cost of system performance degradation.

The objective of this paper is to provide a solution that can *guarantee* the system to converge to a state in which the system throughput is optimized. Specifically, we use a very strong solution concept from game theory, called Strongly Dominant Strategy Equilibrium (SDSE), to guarantee the system convergence. By its definition (see Section 3.2), SDSE ensures that, regardless of other nodes' behavior, a pair of communicating nodes always have incentives to use the equilibrium strategy. Hence, the solution we provide is much stronger than any NE-based solution. The main technical tool we use in this paper is a carefully designed

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payment scheme. The major contributions of this paper are as follows:

- First, we model the channel assignment problem as a strategic game. Our game model applies to the general scenario, where both single-radio devices and multiradio devices can exist. By introducing a carefully designed payment formula (for using channels), we ensure the existence of an SDSE. Furthermore, we show that the SDSE achieves the global optimality in terms of system throughput.
- Second, we extend our game model to a limited tunability system model and prove that one cannot find a similar SDSE in some cases. To deal with limited tunability, we introduce some practical assumptions on the numbers of radios and on the types of channels, etc. With these assumptions, we can again have an SDSE that achieves the global optimality using another carefully designed payment formula.
- Third, we study throughput fairness among the players of this game. Specifically, we provide a bound for the *fairness ratio* (see Section 6.1 for definition). Furthermore, we extend the strategic game to a repeated game so that we can achieve optimal throughput fairness. In our repeated game of channel assignment, not only optimal system throughput is preserved, but also the throughput shared among players is balanced in the long run.
- Fourth, we consider the effect of players joining and leaving the game, and discuss how to deal with possibly inconsistent information caused by such events.
- Finally, we evaluate our solutions using extensive simulated experiments. Numerical results show that, with the proposed payment formula, the system does converge to the globally optimal channel assignment. In addition, the system throughput is significantly higher than that achievable with the random-based and NE-based channel assignment schemes.

The rest of the paper is organized as follows: We briefly review the related works in Section 2 and present the technical preliminaries in Section 3. In Section 4, we describe our strategic game model of channel assignment, prove the existence of SDSE, and propose the algorithm for computing globally optimal channel assignment. We consider the limited tunability system model in Section 5. In Section 6, we present the extension for fairness and discuss how to deal with inconsistent information and multiple collision domains. We present the evaluation results in Section 7. Finally, we conclude the paper and point out potential future works in Section 8.

2 Related Works

In this section, we first review related works on channel assignment that assume cooperation of participants, and then review the works with selfish participants.

2.1 Cooperative Channel Assignment

The channel assignment problem was first studied in cellular networks. We refer to [1] for a comprehensive survey.

Due to explosive growth of wireless LANs (WLANs) in recent years, how to efficiently manage the channels becomes an important problem. For instance, Mishra et al. [2] utilized weighted graph coloring to address channel allocation for WLANs. Mishra et al. [3] used client-driven mechanisms to address the joint problem of channel assignment and load balancing in centrally managed WLANs.

As multiradio devices are becoming more and more useful in wireless mesh networks (WMNs), many researchers devoted themselves to studying channel assignment problems in WMNs. For example, Alicherry et al. [4], Raniwala et al. [5], and Kodialam and Nandagopal [6] considered channel assignment together with routing or scheduling in order to maximize network throughput. While the above works considered omnidirectional antennae, other authors (e.g., those in [7]) considered the channel allocation problem in rural mesh networks that are built using directional antenna.

The channel assignment problem is also studied in other wireless networks, such as ad hoc networks (e.g., [8]) and software-defined radio networks (e.g., [9]).

2.2 Channel Assignment with Selfish Participants

The related works described in Section 2.1 require that all nodes in the network must be cooperative. Here, cooperative means that the nodes unconditionally obey a central control or behave strictly according to the prescribed protocol. However, this assumption is not valid when the network consists of selfish nodes, whose goal is to maximize their utility/profit. With the existence of selfish nodes, assigning radios to channels becomes a *game*.

In an earlier work, Félegyházi et al. [11] studied Nash Equilibria in a static multiradio multichannel allocation game. Their work is restricted to the scenario in which each device is equipped with the same number (>1) of radios. In this paper, we adopt a much stronger solution concept called SDSE and give a scheme to achieve it. Further, our work is applicable to the general case in which each wireless device can be equipped with an arbitrary number (possibly one) of radios.

Another important related work on channel assignment game is [13] in which the authors proposed a graph coloring game model and discussed the price of anarchy under various topology conditions such as different channel numbers and bargaining strategies. Nevertheless, the work is restricted to networks of base stations and requires the assumption that each base station has to choose a channel that has not been used by any other existing base stations. In contrast, our work does not have such assumptions. (e.g., we allow sharing of a physical frequency channel.)

In wireless networks, game-theoretic approaches are also used to study media access problems. For example, MacKenzie and Wicker [14] studied the selfish behavior of nodes in Aloha networks. Later, Čagalj et al. [15] and Konorski [16] used game-theoretic approaches to investigate the media access problem of selfish nodes in CSMA/CA networks. In cognitive radio networks, Nie and Comaniciu [17] proposed a game-theoretic framework to analyze the behavior of cognitive radios for distributed adaptive spectrum allocation, but their main results are for cooperative users only.

There are also other works on incentive compatibility in wireless networks. Examples include those works on packet routing and forwarding in ad hoc networks [18], [19], [20], [21], [22], [23], [24], [25], [26].

3 Technical Preliminaries

3.1 System Model

We consider a wireless network, where each node is equipped with a single or multiple radio(s). Each radio has both a transmitter and a receiver, which may or may not be able to work simultaneously. We assume a wireless network with a common signaling channel and where the nodes communicate with each other without involving other nodes as intermediate relays.

As in paper [11], we assume that each node participates in only one of the communication sessions at a time. To communicate, a pair of nodes allocate one or multiple radios. We assume that the transmission must be between two radios, where one acts as transmitter and the other acts as receiver. So we only consider the case in which each node of the pair allocates the same number of radios in the same channel(s). A pair of nodes can have parallel transmissions between them if they both have multiple radios and allocate multiple radios.

The available frequency band is divided into orthogonal channels (e.g., the IEEE 802.11a protocol [27] has 12 orthogonal channels). We denote the set of available orthogonal channels by C. These channels can be either *fixed-rate* channels C^f or *varying-rate* channels C^v. More specifically, we denote the aggregate throughput of a channel $c \in C$ by $R_c(n)$, where n is the number of pairs of radio transmitter and receiver allocated to the channel c. $R_c(n)$ can be either a constant independent of n or a decreasing function of n, corresponding to a fixed-rate channel or a varying-rate channel. For instance, $R_c(n)$ is independent of n if TDMA-based scheduling scheme is used; and $R_c(n)$ is a decreasing function when using CDMA or CSMA/CA-based protocol (e.g., the IEEE 802.11 standards). As in [11], we assume that the aggregate throughput $R_c(n)$ of a channel c is shared evenly among the radios using the channel. So each radio pair gets throughput $R_c(n)/n$, when $n > 0$.

In this paper, we only consider a single collision domain, wherein all transmissions on the same channel will collide with each other. Extending our work to multiple collision domains will be left for future study.

3.2 Notations and Concepts from Game Theory

Before introducing our game-theoretic model, we need to recall some notations from game theory. In the classic model of strategic game, there are a finite set of players $N = \{1, 2, \dots, n\}$, and for each player $i \in N$, a nonempty set Σ_i of all possible (mixed) strategies. The set of strategy profiles is $\Sigma = \times_{i \in N} \Sigma_i$. Each player i chooses a strategy $s_i \in \Sigma_i$. As a notational convention, s_{-i} represents the strategy profile of all players except player i, i.e., $s_{-i} \in \Sigma_{-i} = \times_{j \neq i} \Sigma_j$. Note that $s = (s_i, s_{-i})$ is a strategy profile in which player i takes strategy s_i and the other players take strategies s_{-i}. A player i's preferences can be determined by a utility function $u_i(s)$. Player i prefers strategy s_i to s_i' when the other players take s_{-i}, if $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$.

The most commonly used solution concept in game theory is *Nash Equilibrium* (NE) [28]:

Definition 1 (Nash Equilibrium). *A Nash Equilibrium of a strategic game is a profile $s^* \in \Sigma$ of strategies with the property that for every player $i \in N$, we have*

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \tag{1}$$

for all $s_i \in \Sigma_i$.

Although the Nash Equilibrium gives a fundamental solution concept to game theory, it relies on knowing all the other players' strategies and beliefs on the other players, and also loses power in the games where multiple NEs exist. A stronger solution concept is SDSE.[1]

Definition 2 (Strongly Dominant Strategy Equilibrium). *A Strongly Dominant Strategy Equilibrium of a strategic game is a profile $s^* \in \Sigma$ of strategies with the property that for every player $i \in N$,*

$$\begin{cases} \forall s_{-i} \in \Sigma_{-i}, \forall s_i \neq s_i^*, & u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}), \\ \exists s_{-i} \in \Sigma_{-i}, \forall s_i \neq s_i^*, & u_i(s_i^*, s_{-i}) > u_i(s_i, s_{-i}). \end{cases} \tag{2}$$

4 Strategic Game of Channel Assignment

We model the channel assignment problem as a strategic game G in which a player is a pair of nodes having packets to exchange.

4.1 Strategic Game Model

In this paper, we consider a set N of players, where each player i knows her identity. A player's identity can be a quite long bit string, like an MAC address. Nevertheless, for simplicity of presentation, in this paper, we assume that the players' identities are 1-n. Note that our results are independent of this simplifying assumption. That is, all our results are still valid if the identities are not 1-n.

Each player $i \in N$ has w_i radio pairs (e.g., each node has w_i radios). In this section, we consider the model where each player has fully tunable radio pair(s). (In Section 5, we will consider an extend model, where players' radios may not have full tunability.) The radio pair distribution vector is denoted by $W = \{w_1, w_2, \dots, w_n\}$.

In this game, the strategy of a player $i \in N$ is just her channel assignment vector $s_i = \{s_{i,1}, s_{i,2}, \dots, s_{i,c}, \dots, s_{i,|C|}\}$, where $s_{i,c}$ is the number of radio pairs that player i assigns to channel c. (In Section 5, we extend our channel assignment game to the limited tunability model in which the strategy of player also includes her claimed tunability.)

The strategy profile s is a matrix composed of all the players' strategies: $s = (s_1, s_2, \dots, s_n)^T$.

1. SDSE is related to, but different from, the well-known concept of dominant strategy equilibrium (DSE). The major difference is that SDSE requires that, compared with any other strategy, the equilibrium strategy is strictly better in some cases. Note that in some literature, SDSE is called *Weakly* Dominant Strategy Equilibrium (WDSE). We choose to call it SDSE rather than WDSE because it is stronger than DSE, not weaker than DSE.

Given a strategy profile s, it is easy to see the total number of radio pairs used by a player i is $m_i(s) = \sum_{c \in C} s_{i,c} \leq w_i$. Here, the inequality indicates that it is not necessary to use up all one's available radios. Similarly, it is also easy to see the total number of radios assigned to a channel c is $n_c(s) = \sum_{i \in N} s_{i,c}$. Hence, the throughput a player i gets from a channel c is

$$r_{i,c}(s) = \frac{s_{i,c}}{n_c} R_c(n_c), \tag{3}$$

and the total throughput a player i gets is

$$r_i(s) = \sum_{c \in C} r_{i,c}(s). \tag{4}$$

Finally, the system throughput is

$$T(s) = \sum_{c \in C} R_c(n_c). \tag{5}$$

In reality, any practical solution to the channel assignment game should satisfy some additional requirements. First of all, there should not be any starvation. Second, we need *social efficiency*, which means that the system throughput should be maximized. We combine these two requirement to define the concept of global optimality of a solution.[2]

Definition 3 (Global Optimality). *In a strategic game of channel assignment, suppose that s^* is a strategy profile or, say, a channel assignment. We say s^* is* globally optimal *if the following two requirements are met:*

1. *No starvation.* $\forall i \in N, r_i > 0$.
2. *Social efficiency.* $\forall s \in A$, $s \neq s^*$, *if s satisfies requirement (1), then* $T(s) \leq T(s^*)$.

We note that the globally optimal channel assignment might not be unique. But all globally optimal channel assignments have the same overall throughput in the system.

4.2 Method to Achieve Global Optimality

It is ideal to have a globally optimal channel assignment. However, achieving the globally optimal channel assignment is a highly challenging task. If we allow the players to choose the channels without giving them any influence, most likely, the system would either not converge at all or converge to an assignment that is not globally optimal [11], [12]. Therefore, we need to introduce a method to influence the strategies of the players. Here, the method we use is to require players to make payments.

Just as in [20], [21], [22], [23], [24], [25], [26], we assume that there is some kind of virtual currency in the system. Each player has to pay some virtual money to the system administrator based on the outcome of the strategy profile. We regard this payment as the fee for using the channels. Note that the system administrator need not to be an online authority. It is just a server connected to the Internet. So the players can pay or receive credit from the system administrator when they have connections to the Internet.

Now let's assume that we have a globally optimal strategy profile s^* (we will explain how to compute s^* in Section 4.3.). We define the payment of player i as follows:

$$p_i(s) = \alpha r_i(s) + \beta \left(D(s_i, s_i^*) - \frac{1}{n-1} \sum_{j \in N, j \neq i} D(s_j, s_j^*) \right) - \epsilon, \tag{6}$$

where $D(s_i, s_i^*)$ is the Manhattan distance (also known as the L1-distance) between strategies s_i and s_i^*; $\alpha > 0$ and $\beta > 0$ are parameters used for converting throughput and the Manhattan distance into virtual currency values, respectively; $\epsilon > 0$ is a very small constant. (Suppose that r_{min} is the minimal reasonable bandwidth with which the basic networking operation can be completed. We require that $\epsilon < \alpha r_{min}$ so that $p_i > 0$ when all players follow the optimal strategy.) Intuitively, the payment is the charge for the player's overall throughput plus a penalty (bonus) for more (less) deviation from the globally optimal strategy than other players. We note that the total payments to the system administrator is

$$P(s) = \sum_{i \in N} p_i(s) = \alpha \sum_{i \in N} r_i(s) - n\epsilon,$$

which is the value of total throughput shared by the players minus $n\epsilon$. We further note that if all the channels are used,

$$P(s) = \alpha \sum_{i \in N} r_i(s) - n\epsilon = \alpha \sum_{c \in C} R_c(n_c) - n\epsilon,$$

which is the value of system throughput minus $n\epsilon$.

We define the utility of player i as the value of throughput she obtains minus her payment to the system administrator:

$$u_i(s_i, s_{-i}) = \alpha r_i(s_i, s_{-i}) - p_i(s_i, s_{-i}). \tag{7}$$

Since each player is selfish and rational, she always wants to maximize her utility.

Theorem 1. *It is an SDSE when each player i takes strategy s_i^*.*

Proof. Combining (6) and (7), we can get the following:

$$u_i(s_i, s_{-i}) = \epsilon - \beta \left(D(s_i, s_i^*) - \frac{1}{n-1} \sum_{j \in N, j \neq i} D(s_j, s_j^*) \right). \tag{8}$$

Then, the utility difference of taking strategy s_i^* and $s_i \neq s_i^*$ is

$$\begin{aligned} & u_i^*(s_i^*, s_{-i}) - u_i(s_i, s_{-i}) \\ &= -\beta \left(D(s_i^*, s_i^*) - \frac{1}{n-1} \sum_{j \in N, j \neq i} D(s_j, s_j^*) \right) \\ &\quad + \beta \left(D(s_i, s_i^*) - \frac{1}{n-1} \sum_{j \in N, j \neq i} D(s_j, s_j^*) \right) \\ &= \beta(D(s_i, s_i^*) - D(s_i^*, s_i^*)) \\ &= \beta D(s_i, s_i^*) \\ &> 0. \end{aligned}$$

So strategy profile s^* is an SDSE. □

By Theorem 1, it is straightforward to see that if s^* is a globally optimal channel assignment, then the SDSE achieved is also globally optimal.

2. Our definition of global optimality is thus slightly different from a traditional definition, which usually considers the optimization of a single metric (e.g., throughput).

4.3 Computing Globally Optimal Channel Assignment

To *implement* the SDSE, each player must have an algorithm for computing the globally optimal assignment s^*. In this section, we present a distributed algorithm that computes a globally optimal channel assignment s^*, if there exists one.

The input of this algorithm is the set of channels C, the set of players N, and the radio distribution vector W. This algorithm requires perfect information of the system. Every player can obtain such information in ad hoc traffic indication message (ATIM) window of multichannel MAC protocol (MMAC) [29] or channel switching function of multiradio unification protocol (MUP) [30], by sending probe signals, which contain player ID and number of radio pairs, and listening to others' probe signals.

Algorithm 1 shows the pseudocode of our algorithm. Intuitively, the algorithm considers three cases: 1) The number of players is less than that of the channels. 2) The number of players is more than that of the channels. 3) The number of players and that of channels are equal. For all the cases, the algorithm first assigns each player with a single channel. Next, in case 1, the algorithm tries to assign each unoccupied channel with a player who still has unused radio pair, until all the channels are occupied or all the radios of players are used. In case 2, for each unassigned player i, the algorithm finds a channel c on which adding a radio pair will cause the least throughput degradation. Then it assigns player i with channel c. In case 3, we are done with channel assignment and the algorithm terminates.

Algorithm 1. Algorithm for Computing Globally Optimal Channel Assignment

Input: Set of channels $C = C^f \cup C^v$, set of players N, radio distribution vector W.

Output: Globally optimal channel assignment s^*.

Initialize all entries of s^* to 0.
$i \Leftarrow 1$; $c \Leftarrow 1$.
while $i \leq n$ and $c \leq |C|$ **do**
    $s^*_{i,c} \Leftarrow 1$; $w_i \Leftarrow w_i - 1$.
    $i \Leftarrow i + 1$; $c \Leftarrow c + 1$.
end while
if $n < |C|$ **then**
    $i \Leftarrow 1$.
    while $c \leq |C|$ and $i \leq n$ **do**
        if $w_i > 0$ **then**
            $s^*_{i,c} \Leftarrow 1$; $w_i \Leftarrow w_i - 1$; $c \Leftarrow c + 1$.
        else
            $i \Leftarrow i + 1$.
        end if
    end while
else if $n > |C|$ **then**
    while $i \leq n$ **do**
        $c \Leftarrow argmin_{c \in C}\Big(R_c\Big(\sum_j s^*_{j,c}\Big) - R_c\Big(\sum_j s^*_{j,c} + 1\Big)\Big)$.
        $s^*_{i,c} \Leftarrow 1$.
        $i \Leftarrow i + 1$.
    end while
end if
return s^*.

It is not hard to see the correctness of Algorithm 1. In cases 1 and 3, since each channel is assigned with at most one radio pair, the assignment causes no throughput degradation. In case 2, since only one radio pair is used for each player and the assignment of each radio pair always causes the least throughput degradation, the overall throughput degradation is minimized. Putting these together, we can easily see that Algorithm 1 always computes a globally optimal channel assignment.

5 Limited Tunability

In previous sections, we have considered the case in which all radios have unlimited tunability, and thus, have full access to all channels. In reality, since the wireless networks usually consist of various devices (e.g., laptop/desktop PC, PDA, and IP phone), the radios of the devices may not have the tunability to access all the channels. In this section, we extend our work to the case in which some players may have limited tunability. Here, we say a player can be tuned to, or can access, a channel if *both* nodes of the player can send/receive signals in that channel. (Recall that the two nodes of each player must assign the same number of radios to each channel.) Note that the problem in limited tunability model is much more challenging than the one in unlimited tunability model. For example, a selfish player may not be willing to reveal its real tunability information, instead, it may conduct probing experiments to determine the tunability that can lead to more favorite channel assignment to the player itself. This complicates the problem a lot. So, it is not surprising that our first result in the limited tunability model is a result of nonexistence of SDSE solution.

Note that, to model limited tunability, we need to extend our channel assignment game. In particular, as we have mentioned in Section 4.1, in the model with possible limited tunability, each player's strategy includes her claimed tunability in addition to her vector of radio pair assignment. In this extended game, the utility of a player is still her rate (times a constant α) minus her payment.

5.1 Nonexistence of SDSE Solution

With possibly limited tunability of radios, the first result we obtain is that we can no longer have an SDSE solution as in the case of unlimited tunability, when nodes could lie their tunability.

Theorem 2. *Assume that players may have limited tunability and can lie about the tunability (i.e., each player can claim arbitrarily which channels it can access). Then, we cannot guarantee to find a deterministic algorithm that, in each game, outputs an SDSE such that the assignment of channels in the equilibrium is globally optimal.*

Proof. First, we make an important observation: Given a strategy profile, the utility of a player does not depend on which game she is in, as long as the involved strategies do not violate any tunability restriction. We say a strategy profile violates the tunability restriction in a game if in that strategy profile, any player places any radio pair in any channel that cannot be accessed by the player. Lying about tunability in the strategy profile is not considered violation of tunability restrictions. In other words, given a strategy profile, a player has the same utility in all games whose tunability restrictions are not violated by the involved strategies. This is because the total rate obtained by a player is determined by the strategy profile as long as there is no violation of tunability restriction, and because the payment is always determined by the strategy profile only. In particular, note that we must compute the same payment for the same assignments of radio pairs and same *claimed* tunability of players, regardless of what the *real* tunability restrictions are.

Then, we prove the theorem by contradiction. Assume that the theorem is not true. That is, we have a deterministic algorithm that outputs an SDSE for each game such that the channel assignment in the SDSE is globally optimal in the game. Then, we study two games S_1 and S_2, both of which have two players (1 and 2) and two channels (c_1 and c_2). In game S_1, both players can access both channels. As we have assumed, our algorithm should output an SDSE s^* for S_1 such that the channel assignment in the SDSE is globally optimal. Clearly, s^* must assign each of the two channels to each of the two players. Without loss of generality, suppose that s^* assigns channel c_1 to player 1 and channel c_2 to player 2.

Next, we construct a different game S_2 based on the above assignment of s^*. In S_2, player 1 can access channel c_2 only (but lies to have full tunability to both c_1 and c_2), while player 2 can still access both channels. By our assumption, in this game, our assumed algorithm should output an SDSE s' such that the channel assignment in the SDSE is globally optimal. Note that s' must assign c_1 to player 2 and c_2 to player 1, since player 1 cannot access channel c_1. Clearly, we can see $s'_2 \neq s^*_2$.

Recall the observation at the beginning of our proof. Consider any strategy s_1 of player 1, which includes a claimed tunability and a channel assignment vector, that does not violate the tunability restriction of S_2 (i.e., does not put any radio in c_1). Given the strategy profile (s_1, s^*_2), the utility of each player in game S_1 is identical to her corresponding utility in game S_2. Hence, when the strategy profile (s_1, s^*_2) is used, we can use $u_1(s_1, s^*_2)$ for the utility of player 1 and $u_2(s_1, s^*_2)$ for the utility of player 2, without mentioning whether the players are in game S_1 or game S_2.

Based on the above observation, now we study the utility of player 2. Since s' is an SDSE in game S_2, there exists a strategy s_1 of player 1 such that

$$u_2(s_1, s'_2) > u_2\big(s_1, s^*_2\big). \tag{9}$$

On the other hand, since s^* is an SDSE in game S_1, and since the strategies s_1, s^*_2, and s'_2 do not violate tunability restrictions in game S_1 (because in S_1, there is simply no tunability restriction to violate), we must have

$$u_2\big(s_1, s^*_2\big) \geq u_2(s_1, s'_2). \tag{10}$$

Clearly, there is a contradiction between (9) and (10). □

5.2 Simplified Model and Solution

Given Theorem 2, to achieve an SDSE solution, we have to simplify our previous model to make the problem more tractable. Consequently, we assume that each player has only one pair of radios, and majority of players can access all channels. Furthermore, if a number of players detect that a player is cheating about her tunability, then the latter player will be punished by an overwhelming penalty.

In this simplified model, again, we assume that we have an algorithm for computing the globally optimal channel assignment. (Note that such an algorithm is different from the one in Section 4.3, since the model is now different. We will discuss this new algorithm in Section 5.3.) We design a scheme (see Fig. 1) that ensures the existence of an SDSE that achieves global optimality. In our scheme, to claim the accessibility to a channel, a player needs to send a test signal in that channel[3] so that other players can verify her claim. In this way, a player has no way to exaggerate her accessible channels. So a player can only claim a subset of her real accessible channels. Recall that the majority of the players have unlimited tunability. Consequently, the above test signal can be verified by most players. The following lemma shows that by claiming a *proper* subset of one's accessible channels, a player will lose her utility. In other words, a player maximize her expected utility only by revealing the true tunability.

1) Each player $i \in N$ sends test signals in each channel it claims to be able to access. We denote the accessible channel set of player i as T_i.
2) After receiving all the test signals, the players with full accessibility compute the globally optimal channel assignment s^* and broadcast it in all channels.
3) Each player i takes strategy s_i and pays two payments:

$$p_i^1(s) = \alpha r_i + \beta\Big(D(s_i, s_i^*) - \frac{1}{n-1}\sum_{j\in N, j\neq i} D(s_j, s_j^*)\Big) - \epsilon, \tag{11}$$

$$p_i^2(s) = \gamma(|C| - |T_i|), \tag{12}$$

here γ is a charge for inaccessibility to a channel and $\gamma > 4\beta$.

Fig. 1. Scheme for achieving SDSE in the simplified limited tunability model.

Lemma 1. *Other things being equal, if our scheme is used, for every player, revealing the true tunability is always better than claiming a proper subset of accessible channels.*

Proof. Suppose that a player i claims her accessible channel set $T'_i \subset T_i$ and gets utility $u'_i(s')$. We show that $u'_i(s')$ is always less than $u_i(s)$, which is the utility when claiming the true accessible channel set T_i:

$$\begin{aligned} &u_i(s) - u'_i(s') \\ &= \alpha r_i - p_i^1 - p_i^2 - (\alpha r'_i - p_i'^1 - p_i'^2) \\ &= -\beta\Big(D(s_i, s_i^*) - \frac{1}{n-1}\sum_{j\neq i} D(s_j, s_j^*)\Big) \\ &\quad + \beta\Big(D(s'_i, s_i'^*) - \frac{1}{n-1}\sum_{j\neq i} D(s'_j, s_j'^*)\Big) \\ &\quad - \gamma(|C| - |T_i|) + \gamma(|C| - |T'_i|). \end{aligned} \tag{13}$$

Since each player only has one radio pair,

$$0 \leq D(s_x, s_y) \leq 2, \tag{14}$$

for all s_x, s_y.

By combining (13) and (14), we get

3. Here, the test signal we mentioned is actually a pair of signals sent by the pair of radios of the player.

$$u_i(s) - u_i'(s') \geq \gamma(|T_i| - |T_i'|) - 4\beta$$
$$\geq \gamma - 4\beta.$$

Since $\gamma > 4\beta$,

$$u_i(s) - u_i'(s') > 0.$$

□

Now it is not hard to show that we have an SDSE in which each player claims her true tunability and uses the (computed) strategy for globally optimal assignment.

Theorem 3. *There exists an SDSE in the simplified model of limited tunability such that each player claims the true tunability and that the channel assignment is globally optimal.*

5.3 Computing Globally Optimal Channel Assignment

To implement the scheme in Section 5.2, we need an algorithm for computing the globally optimal channel assignment in our simplified model of limited tunability. We propose a simple algorithm to deal with the case in which *all the channels are fixed-rate channels*. Just as Algorithm 1, this algorithm also has the player set N and the channel set C as the input. It does not need the radio distribution vector because the radio distribution vector is a constant in our simplified model. In addition, the algorithm takes as input of an accessibility vector ($X = (X_1, X_2, \ldots, X_i, \ldots, X_n)$, where $X_i \subseteq C$) that indicates which player can access which channel(s). Based on this information, the algorithm needs to compute a globally optimal channel assignment.

We convert the problem to a graph-theoretic problem. Construct a vertex set V_1 by having a vertex for each player. Construct another vertex set V_2 by having a vertex for each channel. If a player can access a channel, then these two vertices are connected together by an edge—let E be the set of such edges. In this way, we get a bipartite graph $G = (V_1 \cup V_2, E)$. A channel assignment corresponds to a subset of edges such that each player is associated with only one channel through this subset. We note that we can map each channel assignment to a matching in the graph: in the subset of edges corresponding to the assignment, for each channel assigned to more than one player, we keep one edge and delete the others; in this way, we get a subset of edges that is a matching, and this matching's aggregate throughput is the same as the original assignment. (However, we note that *more than one* assignment may map to the same matching.) Therefore, a globally optimal channel assignment is mapped to a maximum bipartite matching in the bipartite graph. We consider a deterministic algorithm for maximum bipartite matching[4]:

$$(L, R, m) \Leftarrow MBM(V_1, V_2, E),$$

where $L \subseteq V_1$ and $R \subseteq V_2$ are the sets of matched vertices, and m is a binary matrix that represents the matching result. We require all players to use the same MBM algorithm to find a maximum matching. When there is more than one maximum matching, the MBM algorithm should choose to output one of them. Clearly, all players will get the same maximum matching because they are using the same algorithm and the algorithm is deterministic.

Algorithm 2 shows the pseudocode of our algorithm in the simplified limited tunability model. First, the algorithm computes a channel assignment based on the maximum bipartite matching. (Note that we are a little sloppy in the algorithm—in fact, we need to convert N and C into sets of vertices and X into a set of edges connecting vertices in N and C before applying the algorithm MBM. However, since such a conversion is trivial, we skip it in order to make the algorithm easier to read.) Then, to ensure that there is no starvation, for each player that has not been assigned a channel in the matching, the algorithm *deterministically* assigns an arbitrary channel to her.

Algorithm 2. Algorithm for Computing Globally Optimal Channel Assignment in the Simplified Model of Limited Tunability

Input: Set of players N, set of channels C, accessibility vector X.

Output: Globally optimal channel assignment s^*.

1: Initialize all entries of s^* to 0.
2: $(L, R, s^*) \Leftarrow MBM(N, C, X)$.
3: **for all** $i \in N - L$ **do**
4: $c \Leftarrow$ Arbitrary channel that i can access.
5: $s^*_{i,c} \Leftarrow 1$.
6: **end for**
7: **return** s^*.

Since Algorithm 2 is based on maximum bipartite matching, the maximum number of channels is utilized in the computed channel assignment s^*. Consequently, the result s^* is globally optimal.

6 Throughput Fairness, Inconsistent Information, and Multiple Collision Domains

In previous sections, we have designed channel assignment schemes to achieve SDSE in the unlimited and limited tunability models. In this section, instead of designing more schemes for channel assignment, we study the following questions regarding the schemes we have presented:

- For players, how fair on throughput are our schemes? Is there any way to further improve the throughput fairness?
- Our schemes are based on the fact that the information different players have must be consistent. When players join or leave, the information may become inconsistent. How can we deal with that?
- How to extend our work from single collision domain to multiple collision domains?

In the rest of this section, we answer these questions.

6.1 Throughput Fairness

To study how fair on throughput our schemes are, we define a metric called *throughput fairness ratio*. This ratio relates to other well-known measures, such as max-min fairness.

Definition 4. *The throughput fairness ratio of a scheme is defined as*

$$\mathcal{F} = \frac{max_{i \in N}\ r_i(s^*)}{min_{i \in N}\ r_i(s^*)},$$

where s^* is the equilibrium the scheme should converge to.

4. See [31] for example of such algorithms.

Note that the larger $\mathcal{F}$ is, the *less* fair the scheme is. Hence, to show that our schemes are fair on throughput to a certain degree, we need to establish upper bounds for the throughput fairness ratio. Below is such a bound for our scheme in the unlimited tunability model.

Theorem 4. *When algorithm 1 is used, the throughput fairness ratio $\mathcal{F}$ has an upper bound*

$$\mathcal{F} \leq \frac{max_{c\in C}\ R_c(1)}{min_{c\in C}\ R_c(n-|C|+1)} \cdot \max\{|C|-1, n-|C|+1\}.$$

Proof. We distinguish three scenarios:

1. $n < |C|$: In this case, each channel is assigned at most one radio pair. Hence, assuming $i_1 = \arg\max_{i\in N} r_i(s^*)$ and $i_2 = \arg\min_{i\in N} r_i(s^*)$, the throughput fairness ratio in this case is

$$\begin{aligned}\mathcal{F} &= \frac{r_{i_1}(s^*)}{r_{i_2}(s^*)} \\ &= \frac{\sum_{s^*_{i_1,c}=1} R_c(1)}{\sum_{s^*_{i_2,c}=1} R_c(1)} \\ &\leq \frac{max_{c\in C}\ R_c(1)}{min_{c\in C}\ R_c(1)} \cdot (|C|-1).\end{aligned}$$

2. $n = |C|$: In this scenario, the globally optimal channel assignment s^* is that each player gets a separate channel and assigns a single radio pair to it. So the throughput fairness ratio is

$$\mathcal{F} = \frac{max_{i\in N}\ r_i(s^*)}{min_{i\in N}\ r_i(s^*)} \leq \frac{max_{c\in C}\ R_c(1)}{min_{c\in C}\ R_c(1)}.$$

3. $n > |C|$: In the globally optimal channel assignment of this scenario, each player only uses a single pair of radios. Then, assuming that $i_1 = \arg\max_{i\in N} r_i(s^*)$, $i_2 = \arg\min_{i\in N} r_i(s^*)$, and that $s^*_{i_1,c_1} = s^*_{i_2,c_2} = 1$, the throughput fairness ratio is

$$\begin{aligned}\mathcal{F} &= \frac{r_{i_1}(s^*)}{r_{i_2}(s^*)} \\ &= \frac{R_{c_1}(n_{c_1}(s^*))/n_{c_1}(s^*)}{R_{c_2}(n_{c_2}(s^*))/n_{c_2}(s^*)} \\ &\leq \frac{max_{c\in C}\ R_c(1)}{min_{c\in C}\ R_c(n-|C|+1)} \cdot (n-|C|+1).\end{aligned}$$

By combining the above three scenarios, we get

$$\mathcal{F} \leq \frac{max_{c\in C}\ R_c(1)}{min_{c\in C}\ R_c(n-|C|+1)} \cdot \max\{|C|-1, n-|C|+1\},$$

which completes the proof. □

Fig. 2 shows the throughput fairness ratio of our scheme in the unlimited tunability model. In the figure, the solid line shows the upper bound of the throughput fairness ratio, while the plus marks show the ratio of 100 samples for each different size of player set. In the figure, we can see that the samples are well bounded by the upper bound in Theorem 4.

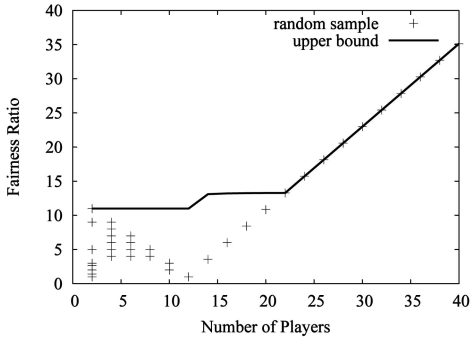

Fig. 2. Throughput fairness ratio of our scheme in the unlimited tunability model.

We do not give a similar bound for the scheme in the limited tunability model because throughput fairness can be strongly affected by the tunability restrictions.

In some applications, throughput fairness may be crucial. Therefore, we extend our work to another game-theoretic model so that we can obtain the maximum amount of throughput fairness without losing global optimality.

This extended model is *infinitely repeated game*.[5] Essentially, this infinitely repeated game models a situation in which players repeatedly engage in the strategic game G as defined in Section 4. There is no limitation on the number of times that G is played; and in each round, the players take their strategies simultaneously. Using the terminology of infinitely repeated game, each round of strategic game here is called a *stage*. All the stages have the same length of time. We treat each stage as a strategic game. Furthermore, we define the cumulative utility of player $i \in N$ from the beginning of the game to stage t as

$$\hat{u}_i[t] = \sum_{j=0}^{t} u_i[j]. \tag{15}$$

Let us assume, as in Section 4, that all radios have unlimited tunability. We further assume that each player has the same number of radios in the infinitely repeated game. Then, we can get a completely fair channel assignment as follows:

In stage t, we define a channel assignment matrix $s^*[t]$, which will be used to compute the payments to the system administrator:

$$s^*[t] = \begin{cases} s^* & \text{if } t = 0 \\ \begin{pmatrix} s^*_2[t-1] \\ \cdots \\ s^*_n[t-1] \\ s^*_1[t-1] \end{pmatrix} & \text{if } t > 0, \end{cases} \tag{16}$$

where s^* is an SDSE we defined in Section 4.

5. Note that our model of infinitely repeated game is slightly different from the one in classic game theory. In the standard model, the utility function is fixed in each stage of the game. However, in our model, the definition of payment is based on the globally optimal channel assignment. Consequently, the payment formula changes along with the globally optimal channel assignment in each stage of the game. So does the utility function.

According to the definition of the utility function, each player's utility in a stage is independent of other stages. So each player gets its cumulative utility maximized if her utility in each stage is maximized. The player can achieve this by taking the strongly dominant strategy $s_i^*[t]$ in each stage t. Since $s_i^*[t]$ is a globally optimal channel assignment, the global optimality is preserved in our repeated game.

In our infinitely repeated game, suppose that player i takes the dominant strategy $s_i^*[t]$ in each stage t. The average throughput that player i gets from the beginning of the game to stage t is

$$\begin{aligned}\bar{r}_i[t] &= \frac{1}{t}\sum_{j=0}^{t} r_i[j] \\ &= \frac{1}{t}\sum_{j=0}^{t} r_{(i+j) \bmod n}[0].\end{aligned} \tag{17}$$

Consider the infinity of the repeated game:

$$\begin{aligned}\lim_{t\to+\infty}\bar{r}_i[t] &= \lim_{t\to+\infty}\frac{1}{t}\sum_{j=0}^{t} r_{(i+j) \bmod n}[0] \\ &= \lim_{t\to+\infty}\frac{1}{t}\cdot\frac{t}{n}\sum_{j=0}^{n-1} r_{i+j}[0] \\ &= \frac{1}{n}\sum_{j=1}^{n} r_j[0].\end{aligned} \tag{18}$$

So in our infinitely repeated game of channel assignment, if the players take the dominant strategy in each stage, then they get the same throughput in the long run. This shows that our infinite repeated game can achieve throughput fairness without losing global optimality.

We note that the infinitely repeated game of channel assignment requires the players to coordinate the channel assignment over stages. Therefore, all clocks in the system must align with a reference clock. Furthermore, techniques that support fast switching among channels will be highly needed. For example, the fast access point switching technique proposed in [32] can be applied to fast channel switching.

6.2 Inconsistent Information

As we have mentioned, the schemes we have presented are based on the assumption that the players have consistent information. When players join or leave, the information may become inconsistent. Therefore, if the wireless network is dynamically changing, the globally optimal channel assignment may need to be recomputed when a node joins or leaves.

Fortunately, a player can notice the signs of information inconsistency by observing the following:

- The obtained throughput is different from the expected throughput.
- The payment to the system administrator is higher than the calculated cost for obtaining the throughput (i.e., the node is penalized by the system administrator).

When a player has inconsistent information, a number of players who share channel(s) with it can be affected. It is not necessary for all of affected players to recompute the globally optimal channel assignment. Here, we propose a simple method to deal with the problem of information inconsistency:

1. When a player i notices that the current throughput is different from the expected throughput, it sets a back-off time $\mathcal{T}_i$:
$$\mathcal{T}_i = \frac{\mathcal{M}}{2^{|r_i - r_i^e|}},$$
where $\mathcal{M}$ is the maximum back-off time, and r_i and r_i^e are obtained and expected throughputs, respectively. After the back-off time $\mathcal{T}_i$, the player rechecks the obtained throughput. If the obtained throughput is still different from the expected throughput, the player needs to probe the channels, recompute the globally optimal channel assignment, and reassign the channel(s) to radio pair(s).
2. If connected to the system administrator, a player checks the payment to the system administrator. If the payment includes a penalty, it probes the channels, recomputes the globally optimal channel assignment, and reassigns the channel(s) to radio pair(s).

In case 1, intuitively, the greater the difference between the obtained and expected throughputs is, the shorter the back-off time needs to be. A player with inconsistent information is likely to have a significant throughput difference. And case 2 is complementary to case 1 in that the player with inconsistent information happens to have little throughput difference. It enables the player who gets a penalty immediately to realize that its information is inconsistent with other players'.

6.3 Multiple Collision Domains

So far, we have presented schemes designed for a single collision domain, wherein all transmissions on the same channel will collide with each other.

Note that the above definition of single collision domain does not imply that all nodes have to hear each others' transmission, so even in a single collision domain, there may be so-called hidden terminals. For example, assume that there are four nodes A, B, C, and D, which are lined up from left to right with certain distance between them, such that A and D can't hear each other but both B and C can hear A and D. Suppose that node A wants to transmit to node B (so they form a player), and node D wants to transmit to node C (so they form another player). We note that A and D are considered as a hidden terminal by each other, but according to our definition above, since A and D cannot transmit on the same channel at the same time without causing collisions at nodes B and C, they are considered in the same collision domain, and accordingly, their optimal channel assignment may be obtained using the schemes described above.

When players are scattered around a relatively large area, we may have multiple collision domains and face different challenges from those addressed in the single collision domain so far. For example, with multiple collision domains, some players which are far apart from each other can transmit on the same channel at the same time.

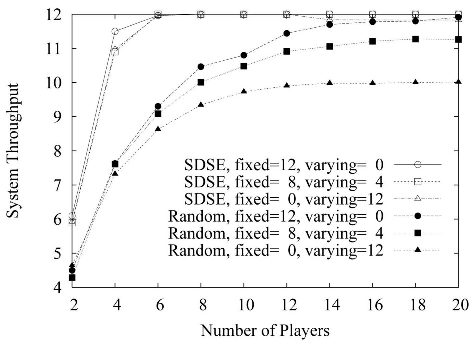

Fig. 3. System throughput achieved by using our SDSE-based scheme and the random-based scheme.

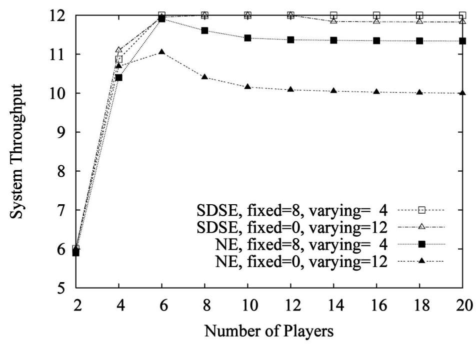

Fig. 4. System throughput achieved by using our SDSE-based scheme and the NE-based scheme.

Achieving global optimality in terms of system throughput in multiple collision domains is a nontrivial task in itself even if we assume that all players are cooperative. One way to make channel assignment incentive compatible is to employ some other techniques, such as link scheduling. More specifically, we partition the time into slots and we allow links to place their radios on different channels in different time slots. For each player, there is a payment for using each channel, which varies with time. This reflects the requirement of link scheduling. In this way, the system may converge to a stable state with globally optimal throughput in multiple collision domains.

We must note that a lot of *nontrivial* works need to be done in order to extend our schemes to multiple collision domains along the line of the above initial thoughts. We will study these problems in our future work.

7 Numerical Results

In this section, we evaluate our schemes using MATLAB. We assume that the available frequency band is divided into 12 orthogonal channels, which consist of fixed-rate channels and varying-rate channels. In the evaluations, a basic CSMA/CA protocol with binary slotted exponential back-off is used for varying-rate channels. We use the same system parameters as those in [33]. For our evaluation, we set $\alpha = \beta = 1$ and $\gamma = 5$.

7.1 Results in the Unlimited Tunability Model

We have performed two sets of simulations in the unlimited tunability model. The first one is to compare the system throughput achieved by using our SDSE-based channel assignment scheme with that of random-based channel assignment scheme and NE-based channel assignment scheme [11]. In random-based scheme, players arbitrarily assign their radios to the channels. Here, random-based scheme and NE-based scheme do not charge the players for using the channels. The second set of simulations is to show that if our scheme is used, deviating from the computed channel assignment cannot increase one's utility (see (7) for definition of utility).

In the first set of simulations, we consider three different channels deployments: 1) no varying-rate channel, 2) eight fixed-rate channels and four varying-rate channels, and 3) no fixed-rate channel. We vary the number of players from 2 to 20. The number of radio pairs each player has is uniformly distributed in $[1, 5]$. We repeat the simulation until the convergence level 10^{-6} is reached.

Fig. 3 shows the result of the comparison on system throughputs between our SDSE-based scheme and the random-based scheme. Generally, the SDSE-based scheme reaches the maximum system throughput as long as there are only a small number of players. As Fig. 3 shows, in all three cases, the SDSE-based scheme reaches a system throughput of 12 Mbit/s with only eight players. On the other hand, when there exist varying-rate channels, the system throughput of the random-based scheme will never reach 12 Mbit/s. Even without any varying-rate channel, the random-based scheme can get 12Mbit/s only when there are at least 20 players in the system. Another advantage of the SDSE-based scheme is that it results in a much less system degradation than the random-based scheme, when there exist varying-rate channels. In Case 2, the SDSE-based scheme achieves a higher (0.68 Mbit/s more) system throughput than the random-based scheme in most cases; while in Case 3, the difference between system throughputs is as high as 1.76 Mbit/s or even more.

Fig. 4 shows the comparison result between the SDSE-based scheme and NE-based scheme. Since there is no system degradation when no varying-rate channel exists, we only show the later two cases here. When the resource (channels) is abundant (less than or equal to four players, each with average of radio radio pairs), the NE-based scheme achieves almost the same system throughput as the SDSE-based scheme. But when the resource is scarce, the greedy nature of the players in NE-based scheme will result in more severe contention for the channels as the number of players increases. Accordingly, the SDSE-based scheme performs much better than the NE-based scheme, when the resource is scarce. When there are 20 players, the system throughput of the SDSE-based scheme is 0.66 Mbps higher than that of the NE-based scheme for case 2, and 1.83 Mbps higher for case 3.

Our second set of simulations demonstrates the effect of some players deviating from our scheme. In this set of simulations, we assume that there are 20 players in the system, and 50 percent of them deviate from our scheme by arbitrarily assigning their radios to the channels. The other setups are the same as the first set of simulations. The simulation is repeated 100 times. We keep track of a player and record her utility in the two cases: following our scheme or deviating from it.

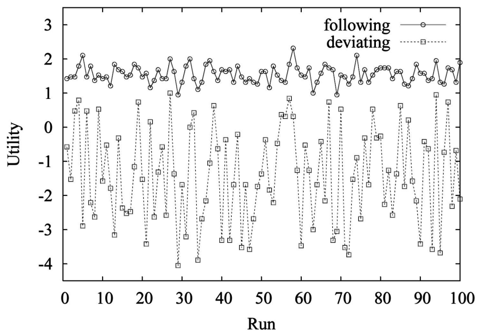

Fig. 5. Utility of following our scheme and deviating.

Fig. 5 illustrates the utility of the tracked player. It is shown that when following our scheme, the player can always obtain nonnegative utility. Furthermore, the utility obtained by following the scheme is always higher than by deviating from it. This will motivate each player to follow our scheme.

7.2 Results in the Limited Tunability Model

We have also performed two sets of simulations in the limited tunability model. The first one compares the system throughput of our SDSE-based scheme with that of the random-based scheme and the NE-based scheme, while the second one studies the effect of some players cheating about their tunability and deviating from the computed channel assignment.

In the first set of simulations, we assume that 60 percent of players have limited tunability. For the players with limited tunability, we restrict her number of accessible channels uniformly between 1 and 11. We vary the number of players from 2 to 20, and repeat each simulation until the convergence level 10^{-6} is reached.

Fig. 6 compares the system throughputs of our SDSE-based scheme, the random-based scheme, and the NE-based scheme in the limited tunability model, when all the channels are fixed-rate channels. The system throughputs of the SDSE-based scheme and the NE-based scheme grow almost linearly when no more than 12 players and remain at the maximum value after that. (Note that in this special case, the NE-based scheme can also have globally optimal throughput because there is no varying rate channel. In this case, the main advantage of our SDSE-based scheme is that it provides much stronger incentives to converge to the equilibrium.)

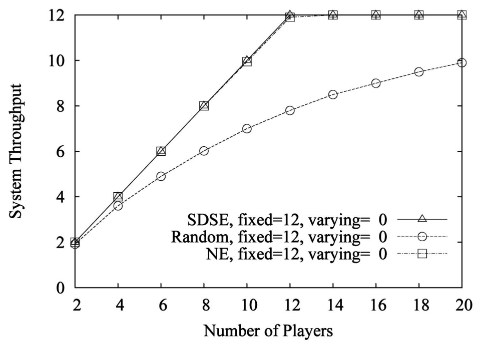

Fig. 6. System throughput of our SDSE-based scheme, the random-based scheme, and the NE-based scheme in the limited tunability model, when all the channels are fixed-rate channels.

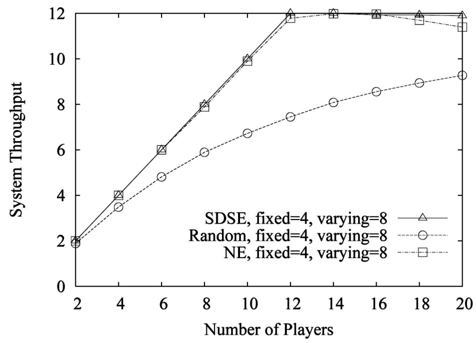

Fig. 7. System throughput of our SDSE-based scheme, the random-based scheme, and the NE-based scheme in the limited tunability model, when there are both fixed-rate channels and varying-rate channels.

Compared with the random-based scheme, which never reaches 12 Mbit/s, the SDSE-based scheme and the NE-based scheme obviously have higher system throughputs.

Fig. 7 compares the three schemes in the limited tunability model, when there are both fixed-rate channels and varying-rate channels. Again, the SDSE-based scheme and the NE-based scheme achieve higher system throughputs than the random-based scheme. In contrast to Fig. 6, the SDSE-based scheme achieves a higher system throughput than the NE-based scheme, when the number of players is more than 16. This shows that the SDSE-based scheme causes less system degradation than the other two schemes.

In the second set of simulations, we observe the effect of some players cheating about tunability and deviating from the computed channel assignment. We calculate the difference between utility a player obtains by following our scheme and the utility by cheating alone and deviating.

In Fig. 8, we can see that the difference in the utilities obtained is always positive, meaning that following our scheme will always result in a higher utility than cheating and/or deviating. In addition, we observe that compared with deviating, cheating is clearly the dominant source of utility loss. Accordingly, it is always better for the players to claim their true tunability and follow the computed channel assignment.

7.3 Results on the Repeated Game

In this set of evaluations, we assume that there are eight fixed-rate channels and four varying-rate channels. The

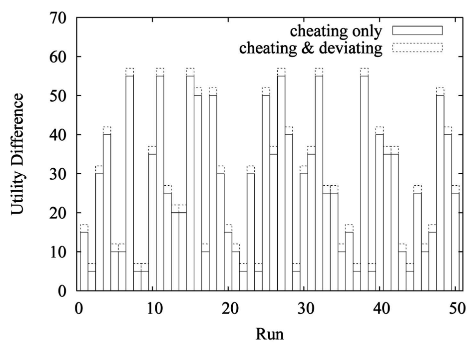

Fig. 8. Utility difference between following our scheme and cheating and/or deviating in the limited tunability model.

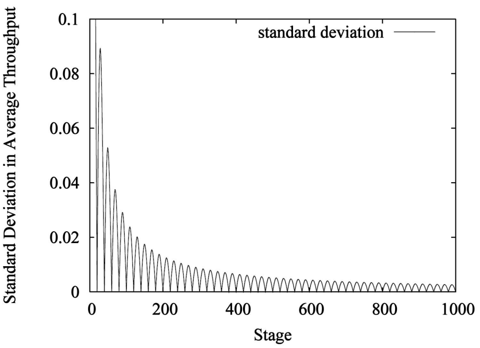

Fig. 9. Standard deviation of players' average throughput with growth of the stage.

number of players is 20. Each player has two radio pairs. We let the repeated game go 1,000 stages and record the standard deviation of players' average throughput in each stage.

In Fig. 9, we observe that there is a cycle of 20 stages. At the end of each cycle, the standard deviation becomes zero. Each cycle has a peak, which goes down toward zero with progress of the repeated game. The length of the cycle is determined by the period of the channel assignment matrix $s^*(t)$ (see (16)), which is equal to the number of players.

7.4 Results on Efficiency

We evaluate the efficiency of our schemes in terms of computational overhead in the unlimited and limited tunability models, respectively. We run the proposed channel assignment schemes on a laptop with 2.0 GHz Intel CPU and 1 GB memory. The setups for the unlimited and limited tunability models are the same as those in Sections 7.1 and 7.2, respectively, except that we vary the number of players from 10 to 40. We repeat each evaluation 10,000 times and calculate the average running time.

Figs. 10a and 10b show the computational overheads of our schemes in the unlimited and limited tunability models, respectively. Generally, the computational overhead increases in the number of players.

In Fig. 10a, we can see that the computational overhead in the case of no fixed-rate channel is significantly higher than that in the other two cases. The extra computational overhead is mainly induced by step 18 of Algorithm 1, which looks for the right channel c. If there are fixed-rate channels, then one of the fixed-rate channels will be quickly chosen as c. But when there is no fixed-rate channel, the algorithm has to search through all the channels to find the channel with the smallest throughput degradation, and thus, needs much more time to finish the computation.

Fig. 10b shows that when all the channels are fixed-rate channels, the computational time in case of 40 percent nodes having limited tunability is longer than that of 20 percent nodes having limited tunability. This is because the complexity of the bipartite matching algorithm used is $O(|V|^2|E|)$, where $|V|$ is the number of players plus the number of channels. When we implement Algorithm 2, we only need to assign channels to the users with limited tunability using this algorithm; after that, those users without limited tunability can be easily assigned. In contrast, when there are both fixed-rate channels and varying-rate channels, the computational time in case of

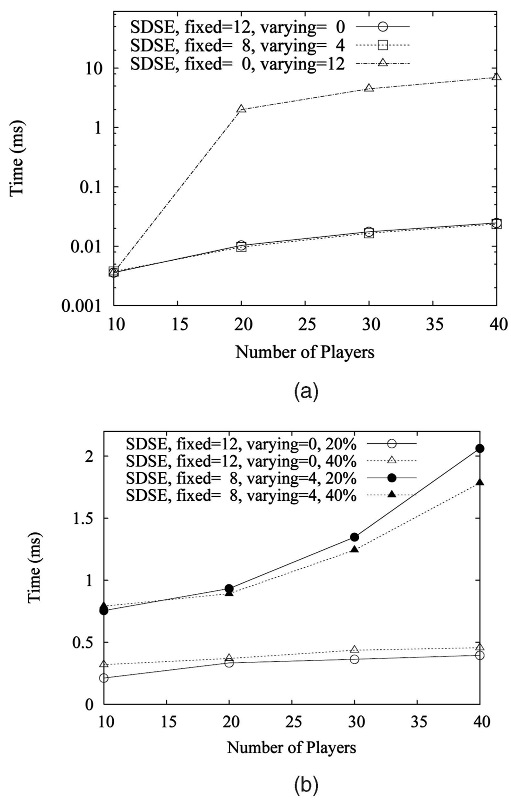

Fig. 10. Computational overhead. (a) Unlimited tunability. (b) Limited tunability (20 and 40 percent nodes with limited tunability).

40 percent nodes having limited tunability is shorter than that of 20 percent nodes having limited tunability. This is because when majority of players have full tunability, Line 9 takes the most significant part of computational time in Algorithm 1. Consequently, the more fully tunable players there are, the longer computational time is needed.

In all the cases we tested, the computational overhead remains very low. The computations are guaranteed to complete in less than 7 milliseconds and 2.1 milliseconds in the unlimited and limited tunability models, respectively. It may seem counterintuitive that the unlimited tunability model can have a higher computational overhead than the limited tunability model. The reason for this phenomenon is that our limited tunability model has restrictions on the players and channels. In particular, it requires that each player has only one pair of radios, which makes channel assignment much faster.

8 Conclusion and Future Work

In this paper, we have studied the channel assignment problem in noncooperative wireless networks. We modeled the channel assignment problem as a strategic game and can guarantee the existence of an SDSE by introducing a payment formula. Furthermore, the SDSE scheme achieves the global optimality in terms of system throughput. We have proved that the above result does not hold in the general limited tunability model. Nevertheless, when we make additional practical assumptions on the numbers of radios and the types of channels, etc., we can achieve an SDSE solution that guarantees globally optimal system throughput again. We've quantified the throughput fairness of our solution by providing bounds onto its throughput fairness ratio. In addition, we have considered the issues of throughput fairness and inconsistent information. We have extended the strategic game to a repeated game, which preserves the global optimality in each stage and achieves throughput fairness in the long run; we also have proposed a simple method to detect and eliminate inconsistent information. Numerical results have demonstrated that our solutions have strong incentives for players to cooperate with low computational overheads.

There are several potential ways to further extend our work. One possibility is to study the trade-offs among system throughput, fairness among the players, and load-balance on the channels. Another possibility is to consider a strategic game of channel assignment in multiple collision domains.

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