Network Coding-Based Multicast in Multi-Hop CRNs under Uncertain Spectrum Availability

Yuben Qu*¶, Chao Dong*, Haipeng Dai†, Fan Wu‡, Shaojie Tang§, Hai Wang*, Chang Tian*¶
*College of Communications Engineering, PLA University of Science and Technology, China
†Department of Computer Science and Technology, Nanjing University, China
‡Department of Computer Science and Engineering, Shanghai Jiao Tong University, China
§Naveen Jindal School of Management, University of Texas at Dallas, USA
¶Corresponding author: quyuben@gmail.com, tianchang126@126.com

***Abstract*—The benefits of network coding on multicast in traditional multi-hop wireless networks have already been demonstrated in previous works. However, most existing approaches cannot be directly applied to multi-hop cognitive radio networks (CRNs), given the unpredictable primary user occupancy on licensed channels. Specifically, due to the unpredictable occupancy, the channel's bandwidth is uncertain and thus the capacity of the link using this channel is also uncertain, which may result in severe throughput loss.**

In this paper, we study the problem of network coding-based multicast in multi-hop CRNs considering the uncertain spectrum availability. To capture the uncertainty of spectrum availability, we first formulate our problem as a chance-constrained program. Given the computationally intractability of the above program, we transform the original problem into a tractable convex optimization problem, through appropriate Bernstein approximation together with relaxation on link scheduling. We further leverage Lagrangian relaxation-based optimization techniques to propose an efficient distributed algorithm for the original problem. Extensive simulation results show that, the proposed algorithm achieves higher multicast rates, compared to a state-of-the-art non-network coding algorithm in multi-hop CRNs, and a conservative robust algorithm that treats the link capacity as a constant value in the optimization.

I. Introduction

Due to the pressing demand of efficient frequency spectrum usage, cognitive radio networks (CRNs) [1] have spurred a wide range of research interests and emerged as a promising technology to improve the spectrum utilization. In CRNs, with the proliferation of computationally powerful wireless devices and compelling demand on diversified services, providing multicast services for secondary users (SUs), especially in a multi-hop fashion, is urgently needed [2]–[5]. Therefore, it is crucial to improve the performance of multicast in multi-hop CRNs.

During the past decade, network coding [6], has been proposed to improve the network resource utilization. By combining multiple input packets into one packet algebraically before forwarding, network coding is able to increase the multicast capacity [6]. The throughput benefits of network coding on multicast in multi-hop wireless networks have been well recognized in previous works, e.g., [7]–[13]. However, most of those works focus on traditional multi-hop wireless networks, and few of them study the impact of network coding on improving the multicast throughput in multi-hop CRNs.

Indeed, most existing works on network coding-based multicast cannot handle new challenges arising from CRNs, including the uncertainty of spectrum availability. Particularly, in the optimization of traditional wireless networks, we often assume that the link capacity is a fixed value [7]–[9], [13]. However, in CRNs, due to unpredictable primary occupancy on licensed channels, the bandwidth of an available channel is uncertain in the sense that it is a random variable satisfying some certain distribution [14]–[17]. That is to say, the capacity of a link using this channel is also a random variable, which invalidates all of the previous approaches. Besides, simply replacing the uncertain link capacity by its expected constant value is not preferable, since the fruitful information of spectrum availability is not exploited, which will incur great performance degradation.

In this paper, we study how to maximize the data rate of a network coding-based multicast session in multi-hop CRNs. To the best of our knowledge, this is the first work to study the problem of network coding-based multicast in multi-hop CRNs, by taking into account *the uncertainty of spectrum availability*. Different from existing works, we model the link capacity as a random variable instead of a fixed value, which captures the uncertainty of spectrum availability in CRNs. To deal with the random variables in the constraints, we cast these constraints as chance constraints [18]–[20], i.e., candidate solutions are required to satisfy the randomly constraints with a given probability. However, chance constraints are usually more difficult to tackle because these constraints are typically nonconvex and sometimes difficult to be expressed in a closed form [18]. Another challenge in our problem is that since the available channels at different nodes could be different in multi-hop CRNs, how to efficiently schedule the links under the channel availability is also a critical concern.

To overcome the above challenges, we first employ Bernstein approximation techniques [18] to translate the aforementioned chance constraints into convex computable constraints, by properly choosing the approximation bounds. Further, we encapsulate the constraints of hyperarc(link)-channel scheduling in a two-dimensional conflict graph (TDCG), and

978-1-4799-8381-0/15/$31.00

eventually obtain an optimization problem that can be solved by common convex optimization approaches. Specifically, we decompose the overall problem into two relatively independent subproblems, i.e., a multiple-shortest-paths problem with channel availabilities constraints and a maximum-weighted-stable-set problem on the TDCG, respectively. Then, we propose an efficient distributed algorithm for network coding-based multicast in multi-hop CRNs, to simultaneously optimize the flow rate and coding subgraph with channel selection.

Our main contributions are summarized as follows:

1) We first consider the uncertainty of spectrum availability and study the problem of network coding-based multicast in multi-hop CRNs. We capture the uncertainty of link capacity in chance constraints, and encapsulate both hyperarc scheduling and channel selection into multiple hyperarc-channel tuples represented by the TDCG.
2) We transform the formulated chance-constrained program into a computationally tractable convex problem, through appropriate Bernstein approximation and relaxation of finding stable sets on the TDCG. We decompose the convex problem into two separate subproblems by using Lagrangian relaxation, and propose an efficient distributed algorithm.
3) Our extensive simulation results show that, our distributed algorithm outperforms the state-of-the-art non-network coding algorithm in multi-hop CRNs and the conservative robust algorithm, in terms of multicast rate. We also demonstrate the fast convergence of our decentralized algorithm.

II. Network Model

In this section, we will first introduce the basic network model used in our work, then interpret the modeling of uncertain spectrum availability in CRNs, and lastly, explain the coding subgraph optimization in multi-hop CRNs.

A. System Model

We consider a multi-hop CRN, where N SUs[1] share C orthogonal channels that can be accessed by SUs when they are not occupied by PUs. The set of nodes is denoted by $\mathcal{N} = \{1, 2, ..., N\}$, and the set of channels is denoted by $\mathcal{C} = \{1, 2, ..., C\}$. Each node is equipped with two radios: one is a half-duplex cognitive radio that can switch among the channels for data transmissions, and the other is a half-duplex normal radio working on a common control channel (CCC) for exchanging control messages [5]. Following most previous works [3], [5], we assume that the available channels at a SU may be different from those at another one in the network, considering the geographical location differences of the SUs. Let $\mathcal{C}_i \subseteq \mathcal{C}$ represent the set of available channels at SU $i \in \mathcal{N}$, and $\mathcal{C}_J \subseteq \mathcal{C}$ denote the set of common available channels at a set of nodes J, i.e., $\mathcal{C}_J = \bigcap_{j \in J} \mathcal{C}_j$. We assume that set $\mathcal{C}_i$ varies with time. However, we suppose that the channel availability at SUs is quasistatic, i.e., does not change in a short period of time [5].

[1]SU and node will be used interchangeably in the rest of this paper.

For flow service, we consider there is a multicast connection of rate R with a source $s \in \mathcal{N}$ and sinks $\mathcal{D} = \{1, 2, ..., D\} \subset \mathcal{N}$. All multicast sinks request the same information and thus R can be defined as the total amount of data that are successfully delivered to all sinks per unit time. Table I provides a list for the major notations used in this paper.

TABLE I: A List of Notations

Notation	Definition
$\mathcal{N}$	set of SU nodes
$\mathcal{A}$	set of hyperarcs
$\mathcal{C}$	set of licensed channels
R	rate of multicast session
$\mathcal{C}_i$	set of available channels at node i
$\mathcal{C}_J$	set of common available channels at node set J
$\mathcal{D}$	set of sinks
B^c	bandwidth of channel c
$z_{iJ}(c)$	rate restricted by coding subgraph on (i, J) through channel c
$z_{iJK}(c)$	average rate on (i, J) exactly received by K on channel c
d_{ij}	distance between node i and node j
g_{ij}	power propagation gain between node i and node j
Q	power spectral density from the transmitter
η	ambient Gaussian noise density
$f^d_{iJj}(c)$	flow rate for sink d on channel c from node i to node $j \in J$
$\mathcal{T}^c_i$	set of nodes that can communicate with i through channel c
$u_{ij}(c)$	indicator of whether i communicates to j on channel c
$u_{iJ}(c)$	indicator of whether i communicates to J on channel c
$h_{ij}(c)$	capacity of link (i, j) using channel c
$h_{iJ}(c)$	capacity of hyperarc (i, J) using channel c

B. Modeling of Uncertain Spectrum Availability

In CRNs, due to the unpredictable bandwidth occupancy of PUs, the bandwidth of an available channel is uncertain in the frequency domain [17]. In other words, SUs do not know what the exact bandwidth is even if the channel is sensed idle. To model this unique feature, let B^c denote the unoccupied bandwidth of an available channel $c \in \mathcal{C}$, where B^c is a *random variable*. According to the well known Shannon-Hartley theorem, the capacity of a link has strong positive relationship with the bandwidth of the channel it uses. Since the channel bandwidth is random in CRNs, the link capacity is also a *random variable*, which may pose great challenges to the optimization of network performance.

C. Coding Subgraph Optimization in Multi-Hop CRNs

We leverage the broadcast nature for efficient multicast in wireless CRNs, i.e., a node can transmit data simultaneously to multiple nodes in its transmission range by switching to the same available channel [5]. To illustrate the broadcast relationship, we model the network by a hypergraph $\mathcal{H} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ is the set of nodes and $\mathcal{A}$ is the set of hyperarcs. A *hyperarc* is a pair (i, J) (at least one common channel available in node i and set J), where $i \in \mathcal{N}$ is the start node, and $J \subset \mathcal{N}$ is the set of end nodes. Every hyperarc (i, J) represents a broadcast link from node i to the nodes in the nonempty set J. Let $z_{iJ}(c)$ denote the rate at which coded packets are injected into hyperarc (i, J) through channel c. The rate vector

$\mathbf{z} = (z_{iJ}(c))_{(i,J)\in\mathcal{A},c\in\mathcal{C}_i\bigcap\mathcal{C}_J}$ is termed as the *coding subgraph* [8]. Denote $z_{iJK}(c)$ the rate at which packets are received by precisely the set of nodes $K \subset J$ through hyperarc (i, J). Hence, we have $z_{iJ}(c) = \sum_{K\subset J} z_{iJK}(c)$.

III. Problem Formulation

In this section, we formulate the problem of network coding-based multicast in multi-hop CRNs. The objective is to find the flow rates and coding subgraphs with channel selection, with the consideration of *uncertain spectrum availability*, such that the multicast rate R is maximized. In general, we should consider the following constraints: flow rate constraints, coding subgraph constraints, and link capacity constraints. We now specifiy these constraints separately in details.

A. Flow Rate Constraints

A flow rate constraint requires that the amount of incoming traffic rate equals to the amount of outgoing traffic at every node in the network. We denote by $f^d_{iJj}(c)$ the data rate for sink d from node i to node $j \in J$, when packets are actually transmitted on virtual wireless broadcast link (i, J), using channel c. To explicitly demonstrate the dependency of a broadcast link on a specific channel, we define

$$\mathcal{T}^c_i = \{j|d_{ij} \le r_T, j \ne i, c \in \mathcal{C}_i \bigcap \mathcal{C}_j\}, \tag{1}$$

where d_{ij} is the distance between node i and j, and r_T is the transmission range of node i. $\mathcal{T}^c_i$ is thus the set of nodes that node i can simultaneously transmit data to through channel c.

According to the max-flow min-cut theorem in [6], there exists a flow vector $f^d_{iJj}(c)$ for each sink d, which satisfies

$$\sum_{J\subset\mathcal{T}^c_i}\sum_{j\in J} f^d_{iJj}(c) - \sum_{\{j|I\subset\mathcal{T}^{c'}_j, i\in I\}} f^d_{jIi}(c') = \begin{cases} R & i = s \\ -R & i \in \mathcal{D} \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

for all $d \in \mathcal{D}$. The first and second parts in the left of Eq. (2) correspond to the total outgoing rate and total incoming rate, respectively. If node i is the source node, its outgoing rate is R and incoming rate is 0. If i is one of the sinks, its outgoing rate is 0 and incoming rate is R. Otherwise, node i is an intermediate node, and its outgoing rate should be equal to its incoming rate.

B. Coding Subgraph Constraints

Similar to [7]–[9], [11], the constraints of coding subgraph include two types. One is that the link rate for any sink cannot exceed the corresponding injection rate restricted by the coding subgraph. The other is about the scheduling of hyperarcs, i.e., two hyperarcs cannot simultaneously transmit if they conflict.

We define $b_{iJK}(c)$ as the ratio of the sum of any successful reception rate that related to subset $K \subset J$, to the injection rate on hyperarc (i, J) through channel c, i.e.,

$$b_{iJK}(c) = \frac{\sum_{\{S|S\subset J, S\bigcap K\ne\emptyset\}} z_{iJS}(c)}{z_{iJ}(c)}. \tag{3}$$

If channel c is lossless, we have $b_{iJK}(c) = 1$ for all nonempty $K \subset J$ and $b_{iJ\emptyset}(c) = 0$. Then, for the link rate requirement on the coding subgraph [8], [11], i.e., the flow rate for any sink cannot exceed the rate determined by both coding subgraph and channel quality, we have

$$\sum_{j\in K} f^d_{iJj}(c) \le z_{iJ}(c) b_{iJK}(c). \tag{4}$$

Next, we deal with more complex scheduling constraints on hyperarcs. According to [8], $\mathbf{z}$ should be restricted with a convex subset. The core is to decide which sets of hyperarcs can transmit simultaneously without conflict. However, this depends on the interference model of the network. In this study, we consider a more general secondary interference model [11], i.e., each node is constrained to receive from at most one other node, and can only successfully receive if all other neighbors on the same channel are silent.

One may just follow the approach in [8], [11] using a conflict graph on the hypergraph only to capture the conflict of hyperarcs. Yet, this cannot be directly applied because there are multiple channels in the network and channel availabilities differ at different nodes. Instead, we utilize a two-dimensional conflict graph (TDCG) to describe the conflicts over hyperarcs and channels jointly, which can be used in multiradio multichannel (MR-MC) networks [21]. We first define a tuple w in the format:

$$\text{hyperarc-channel tuple:} \quad [(i, J), c],$$

where $(i, J) \in \mathcal{A}$ and $c \in \mathcal{C}_i \bigcap \mathcal{C}_J$. The tuple indicates that the hyperarc (i, J) operates on channel c.

Consider two tuples $w_1 = [(i_1, J_1), c_1]$ and $w_2 = [(i_2, J_2), c_2]$. We say that w_1 and w_2 do *not conflict* if the following conditions hold: 1) $i_1 \ne i_2$; 2) $i_1 \notin J_2$; 3) $i_2 \notin J_1$; 4) $J_1 \bigcap J_2 = \emptyset$; 5) $J_1 \bigcap \mathcal{T}^{c_2}_{i_2} = \emptyset$ and $J_2 \bigcap \mathcal{T}^{c_1}_{i_1} = \emptyset$ when $c_1 = c_2$.

Definition 1: The TDCG of a hypergraph $\mathcal{H}$ with the channel availability $(\mathcal{C}_i)_{i\in\mathcal{N}}$ is an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with $\mathcal{V}$ corresponding to the set of all hyperarc-channel tuples, and an edge (w_1, w_2) belongs to the edge set $\mathcal{E}$ if w_1 and w_2 interfere, for all $w_1, w_2 \in \mathcal{V}$.

Given the TDCG, a stable (independent) set $\mathcal{I}$ is a set of vertices belonging to $\mathcal{V}$, any two of which are nonadjacent. $\mathcal{I}$ includes the tuples that can be active simultaneously in the network. If adding any one more vertex into a stable set $\mathcal{I}$ results in a non-stable set, $\mathcal{I}$ is a maximal stable set (MLSS). A maximum stable set (MMSS) is one that is not contained in any other stable set. Similar to [11], we define $\pi^v_{\mathcal{I}}$ as the incidence vector of $\mathcal{I}$, which is given by

$$\pi^v_{\mathcal{I}} = \begin{cases} 1 & \text{if node } v \in \mathcal{I}, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Then, the stable set polytope of $\mathcal{G}$, i.e., $\mathcal{P}_{\text{STAB}}(\mathcal{G})$, is thus the convex hull of the incidence vectors of all MMSSs of $\mathcal{G}$.

Finally, the constraints of scheduling on the coding subgraph can be obtained as follows

$$\mathbf{z} = (z_{iJ}(c))_{(i,J)\in\mathcal{A},c\in\mathcal{C}} \in \mathcal{P}_{\text{STAB}}(\mathcal{G}). \tag{6}$$

The above constraint not only captures the classical coding subgraph requirements including the scheduling of hyperarcs, but also restricts the channel selection of different hyperarcs, which significantly differentiates our study from previous works.

C. Link Capacity Constraints

Since we consider wireless broadcasting nature, we should take the capacity of a broadcast link rather than a point-to-point link into consideration. We define the capacity of a broadcast link as the minimum of the capacities of its component point-to-point links [13].

Because the link capacity is significantly affected by the channel selection, we define $u_{ij}(c)$ as the 0-1 indicator representing whether node i and j operate on channel c

$$u_{ij}(c) = \begin{cases} 1 & \text{if } i \text{ transmits data to } j \text{ on channel } c \\ 0 & \text{otherwise,} \end{cases} \quad (7)$$

where $c \in \mathcal{C}_i \bigcap \mathcal{C}_j$. We note that the value of $u_{ij}(c)$ is determined by the aforementioned coding subgraph constraint. Based on the channel selection, we can define the capacity of a link (i,j) on channel c [17], i.e., $h_{ij}(c)$, which is given by

$$h_{ij}(c) = u_{ij}(c) B^c \log_2 \left(1 + \frac{g_{ij} Q}{\eta}\right), \quad (8)$$

where g_{ij} is the power propagation gain between node i and node j, Q is the power spectral density from the transmitter [2], and η is the ambient Gaussian noise density. Accordingly, the capacity of a broadcast link (i,J) on channel c can be given by

$$h_{iJ}(c) = \min_{j \in J} h_{ij}(c) = \min_{j \in J} \left\{ u_{ij}(c) B^c \log_2 \left(1 + \frac{g_{ij} Q}{\eta}\right) \right\}. \quad (9)$$

By defining $u_{iJ}(c) \triangleq \prod_{j \in J} u_{ij}(c)$, we further have

$$h_{iJ}(c) = u_{iJ}(c) B^c \min_{j \in J} \log_2 \left(1 + \frac{g_{ij} Q}{\eta}\right), J \subset \mathcal{T}_i^c. \quad (10)$$

When $u_{iJ}(c)$ = 0, we have $h_{iJ}(c)$ = 0.

Intuitively, one may easily have the following constraints for the flow rate requirement, i.e., the aggregate data rates on each broadcast link (i, J) cannot exceed the link's capacity on channel c [11], [13]

$$\sum_{d \in \mathcal{D}} \sum_{j \in K} f_{iJj}^d(c) \leq h_{iJ}(c), c \in \mathcal{C}_i, , K \subset J \subset \mathcal{T}_i^c. \quad (11)$$

However, the link capacity $h_{iJ}(c)$ in (11) is actually a random variable and thus uncertain — not knowing what the exact value is. The standard way to treat such uncertainty is to pass from the uncertain constraint to its *chance-constrained* version [18]–[20]

$$\Pr \left\{ \sum_{d \in \mathcal{D}} \sum_{j \in K} f_{iJj}^d(c) - h_{iJ}(c) \leq 0 \right\} \geq 1 - \varepsilon_{iJ}(c), \quad (12)$$

where $\varepsilon_{iJ}(c) \in (0,1)$ is the corresponding violation tolerance. Define $\varepsilon \triangleq \sum_{i \in \mathcal{N}} \sum_{c \in \mathcal{C}_i} \sum_{J \subset \mathcal{T}_i^c} \varepsilon_{iJ}(c)$. By setting $\varepsilon \ll 1$, we can guarantee that for all $c \in \mathcal{C}_i, J \subset \mathcal{T}_i^c, K \subset J$, all the constraints like (11) hold with a high probability 1 - ε [3].

D. Network Coding-Based Multicast in Multi-Hop CRNs

Putting the three types of constraints together, we have the following *chance-constrained program*:

$$\begin{aligned} &\text{maximize} && R \\ &\text{subject to} && (2), (4), (6), (12). \end{aligned}$$

In the program, $f_{iJj}^d(c) \geq 0$ and $z_{iJ}(c) \geq 0$ are optimization variables, corresponding to flow rate and coding subgraph, respectively.

The above optimization problem is intractable, owing to two main reasons. One is that the constraint in (6) requires to find all MMSSs in a given TDCG. However, searching all MMSSs is NP-hard [11], [25], due to the combinatorial difficulty encapsulated in $\mathcal{P}_{\text{STAB}}(\mathcal{G})$. The other, which is more challenging, is the probabilistic constraint in (12), rendering the convexity of the feasible set defined by (12) difficult to verify. Specifically, the feasible set of (12) can be either convex or non-convex, which depends on the distribution of $h_{iJ}(c)$ [18]. Typically, if $[\mathbf{a}^T \mathbf{b}^T]^T$ has a symmetric logarithmically concave density, $\Pr\{\mathbf{a}^T \mathbf{x} < \mathbf{b}\} \geq 1 - \epsilon$ is convex for $\epsilon <$ 1/2 [18]. However, it is still unclear how to compute the closed form of the probability even if it is convex [18], making the problem intractable.

IV. Bernstein Approximation to the Chance-Constrained Program

In this section, we propose a *conservative* convex approximation approach. Bernstein approximations are known as a useful class of approximation techniques for chance constraints [18]–[20]. The key idea is utilizing Bernstein approximation and judiciously choosing the approximation form, so as to obtain a series of *safe* and *tractable* constraints. Specifically, we replace the chance constraints with a series of constraints Υ, such that (i) any solution satisfies (12) when it is feasible for Υ (*safe approximation*), and (ii) the constraints in Υ are convex and efficiently computable (*tractable approximation*). In the rest of this section, we describe the approximation approach in two steps, presented by Theorem 1 and Theorem 2, respectively.

For simplicity, we define $f_{iJj}(c) \triangleq \sum_{d \in \mathcal{D}} f_{iJj}^d(c)$. Then, (12) turns to be

$$\Pr \left\{ \sum_{j \in K} f_{iJj}(c) - h_{iJ}(c) \leq 0 \right\} \geq 1 - \varepsilon_{iJ}(c). \quad (13)$$

[2] We here assume that all nodes use the same power for transmission.

[3] Note that there exists a small risk that no solution is feasible to (12). If this happens, we can enlarge the parameter ε and corresponding $\varepsilon_{iJ}(c)$ to obtain a feasible solution.

Enlightened by the Bernstein approximation [18], we construct the following function

$$\Phi(\mathbf{f}) \triangleq \inf_{\rho>0} \left\{ \sum_{j\in K} f_{iJj}(c) + \rho\Omega(-\rho^{-1}) - \rho \log(\varepsilon_{iJ}(c)) \right\}, \tag{14}$$

where $\mathbf{f} = (f_{iJj}(c))_{j\in K} \geq \mathbf{0}$ and $\Omega(y) \triangleq \log \mathbb{E}[\exp(y h_{iJ}(c))]$. Here $\mathbb{E}[\cdot]$ is excuted on the distribution information of $h_{iJ}(c)$. For a given $\varepsilon_{iJ}(c) > 0$, consider the following inequality

$$\Phi(\mathbf{f}) \leq 0. \tag{15}$$

We can prove that any solution $\mathbf{f}$ that is feasible for (15) is also feasible for the chance constraint (13) through the following theorem.

Theorem 1. *Assume that $\Phi(\mathbf{f})$ is defined as in (14) and $h_{iJ}(c)$ is a random variable following a specific probability distribution. For a given $\varepsilon_{iJ}(c) > 0$, suppose that there exists $\mathbf{f} = (f_{iJj}(c))_{j\in K} \geq \mathbf{0}$, such that (15) holds. Then, $\mathbf{f}$ satisfies (13).*

Proof: (*Sketch*) Inspired by the Bernstein approximation [18], let $F(\mathbf{f}, h) = \sum_{j\in K} f_{iJj}(c) - h_{iJ}(c)$, then (15) is equivalent to

$$\inf_{\rho>0} \left\{ \rho \mathbb{E}[\exp(\rho^{-1} F(\mathbf{f}, h))] - \rho \varepsilon_{iJ}(c) \right\} \leq 0.$$

If $\Pr\{F(\mathbf{f}, h) > 0\} \leq \mathbb{E}[\exp(\rho^{-1} F(\mathbf{f}, h))]$ holds and $\mathbb{E}[\exp(\rho^{-1} F(\mathbf{f}, h))] \leq \varepsilon_{iJ}(c)$, then we can have (13). In fact, the former equality can be obtained by the property of $\exp(\cdot)$ and the latter one can be derived from (15). The detailed proof is provided in [29]. ■

In light of Theorem 1, we can use a series of constraints

$$\begin{aligned} \Upsilon : \inf_{\rho>0} \{ &\sum_{d\in\mathcal{D}} \sum_{j\in K} f^d_{iJj}(c) + \rho \log \mathbb{E}[\exp(-\rho^{-1} h_{iJ}(c))] \\ &- \rho \log(\varepsilon_{iJ}(c)) \} \leq 0, i \in \mathcal{N}, J \subset \mathcal{T}^c_i, c \in \mathcal{C}_i \end{aligned} \tag{16}$$

to replace (12) in the chance-constrained program, so that an optimal solution to the new problem is a feasible suboptimal solution to the original problem. However, the closed form of Υ is still hard to compute, owing to the uncertain expectation value and the $\inf(\cdot)$ function.

Next, we exploit some useful distribution features including the support, unimodality, symmetry of the distribution, to obtain a computationally tractable approximation, which is presented by the following theorem.

Theorem 2. *Suppose that for a given $\varepsilon_{iJ}(c) > 0$, there exists $\mathbf{f} = (f_{iJj}(c))_{j\in K} \geq \mathbf{0}$ such that (15) holds, where $\Phi(\mathbf{f})$ is defined by (14). If the distribution of $h_{iJ}(c)$ has bounded support $[a_{iJ}(c), b_{iJ}(c)]$, we then have*

$$\sum_{d\in\mathcal{D}} \sum_{j\in K} f^d_{iJj}(c) \leq \beta_{iJ}(c) - \alpha_{iJ}(c)\nu^- + \alpha_{iJ}(c)\sigma \sqrt{2 \log(\frac{1}{\varepsilon_{iJ}(c)})}, \tag{17}$$

which implies (13), where $\alpha_{iJ}(c) = \frac{1}{2}(b_{iJ}(c) - a_{iJ}(c)) \neq 0$, $\beta_{iJ}(c) = \frac{1}{2}(b_{iJ}(c) + a_{iJ}(c))$, and ν^-, σ are the constants related to the probability distribution of $h_{iJ}(c)$.

Proof: (*Sketch*) The proof of this theorem mainly utilizes the following inequality $\Omega(y) \leq \max\{\nu^- y, \nu^+ y\} + \frac{\sigma^2}{2} y^2$, where $\nu^-, \nu^+ \in [-1, 1]$ and $\sigma \geq 0$ are constants that depend on the given families of probability distributions of y with bounded support [-1, 1]. Since $h_{iJ}(c)$ may not be bounded in [-1, 1], e.g., $h_{iJ}(c) \in [a_{iJ}(c), b_{iJ}(c)]$, we need to normalize $h_{iJ}(c)$ as follows: $\zeta_{iJ}(c) \triangleq \frac{h_{iJ}(c) - \beta_{iJ}(c)}{\alpha_{iJ}(c)}$, where $\alpha_{iJ}(c) \triangleq \frac{1}{2}(b_{iJ}(c) - a_{iJ}(c)) \neq 0$ and $\beta_{iJ}(c) \triangleq \frac{1}{2}(b_{iJ}(c) + a_{iJ}(c))$. And substituting $\zeta_{iJ}(c)$ in the inequality can derive the final approximation. Please refer to [29] and find the detailed proof. ■

V. Decentralized algorithm

We note that (17) is linear and thus convex, which may enable the whole problem to be solved by convex optimization techniques. However, the new problem is still NP-hard in general, owing to the aforementioned combinatorial difficulty in (6). Fortunately, enlightened by [11], after relaxing (6) of finding all MLSSs rather than MMSSs in the TDCG, we can employ the dual decomposition method to obtain an efficient and decentralized solution.

Let $f^d_{ij}(c) \triangleq \sum_{J\subset\mathcal{T}^c_i} f^d_{iJj}(c)$, where $\forall i \in \mathcal{N}$ and $j \in \mathcal{T}^c_i$, with the clear understanding that $f^d_{iJj}(c) = 0$ if $j \notin \mathcal{T}^c_i$. Therefore, $f^d_{ij}(c)$ represents the flow rate of innovative packets on link (i, j) using channel c ($c \in \mathcal{C}_i \bigcap \mathcal{C}_j$) for sink d.

The optimization problem is converted into

$$\begin{aligned} &\text{maximize} \quad R \\ &\text{subject to} \quad (2), (4), (6), \text{and} \\ &\sum_{d\in\mathcal{D}} \sum_{j\in K} f^d_{ij}(c) \leq \sum_{J\subset\mathcal{T}^c_i} \hat{h}_{iJ}(c), i \in \mathcal{N}, K \subset \mathcal{T}^c_i, \end{aligned} \tag{18}$$

where $\hat{h}_{iJ}(c) \triangleq \beta_{iJ}(c) - \alpha_{iJ}(c)\nu^- + \alpha_{iJ}(c)\sigma \sqrt{2 \log(\frac{1}{\varepsilon_{iJ}(c)})}$.

A. Dual Decomposition and Subgradient Method

We form the dual problem by introducing the Lagrange multipliers for constraints in (4) and (18). By introducing dual variables $\boldsymbol{\lambda} \triangleq (\lambda^d_{iK}(c)) \geq \mathbf{0}$, $\boldsymbol{\mu} \triangleq (\mu_{iK}(c)) \geq \mathbf{0}$ to relax (4) and (18) respectively, and moving them to the objective function, we have

$$\begin{aligned} \theta(\boldsymbol{\lambda}, \boldsymbol{\mu}) = &\max_{R,\mathbf{f},\mathbf{z}} L(R, \mathbf{f}, \mathbf{z}, \boldsymbol{\lambda}, \boldsymbol{\mu}) \\ &\text{subject to} \quad (R, \mathbf{f}) \in Q_F \quad (19) \\ &\qquad \mathbf{z} \in \mathcal{P}_{\mathbf{STAB}}(\mathcal{G}), \quad (20) \end{aligned}$$

where $\mathbf{f} = (f^d_{ij}(c)) \geq \mathbf{0}$, $\mathbf{z} = (z_{iJ}(c)) \geq \mathbf{0}$, and the flow polytope Q_F is defined by (2). The corresponding Lagrangian is given by

$$\begin{aligned}
&L(R,\mathbf{f},\mathbf{z},\boldsymbol{\lambda},\boldsymbol{\mu})\\
&= R\\
&+\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{N}}\sum_{c\in\mathcal{C}_i}\sum_{K\subset\mathcal{T}_i^c}\lambda_{iK}^d(c)\left(\sum_{J\subset\mathcal{T}_i^c} z_{iJ}(c)b_{iJK}(c)-\sum_{j\in K} f_{ij}^d(c)\right)\\
&+\sum_{i\in\mathcal{N}}\sum_{c\in\mathcal{C}_i}\sum_{K\subset\mathcal{T}_i^c}\mu_{iK}(c)\left(\sum_{J\subset\mathcal{T}_i^c}\hat{h}_{iJ}(c)-\sum_{d\in\mathcal{D}}\sum_{j\in K} f_{ij}^d(c)\right)\\
&=\Bigg(R-\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{N}}\sum_{c\in\mathcal{C}_i}\sum_{K\subset\mathcal{T}_i^c}\lambda_{iK}^d(c)\sum_{j\in K} f_{ij}^d(c)\\
&\quad-\sum_{i\in\mathcal{N}}\sum_{c\in\mathcal{C}_i}\sum_{K\subset\mathcal{T}_i^c}\mu_{iK}(c)\sum_{d\in\mathcal{D}}\sum_{j\in K} f_{ij}^d(c)\Bigg)\\
&\quad+\left(\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{N}}\sum_{c\in\mathcal{C}_i}\sum_{K\subset\mathcal{T}_i^c}\lambda_{iK}^d(c)\sum_{J\subset\mathcal{T}_i^c} z_{iJ}(c)b_{iJK}(c)\right)\\
&\quad+\sum_{i\in\mathcal{N}}\sum_{c\in\mathcal{C}_i}\sum_{K\subset\mathcal{T}_i^c}\mu_{iK}(c)\sum_{J\subset\mathcal{T}_i^c}\hat{h}_{iJ}(c). \qquad (21)
\end{aligned}$$

The first term in the above equation depends only on the flow rate variables $f_{ij}^d(c)$, while the second term relies solely on the coding subgraph variables $z_{iJ}(c)$. Therefore, the dual function can be computed by decomposing the optimization problem into following subproblems:

$$\begin{aligned}
\theta(\boldsymbol{\lambda},\boldsymbol{\mu}) &= \max_{\{R,\mathbf{f},\mathbf{z}\}} L(R,\mathbf{f},\mathbf{z},\boldsymbol{\lambda},\boldsymbol{\mu})\\
&= \theta_1(\boldsymbol{\lambda},\boldsymbol{\mu})+\theta_2(\boldsymbol{\lambda})+\theta_0, \qquad (22)
\end{aligned}$$

where

$$\begin{aligned}
&\theta_1(\boldsymbol{\lambda},\boldsymbol{\mu})\\
&=\max_{(R,\mathbf{f})\in Q_F, 0\le R\le 1}\Bigg(R-\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{N}}\sum_{c\in\mathcal{C}_i}\sum_{K\subset\mathcal{T}_i^c}\lambda_{iK}^d(c)\sum_{j\in K} f_{ij}^d(c)\\
&\qquad -\sum_{i\in\mathcal{N}}\sum_{c\in\mathcal{C}_i}\sum_{K\subset\mathcal{T}_i^c}\mu_{iK}(c)\sum_{d\in\mathcal{D}}\sum_{j\in K} f_{ij}^d(c)\Bigg) \qquad (23)\\
&\theta_2(\boldsymbol{\lambda})\\
&=\max_{\mathbf{z}\in\mathcal{P}_{\mathbf{STAB}}(\mathcal{G})}\left(\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{N}}\sum_{c\in\mathcal{C}_i}\sum_{K\subset\mathcal{T}_i^c}\lambda_{iK}^d(c)\sum_{J\subset\mathcal{T}_i^c} z_{iJ}(c)b_{iJK}(c)\right) \qquad (24)
\end{aligned}$$

and

$$\theta_0=\sum_{i\in\mathcal{N}}\sum_{c\in\mathcal{C}_i}\sum_{K\subset\mathcal{T}_i^c}\mu_{iK}(c)\sum_{J\subset\mathcal{T}_i^c}\hat{h}_{iJ}(c). \qquad (25)$$

In the first subproblem, similar as in [11], we add a redundant constraint on R ($0\le R\le 1$), since by the definitions of $z_{iJ}(c)$ and $b_{iJK}(c)$, and constraint (4), the rate R cannot be larger than one. This constraint is to avoid handling possibly unbounded solutions.

Then, we have the following dual problem

$$\begin{aligned}
&\text{minimize} \quad \theta(\boldsymbol{\lambda},\boldsymbol{\mu})\\
&\text{subject to} \quad \boldsymbol{\lambda}\ge\mathbf{0},\boldsymbol{\mu}\ge\mathbf{0}.
\end{aligned}$$

Since $\theta(\boldsymbol{\lambda},\boldsymbol{\mu})$ may not be differentiable, we apply the well-known subgradient projection method to solve the above dual problem. And the update rules are given by

$$\boldsymbol{\lambda}[t+1]=\{\boldsymbol{\lambda}[t]-\eta_1[t]\boldsymbol{\Delta}_1[t]\}^+ \qquad (26)$$

$$\boldsymbol{\mu}[t+1]=\{\boldsymbol{\mu}[t]-\eta_2[t]\boldsymbol{\Delta}_2[t]\}^+, \qquad (27)$$

where $\{x\}^+ \triangleq \max(x,0)$, $\eta_1[t]$ and $\eta_2[t]$ are the corresponding appropriate stepsizes, and $\boldsymbol{\Delta}_1[t] = (\sum_{J\subset\mathcal{T}_i^c}\hat{z}_{iJ}(c)[t]\hat{b}_{iJK}(c) - \sum_{j\in K}\hat{f}_{ij}^d(c)[t])$, $\boldsymbol{\Delta}_2[t] = (\sum_{J\subset\mathcal{T}_i^c}\hat{h}_{iJ}(c) - \sum_{d\in\mathcal{D}}\sum_{j\in K}\hat{f}_{ij}^d(c)[t])$. Here, $\hat{\mathbf{f}}[t] = (\hat{f}_{ij}^d(c)[t])$ and $\hat{\mathbf{z}}[t] = (\hat{z}_{iJ}(c)[t])$ are the solutions of subproblems (23) and (24) respectively, at step t. And $\hat{b}_{iJK}(c)$ can be locally calculated by node i. We will show how to solve these subproblems in a decentralized way in Section V-B.

B. Flow Optimization and Scheduling Subproblem Solution

Rearranging (23) yields

$$\begin{aligned}
&(\hat{R},\hat{\mathbf{f}})\\
&=\operatorname*{arg\,max}_{(R,\mathbf{f})\in Q_F,0\le R\le 1}\left(R-\sum_{d\in\mathcal{D}}\left(\sum_{i\in\mathcal{N}}\sum_{c\in\mathcal{C}_i}\sum_{j\in\mathcal{T}_i^c} q_{ij}^d(c)f_{ij}^d(c)\right)\right), \qquad (28)
\end{aligned}$$

where $q_{ij}^d(c) = \sum_{\{K|K\subset\mathcal{T}_i^c, j\in K\}}(\lambda_{iK}^d(c)+\mu_{iK}(c))$. This subproblem is a multiple-shortest-paths problem owing to the conservation constraints in the flow polytope Q_F. Additionally, the subproblem is similar to the subgraph optimization problem in [11], except that we should consider the channel selection for every link under a given set of available channels. Solving (28) is equivalent to searching for each sink $d\in\mathcal{D}$, the shortest path with respect to the cost $q_{ij}^d(c)$ from the source s to sink d, ensuring that there is at least a common channel available for each link in the path. Moreover, to maximize (28), we need to increase R and make the right expression small. Nevertheless, R and $\mathbf{f}$ are coupled by Q_F. Fixing R, we should minimize the cost:

$$\sum_{d\in\mathcal{D}}\min_{\mathbf{f}}\sum_{i\in\mathcal{N}}\sum_{c\in\mathcal{C}_i}\sum_{j\in\mathcal{T}_i^c} q_{ij}^d(c)f_{ij}^d(c), \qquad (29)$$

for s to each d supporting end-to-end throughput R. (29) can be seen as a minimum-cost network problem without capacity constraints [11]. Consequently, it is sufficient for each of the D parallel optimization problems to find a single minimum-cost path from s to d with an available channel for each link. Since each of the flows in the minimum-cost path carries the total throughput R as there is only one active path from s to d and one channel for each link, (28) can be reformulated as

$$\max_{R,0\le R\le 1}\left(R(1-\sum_{d\in\mathcal{D}}\sum_{(i,j,c)\in\mathcal{P}_{sd}} q_{ij}^d(c))\right), \qquad (30)$$

where $\mathcal{P}_{sd}$ denotes the collection of links (i, j) with channel c ($c \in \mathcal{C}_i \bigcap \mathcal{C}_j$) in the minimum-cost path from s to d. If the sum of costs in all paths from source s to all sinks is less than 1, the maximum is 1 for $\hat{R}$ and for those link flows $\hat{f}_{ij}^{d}(c)$ with $(i, j, c) \in \mathcal{P}_{sd}$. Otherwise, the maximum is zero by setting both R and $\mathbf{f}$ to zero.

To solve (30), we have to determine D shortest paths, which can be distributively solved by the asynchronous Bellman-Ford algorithm [26]. Since all $q_{ij}^{d}(c)$ are greater than zero, there exist no cycles with negative costs and thus the shortest path algorithms converge [26] in $O(MN)$ time, where M and N are the number of arcs and nodes, respectively. In fact, in our problem, the time complexity is $O(MNC)$, where C is maximal number of available channels.

We now consider the second subproblem, which can be rewritten as

$$\hat{\mathbf{z}} = \arg\max_{\mathbf{z}\in\mathcal{P}_{\mathbf{STAB}}(\mathcal{G})} \left(\sum_{i\in\mathcal{N}} \sum_{c\in\mathcal{C}_i} \sum_{J\subset\mathcal{T}_i^c} w_{iJ}(c) z_{iJ}(c) \right), \tag{31}$$

where weights $w_{iJ}(c) = \sum_{d\in\mathcal{D}} \sum_{K\subset\mathcal{T}_i^c} (\lambda_{iK}^{d}(c) b_{iJK}(c))$. The weight $w_{iJ}(c)$ of a node i is just a function of local variables $\lambda_{iK}^{d}(c)$ and $b_{iJK}(c))$, which enables us to design a distributed scheduling algorithm. This is a standard maximum-weight-stable-set (MWSS) problem in the scheduling literature [27], which is NP-hard in general. A worth emphasizing point is that the weight is not only related to the hyperarc but also the channel they choose. We have encapsulated both the hyperarc and channel in a tuple in the proposed TDCG. To obtain an efficient solution, we relax the *maximum* weight stable set constraint and instead search a *maximal* stable set. The relaxed problem can be solved using the distributed algorithm in [27]. The input of the algorithm is a weighted indirected graph and the output is a stable set with high weight. Further, the output stable set is guaranteed to be maximal too. And the algorithm converges in just $2\delta(\mathcal{G})$ steps, where $\delta(\mathcal{G})$ is the stability number of the conflict graph $\mathcal{G}$ [27].

The sequence of $\hat{\mathbf{f}}[t]$ and $\hat{\mathbf{z}}[t]$ iterated by subgradient optimization methods might be infeasible [28]. To recover the primal solution, we employ a classical primal recovery method [28], which averages the intermediate optimal values, i.e.

$$R^* = \frac{1}{T}\sum_{t=1}^{T}\hat{R}[t], \quad \mathbf{f}^* = \frac{1}{T}\sum_{t=1}^{T}\hat{\mathbf{f}}[t], \quad \mathbf{z}^* = \frac{1}{T}\sum_{t=1}^{T}\hat{\mathbf{z}}[t]. \tag{32}$$

This approach can coverage to the primal optimal solution for a stepsize $\frac{a'}{b'+c't}$ with $a' > 0$, $b' \geq 0$ and $c' > 0$ [28].

VI. Evaluation

In this section, we present evaluation results for our proposed solution. The main goal is to demonstrate the superior performance of our algorithm over existing approaches.

A. Objects of Comparison

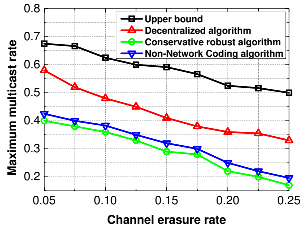
Our decentralized algorithm offers a feasible solution to the chance-constrained program. To evaluate the gap between our algorithm and the optimal solution, we remove constraint (12) in the program and obtain the new solution as an "upper bound". We also incorporate a naive approach called as "conservative robust algorithm" that simply substitutes the bandwidth B^c to the expectation value of B^c in the link capacity $h_{iJ}(c)$, which does not consider the uncertainty of spectrum availability. Besides, since [3] is the first to study multicast in multi-hop CRNs, we implement their approach (labeled "non-network coding algorithm" in the figs) to show the benefits of network coding on multicast in multi-hop CRNs when considering the uncertain spectrum availability.

B. Setup

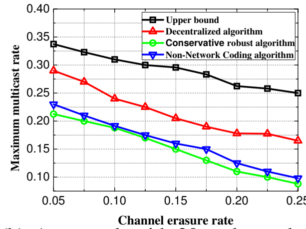
We conduct simulations with a CRN over random network topologies, where nodes are randomly placed on a square region. Concretely, we evaluate all algorithms on two scenarios, 10 nodes on a 56×56 m^2 square and 20 nodes on a 80×80 m^2 square, respectively. We consider the leftmost node to always be the source s, and the two rightmost nodes to be the sinks. Following previous works [3], [17], the explicit settings of each node are as follows. The power propagation gain between node i and node j is $g_{ij} = \gamma d_{ij}^{-n}$, where d_{ij} denotes the distance of node i and node j, γ is an antenna related constant, and n is the path loss factor. We set γ and n to be 1 and 4, respectively. The power spectral density from the transmitter is $Q = 1.6 \times 10^5\eta$, where η is the ambient Gaussian noise density. The transmission range for a node is thus $r_T = (\gamma Q/\eta)^{\frac{1}{n}} = (1.6 \times 10^5)^{\frac{1}{4}} = 20$ (m). As for the erasure model, for simplicity, we assume that any receiver successfully receives a packet from its neighbor node with identical probability 1 - p, where we change p from 0.05 to 0.25 in the simulations. We also suppose that the erasures across different receivers are independent. For the tolerance level, we set $\varepsilon_{iJ}(c) = 0.01$ for all $i \in \mathcal{N}$, $c \in \mathcal{C}_i$, $J \subset \mathcal{T}_i^c$.

For the channel availabilities, we assume that there are C = 3 and C = 5 licensed channels for the first network and second network that can be opportunistically used by SUs, respectively. Based on the data collection as well as statistical analysis on spectrum utilization in [14], the bandwidth of a channel c, B^c, can be exponential, i.e., $B^c \sim E(\upsilon^c)$, where $\upsilon^c \in (0, 3]$. Note that different channel c may have different distribution parameter υ^c. In our simulation study, we randomly choose υ^c from (0, 3] for each channel c to simulate heterogeneous channels. The set of available channels at each node is randomly selected from the channel pool. Besides, to evaluate the performance of our algorithm in various spectrum availabilities, we conduct the similar simulation study on an another set of bandwidth distribution, i.e., B^c follows a Normal distribution, $B^c \sim N(\mu', \sigma'^2)$. In the simulations, μ' and σ' are set as 2 and 1, respectively.

Due to the inherent spectrum scarce and consequent disconnectionless, there may exist no feasible solution for some specific data set. In this study, we only focus on the data sets with feasible solutions and present the corresponding results in the following subsection.

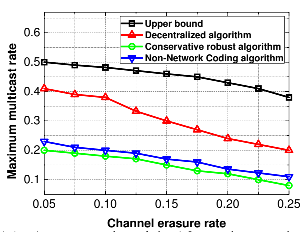


(a) A network with 10 nodes and 3 channels.

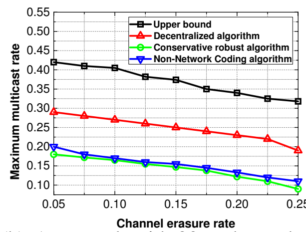


(b) A network with 20 nodes and 5 channels.

Fig. 1: Maximum multicast rate as a function of channel erasure rate for two different sized networks under *Exponential* distribution, i.e., $B^c \sim E(\upsilon^c), \upsilon^c \in (0, 3]$.



(a) A network with 10 nodes and 3 channels.



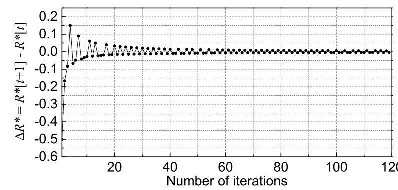
(b) A network with 20 nodes and 5 channels.

Fig. 2: Maximum multicast rate as a function of channel erasure rate for two different sized networks under *Normal* distribution, i.e., $B^c \sim N(2, 1)$.

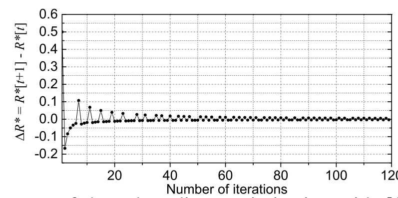
C. Results

Fig. 1 (a) shows the throughput performance of our decentralized algorithm in a network with 10 nodes and 3 channels under Exponential distribution, compared to the upper bound, conservative robust algorithm and non-network coding algorithm, when the channel erasure rate p varies. According to the figure, firstly, our algorithm outperforms the non-network coding algorithm, no matter when the channel quality deteriorates. This is because with network coding, both wireless broadcast nature and multipath benefits are effectively exploited in multi-hop CRNs, while without network coding, packets are transmitted for each sink on one single path only. Moreover, we find that the performance of the conservative robust algorithm is a little worse than the non-network coding algorithm, due to the overly conservative constraints on link capacity. And this shows the importance of considering the uncertainty of spectrum availability. Also, our decentralized algorithm achieves a competitive performance compared with the upper bound. The above phenomenons are similar in a larger network with 20 nodes and 5 channels, as shown in Fig. 1 (b). This illustrates that our algorithm is effective facing with a larger network and bigger channel set.

In Figs. 2 (a) and (b), we evaluate the performance of all algorithms under Normal distribution, when the channel erasure rate p changes from 0.05 to 0.25. As shown in the figures, similar to Figs. 1 (a) and (b), compared to the conservative robust algorithm and the non-network coding algorithm, our proposed algorithm always achieves higher maximum multicast rates when the channel erasure rate varies. Similarly, the performance of the conservative robust algorithm is as low as that of non-network coding algorithm. All in all, the performance of our algorithm is relatively stable when the type of spectrum availability changes.



(a) Convergence of the subgradient optimization with N=10 and C=3.



(b) Convergence of the subgradient optimization with N=20 and C=5.

Fig. 3: Convergence of the distributed algorithm.

In Figs. 3 (a) and (b), we illustrate the convergence of the decentralized algorithm for the two different sized networks. Since the optimal value of the maximum multicast rate, i.e., R^*, cannot be efficiently obtained, we use instead the improvement in the maximum multicast rate in each iteration, i.e., $\triangle R^* = R^*[t+1] - R^*[t]$. From the figures, we note that our algorithm achieves fast convergence in both cases, where $\triangle R^*$ quickly converges close to zero within ~60 iterations.

VII. Related Work

In this section, we review the related work in the following two aspects: network coding-based multicast in MR-MC networks, and multicast in multi-hop CRNs, which is further divided into two parts according to whether network coding is used.

Network Coding-Based Multicast in MR-MC Networks: Zhang and Li [12] showed that network coding can further increase the capacity of multichannel mesh networks. With opportunistic overhearing, Chieochan and Hossain [13] proposed a suboptimal, auction-based solution for the overall network throughput optimization with network coding in MR-MC networks. Lin and Yang [24] studied the problem of multirate multichannel multicast with intraflow network coding. However, these studies for MR-MC networks cannot be applied to address our problem. This is because in MR-MC networks, the set of common available channels for every node in the network is assumed fixed and corresponding link capacity is definite, which is impractical for CRNs.

Multicast in Multi-Hop CRNs:

Without network coding: Kim *et al.* [22] improved the scalability of the classical routing protocol ODMRP to support multicast services in a cognitive radio ad hoc wireless network. Almasaeid *et al.* [23] proposed an on-demand multicast rouging algorithm for cognitive radio mesh networks. In [4], a reactive joint channel allocation and multicast scheme is proposed for a multi-hop CRN. Gao *et al.* [3] formulated the problem of multicast in multi-hop CRNs as mixed linear program to minimize the required resources. However, none of the aforementioned works utilizes network coding to pursue higher throughput performance.

With network coding: Jin *et al.* [2] studied multicast scheduling with cooperation and network coding in CRNs, where network coding is used to reduce overhead and perform error control. Although [2] exploits network coding to improve the network performance in multi-hop CRNs, network coding is actually not utilized in SUs' data transmissions. Using network coding as an assistance strategy, Almasaeid and Kamal [5] proposed an algorithm to reduce the effect of the channel heterogeneity property on the multicast throughput in cognitive radio wireless mesh networks. In [5], network coding is employed to improve the performance of multicast in one cell, which is limited to single hop only and thus the approach cannot be utilized in multi-hop CRNs. Specifically, in single hop scenarios, there are no forwarders, and thus the coding subgraph selection is not involved, while it it not the case for multi-hop CRNs. And both [2] and [5] do not consider the uncertainty of spectrum availability in CRNs.

VIII. Conclusion

In this paper, we for the first time studied the network coding-based multicast problem in multi-hop CRNs, while considering the uncertain spectrum availability. We formulated the overall problem as a chance-constrained program. We constructed a TDCG to encapsulate both hyperarc scheduling and channel selection into hyperarc-channel tuples. Though appropriate Bernstein approximation and relaxation of finding stable sets in the TDCG, we proposed an efficient distributed algorithm for the original problem. Simulation results showed that our algorithm obtains higher multicast rates than the non-network coding algorithm in CRNs and the conservative robust algorithm. Although our algorithm is provably efficient, we found that the algorithm works not well in larger networks as the number of possible stable sets increases very quickly with the increased network size, which will be studied in future.

Acknowledgment

This work is supported in part by the State Key Development Program for Basic Research of China (973 project 2014CB340303), in part by the National NSF of China under Grant No.61103224, No.61371124, No.61472445, No. 61422208 and No. 61272443, in part by the NSF of Jiangsu Province under Grant No.BK2011118 and No.BK20140076, and in part by CCF-Intel Young Faculty Researcher Program and CCF-Tencent Open Fund.

References

[1] J. Mitola III, "Cognitive radio: an integrated agent architecture for software defined radio," Ph.D. dissertation, KTH Royal Institute of Technology, 2000.

[2] J. Jin, H. Xu, and B. Li, "Multicast scheduling with cooperation and network coding in cognitve radio networks," *IEEE INFOCOM*, 2010.

[3] C. Gao, Y. Shi, Y. Thomas Hou, H. Sherali, and H. Zhou, "Multicast communications in multi-hop cognitive radio networks," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 4, pp. 784-793, 2011.

[4] Z. Shu, Y. Qian, Y. Yang, and H. Sharif, "Channel allocation and multicast routing in cognitive radio networks," *IEEE WCNC*, 2013.

[5] H. Almasaeid and A. Kamal, "Exploiting multichannel diversity for cooperative multicast in cognitive radio mesh networks," *IEEE/ACM Transactions on Networking*, vol. 22, no. 3, pp. 770-783, 2014.

[6] R. Ahlswede, N. Cai, S. Li, and R. Yeung, "Network information flow," *IEEE Transactions on Information Theory*, vol. 46, no. 4, pp. 1204-1216, 2000.

[7] T. Ho, R. Kotter, M. Médard, M. Effros, J. Shi, and D. Karger, "A random linear network coding approach to multicast," *IEEE Transactions on Information Theory*, vol. 52, no. 10, pp. 4413-4430, 2006.

[8] D. Lun, N. Ratnakar, M. Médard, R. Koetter, D. Karger, T. Ho, E. Ahmed, and F. Zhao, "Minimum-cost multicast over coded packet networks," *IEEE Transactions on Information Theory*, vol. 52, no. 6, pp. 2608-2623, 2006.

[9] T. Ho and H. Viswanathan, "Dynamic algorithms for multicast with intra-session network coding", *IEEE Transactions on Information Theory*, vol. 55, no. 2, pp. 797-815, 2009.

[10] A. Khreishah, I. Khalil, and J. Wu, "Distributed network coding-based opportunistic routing for multicast,", *ACM MobiHoc*, 2012.

[11] D. Traskov, M. Heindlmaier, M. Médard, and R. Koetter, "Scheduling for network-coded multicast," *IEEE/ACM Transactions on Networking*, vol. 20, no. 5, pp. 1479-1488, 2012.

[12] X. Zhang and B. Li, "On the benefits of network coding in multi-channel wireless networks," *IEEE SECON*, 2008.

[13] S. Chieochan and E. Hossain, "Channel assignment for throughput optimization in multichannel multiradio wireless mesh networks using network coding," *IEEE Transactions on Mobile Computing*, vol. 12, no. 1, pp. 118-135, 2013.

[14] D. Chen, S. Yin, Q. Zhang, M. Liu, and S. Li, "Mining spectrum usage data: a large-scale spectrum measurement study," *ACM MobiCom*, 2009.

[15] M. Mchenry, "Spectrum white space meansurements," New America Foundation Broadband Forum, June 20, 2003.

[16] M. Wellens, J. Riihijärvi, and P. Mähönen, "Evaluation of adaptive MAC-layer sensing in realistic spectrum occupancy secnarios," *IEEE DySPAN*, 2010.

[17] M. Pan, C. Zhang, P. Li, and Y. Fang, "Spectrum harvesting and sharing in multi-hop CRNs under certain spectrum supply," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 2, pp. 369-378, 2012.

[18] A. Nemirovski and A. Shapiro, "Convex approximations of chance constrained programs," *SIAM Journal on Optimization*, vol. 17, no. 4, pp. 969-996, 2006.

[19] A. Ben-Tal and A. Nemirovski, "Selected topics in robust convex optimization," *Mathematical Programming*, vol. 112, pp. 125-158, 2008.

[20] A. Nemirovski, "On safe tractable approximations of chance constraints," *European Journal of Operational Research*, vol. 219, no. 2012, pp. 707-718, 2011.

[21] Y. Cheng, H. Li, and P. Wan, "A theoretical framework for optimal cooperative networking in multiradio multichannel wireless networks," *IEEE Wireless Communcations*, vol. 19, no. 2, pp. 66-73, 2012.

[22] W. Kim, S. Oh, M. Gerla, and J.-S. Park, "COCAST: multicast mobile ad hoc networks using cognitive radio," *IEEE MILCOM*, 2009.

[23] H. Almasaeid, T. Jawadwala, and A. Kamal, "On demand multicast routing in cognitive radio mesh networks," *IEEE GLOBECOM*, 2010.

[24] K. Lin and D. Yang, "Multicast with intraflow network coding in multirate multichannel wireless mesh networks," *IEEE Transactions on Vehicular Technology*, vol. 62, no. 8, pp. 3913-3927, 2013.

[25] M. Garey and D. Johnson, "Computers and intracctability: a guide to the theory of NP completness," W. H. Freeman and Company, 1979.

[26] D. Bertsekas and R. Gallager, "Data networks (2nd ed.)", Upper Saddle River, NJ: Prentice-Hall, 1992.

[27] S. Basagni, "Finding a maximal weighted indepedent set in wireless networks," *Telecommunication System*, vol. 18, no. 1-3, pp. 155-168, 2001.

[28] T. Larsson, M. Patriksson, and A.-B. Stroemberg, "Ergodic, primal convergence in dual subgradient schemes for convex programming," *Mathematical Programming*, vol. 86, no. 2, pp. 283-312, 1999.

[29] Technical report, Network coding-based multicast in multi-hop CRNs under uncertain spectrum availability, available at: http://cl.ly/Wom9.