On Designing Protocols for Noncooperative, Multiradio Channel Assignment in Multiple Collision Domains

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Abstract—Channel assignment is a crucial problem for wireless networks, especially for noncooperative wireless networks, in which nodes are selfish. While there have been a few studies of noncooperative, multiradio channel assignment, most existing studies are restricted to single collision domains only. In this paper, we study the design of incentive-compatible protocols for noncooperative, multiradio channel assignment in *multiple collision domains*. First, we show the necessity of designing incentive-compatible protocols for this problem. Specifically, we show that, if no incentive-compatible protocol is deployed, Nash Equilibria (NEs) may have undesired properties, such as Pareto suboptimality and low throughput. To prevent the system from converging to the NEs with undesired properties, we propose an incentive-compatible protocol for channel assignment in multiple collision domains. We rigorously show that our protocol guarantees that the system converges to NEs that are Pareto-optimal and have the maximum system-wide throughput. Our simulation results also verify that our protocols are effective in ensuring that the system converges to the desired NEs.

Index Terms—Wireless access, channel assignment, mechanism design

1 Introduction

Frequency division multiplexing access (FDMA) is a frequently used multiplexing technique in wireless networks. FDMA divides the carrier bandwidth into a number of subbands, called channels. The wireless devices need to assign their radio transmitters to these channels, so that they can transfer signals simultaneously. This classical problem of channel assignment is of great importance to wireless communications and, thus, has been studied extensively [1], [2], [3], [4], [5], [6], [7]. In particular, when the involved mobile devices have multiple interfaces, this problem becomes the *multiradio channel assignment*, which has been addressed in some existing works (e.g., [5], [6], [7]).

Recently, a new variant of the multiradio channel assignment problem, *noncooperative, multiradio channel assignment* [8], has attracted a lot of attention. When wireless devices are noncooperative (i.e., *selfish*), traditional channel assignment protocols, which have been designed for cooperative devices, can no longer be used. The reason is that selfish devices may deviate from the protocols for their own benefits.

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While a number of interesting results have been obtained on noncooperative, multiradio channel assignment, existing studies are restricted to single collision domains only. For example, Felegyhazi et al. [8] are the first to study noncooperative, multiradio channel assignment in a single collision domain. They assume that the involved wireless devices are all within a single hop from each other. Wu et al. [9], [10] work in a similar setting and design a channel assignment protocol that can achieve globally optimal throughput. Gao and Wang [11] remove the single hop assumption and obtain very nice results by modeling the multiple hop channel allocation problem as a static cooperative game. We note that removing the assumption of single hops is *not* identical to removing the assumption of single collision domain, because of the difference between transmission range and sensing range. In particular, in [11], Gao and Wang still keep assumption that players reside in a single collision domain.

More Recently, [12] studied the noncooperative channel assignment problem in multiradio networks with multiple collision domains. Without introducing any incentive-compatible protocols, the authors of [12] obtained nice results on the properties of system stable states with an emphasis on fairness. Different from [12], our goal in this paper is to guarantee system-wide optimality and maximal throughput in the multi-collision-domain systems that use noncooperative multiradio channel assignment.

First, we find that it is necessary to design incentive-compatible protocols if system-wide optimality and maximal throughput is desired. Specifically, we investigate the possible stable states, namely *Nash Equilibria* (NEs), that the system could converge to, if no incentive-compatible channel assignment protocol is deployed. (In practice, the system should evolve to one of the NEs and then permanently stay in that state.) We obtain quantified results on the economic efficiency and throughput of these NEs. Our results indicate that these NEs may have undesired properties. For example, some NEs can be Pareto suboptimal, which means that there are better states of the system giving more payoffs to some devices than these NEs without decreasing other devices' payoffs. Hence, if the system finally evolves to one of such NEs, then some devices lose part of their payoffs unnecessarily. Moreover, we show that some NEs may have low system-wide throughputs.

To prevent the system from converging to these NEs having undesired properties, we propose an incentive-compatible channel assignment protocol for multiple collision domains. This protocol guarantees the Pareto optimality of all NEs and maximizes the system-wide throughput of them. The main tool we use to build this protocol is payment—we require a user to pay an amount of virtual currency for her devices' use of the channels. We argue that this is a *natural* requirement since communication bandwidth is a type of resource and it is reasonable to request the users to pay for their usage of resources. Furthermore, these payments can be collected in a *secure* and *efficient* way, and may *not* require an online central authority, as discussed in [13], [14], [15].

In summary, we make the following contributions in this paper:

- We study the problem of noncooperative, multiradio channel assignment in multiple collision domains, using a mechanism design approach.
- We analyze the NEs of the multiradio channel assignment game in multiple collision domains and obtain quantified results on economic efficiency and throughput. Our results indicate that designing incentive-compatible protocols is necessary, because otherwise the system may converge to a NE that is Pareto suboptimal or has low system-wide throughput.
- To guarantee that stable states (i.e., NEs) of the system always have the desired properties, we propose an incentive-compatible channel assignment protocol for noncooperative, multiradio channel assignment. This protocol guarantees that the NEs maximize the system-wide throughput, and that all the NEs are Pareto-optimal. We show the properties of this protocol with rigorous analysis.
- We perform extensive evaluations on GloMoSim [16]. The results show that our protocol is effective in ensuring that the system converges to the desired states.

The rest of this paper is organized as follows: First, we introduce the technical preliminaries and our game model for multiradio channel assignment in multiple collision domains in Section 2. In Section 3, we analyze the properties of NEs in this game. Then, we propose an incentive-compatible protocol to maximize the system throughput and achieve Pareto optimality in Section 4. We present the evaluation results in Section 5. Finally, after briefly reviewing the related work in Section 6, we conclude our paper in Section 7.

2 Preliminaries

In this section, we first present our system model, then describe the channel assignment game that we study, and finally review the definitions we use in this paper.

2.1 System Model

In our model, we assume a network that consists of a number of node pairs. Let P denote the set of node pairs in the network. For the entire network the available frequency band is divided into orthogonal channels (e.g., eight orthogonal channels in IEEE 802.11a protocol), the set of which is denoted by C. The channels are assumed to have the same characteristics. Each node has K transceivers to use. We assume that the MAC layer coordination function is turned off. The two nodes in each pair are within the transmission range of each other. They can establish a bidirectional communication, by tuning a pair of transceivers (one transceiver from each node) to the same channel. There is a mechanism that enables each node pair to simultaneously transmit packets using multiple channels. Each node is only involved in one such node pair.

We consider multiple collision domains. That is, some node pairs cannot interfere with the communications of some other pairs, even if they are all using the same channel. Two node pairs can interfere with each other's communication only when they are within the *interference range* of each other.

2.2 Multiradio Channel Assignment Game in Multiple Collision Domains

In this paper, our goal is to design incentive-compatible channel assignment protocols for multiple collision domains, to achieve desirable system properties. Here by incentive compatible, we mean that even though each node in the system can control his radios, it is still to his best interest to assign his radios to channels in a way such that desirable system performance can be achieved. To provide incentives to each node, we design suitable payments for the channel usage. This can be viewed as an application of mechanism design to the wireless network channel assignment problem in multiple collision domains. For a general introduction to the mechanism design literature, please refer to [17]. In this paper, we take game-theoretic approach to mechanism design.

We model the multiradio channel assignment problem in multiple collision domains as a noncooperative strategic game, in which each pair of communicating nodes is a selfish player. The set of players is thus P. The objective of each player is to maximize its own communication throughput and to minimize the cost at the same time. Note that the attempt to transmit packets may not be successful due to interference. We use the interference model (e.g., in [18]) that if two players within each other's interference range are transmitting packets on the same channel at the same time, no one can successfully transmit any useful data. Under this interference model, each player will not put more than one radio on the same channel at the same time, to avoid the interference with himself.

Each player's strategy in the game is to decide whether to use its radios and which channels to put radios on.

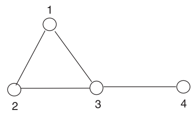

Fig. 1. An example of flow contention graph.

Formally, the strategy of player i is defined as

$$s_i = \{S_i^c | c \in C\},$$

where

$$S_i^c = \begin{cases} 1 & \text{if player } i \text{ has one radio on channel c} \\ 0 & \text{if player } i \text{ has no radio on channel c.} \end{cases}$$

Since each player only has K radios, the number of channels used by player i (denoted by k_i), cannot exceed K. (i.e., $\forall i, k_i = \sum_{c \in C} S_i^c \leq K$). The strategy profile is a matrix composed of all players' strategies, $s = (s_1, s_2, \ldots, s_{|P|})$. The strategy profile except for i's strategy is denoted as s_{-i}.

Whether players can successfully transmit packets depends on their strategies as well as those of others, which may cause interference to them. We use flow contention graph[1] to illustrate the interference relationship between players. In the flow contention graph, each node represents a player. If and only, if two players are within each other's interference range, there is an edge between the two nodes in the flow contention graph. Fig. 1 shows an example of flow contention graph. The topic of how to obtain the flow contention graph is closely related to the wireless network topology discovery problem which has been well studied (e.g., [22], [23]). We can adopt some of the available adaptive topology discovery algorithms (e.g., [24]), but since the topic of topology discovery is already beyond the scope of this paper, we will not explore it in detail.

For player i, the set of players who are connected with i (including i itself) in the flow contention graph is called i's interference set, denoted by N_i. We also define $n_{max} = \max_{i \in P} |N_i|$.

Now, we define the payoff function of player i as the amount of data that i successfully transmits, minus the cost of transmission. Formally,

$$u_i = \sum \left(r \sum_{c \in C} \left(S_i^c \cdot \prod_{j \in N_i} (1 - S_j^c) \right) - \beta \cdot k_i \right), \quad (1)$$

in which r is the throughput that a player can obtain through one radio, and $\beta (\beta < r)$ is a constant number representing the energy cost rate for one radio. Given the interference model described above, we know that each player can perform successful transmissions on one channel only when all the players in his interference set do not use that channel. If any of the players in the interference set is attempting to transmit, there will be collisions. Correspondingly, in (1), if none of the neighbor players of i use channel c, then $\prod_{j \in N_i} (1 - S_j^c) = 1$. $\prod_{j \in N_i} (1 - S_j^c) = 0$ implies that at least one neighbor player of i has a radio on channel c. In this case, even if i puts one radio on channel c (i.e., $S_i^c = 1$), he will not successfully transmit data and as a result he will lose the corresponding share of payoff.[2] We summarize the important notations used in this paper in Table 1.

TABLE 1
Table of Notations

P	Set of players
s_i	Strategy of player i
K	The number of radios that each player has
S_i^c	The number of radio that player i has on channel c
k_i	The number of channels used by player i
s	Strategy profile of all players
s_{-i}	The strategy profile except for i's strategy
N_i	Player i's interference set in the flow contention graph
n_{max}	The size of the largest interference set
r	the amount of data that a player can transmit through one radio
β	Energy cost parameter
u_i	Player i's utility

2.3 Definitions

To analyze the channel assignment game, we use some of the definitions (as described below) from game theory. For completeness, we include these definitions below. (Readers interested in these definitions can refer to, e.g., [25] for detailed discussions.)

Definition 1 (NE). *Let (S, U) be a game with the player set P, where S_i is the strategy set for player i, $S = S_1 \times S_2 \times \cdots \times S_{|P|}$ is the set of strategy profiles, and $U = (u_1(s), u_2(s), \ldots, u_{|P|}(s))$ is the utility functions for $s \in S$. The strategy profile $s^* = \{s_1^*, s_2^*, \ldots, s_{|P|}^*\}$ is a (NE) if for every player $i \in P$, we have*

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \quad (2)$$

for all strategy $s_i \in S_i$.

NEs are the *stable states* of the system, because no single player has incentives to leave them. Normally, the system should converge to a NE and then permanently stay there. Consequently, it is important to guarantee that NEs have good properties such as economic efficiency. A definition often used for economic efficiency is Pareto optimality:

Definition 2 (Pareto Optimality). *Let (S, U) be a game with the player set P, where S_i is the strategy set for player i, $S = S_1 \times S_2 \times \cdots \times S_{|P|}$ is the set of strategy profiles, and $U = (u_1(s), u_2(s), \ldots, u_{|P|}(s))$ is the utility functions for $s \in S$. A strategy profile s^{po} is Pareto optimal if for every strategy profile s such that there exists player $i \in P$,*

$$u_i(s^{po}) < u_i(s),$$

there must exist another player $j \in P$,

$$u_j(s^{po}) > u_j(s).$$

Intuitively, in a Pareto-optimal state, no player can get more payoff without hurting another player. Clearly, it is desirable to guarantee that all NEs are Pareto optimal.

1. All flows are single-hop flows in our game and each node in flow contention graph represents a player or his flow.

2. We note that there could be some DoS attackers who are willing to sacrifice payoff initially by jamming other users until some of them drop out. We assume that this type of DoS attacks can be detected by the network administrators and once detected, the attackers will be removed away from the network service.

3 Necessity of Designing Incentive-Compatible Protocols

In this section, we show the necessity of designing incentive-compatible protocols for noncooperative, multiradio channel assignment. In particular, we rigorously analyze the NEs in a system without incentive-compatible protocols, and study their economic efficiency and throughput.

Before we analyze the properties of NEs in the channel assignment game, we first characterize them by providing a necessary and sufficient condition for strategy profiles to become NEs.

Theorem 1. *s^* is a NE if and only if the following two conditions hold:*

1. $\forall i, \forall c, \text{if } S_i^{c*} = 1, \text{then} \prod_{j \in N_i}(1 - S_j^{c*}) = 1;$
2. *in any channel c, there does not exist player i, s.t.*

$$\sum_{j \in N_i} S_j^{c*} + S_i^{c*} = 0, \text{and } k_i^* < K.$$

We first introduce two Lemmas to help the proof of Theorem 1.

Lemma 1. *If s^* is a NE, then*

$$\forall i, c, \text{if } S_i^{c*} = 1, \prod_{j \in N_i}(1 - S_j^{c*}) = 1.$$

Due to the limited space, please find the proof of Lemma 1 in the technical report [27].

Another straightforward necessary condition of NEs is that players will put as many radios as possible on channels to increase their utilities as long as there is no interference with others. Formally, we have Lemma 2.

Lemma 2. *If s^* is a NE, then there does not exist i, s.t.*

$$\sum_{j \in N_i} S_j^{c*} + S_i^{c*} = 0, \text{and } k_i < K.$$

Proof of Theorem 1

Proof. Since we already have Lemma 1 and Lemma 2, all we need to prove here is that if the two conditions hold, s^* is a NE.

Suppose that under the two conditions above, a player i can unilaterally increase his utility by changing his strategy to u_i'. He has two possible ways in total to do so:

- Changing some S_i^{c*} from 1 to 0.

If $S_i^{c*} = 1$, from condition (1), we know that $\prod_{j \in N_i}(1 - S_j^{c*}) = 1$. In this case, $u_i' - u_i^* \leq 0$. Therefore, by changing some S_i^{c*} from 1 to 0, i cannot increase his utility.

- Changing some S_i^{c*} from 0 to 1.

We now consider two cases.

If $\sum_{j \in N_i} S_j^{c*} + S_i^{c*} > 0$, then $\prod_{j \in N_i}(1 - S_j^{c*}) = 0$. In this case, if i changes S_i^{c*} to 1, it will decrease his utility by β.

If $\sum_{j \in N_i} S_j^{c*} + S_i^{c*} = 0$, from condition (2) we know that, it must be the case that $k_i = K$, which means i has no spare radios to improve his utility.

Therefore, there is no way for i to unilaterally increase his utility with others strategies being equal. Hence, s^* is a NE. □

Condition (1) suggests that players will avoid interference to maximize their payoffs. Condition (2) says no player wants to spare their radios if they could successfully transmit packets. If both (1) and (2) are satisfied, the system is in its NE and vice versa. If in the system each node always tries to change his channel assignment for better utility in a distributed fashion, the system will converge to NE status as described in Theorem 1. This is due to the definition of NE and that the status in Theorem 1 is within the system capacity. Although the system will always converge, it is still nontrivial to determine whether these NEs can guarantee desired system properties. Hereafter, we will use Theorem 1 in the analysis of NEs' properties.

3.1 Economic Efficiency

In this section, we study the property of NEs from a system-wide perspective, *economic efficiency*,[3] using Pareto optimality as the criterion. If the system converges to a NE that is not Pareto optimal, then some players lose the opportunities of increasing their own payoffs without hurting anyone else, which immediately implies that some resources in the system are wasted. Therefore, it is important to identify whether all NEs in the channel assignment game are Pareto optimal.

First, we observe an example.

Example 1. Consider a network with three players and the flow contention graph is shown as Fig. 1. Each player has two radios and there are three channels, a, b, c, available.

Consider a strategy profile

$$\begin{aligned} s &= \{s_1, s_2, s_3\}, \\ s_1 &= \{\forall t, S_1^a = 1,\ S_1^b = 0, S_1^c = 1\}, \\ s_2 &= \{\forall t, S_2^a = 0,\ S_2^b = 0,\ \ S_2^c = 0\}, \\ s_3 &= \{\forall t, S_3^a = 0, S_3^b = 1,\ S_3^c = 1\}. \end{aligned}$$

In words, player 1 is using channel a and c; player 3 is using channel b and c; player 2 has no radio in use. Here, s achieves a NE, because player 1 and 3 both have obtained their best possible payoffs and player 2 has no way to improve his payoff given the fact that no matter which channel (a or b) he tries to use there will be an interference. However, s is not Pareto optimal. In fact, if player 1 moves one of his radios from channel a to channel b, player 2 can start using one of his radios to transmit packets on channel a without any interference. In this way, player 2 increases his payoff without decreasing any other player's payoff, which implies that s is Pareto suboptimal.

This example shows that NEs may not be Pareto optimal in the noncooperative, multiradio channel assignment. But what are the exact conditions for NEs to be Pareto optimal or Pareto suboptimal? Is there a system that has all its NEs being Pareto-optimal?

Our main observation is that Pareto optimality depends on the values of K, $|C|$, and n_{max}. More precisely, Pareto optimality can be guaranteed in all NEs when $|C|$ is not less than $n_{max} \cdot K$ (Proposition 1), or not more than K (Proposition 2). If the value of $|C|$ is between these two thresholds,

3. Note that economic efficiency is a standard term for resource allocation in economic theory, even though in many cases real money is not involved.

there can be some NEs that are Pareto suboptimal, as we have shown in Example 1.

Proposition 1. *If $|C| \geq n_{max} \cdot K$, all the NEs are Pareto-optimal.*[4]

Proposition 2. *If $|C| \leq K$, all the NEs are Pareto optimal.*[5]

Due to limited space, please refer to [27] for the proofs of Propositions 1 and 2.

An intuitive explanation of Proposition 2 is that when $|C| \leq K$, since the channel resource is so limited, in a NE if a player wants to increase its payoff by employing one more radio in some channel, at least one of its neighbors must remove its radio from that channel. Because in a NE, for each player, there is no more available channel to use, the change that a player uses one more channel must result in the consequence that some other player loses part of its utility due to the decreased number of occupied channels.

The above two propositions tell us that if the number of channels available is large enough ($|C| \geq n_{max} \cdot K$) or small enough ($|C| \leq K$), any NE channel allocation is Pareto optimal. It implies that in these two cases, the system administrators do not have to consider economic efficiency when choosing channel assignment protocols and, thus, can focus on other properties such as throughput. But note that considering the current real applications, both $|C| \geq n_{max} \cdot K$ and $|C| \leq K$ are minor cases.

Now, we study the remaining cases, in which $K < |C| < n_{max} \cdot K$. Let us revisit Example 1 in which NEs are not Pareto optimal. In Example 1, $K = 2$, $|C| = 3$, $n_{max} = 3$. (The size of interference set of Node 2 (containing Node 2 itself) is 3.) We have $K < |C| < n_{max} \cdot K$. So if $K < |C| < n_{max} \cdot K$, there may be some NEs which are not Pareto optimal.

3.2 Throughput

The second property of NEs that we study is system-wide throughput. Let $I(i)$ denote the interference degree of i—the number of players in the interference set of i that can transmit packets simultaneously without interfering with each other. Let $I(G)$ denote the maximum interference degree among all the players. Let r^* denote the maximum system-wide throughput that a network can achieve. In fact, the system-wide throughputs of some NEs can be as low as $r^*/I(G)$. Below, we give an example of low throughput NEs.

Example 2. Consider a network with the flow contention graph shown in Fig. 3, where $|C| = 2$ and $K = 2$. Clearly, the maximum system-wide throughput is achieved when player 1 through n use the two channels. However, the system could converge to a NE, in which only player 0 transmits packets using two radios. In this case, the system only obtains $1/n$ of the maximum system-wide throughput.

Above, we have obtained a number of results on NEs. In particular, we see that in some cases some NEs can be Pareto suboptimal or result in low system-wide throughput. If we let the system evolve by itself, the system may converge to a NE that is not desirable. To solve this problem, we propose to design an incentive-compatible channel assignment protocol that can achieve maximal throughput and Pareto optimality.

4 Protocol for Maximum System-Wide Throughput and Pareto Optimality

In different games, the NEs may have different properties. In Section 3, we show that without incentive-compatible schemes, in the multi-collision-domain noncooperative channel assignment game, some NEs can be Pareto suboptimal or result in low system-wide throughput. Our findings raise the need for incentive-compatible channel assignment protocols to achieve NEs with desirable Pareto optimality and system-wide throughput.

In this section, we design *PMT*, an incentive-compatible channel assignment protocol that guarantees that all the NEs have the maximum system-wide throughput and are Pareto optimal.

4.1 The PMT Protocol

Maximizing the system-wide throughput in multiple collision domains is not a trivial task even if all involved players are cooperative ([5], [26]). Given the selfishness of the players, it is even more challenging to ensure that all players use the channels in such a way that the maximum system-wide throughput is achieved. To solve this problem, we use an economic tool, payment, to stimulate players to choose channels cooperatively, so that all the NEs that the system can converge to have the maximum system-wide throughput. In the following, we first introduce *Independent Set* IDs, which play an important role in our protocol. Then, we present the design of our payment function and the entire PMT.

Independent set ID $i._{\mathrm{MISID}}$. Before the channel assignment game starts, for each player an independent set ID in the flow contention graph is assigned as an input. Denote $i._{\mathrm{MISID}}$ the ID of the independent set that player i belongs to, which can be obtained by running an algorithm for maximal independent sets (MIS). To compute the independent set IDs, we can adopt some approximation algorithms for MIS partition (e.g., [29]), which provide good performance as well as time efficiency.[6] Protocol 1 shows the pseudocode for MIS partition using the algorithm in [29], where $d(i)$ is the degree of the node, and N_i is the set of i's neighbors in the remaining graph.

Protocol 1. MIS: Maximal independent sets partition [29]

INPUT: Interference graph G, with the vertex set $V(G)$.
OUTPUT: $i._{\mathrm{MISID}}$, $\forall\ i \in V(G)$.
$\mathrm{MISID} = 1$.
$G' \leftarrow G$.
while $G' \neq \Phi$ **do**
  $I \leftarrow G$.
  while $I \neq \Phi$ **do**
    Choose i such that $d(i) = \min_{v \in V(I)} d(v)$.

4. The bound for $|C|$ is tight, i.e., this proposition holds when $|C| = n_{max} \cdot K$.

5. The bound for $|C|$ is tight in this proposition. Please see the proof for details.

6. One may notice that computing the MIS is NP-hard. However, because the size of flow contention graph is usually small, it is *practical* to use exponential time algorithms.

$\quad\quad i._{\mathrm{MISID}} = \mathrm{MISID}$.
$\quad\quad I \leftarrow I - \{i\} - N_i$.
$\quad\quad G' \leftarrow G' - \{i\}$.
$\quad$ **end while**
$\quad \mathrm{MISID} = \mathrm{MISID} + 1$.
end while

Each $i._{\mathrm{MISID}}$ is known when our PMT starts. We assume that the MIS are sorted according to its size, and a smaller independent set ID means a larger size. We also assume that after the MIS partition, the number of MIS is greater than $\lceil \frac{|C|}{K} \rceil$, because otherwise it is trivial to have all radios assigned without any interference. Actually, in a same independent set, radios from different players do not interfere with each other. Since each player has K radios, it means that the players in one independent set can utilize all their radios on K channels without interference. If the number of independent sets is less than or equal to $\lceil \frac{|C|}{K} \rceil$, the total number of channels for all the players without interference is less than or equal to $|C|$. In this case, we can just assign K channels to the players in each independent set. This simple assignment solution will not cause any interference. Hence, in this paper, we mainly focus on the nontrivial case that the number of MIS greater than $\lceil \frac{|C|}{K} \rceil$.

Virtual currency. As in many existing works (e.g., [30], [31], [32], among many others), we assume that there is a kind of virtual currency in the system.

There is a system administrator in the network, which can simply be a server connected to the Internet. The system administrator maintains an account for each player. Initially, each player can buy some virtual currency, for example, using real money. Whenever a player needs the access to some channels, the system administrator charges him a certain amount of fee and updates his account. If a player does not have enough virtual currency to access the channel, it can always buy some using real money. All transactions are cleared in the system administrator. We believe it is natural to ask the channel users to pay for their use of network resources.

Design of payment function. In this paper, we assume that all players have enough budgets to make payments and we leave the consideration of budget balance with a limited budget to our future work. To achieve the maximum system-wide throughput, we need to have as many radios as possible to successfully transmit packets. However, not all players can place all their radios in use at the same time due to interference. The most important part of our PMT protocol is a carefully designed payment function, which gives players incentives to use channels in such a way that the system has the maximum throughput. In particular, we consider a special independent set τ, which ranks $\lceil \frac{|C|}{K} \rceil$ among all the independent sets in the decreasing order of sizes. We call τ the threshold independent set. By our payment function, only the players in independent sets larger than τ are encouraged to employ as many radios as possible into channels. We make the independent set that ranks $\lceil \frac{|C|}{K} \rceil$ among the other independent sets, because our goal here is to achieve maximum system-wide throughput. In particular, we want the $|C|$ channels to be allocated to as many radios as possible. Since each independent set can use K radios without interference, the $|C|$ channels can be assigned to at most $\lceil \frac{|C|}{K} \rceil$ independent set. Hence, we encourage the $|C|$ channels to be assigned to the top $\lceil \frac{|C|}{K} \rceil$ largest independent set. In this way, maximum system-wide throughput can be achieved. Compared with other methods of determining threshold independent set, ours can guarantee maximum system-wide throughput.

More precisely, the payment of player i is designed as

$$p_i = \frac{(r-\beta) \cdot k_i \cdot (n_\tau - \epsilon)}{n_{i.\mathrm{MISID}}}. \tag{3}$$

In (3), recall that k_i is the number of channels on which player i is transmitting packets and $n_{i.\mathrm{MISID}}$ is the size of the independent set that i is in. The parameter ϵ is a constant positive number smaller than 1; n_τ denotes the size of the threshold independent set τ. The introduction of ϵ guarantees that when the player is in the threshold independent set, i.e., $n_{i.\mathrm{MISID}} = n_\tau$, the player is encouraged to use their radios as many as possible (as shown in the utility function later). (Here, we assume the threshold independent set is unique. In some cases, there may be more than one threshold independent set of size n_τ in the result of the MIS partition algorithm. If so, the system administrator can arbitrarily choose one of them.)

From this payment formula, we can see that if a player employs more radios (i.e., k_i is larger), its payment is correspondingly higher. Moreover, when the network system can provide better communication services (i.e., $(r-\beta)$ is higher), the players need to pay more. More importantly, in order to control how the nodes employ their radios and, thus, achieve maximum throughput, in the payment formula we have that nodes in larger independent set can pay less (when $n_{i.\mathrm{MISID}}$ is smaller). In this way, we encourage the nodes to employ as many as possible radios at the same time.

Plugging the payment formula into the payoff of each player defined in Section 2.2, we can get the following equation:

$$u_i = \sum_{c \in C} r \cdot \left(S_i^c \cdot \prod_{j \in N_i} (1 - S_j^c) \right) - \beta k_i - \frac{(r-\beta) \cdot k_i \cdot (n_\tau - \epsilon)}{n_{i.\mathrm{MISID}}}.$$

Assuming that there is no collision, the above equation of utility becomes[7]:

$$u_i = (r-\beta)\left(1 - \frac{n_\tau - \epsilon}{n_{i.\mathrm{MISID}}}\right) k_i. \tag{4}$$

In the payoff function, we let one unit of throughput, one unit of energy cost, and one unit of payment all equal to one unit of utility when counting the total payoff. This assumption does not affect our analysis of players' payoffs. We can always adjust the coefficient of unit conversion if necessary.

As we can easily see, each player in independent sets larger than n_τ will get higher payoff if he increases his number of radios to transmit packets (because n_τ and $n_{i.\mathrm{MISID}}$

7. Note that collisions may occur. We have shown that when collisions occur the system state is not a NE and the players unnecessarily lose payoffs. Our goal is to design protocol which maximizes the throughput and thus here we only focus on the cases when there is no collision for simplification and clarity.

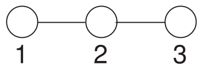

Fig. 2. The flow contention graph in example of pareto suboptimality.

are both integers and $0 < \epsilon < 1$). On the other hand, players with $n_{i \cdot \text{MISID}} < n_\tau$ will decrease their payoffs if they use more radios to transmit packets.

PMT protocol. We now provide the pseudocode of our PMT protocol (see Protocol 2), which guarantees that all NEs maximize the system-wide throughput. Our PMT protocol is a distributed protocol with imperfect information, which does not assume that each node has the perfect information about other nodes' channel assignment. In this paper, we assume that after running the MIS partition algorithm, the central authority sends the information $n_{i \cdot \text{MISID}}$ and n_τ to each node in the system, before the nodes can assign their radios to channels. Each node does not need to know who is in its interference set, but he can sense the interference when at least one of his neighbors and him are using the same channel at the same time. It is due to the broadcasting nature of wireless network communications. By trying to assign required number of radios to different channels and avoid interference from a local view (as shown in line 8 and line 11 in PMT protocol), the system will gradually converge to NE with desirable system properties without interference. As long as there is no change in the system topology, there is no need to communicate to each node at every time period.

Protocol 2. PMT: Multiradio channel assignment protocol for maximum system-wide throughput and Pareto optimality

1: **INPUT:** number of radios per player K; the set of available channels C; independent set size $n_{i \cdot \text{MISID}}$ for each player i; size of the threshold independent set: n_τ.
2: RandomChannelAssignment();
3: **if** $|C| < n_{max} \cdot K$ **then**
4:   **while** there is any change compared with last round **do**
5:     **for** each player i **do**
6:       **if** backoff counter is 0 **then**
7:         **if** ($n_{i \cdot \text{MISID}} > n_\tau$ **or** $n_{i \cdot \text{MISID}} = n_\tau$ **and** $|C| \bmod K = 0$) **and** the number of spare radios (k_i^s) is greater than 0 **then**
8:           Assign the all radio(s) to channels such that no interference exists from player i's local view;
9:         **end if**
10:         **if** $n_{i \cdot \text{MISID}} = n_\tau$ **and** $k_i^s > K - (|C| \bmod K)$ **then**
11:           Assign the spare radio(s) to other channels such that no interference exists from player i's local view, to achieve that $k_i^s = K - (|C| \bmod K)$;
12:         **end if**
13:         **if** $n_{i \cdot \text{MISID}} < n_\tau$ **and** $k_i^s < K$ **then**
14:           Do not assign any radio to any channel.
15:         **end if**
16:         Reset the backoff counter to a new value;
17:       **else**
18:         Decrease the backoff counter value by 1;
19:       **end if**
20:     **end for**
21:   **end while**
22: **end if**

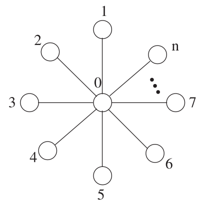

Fig. 3. Flow contention graph in the example of low throughput NEs.

At the beginning of each game, players execute the PMT protocol and keep the obtained channel assignment until some players change their strategies. The PMT protocol is a distributed protocol that works in a round-based fashion. After the initial random assignment, each player tries to change the channel assignment to his radios for better utility. To guarantee that there is only one player changing his strategy in one round, we use the mechanism of backoff window (as explained later in this paragraph). The players change the channel assignment in the following way. i in independent sets larger than n_τ checks whether all his radios are successfully transmitting packets (i.e., the number of spare radios (k_i^s) is 0). If not, he assigns the spare radio(s) to other channels in order to improve his total rate (line 5-7). Here, by a spare radio we mean a radio that is not successfully transmitting packets. Each player in the threshold independent set will stop changing his channel assignment once he has $|C| \bmod K$ radios successfully transmitting packets (line 8-9). For players in independent sets smaller than n_τ, not using any radio is the best strategy. We implement the backoff window as follows: Each player randomly chooses an initial value for his backoff counter from $\{1, 2, \ldots, W\}$, where W is the size of back-off window, with uniform probability. In this way, the backoff counter of each player is different from that of any other player. Since the backoff counter only decreases by 1 in one round, there is only one backoff counter becomes 0 each time PMT runs. Therefore in one round, there is at most one player who changes his strategy.

Now, we take the system shown in Fig. 2 as an example input to protocol PMT and see how PMT runs. With the example shown in Fig. 2, there are two channels available and each player has two radios. There are two independent set in the system: $\{1, 2, \ldots, n\}$ and $\{0\}$, The threshold independent set ranks $\lceil \frac{|C|}{K} \rceil$ in terms of size, and hence $n_\tau = n$. With any random initial assignment, for any player in $\{1, 2, \ldots, n\}$, if he has any spare radio(s), it satisfies that $n_{i \cdot \text{MISID}} = n_\tau$ and $|C| \bmod K = 0$, and line 8 in PMT Protocol will be executed, i.e., he assigns spare radio(s) to the available channels. If he has no spare radios, then there is nothing to change in the channel assignment. For the player 0, it satisfies the conditions in line 13, i.e., $n_{i \cdot \text{MISID}} < n_\tau$. He does not assign any radios to any channel. The PMT protocol in this simple example stops after two rounds. Then, the output of the protocol PMT is that player 0 does not assign any radio to any channel and all players in $\{1, 2, \ldots, n\}$ assign both radios to the two channels.

4.2 Analysis of PMT Protocol

4.2.1 Incentive Compatibility

Theorem 2. *If PMT is used, all the NEs satisfy that* $\forall i$,

$$k_i = \begin{cases} K & \text{if } n_{i\cdot\text{MISID}} > n_\tau \\ |C| \mod K & \text{if } n_{i\cdot\text{MISID}} = n_\tau \\ 0 & \text{if } n_{i\cdot\text{MISID}} < n_\tau. \end{cases} \quad (5)$$

Proof. We first show that PMT will reach the state s^* which satisfies (5). Then, we show s^* is a NE, i.e., the system converges at s^*.

To show that PMT will always reach the state that satisfies (5), first, we notice that (5) is achievable within the system capacity. Recall, that there are $\lceil\frac{|C|}{K}\rceil - 1$ independent sets that have sizes greater than n_τ and one independent set of size n_τ. When players in the same independent set allocate their radios on the same set of channels, the total number of channels without interference required by (5) is $(\lceil\frac{|C|}{K}\rceil - 1)K + |C| \mod K$, which is exactly the number of channels in the system $|C|$. On the other hand, the system will not stabilize in any state that does not satisfy (5), due to the strategy changing conditions in PMT (line 5 and 8). In each round of PMT we only have one player who changes his assignment so that oscillation of strategy changes can be avoided. Hence, there is no possible state in the system that has 0 probability to lead to the state satisfying (5). Therefore, with PMT, the system will always reach the state that satisfies (5).

Now, we show that state s^* which satisfies (5) is a NE, which guarantees that players do not have incentives to deviate from s^* unilaterally.

Let u_i' denote the payoff of i by taking other strategy s_i' that does not satisfy (5). k_i' is used in s_i'. Given $s*_{-i}$, we distinguish two possible types of s_i', i.e., (1) those result in interference and (2) those do not. Since those s_i' that result in interference will clearly bring lower payoffs for player i than those that avoid interference, in the proof, we only consider those s_i' of type (2). If we can prove that even the second type of s_i' cannot increase the payoff of i, then all possible s_i' cannot either.

There are three possible cases as follows:

- Case 1. $n_{i\cdot\text{MISID}} < n_\tau$.

$$u_i' - u_i^* = (r - \beta)\left(1 - \frac{n_\tau - \epsilon}{n_{i\cdot\text{MISID}}}\right)k_i' \leq 0,$$

because $\frac{n_\tau-\epsilon}{n_{i\cdot\text{MISID}}} > 1$.

- Case 2. $n_{i\cdot\text{MISID}} > n_\tau$.

$$u_i' - u_i^* = (r - \beta)\left(1 - \frac{n_\tau - \epsilon}{n_{i\cdot\text{MISID}}}\right)(k_i' - K) \leq 0,$$

since

$$\frac{n_\tau - \epsilon}{n_{i\cdot\text{MISID}}} < 1 \text{ and } k_i' - K \leq 0.$$

- Case 3. $n_{i\cdot\text{MISID}} = n_\tau$. If the players not in the threshold-independent set keep the channel assignment results as in (5), the number of channels that player i can use without interference is at most $|C| \mod K$ (i.e., $k_i' \leq |C| \mod K$). Also from $\frac{n_\tau-\epsilon}{n_{i\cdot\text{MISID}}} < 1$. We can obtain that $u_i'(s_i', s_{-i}^*) - u_i^*(s_i^*, s_{-i}^*) \leq 0$.

Therefore, if PMT is used, all the NEs satisfy (5). □

We note that from the case 3 in the proof of Theorem 2, the PMT protocol is expost incentive compatible. The PMT protocol is not dominant strategy incentive compatible. This is because for the case that $n_{i\cdot\text{MISID}} = n_\tau$, i.e., player is in the threshold independent set, the player can obtain higher utility by assigning more radios than $|C| \bmod K$, when players in larger independent set assign less radios than K.

4.2.2 Throughput and Optimality

Theorem 3 (Throughput Maximization and Pareto Optimality). *If the PMT protocol is used, all the NEs achieve the maximum system-wide throughput. Furthermore, all the NEs are Pareto optimal.*

Proof. We denote system-wide throughput as the sum of the throughput in each channel. $\sum_{c\in C} R_c = \sum_{c\in C} k_c r$, where k_c is the number of radios using channel c in the system. In the convergence state of PMT, $\sum_{c\in C} k_c$ cannot be increased by other ways of channel assignment, because 1) for players in independent sets smaller than or equal to the threshold independent set, it is impossible to put their spare radios on channels that are used by other players in larger independent sets without any interference, since otherwise it will contradict with the definition of MIS, 2) for players in independent sets larger than the threshold independent set, they do not have spare radios to increase throughput (see (5)). Hence, the system-wide throughput $\sum_{c\in C} R_c = \sum_{c\in C} k_c r$ is maximized.

The NEs that guarantee system-wide maximum throughput are also Pareto optimal. This can be proved by contradiction. Note that in any NE, the throughput of each player is proportional to its payoff. If it is not Pareto optimal, it implies that some players can increase their throughputs without decreasing any other's throughput. Consequently, the system-wide throughput can be better off, which contradicts the throughput maximization. □

Our PMT guarantees that all NEs are Pareto optimal, which means that the outcomes of the noncooperative channel assignment achieve social optimality.

When using different MIS partition approximation algorithms, it does not affect the incentive compatibility of our PMT protocol. From the proof of Theorem 2, no matter how MIS are partitioned, as long as there is a $n_{i\cdot\text{MISID}}$ for each player i, our carefully designed payment formula will make sure that PMT protocol is incentive compatible, i.e., the system will always converge to a desirable NE. Different MIS partition approximation algorithms do have effects on achieving system throughput maximization in the system. We would like to note that it is NP-hard to solve the throughput maximization problem for multiple collision domains in general, and different MIS approximation algorithms may well lead to different throughputs in the system. Our PMT algorithm theoretically guarantees that for each system, as long as the MIS partition result is correct, the PMT protocol will produce the maximum system-wide throughput.

4.2.3 Fairness Issue

In the PMT protocol, in order to achieve maximum system throughput, the individual throughput of the players in the independent sets smaller than the threshold set is sacrificed. In particular, those players are not assigning any of their radios to any channel. This causes a fairness issue for the system. Here, we first theoretically analyze the upper bound of the ratio of such silent players and then we discuss possible solutions for this fairness issue.

Theorem 4. *In PMT protocol, the upper bound for the ratio of silent players is* $1 - \frac{\lceil\frac{|C|}{K}\rceil}{n}$, *where* n *is the number of MIS in the system after the MIS partition.*

Proof. Since the threshold independent set ranks $\lceil\frac{|C|}{K}\rceil$ in terms of set size, the ratio of players in the threshold set and larger independent set is greater than $\frac{\lceil\frac{|C|}{K}\rceil}{n}$. Hence, the ratio of players in the independent set (i.e., silent players) is smaller than the threshold set is smaller than $1 - \frac{\lceil\frac{|C|}{K}\rceil}{n}$. □

We observe that the silent players are in smaller MIS. It is because compared with other players, they will interfere with more players if using the channels. So in order to achieve high system throughput, these players need to turn off their radios and let more others use the channel resources. However, long term starvation should be avoided in the wireless networks. To solve this issue, one possible solution is to periodically re-compute the independent set ID for each player to allow the silent node changing its independent set to a larger one, increasing the probability to have more channel access.

4.3 Implementation Issues

Our PMT protocol works in wireless systems that have a protocol or mechanism that enables the wireless devices to use multiple channels to communicate at the same time. For example, [28] is one of such mutiradio protocols for IEEE 802.11 wireless networks. Our protocol let the nodes coordinate to achieve a channel allocation of their radios. To perform the PMT protocol, the system administrator sends a message to each node i whose radio needs to be reconfigured, which contains the n_τ and $n_{i_{\mathrm{MISID}}}$. After receiving the acknowledgment from each node, the system administrator sends a synchronization message, and it invokes the PMT protocol described in Section 4.1.

The computational overhead of our channel assignment is mainly from two parts, i.e., computing the MIS and the time required for system convergence. For the first part of overhead, as the system grows larger, further performance optimization is needed (e.g., by utilizing more efficient heuristic algorithm to compute MIS and by using smaller amount of time in each round of nodes coordination). In Section 5.4, we will investigate the system convergence time in greater details.

4.4 Advanced Model and Analysis

With the considerations of more complicated conditions, noncooperative channel assignment problem in multiradio multichannel wireless networks can be modeled in more advanced game model. For example, in each round of channel assignment, each node can observe the action of his neighbors (by sensing the interference), and accordingly change his own action in the next round of channel assignment, to avoid interference. This sequential nature can be modeled by a sequential game (or dynamic game) in the extensive form. In this model, our payment scheme needs to be extended for each possible action in the sequential game, so that a subgame NE can be achieved. Here, a subgame NE is a solution concept in dynamic games, which guarantees a NE for every subgame of the original dynamic game.

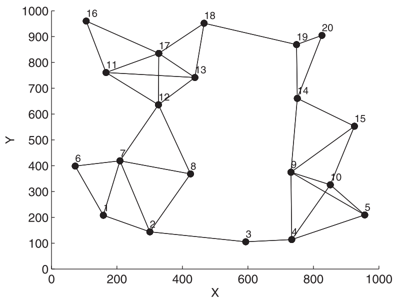

Fig. 4. The flow contention graph of the network.

5 Evaluations

In this section, we carry out a number of experiments in GloMoSim [16] to verify the effectiveness of PMT. In the implementation, we use the approximation algorithm in [29] to compute the MIS before the game starts.

We first generate a network of 20 pairs of nodes with a random topology in a $1{,}000 \times 1{,}000\ \mathrm{m}^2$ region. In each pair, the two nodes are 20 meters away from each other. The flow contention graph is shown in Fig. 4. There is a bidirectional single-hop flow between the two nodes in each pair at a constant bit rate, and we vary the traffic demand rate in the experiments.

We test our protocols with two sets of parameters. In one set, we let the number of channels $|C| = 12$, channel capacity $R = 54$ Mbps, and the number of radios $K = 4$. In the other set, we let $|C| = 3$, $R = 11$ Mbps, and $K = 2$. Each result is obtained by averaging over 500 runs. We set $r = 2$, $\beta = 0.25$, and $\epsilon = 0.1$.

In Section 5.1 we evaluate the payment for each node in the system in two different settings. In Section 5.2, we evaluate the effectiveness of PMT in achieving maximum system throughput. In particular, we measure the system throughput in the stable states when PMT is used, and compare the results with the situation when no incentive-compatible protocol is used. In Section 5.3, we study the fairness property in the system when running PMT. Furthermore, in Section 5.4 we investigate the system convergence process, and find that the protocol can make the system converge to the stable state fairly quickly. We evaluate the system efficiency for PMT in Section 5.5. The experiments in the above sections are performed using the system topology shown in Fig. 4, in Section 5.6, we randomly place the 20 nodes in the $1{,}000 \times 1{,}000\ \mathrm{m}^2$ region, and analyze the average results of throughput and fairness for different system topologies.

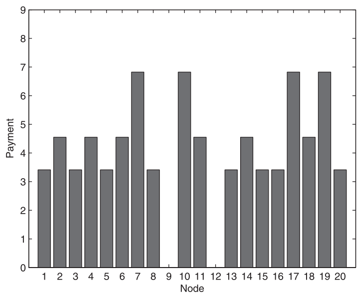

Fig. 5. Payment of each node in the system. ($|C| = 12$, $K = 4$, $R = 54$ Mbps.)

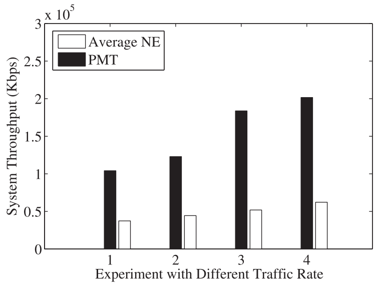

Fig. 7. Aggregate system throughput. ($|C| = 3$, $K = 2$, $R = 11$ Mbps. The traffic demand rates of experiments 1, 2, 3, and 4 are 8,10,16, and 20 Mbps, respectively.)

5.1 Payment

In this section, we closely observe the payment that each node makes in our PMT protocol, in two different settings. After running the MIS partition algorithm, we find that when $|C| = 12$ and $K = 4$, the size of threshold independent set $n_\tau = 4$; when $|C| = 3$ and $K = 2$, we have $n_\tau = 6$.

Fig. 5 plots the payment of each node in the system, when $|C| = 12$ and $K = 4$. We notice that node 9 and node 12 are making 0 payments. It is because they are not using any channels and correspondingly, they do not need pay anything. Similar results for the setting that $|C| = 12$ and $K = 4$ are shown in Fig. 6. We observe that when the system setting changes, the threshold independent set and the number of radios that each node uses may correspondingly change. Consequently, the payment of the same node is different for different system settings.

5.2 Evaluation of Throughput

We measure the system throughputs of PMT, as well as the average system throughput of random NEs (which will be reached when the system has no incentive-compatible protocol). Our objective is to design a channel assignment protocol such that the channel assignment, which leads to maximal throughput also meets the interest of each player. In this paper, as for the degree of incentive compatibility, we use the notion NE, and thus by incentive-compatible protocol, we mean that the protocol by which the maximum throughput channel assignment is the NE strategy for each player. Therefore, from Theorem 2, we know that PMT is an incentive-compatible protocol. We note that the NE convergence algorithm in [8] do not use any incentive-compatible scheme to influence the NEs that the system will converge to, and, hence, by using the algorithm in [8], the system can converge to any random NE. Thus, the comparison results shown in Sections 5.2 and 5.3 are actually the comparison between PMT and [8]. The results of the system throughput are illustrated in Figs. 7 and 8.

In Fig. 7, we show the system throughputs of four different experiments with different traffic demand rates, when we set $|C| = 3$, $K = 2$, and $R = 11$ Mbps. As we can see, for each experiment, the system-wide throughput achieved by PMT is much higher than the average of random NEs with no incentive-compatible protocols. It implies that, compared with the systems without incentive-compatible channel assignment protocols, PMT greatly improves the system-wide throughput.

Similar conclusions can be drawn from Fig. 8, when we set $|C| = 12$, $K = 4$, and $R = 54$ Mbps.

5.3 Fairness

Now, we examine the fairness property of our PMT protocol. As in the design of PMT, our objective is to maximize the throughput. Then, it is important to make sure that the throughput maximization does not sacrifice too much fairness in the system. To this end, we measure the fairness in terms of individual throughput. We utilize the Jain's fairness index [33] as a quantitative metric. Fairness index is a real number, ranging from 0.05 (worst) to 1 (best) for the system of 20 players. We measure the fairness indices of the system's stable states achieved by the PMT, and also the average fairness indices of random NEs, which are reached when there is no incentive-compatible channel assignment protocol. According to the definition, the fairness index of the system cannot be guaranteed to be 1 unless the unless all the nodes in the system obtain equal throughput. When the fairness index equals to 1, the system will obtain perfect fairness, which is a strong requirement and, thus, is difficult to achieve. We repeat the experiments

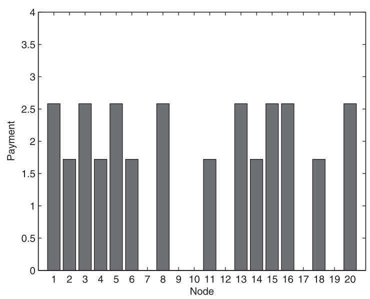

Fig. 6. Payment of each node in the system. ($|C| = 3$, $K = 2$, $R = 54$ Mbps.)

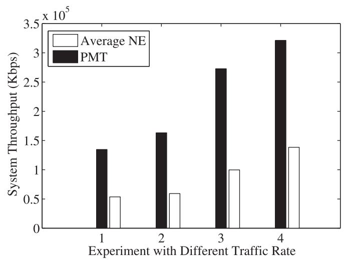

Fig. 8. Aggregate system throughput. ($|C| = 12$, $K = 4$, $R = 54$ Mbps. The traffic demand rates of experiments 1, 2, 3, and 4 are 8,10,16, and 20 Mbps, respectively.)

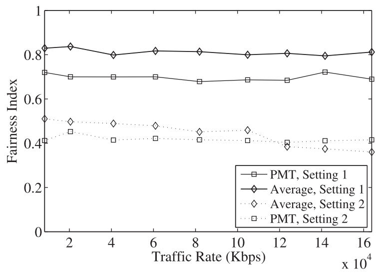

Fig. 9. Fairness index of PMT.

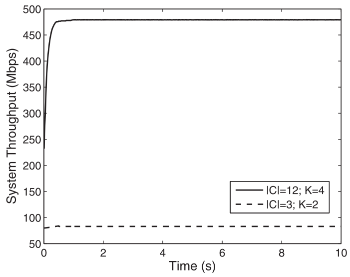

Fig. 10. Convergence of system throughput of PMT.

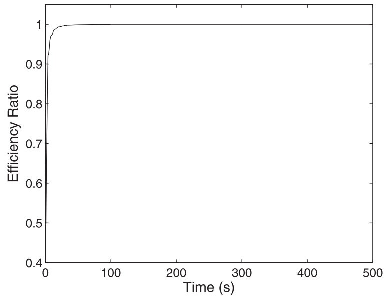

Fig. 11. Convergence of system efficiency ratio of PMT.

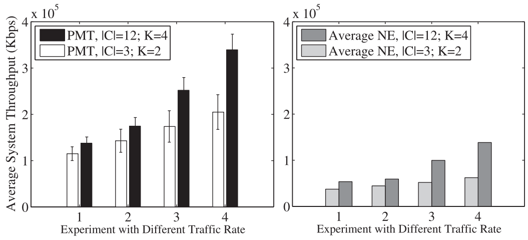

Fig. 12. Average system throughput of PMT in 10 randomly generated network topologies.

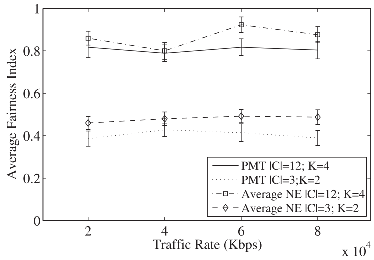

Fig. 13. Average fairness index of PMT in 10 randomly generated network topologies.

with different traffic rates and in two different settings (Setting 1: $|C| = 12K = 4R = 54$ Mbps; Setting 2: $|C| = 3K = 2R = 11$ Mbps). The results are shown in Fig. 9. In the figure, we can see that When $|C| = 12$ and $K = 4$, the average fairness index of random NEs is better than that achieved by the PMT. But there is little difference between the fairness indices achieved by the PMT and the average fairness indices of random NEs.

We noticed that results in [8, Fig. 9] show very high efficiency for the algorithm in [8]. This is not contradictory to the results shown in Fig. 9 in this paper. As defined in [8, Definition 6 on page 7], the *efficiency* measured in [8] is the proportion between the worst case and the NE channel allocation. When the efficiency is 1 in [8], it means that the channel allocation is the same as the NE case. NE cases in [8] balance the channel allocation very well, but cannot always guarantee the fairness index to be 1, unless all the nodes in the system obtain equal throughput.

In the experiments, we found that in setting of $|C| = 12$ and $K = 4$, there are two silent nodes in the system and when the channel resource becomes more limited, i.e., $|C| = 3$ and $K = 2$, there are more silent nodes in the network, six in total.

5.4 System Convergence

The results stated above are on the system performance in the stable states. In this section, our goal is to examine the process the system converges to the stable states. We take records of the system-wide throughput for PMT when the systems are evolving, and show them in Fig. 10. The traffic demand rate is set to 80 Mbps in this experiment. We can see that PMT converges in 0.5 seconds. Therefore, PMT can successfully make the system converge to the stable states and the convergence is very fast.

5.5 Economic Efficiency

In this section, we study the economic efficiency of the system using our PMT protocol. We use an *efficiency ratio* to characterize the efficiency of the system. In particular, the efficiency ratio is defined as the ratio between the sum of payoffs of all players in the Pareto-optimal solution and the sum of payoffs by the current strategy profile. We present an example run of PMT protocol for 10 s in Fig. 11 in the setting of $|C| = 12$, $K = 4$. We can observe that PMT protocol quickly converges to a NE. When the system is stable, the efficiency ratio stays at the value of 1. Hence, we conclude that our PMT protocol makes the system converge to a state with high efficiency.

5.6 Results over Different System Topologies

In this section, we change the system topology and evaluate the system throughput and fairness in different network topologies when PMT is used. In particular, we generate 10 network topologies of 20 nodes. In each topology, we randomly place the 20 nodes in the $1{,}000 \times 1{,}000$ m^2 region and make sure that the maximum degree of nodes is no more than 6. We measure the system throughput and fairness for each network topology and show the average results and standard deviation in Figs. 12 and 13, respectively. Figs. 12 and 13 demonstrate that our PMT protocol works well for different network topologies and the performance is stable. Similar conclusions can be obtained when comparing our PMT protocol with average NEs, as discussed in Sections 5.2 and 5.3.

TABLE 2
Comparisons between PMT Protocol and Existing Protocols

	Advantage of PMT	Evidence
[37]	Multiple Radios Multiple Collision domain Maximized throughput	Section II.A Section II.B Theorem 3 in Section IV.B
[8]	Multiple Collision domain Maximized throughput	Section II.B Theorem 3 in Section IV.B Evaluations in Section V.B,V.F
[9], [11]	Multiple Collision domain Maximized throughput	Section II.B Theorem 3 in Section IV.B
[12]	Maximized throughput	Theorem 3 in Section IV.B

6 Related Work

In WMNs, channel assignment problem has been considered for multiradio devices [5], [6], [7], [28], [34], [35], [36]. It is important because simultaneously transmitting packets with multiple radios on orthogonal channels can significantly increase the system capacity. Both centralized algorithms [6] and distributed protocols [7] have been developed for multiradio channel assignment for WMNs. Alicherry et al. [5] jointly consider channel assignment and routing to optimize the system throughput.

All these channel assignment protocols above are under the same assumption that devices in the network are cooperative in that they never deviate from the protocol. As devices can be selfish when accessing the channels, recently researchers begin to study the noncooperative channel assignment problem [37], [38], [39], especially for cognitive radio networks. For example, Nie and Comaniciu [37] propose a dynamic spectrum allocation scheme based on a potential game model and introduce some learning algorithms for different payoff functions. Thomas et al. [38] also utilize a potential game model to study how to minimize transmission power while maintaining connectivity by channel assignment. The major difference between these works and ours is that they assume that the selfish player has only a single radio, while we study the noncooperative, multiradio channel assignment problem in which selfish devices have multiple radios to manipulate. Since the players and their objectives (i.e., payoff functions) in the game are significantly different, their solutions cannot be applied in the more general case that we are focusing on in this paper.

For wireless networks in which devices have multiple radios, Felegyhazi et al. [8] are the first to introduce the strategic game model to study the noncooperative channel assignment problem. They study the NEs in this game and find out that despite of noncooperative behavior of the players, the NEs result in load balancing. The differences between our work and [8] are in three aspects: First, in [8], only the scenario that the all transmitting nodes are in a single collision clique is considered, while in our papers, we consider the noncooperative channel assignment in more general and complicated cases of the system topology, i.e., it contains multiple collision domains. The elegant results of load balancing by Felegyhazi et al. for single collision domains is based on the fact that each pair of nodes in the system will interfere each other and, thus, cannot be applied to multiple collision domains. Second, in our work, our goal is to design incentive-compatible channel assignment protocols which can achieve maximum system throughput and Pareto optimality. In [8], the maximum system throughput is not guaranteed by their noncooperative channel assignment algorithms. Third, in [8], central and distributed algorithms are designed for system convergence to NE, but no mechanism design approach is used to influence the convergence. In this paper, we use payment-based approach to make sure that the NE that the system will converge to is a desirable one with maximum system throughput and Pareto optimality. In a later work, Wu et al. [9], [10] propose a stronger solution for this game, which is strictly dominant and extend the model such that players can have different number of radios. Gao and Wang [11] go one step further to consider the noncooperative channel allocation for multihop wireless networks. However, all the three works (including [11]) assume that all the nodes in the network are within a single collision domain. In this paper, we remove this assumption and study the noncooperative, multiradio channel assignment problem in multiple collision domains, with a focus on system throughput.

Recently, Vallam et al. [12] also studied the problem of noncooperative channel assignment in multichannel multiradio networks with multiple collision domains. They have obtained nice and solid results. In particular, a new fairness measure in multiple collision domains is proposed and fair equilibrium conditions are derived. Based on the conditions, three nice channel assignment algorithms are also proposed. In fact, they are the *first* to study the problem of noncooperative channel assignment in multichannel multiradio networks with multiple collision domains. The differences between [12] and our work are twofold. First, for system performance, our main focus is on throughput, while [12] has more significant contributions in system fairness. Second, in our paper, we use a payment-based approach to achieve players' incentive compatibility with the objective of maximum system throughput, while [12] leverages advanced learning algorithm in the system convergence.

We compare our PMT protocol with selected existing protocols and summarize the results in Table 2.

7 Conclusion

In this paper, we have systematically studied the problem of noncooperative, multiradio channel assignment in multiple collision domains, and obtained quantified results on economic efficiency, and throughput. Our results show that, without an incentive-compatible channel assignment protocol, the system is likely to converge to NEs with undesired properties like low throughput and Pareto suboptimality. To avoid this, we propose an incentive-compatible protocol for multiradio channel assignment in multiple collision domains. This protocol guarantees that the system converges to NEs that have the maximum throughput and Pareto optimality.

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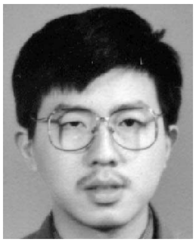

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