Multicast Scaling Law in Multichannel Multiradio Wireless Networks

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Abstract—This paper addresses the issue of multicast scaling performance in multichannel multiradio (MC-MR) networks. Under the assumption of both limited bandwidth and node tunability, a total fixed bandwidth W is equally split into c channels with $0 < m \leq c$ interfaces equipped on each node for channel switching. The network contains totally n nodes, each serving as a source with k randomly and uniformly selected destinations. We try to give a comprehensive picture of multicast scalings by investigating both the static and mobile networks, with the metrics being capacity and delay. Previous literature [9] has indicated that unicast capacity is solely determined by the ratio of channels to interfaces c/m in MC-MR networks. However, in multicast our problem is made more complicated by the interplay among k, c/m and node mobility (if considered in mobile scenario). We characterize their impact on multicast scaling and obtain three remarkable findings from our results. First, we find capacity loss exists in static networks even if the ratio $c/m = O(\log n)$ (We use the following notation throughout our paper: $f(n) = O(g(n)) \Leftrightarrow \limsup_{n \to \infty} \frac{f(n)}{g(n)} < \infty$, $f(n) = \Omega(g(n)) \Leftrightarrow \liminf_{n \to \infty} \frac{f(n)}{g(n)} < \infty$, $f(n) = \Theta(g(n)) \Leftrightarrow f(n) = O(g(n))$ and $g(n) = O(f(n))$, $f(n) = \tilde{\Theta}(\cdot)$: The corresponding order $\Theta(\cdot)$ which contains a logarithmic order.) when k is close to $\Theta(n)$. This differs from unicast that is free of capacity loss as long as $c/m = O(\log n)$. Second, mobility is manifested to improve multicast capacity in MC-MR networks, where two major capacity bottlenecks, i.e., connectivity and interference constraints, in static networks can be effectively broken. Third, a largely reduced delay is possible by simply seeking for multichannel reuse in 2-hop algorithm without redundancy. This even outperforms the delay scaling in single channel framework [26], where a delay smaller than $\Theta(\sqrt{n \log k})$ is not achievable even with more than $\Theta(\sqrt{n \log k})$ relay nodes involved in 2-hop mode. As a high-level summary of our results, our work discloses analytically where the performance improvement and degradation exhibit in MC-MR networks, meanwhile unifying the previous bounds on unicast (setting $k = 1$) in [9].

Index Terms—Multicast, multichannel multiradio, scaling

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1 INTRODUCTION

THE availability of multiple unlicensed spectral bands has recently spawn intense interest in exploiting multiple channels in wireless networks [1], 2], [3], [4], [5], [6], [7]. The capacity of such networks has also been studied under various assumptions on availability/capability of radio interfaces. A corresponding landmark work dates back to year 2000, when Gupta and Kumar [8] indicated that for a single-channel single-interface scenario, in a randomly deployed network, per-flow capacity scales as $\Theta(W/\sqrt{n \log n})$ under protocol interference model. And the capacity scaling remains the same even when the total bandwidth W available is divided into c channels, provided that each node is also equipped with one dedicated interface per channel.

Although many existing standards, for example, IEEE 802.11a, 802.11b, and 802.15.4 allow for multiple channels, nodes are typically hardware constrained and have much fewer interfaces. This problem was addressed in [9], under the assumption of c available channels with bandwidth of W/c for each and $1 \leq m \leq c$ interfaces per node. Interest-

ingly, the finding implied that the corresponding capacity depends solely on the ratio c/m, as long as nodes are capable of switching their interface(s) to any channel. Following this, and motivated on the basis of future low-cost transceiver designs involving limited tunability, Bhandari and Vaidya [10], [11] attempted to quantify the impact of switching constraints on capacity performance. Particularly, their investigation was under random (c, f) assignment, where each node is preassigned a random subset of f channels out of c and may only switch on these. Another relevant body of work includes seeking for the way of capacity enhancement in multichannel multi-radio (MC-MR) scenario, either through using directional antenna [12] or power exploitation [13].

All previous works in MC-MR networks [9] [10], [11], [12], [13] merely characterize capacity scaling for unicast. However, multicast traffic is appearing to be more predominant in many applications such as Battlefield networks, disaster management scenarios and online video viewing, and so on. So far a plethora of literatures have investigated multicast capacity under a wide variety of network settings, such as static networks, mobile ad hoc networks, hybrid networks, hierarchically cooperative networks and clustered networks, and so on, but all confined to the single channel framework. It remains unclear how multicast performs if operated in multichannel networks. Also note that all the analysis of [9], [10], [11], [12], [13] is restricted to static networks whereas most realistic traces demonstrate that nodes (users) are usually moving around different areas rather than staying static. All these facts, on one hand, suggest it is essential to measure multicast performance in a more general multichannel framework

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from a more comprehensive perspective, not simply limited to capacity metric and the static scenario; on the other hand, they pose a notable challenge on multicast analysis, which is made more complicated by the interplay among multiple receivers requesting for the same information, channel assignment, and node mobility.

That motivates us to study mulitcast scaling law of MC-MR networks in this paper. We mainly focus on the bandwidth-limited case where a total fixed bandwidth is available for supporting small data rate equally distributed to multiple channels. And the results can be easily extended to the environment where new channels are created by utilizing additional frequency spectrum. We address the challenge by first focusing on static networks as the startpoint and then turning our attention to mobile case for further investigation. Precisely speaking, our major contributions are threefold. First, we characterize the capacity region in static network, with regard to the impact of both the number of destinations k per multicast session and the channel/interface ratio c/m. Second, we present a first look into the multicast scaling under mobile MC-MR networks, deriving the maximum capacity that can be achieved as well as the corresponding delay. Third, we establish efficient routing in both static and mobile scenarios to reach the capacity bounds obtained.

Specifically, some significant and interesting findings are also obtained from our results, which are briefly outlined as follows:

- In static multicast MC-MR network, there exists capacity loss even if the ratio $c/m = O(\log n)$, given that k is close to the total number of nodes n (except for a polylogarithmic factor). This differs from the unicast traffic that does not suffer from capacity loss as long as $c/m = O(\log n)$. Additionally, extra capacity loss will be incurred by k in the range $c/m = \Omega(\log n)$ and $c/m = O(n)$, when k is larger than $\Theta(\frac{n}{cm})$ (except for a polylogarithmic factor).
- We find capacity improvement for multicast when mobility is introduced into MC-MR networks. In particular, mobility improves the capacity regions exhibited under $c/m = O(n)$ to $\Theta(\frac{nm}{ck})$ (except for a polylogarithmic factor). The benefit stems from the fact that both the connectivity and interference bottlenecks can be broken through node mobility whereas they construct the main obstacles for capacity upper bound in static networks.
- In mobile MC-MR networks, delay can be largely reduced through frequency reuse without seeking for redundancies. This renders us a new perspective of improving delay other than solely relying on relay nodes, as is commonly adopted in conventional single channel framework. Surprisingly, the result even fills in the blank in single channel MANETs, where a delay smaller than $\Theta(\sqrt{n \log k})$ in two-hop scheme is not achievable even with more than $\Theta(\sqrt{n \log k})$ relays introduced.

We note this paper is not merely a generalization and an extension of results from previous works. Our work shows the fact that MC-MR is a handicap to multicast performance scaling in static networks, but brings about potential benefits in mobile cases. Such insight is fundamental and

delivers useful twofold directions to us: we should address major concern with the obstacles that will be encountered in real network design and meanwhile take advantage of the network architecture for performance enhancement.

The roadmap of this paper is as follows: Section 2 lists literature review of some existing scaling law analysis in single-channel single-radio (SC-SR) framework. We introduce the network model and list the definitions in Section 3. The main results of this paper are briefly introduced in Section 4. We give detailed analysis of multicast scaling in Sections 5 and 6. Section 7 is contributed to some discussion on our results and their implications. We conclude the paper in Section 8.

2 RELATED WORKS

Scaling law analysis has for long been under intensive study within the networking research community. A flurry of theoretical studies target for large-scale ad hoc networks where the number of nodes can go to infinity. The seminal work of asymptotic capacity study is initiated by Gupta and Kumar [8], who show that the maximal per-node unicast throughput achievable in wireless networks is $\Theta(1/\sqrt{n \log n})$ for a uniformly distributed destination.

Grossglauser and Tse [14] later introduce mobility to the nodes and show that by employing a store-carry-forward paradigm, capacity can be improved to $\Theta(1)$, at the expense of increased delay. Neely and Modiano [15] further demonstrate that there exists a tradeoff between capacity and delay. A series of works [16], [17], [18], [19], [20], [21], [22], [23] have then followed, focusing on the analysis of throughput-delay tradeoffs under different mobility scenarios, through either the carry-and-forward mode [16], [17], [18], coding techniques [19], [20], [21], or infrastructure support [22], [23].

Multicast capacity is also under extensive study in the literature recently. Li [24] studies the multicast capacity in wireless networks of side length a, with n nodes randomly deployed in it. Their analytical results claim that the per-flow multicast capacity (of n multicast flows, each flow with k destinations) is $\Theta(1/\sqrt{nk \log n})$ when $k = O(n/\log n)$ and $\Theta(1/n)$ when $k = \Omega(n/\log n)$. Later on, Li et al. [25] extend multicast analysis to a more general interference model, say, Gaussian channel model, and derive the capacity from the perspective of percolation theorem. Wang et al. [26] further consider multicast capacity in mobile networks, presenting capacity-delay tradeoffs under different routing schemes.

The numerous papers under Gupta and Kumar's framework are mostly based on SC-SR networks for scaling law investigation. With more unlicensed spectrum available, we anticipate to take a look into the scaling performance in multichannel situations. Based on the fundamental scaling results in MC-MR scenarios [9], [10], more interesting results are further disclosed from the work [2], [12], [13]. Specifically, Wan et al. [2] conduct comprehensive studies on maximum multiflow and maximum concurrent multiflow in multichannel multiradio multihop wireless networks under the 802.11 interference model. By equipping each node with directional antenna, Dai et al. [12] demonstrate that the capacity of MC-MR

networks can be improved by constant factors. The critical role of transmission power in wireless networks is emphasized by Shila et al. [13], who utilize it to improve capacity performance in MC-MR networks. Although providing useful insights, all these works focus on unicast, with capacity as the only performance metric. The emerging applications of multicast motivates us to take the initiative on its performance investigation in MC-MR networks, for both capacity and delay scalings.

3 SYSTEM MODEL

3.1 Network Topology

We consider a static dense network $\mathcal{O}$ as a unit square. The size normalization and wrap-around conditions are also introduced here, which are common technical assumptions adopted in previous works to avoid tedious technicalities. Note that these assumptions will not change the main results of this paper. There are totally c channels available in the network. n nodes with wireless communication capability spread in the network, each equipped with m interfaces. We assume that an interface is capable of transmitting or receiving data on any one channel at a given time. The locations of nodes are denoted by X_i ($i \in [1, n]$), which are a series of independent random variables uniformly distributed in $\mathcal{O}$.

3.2 Channel Model

We assume that the total data rate possible by using all channels is W. The total data rate is divided equally among the channels, and, therefore, the data rate supported by any one of the c channels is W/c. This was the channel model used by Gupta and Kumar [8], and we primarily use this model in our analysis. In this model, as the number of channels increases, each channel supports a smaller data rate. This model is applicable to the scenario where the total available bandwidth is fixed, and new channels are created by partitioning existing channels.

3.3 Communication Model

The well-known protocol model is introduced here to roughly represent the behavior of transmission constrained by interference. The model indicates under a fixed total bandwidth W, a generic node i is allowed to transmit to j if the positions of i and j, denoted by X_i and X_j, satisfy $\| X_i - X_j \| < r(n)$, where r is a common transmission range employed by all the nodes and for every other node k transmitting, $\| X_j - X_k \| > (1 + \Delta)r(n)$, being Δ a guard factor.

3.4 Traffic Pattern

We consider multicast traffic pattern in present work. Each of the n nodes in the network acts as a source, sending packets to k randomly and uniformly chosen destinations. Once the source-destinations pairs are selected, the relationship remains unchanged.

3.5 Capacity and Delay Definition

Definition 1 (Feasible Multicast Capacity). *Given n source nodes, a multicast rate of $g(n)$ bits/s is said to be feasible if there is a spatial and temporal scheme for scheduling transmissions, such that by operating the primary network in a multihop fashion and buffering at intermediate nodes*

when awaiting transmission opportunities, each source can transmit $g(n)$ bits/s on average to its k destinations. That is, there is a $T < \infty$ such that in every time interval $[(i - 1) \cdot T, i \cdot T]$, every source node can send $T \cdot g(n)$ bits of data from class k to each of its k destinations.

Definition 2 (Asymptotic Per-Node Multicast Capacity). λ *of the network is said to be of order $\Theta(g(n))$ if there exist two positive constants c_1 and c_2 such that:*

$$\begin{cases} \lim_{n \to \infty} \mathbb{P}r\{\lambda = c_1 g(n) \text{ is feasible}\} = 1, \\ \lim_{n \to \infty} \mathbb{P}r\{\lambda = c_2 g(n) \text{ is feasible}\} < 1. \end{cases}$$

Definition 3 (The Average Aggregate Multicast Capacity). Λ *can be obtained through taking the average on λ for all n sources, i.e., $\Lambda = n\lambda$.*

Definition 4 (Average Packet Delay). *The delay for a packet is defined as the time it takes the packet to reach all its k destinations after it leaves at the source. The average packet delay D of a network is obtained by averaging over all transmitted packets in the network. Besides, we also assume the packet size scales as the per-node capacity.*

4 MAIN RESULTS

The goal of this work is to characterize the joint impact of the ratio of channels to interfaces c/m, the number of destinations k per source as well as node mobility, on the scaling performance in random wireless networks. We divide the results based on static and mobile networks and present the respective findings.

4.1 Scaling Performance in Static MC-MR Networks

In static MC-MR networks, multicast capacity yields distinctive results based on different relationship between c/m and k.

1. The case of $c/m = O(\log n)$:

    a. when $k = \Omega(\frac{n}{\text{polylog}(n)})$, then the per-node multicast capacity λ is

    $$\Theta\left(\frac{m \log \log n}{ck \log n}\right).$$

    b. when $k = O(\frac{n}{\text{polylog}(n)})$, then the corresponding multicast capacity λ is

    $$\Theta\left(\frac{1}{\sqrt{nk \log n}}\right).$$

2. The case of $c/m = \Omega(\log n)$ and $c/m = O(n \frac{\log \log n}{\log n})$:

    a. when $k = \Omega(\frac{nm}{c\text{polylog}(n)})$, then the multicast capacity λ is $\Theta(\frac{m \log \log n}{ck \log n})$.

    b. when

    $$k = O\left(\frac{nm}{c\text{polylog}(n)}\right),$$

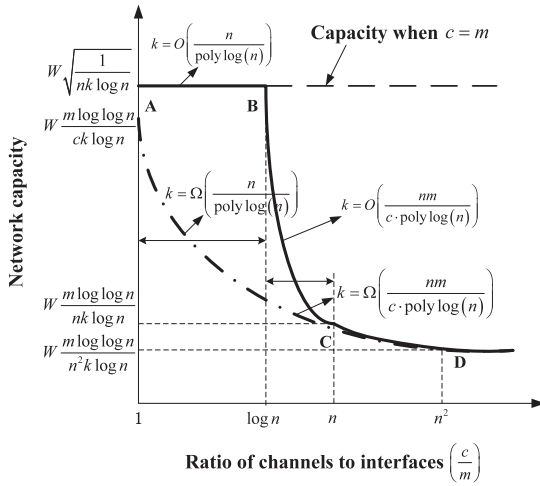   then λ is $\Theta(\sqrt{\frac{m}{nck}})$.

Fig. 1. Impact of the ratio of channels to interfaces c/m and the number of destinations k, on capacity scaling.

3.　The case of $c/m = \Omega(n\frac{\log\log n}{\log n})$: in this case, the per-node multicast capacity $\lambda = \Theta(\frac{m\log\log n}{ck\log n})$. In this case, there is a larger capacity degradation than that in case 2.

A graphical representation of our results is reported in Fig. 1. We adopt the notation $\text{poly}\log(n)$ to hide the detailed poly logarithmic factors for better readability. Refined results are available in Section 5. A notable observation from the figure is that multicast incurs more capacity loss than unicast.

4.2　Scaling Performance in Mobile MC-MR Networks

In mobile MC-MR networks where nodes move according to i.i.d. mobility, multicast capacity can be maximized to $\widetilde{\Theta}(nm/ck)$, where the corresponding delay D satisfying

- $D = \Theta(\frac{nm\log k}{c})$, when $k = O(nm\log\log n/c\log n)$.
- $D = \widetilde{\Theta}(k)$, when $k = \Omega(nm\log\log n/c\log n)$.

We find the capacity improvement compared to the static networks, since both the connectivity and interference constraints can be effectively eliminated through node mobility. Moreover, delay is also largely reduced compared to single channel scenario, providing us a new perspective of improving delay. Detailed results can be found in subsequent sections.

5　MULTICAST CAPACITY ANALYSIS IN STATIC MC-MR NETWORKS

In this section, we will give multicast capacity analysis in static MC-MR network. We first derive the upper bound of multicast capacity and then propose a routing-scheduling scheme to achieve this bound.

5.1　Upper Bound of Multicast Capacity

Recall that each node picks a destination node randomly, and so a node may be the destination of multiple flows. Let $F(n)$ be the maximum number of flows for which a node in the network is a destination. We use the following result to bound $F(n)$.

Lemma 1. *In multicast traffic pattern, let $F(n)$ denote the maximum number of flows for which a node in the network is a destination, then we have*

$$F(n) = \begin{cases} \Theta\left(k\frac{\log n}{\log\log n}\right), & k = O(n), \\ \Theta(n), & k = \Theta(n), \end{cases} \tag{1}$$

with high probability.

Proof. Provided in Section 1.1 of the supplementary file, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TPDS.2012.334.　□

The capacity of multichannel random networks is limited by three constraints, and each of them is used to obtain a bound on the network capacity. The minimum of the three bounds (the bounds depend on ratio between the number of channels c and the number of interfaces m) is an upper bound on the network capacity.

Constraint 1. (Connectivity Constraint): The capacity of random networks is constrained by the need to ensure the network is connected, so that every source-destination pair can successfully communicate. Li [24] has presented a bound on the network capacity of $O(\frac{1}{\sqrt{n\log n}})$ bits/s based on this requirement. This bound is applicable to multi-channel networks as well.

Constraint 2. (Interference Constraint): The capacity of multichannel random networks is also constrained by interference. The bound can be obtained by modifying the techniques presented in [8] to account for multiple channels, interfaces as well as multiple destinations. We present the upper bound in the following lemma.

Lemma 2. *In multicast, the capacity with interference constraint is upper bounded by $O(\sqrt{\frac{m}{cnk}})$ in the network with c channels and m interfaces per node.*

Proof. Provided in Section 1.2 of the online supplementary file.　□

Constraint 3 (Destination Bottleneck Constraint): The capacity of a multichannel network is also constrained by the packets that can be received by a destination. Adopting the techniques in [9], we can derive the network capacity is at most $O(\frac{Wm}{cF(n)})$ bits/s. Since we have shown in Lemma 1 that $F(n) = \Theta(k\frac{\log n}{\log\log n})$ for multicast traffic, we obtain the capacity not more than $O(\frac{Wm\log\log n}{ck\log n})$ bits/s.

Now, we can obtain the capacity upper bound by combining the three bounds above together, i.e.,

$$\lambda = O\left(\min\left\{\frac{1}{\sqrt{nk\log n}}, \sqrt{\frac{m}{cnk}}, \frac{m\log\log n}{ck\log n}\right\}\right).$$

The upper bound can be further rewritten according to the following theorem.

Theorem 1. *The upper bound of multicast capacity in (m,c) network can be presented as follows:*

1.　*When $c/m = O(\log n)$:*

　　a.　*when $k = \Omega(n \cdot (\frac{m}{c})^2 \cdot \frac{(\log\log n)^2}{\log n})$, then the per-node multicast capacity is*

$$O\left(\frac{m\log\log n}{ck\log n}\right).$$

b. *when $k = O(n \cdot (\frac{m}{c})^2 \cdot \frac{(\log\log n)^2}{\log n})$, then the corresponding multicast capacity is $O(\frac{1}{\sqrt{nk\log n}})$.*

2. *When $c/m = \Omega(\log n)$ and*

$$c/m = O\left(n\frac{\log\log n}{\log n}\right).$$

a. *when $k = \Omega(n \cdot (\frac{m}{c}) \cdot (\frac{\log\log n}{\log n})^2)$, then the multicast capacity is*

$$O\left(\frac{m\log\log n}{ck\log n}\right).$$

b. *when $k = O(n \cdot (\frac{m}{c}) \cdot (\frac{\log\log n}{\log n})^2)$, then the multicast capacity is*

$$O\left(\sqrt{\frac{m}{nck}}\right).$$

3. *When $c/m = \Omega(n\frac{\log\log n}{\log n})$, the per-node multicast capacity is at most $O(\frac{m\log\log n}{ck\log n})$.*

5.2 Capacity Achieving Scheme

In this section, we will propose a routing and scheduling scheme which suffices to achieve the capacity upper bound in Theorem 1.

5.2.1 Multicast Routing Scheme

In multicast, routing becomes a major issue. Unlike the case of unicast, where a network-partition approach dominates, an optimal multicast routing tree should also be established. A routing tree is said to be "optimal" in the sense that the capacity achieved under the scheduling scheme based on the tree can reach the upper bound. And we will demonstrate this in Section 5.2.2. Our main idea is to first divide the whole network using the cell-based approach in [9], and then construct a euclidean spanning tree using Prim's algorithm.

The cell-based approach [9] is to partition the network into cells with side-length $r(n)$ satisfying

$$\Theta\left(\min\left\{\max\left\{\sqrt{\frac{\log n}{n}}, \sqrt{\frac{c}{nm}}\right\}, \sqrt{\frac{\log\log n}{k\log n}}\right\}\right).$$

The three values that influence r are based on the three constraints described in the proof of upper bound. Based on such cell size, we proceed to propose an optimal multicast routing tree for each multicast session, shown as follows.

Algorithm 1: Optimal Multicast Routing Tree
STEP 1. Construct a spanning tree using Prim's algorithm:

1. Initially, nodes in each multicast session form k components.
2. The network is partitioned into $k - g$ squares with side length of each square being $1/\lceil\sqrt{k-g}\rceil$. ($g = 1, 2, \ldots, k-1$.)

3. Find a square that contains two nodes from two different connected components. Merge the two components by adding an edge between the two nodes.
4. For each $g \in [1, \ldots, k-1]$, repeat step (b) and (c) until $g = k-1$. Return the multicast routing tree, denoted by MRT(k) for each multicast session.

STEP 2. Consider the network divided into cells with side length r. For each edge uv in MRT(k), randomly select a point w that is in the same row as u and the same column as v. Then select a node in each of the cells which uw and wv are crossed by. Connect those users to form a path from u to v.

STEP 3. Combine the paths and remove cycles. Return the obtained multicast routing tree MRT(k) for each multicast session.

The length of MRT in the algorithm above is at most $c\sqrt{k}$ (c is a constant), which can be proved by using the techniques in [24]. Furthermore, we can use these cells (with area $r(n)^2$) as scheduling units and employ the TDMA scheduling scheme, and route the packets along tree MRT for each multicast session. To analyze capacity, it is important to study the "flow" of each cell under these schemes.

Lemma 3. *Given a cell c, the probability that the flow for a multicast session is routed through c is upper bounded by $\kappa\sqrt{k}r(n)$.*

Proof. Provided in Section 1.3 of the online supplementary file. □

Lemma 4. *Denote $N(c)$ as the number of multicast sessions that invoke c for routing, then uniformly over all cells, it follows:*

$$\mathbb{Pr}\{\forall \text{ cell } c, N(c) \leq \theta n\sqrt{k}r(n).\} \to 1. \quad (2)$$

Proof. For a specific squarelet c, we have $N(c) = \sum_{k=1}^{K} I_c$, where I_c represents the indicator function that squarelet c is invoked by transmission of data class k. According to Lemma 3, I_c is *i.i.d.* Bernoulli random variables with probability $p \leq \kappa\sqrt{k}r(n)$. By Chernoff bounds, we have

$$\mathbb{Pr}\left(N(c) > 2\mathbb{E}\left[\sum_{k=1}^{K} I_c\right]\right) < \mathbb{Pr}\left(\sum_{k=1}^{K} I_c > 2\mathbb{E}\left[\sum_{k=1}^{K} I_c\right]\right)$$
$$< \left(\frac{e}{4}\right)^{n\cdot\kappa\sqrt{k}r(n)} \quad (3)$$
$$< (e)^{-n\cdot\kappa\sqrt{k}r(n)/8}.$$

Since $r(n) > \Theta(\sqrt{\log n/n})$, we can further get

$$\mathbb{Pr}\left(\bigcap_{c=1}^{K}\{N(c) \leq \kappa'Kn\sqrt{k}r(n)\}\right)$$
$$\geq 1 - \sum_c \mathbb{Pr}\left(N(c) > 2\mathbb{E}\left[\sum_{k=1}^{K} I_c\right]\right) \quad (4)$$
$$\geq 1 - ne^{-\sqrt{n\log n}/8} \to 1.$$

Note that the last row of (4) holds as long as n goes to infinity. This completes our proof. □

5.2.2 Scheduling Issue

For each node on MRT, we show that every node can be scheduled to transmit once every Δ time slots, where constant Δ depending only on R and $r(n)$. For each node v, consider a node u whose transmission will interfere with the transmission of node v. Clearly node u will be completely inside the disk centered at v with radius $R + r$. Thus, the squarelet containing u must be inside the disk centered at v with radius $R + r(n) + \sqrt{2}r(n) < R + 3r(n)$. Let Δ be the maximum number of nodes in MRT whose transmission will interfere with the transmission of a node v. Using the area argument, we have

$$\Delta \leq \frac{\pi \cdot (R + 3r(n))^2}{r^2(n)} = \pi \cdot \left(3 + \frac{R}{r(n)}\right)^2. \qquad (5)$$

This property ensures that we can schedule the transmissions of all nodes in MRT by a TDMA manner such that all nodes will be able to transmit at least once in every Δ time slots. Notice that here Δ is a constant.

In previously proposed constructions for proving lower bound on capacity [8], it was immaterial which node in a chosen cell forwarded packets for some flow. However, such an approach may "overload" certain nodes and, therefore, causes capacity degradation, when the number of interfaces per node is smaller than the number of channels. Consequently, it is important to ensure that the routing load is distributed among the nodes in a cell. Through load balancing [9], each flow is assigned to a node within a cell that has been assigned the least number of flows. Thus, each node will have nearly the same number of flows. Since each cell has $\Theta(n \cdot r^2(n))$ nodes with high probability ([9, Lemma 4]), and each cell has at most $O(n\sqrt{k}r(n))$ flows based on Lemma 4, each node is, therefore, assigned at most $O(\frac{\sqrt{k}}{r(n)})$ flows due to load balancing. Also noting that each node in the cell is simultaneously a source, a potential destination as well as a relay for other source-destination pairs, the total flows assigned to every node can be bounded as

$$O\left(1 + F(n) + \frac{\sqrt{k}}{r(n)}\right) = O\left(1 + k\frac{\log\log n}{\log n} + \frac{\sqrt{k}}{r(n)}\right).$$

Recall the choice of cell size $r(n)$, we know it is at most $\sqrt{\frac{\log\log n}{k\log n}}$, which means $\frac{\sqrt{k}}{r(n)}$ is at least $k\sqrt{\frac{\log n}{\log\log n}}$. Hence, the total flows assigned to any node are always asymptotically dominated by the term $\frac{\sqrt{k}}{r(n)}$.

The transmission scheduling scheme is responsible for generating a transmission schedule for each node in MC-MR networks that satisfy two constraints: 1) each interface only allows one transmission/reception at the same time; and 2) any two transmissions on any channel should not interfere with each other. We meet the two constraints by proposing a TDMA scheme to schedule transmissions, which is shown as follows.

Algorithm 2: TDMA Scheduling Scheme:

STEP 1. One second is divided into multiple slots and at most one transmission/reception is scheduled at each node during slot satisfying constraint 1. Since the total flows assigned to any nodes is $O(\frac{\sqrt{k}}{r(n)})$ and each interface allows only one transmission/reception at the same time, we divide every second into $O(\frac{\sqrt{k}}{r(n)})$, with each having a length of $\Omega(\frac{r(n)}{\sqrt{k}})$ seconds.

STEP 2. Each time slot is further divided into mini-slots to satisfy constraint 2. Since we have already shown that there is a constant number of interfering cells and each cell has $\Theta(nr^2(n))$ nodes, the total number of mini-slots on all the c/m channels is, therefore, $O(\Delta nr^2(n))$. Two nodes will not interfere with each other if they are scheduled to transmit either on the same channel at the same time, or on different channels, or at different time slots on the same channel. This can be guaranteed by dividing each slot into $\lceil \frac{\Delta nr^2(n)}{c/m} \rceil$ mini-slots, each with a length of

$$\Omega\left(\frac{r(n)/\sqrt{k}}{\lceil \frac{\Delta nr^2(n)}{c/m} \rceil}\right)$$

seconds.

Considering that each channel can transmit at the rate of Wm/c bits/s, our multicast routing and TDMA scheduling schemes ensure that

$$\lambda = \Omega\left(\frac{Wmr(n)/\sqrt{k}}{c\lceil \frac{\Delta nr^2(n)}{c/m} \rceil}\right)$$

bits can be transported in each mini-slot. Moreover, since $\lceil \frac{\Delta nr^2(n)}{c/m} \rceil \leq \frac{\Delta nr^2(n)}{c/m} + 1$, we can get

$$\lambda = \Omega\left(\frac{Wr(n)}{\Delta n\sqrt{k}r^2(n) + c\sqrt{k}/m}\right)$$

bits/s. With further rewriting, we can represent λ as $\Omega(\min\{\frac{W}{n\sqrt{k}r(n)}, \frac{Wr(n)}{c\sqrt{k}/m}\})$ bits/s, when the denominator is either dominated by $n\sqrt{k}r^2(n)$ or $c\sqrt{k}/m$.

Substituting for the three values of $r(n)$, we have:

Theorem 2. *The multicast capacity of multichannel multiradio network yields distinctive results based on different relationship between c, m, and k.*

    1.   *The case of $c/m = O(\log n)$:*

        a.   *when $k = \Omega(\frac{n\log\log n}{\log^2 n})$, then $r(n) = \sqrt{\frac{\log\log n}{k\log n}}$ and the per-node multicast capacity λ is*

$$\Omega\left(\frac{m\log\log n}{ck\log n}\right).$$

        b.   *when $k = O(\frac{n\log\log n}{\log^2 n})$, then $r(n) = \sqrt{\log n/n}$ and the corresponding multicast capacity λ is*

$$\Omega\left(\frac{1}{\sqrt{nk\log n}}\right).$$

    2.   *The case of $c/m = \Omega(\log n)$ and*

$$c/m = O\left(n\frac{\log\log n}{\log n}\right):$$

        a.   *when $k = \Omega(\frac{nm\log\log n}{c\log n})$, then $r(n) = \sqrt{\frac{\log\log n}{k\log n}}$ and the multicast capacity λ is $\Omega(\frac{m\log\log n}{ck\log n})$.*
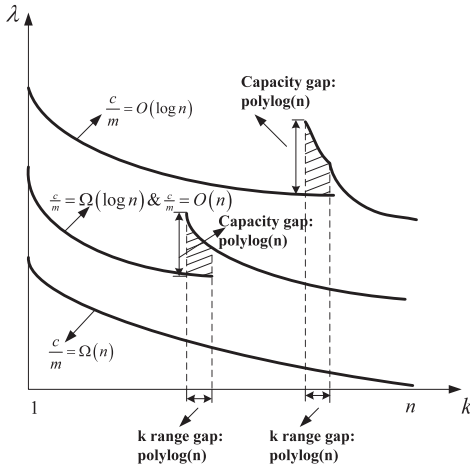
Fig. 2. Capacity gap and the corresponding k range gap between upper bounds and lower bounds.

    b.   *when $k = O(\frac{nm \log \log n}{c \log n})$, then $r(n) = \sqrt{c/nm}$ and λ is $\Omega(\sqrt{\frac{m}{nck}})$.*

  3. *The case of $c/m = \Omega(n \frac{\log \log n}{\log n})$: in this case,*

$$r(n) = \sqrt{\frac{\log \log n}{k \log n}}$$

and $\lambda = \Omega(\frac{m \log \log n}{ck \log n})$. In this case, there is a larger capacity degradation than that in case 2.

Corollary 1. *The lower bounds in Theorem 2 are tight up to a difference of logarithmic factor in the ranges of k.*

Remark 1. With comparison between the upper bounds and lower bounds, it can be seen that there is a difference of logarithmic factor in the range of k when $c/m = O(n)$. As can be seen from the shadow areas in Fig. 2, the turning points of capacity regions exhibit a slight difference when $c/m = O(\log n)$ and $[\Omega(\log n), O(n)]$, respectively. The difference is due to the simplicity of the cell tessellation scheme that employs an almost uniform transmission range. However, this slight performance drawback can be eliminated by adopting a more sophisticated tessellation scheme. Though it is not our main focus, we remark that it is not difficult to extend such schemes to our framework to make the ranges of k strictly meets those in the upper bounds.

6 MULTICAST CAPACITY IN MOBILE MC-MR NETWORKS

6.1 Mobility Model

Time is divided into slots of equal constant duration. We consider a 2D *i.i.d.* mobility model, according to which the positions of the nodes are totally reshuffled after each slot, independently from slot to slot and among the nodes. With the network divided into n nonoverlapping cells, at the beginning of each slot, a node jumps in zero time to a new cell, and remains in the new cell for the entire duration of a slot. Although the *i.i.d.* mobility model may appear to be unrealistic, it has been widely adopted in the literature because of its mathematical tractability. Note that the *i.i.d.*

model also characterizes the maximum degree of mobility. With the help of mobility, packets can be carried by the nodes until they reach the destinations.

6.2 Maximum Capacity

Theorem 3. *Under i.i.d. mobility model, the maximum multicast capacity in multichannel multiradio network is $O(\frac{m \log \log n}{ck \log n})$.*

Proof. As is already mentioned in static networks, multicast capacity is constrained by destination bottleneck, under which the per-node capacity is at most $O(\frac{m \log \log n}{ck \log n})$ in MC-MR networks. Unlike static networks, there is no connectivity constraint in mobile networks. Thus, other than destination bottleneck, another constraint that will impact capacity is the interference. And we will show later that the capacity derived under interference constraint will be least $O(\frac{m \log \log n}{ck \log n})$. Therefore, the maximum multicast capacity is upper bounded by $O(\frac{m \log \log n}{ck \log n})$ in multichannel multiradio mobile networks. □

6.3 Maximum Capacity Achieving Scheme and Corresponding Delay

The multichannel construction differs from the mechanisms used in single-channel in that the scheduling is on a per-node basis since flows are distributed among nodes, whereas in the past work it was sufficient to schedule on a per-cell basis. Moreover, to achieve the maximum capacity, it requires that at most one redundancy is used to relay the packets from source to destinations. Also note that there are k destinations, the source has to duplicate a packet at least k times to make the packet reach all k destinations. Recall that in multicast traffic with sources randomly selecting destinations, each node is the destinations of at most $F(n)$ sources. We will propose the following scheduling scheme which distributes channels among $F(n)$ flows for each node so that the maximum capacity can be achieved.

Algorithm 3: Multi-Channel Scheduling Scheme without Redundancy

    STEP 1. The network is divided into nonoverlapping cells, with each cell of the area $r^2(n)$.

    STEP 2. Each cell becomes active once in every $1 + c_3$ cell time slots.

    STEP 3. In an active cell, only one transmission is allowed between two nodes within the same cell in the same channel. And multiple transmissions can be conducted simultaneously if they are scheduled on different channels.

    STEP 4. An active packet time-slot is divided into two subslots A and B.

- In subslot A, *source-to-relay transmission:* If the sender has a new packet, one that has never been transmitted before, send the packet to the receiver and delete it from the buffer. Otherwise, stay idle. In subslot A, totally $\min\{nr^2(n), c/m\}$ source-relay pairs can transmit simultaneously in an active cell.
- In subslot B, *relay-to-destination transmission:* If the sender has packets received from other nodes that are destined for the receiver and have not been transmitted to the receiver yet, then choose the latest one, transmit. If all the destinations that want to get this packet have received it, it will be dropped from

the buffer in the sender. Otherwise, stay idle. Similarly, totally $\min\{nr^2(n), c/m\}$ relay-destination pairs can transmit simultaneously in an active cell.

We now prove that this scheme achieves the maximum capacity derived in Section 6.2.

Lemma 5. *Adopting multichannel scheduling schemes without redundancy, the multicast capacity λ is at least $\Omega(\min\{\frac{\log\log n}{nr^2(n)k\log n}, \frac{m\log\log n}{ck\log n}\})$.*

Proof. One second is divided into mini-slots, each with the length of $\frac{1}{F(n)\lceil\frac{\Delta nr^2(n)}{c/m}\rceil}$. Considering that each channel can transmit at the rate of Wm/c bits/s, our multicast routing and TDMA scheduling scheme ensures that

$$\lambda = \Omega\left(\frac{Wm}{cF(n)\lceil\frac{\Delta nr^2(n)}{c/m}\rceil}\right)$$

bits can be transported in each mini-slot. Moreover, since $\lceil\frac{\Delta nr^2(n)}{c/m}\rceil \leq \frac{\Delta nr^2(n)}{c/m} + 1$, we can get

$$\lambda = \Omega\left(\frac{W}{F(n)\Delta nr^2(n) + cF(n)/m}\right)$$

bits/s. With further rewriting, we can represent λ as

$$\Omega\left(\min\left\{\frac{W}{nr^2(n)F(n)}, \frac{W}{cF(n)/m}\right\}\right)\text{bits/s},$$

when the denominator is either dominated by $nr^2(n)F(n)$ or $cF(n)/m$. Substituting $F(n) = \Theta(\frac{k\log n}{\log\log n})$, we can derive the capacity lower bound shown in the lemma. □

Note that here $r(n)$ should be chosen as

$$\min\left\{\sqrt{\frac{c}{nm}}, \sqrt{\frac{\log\log n}{k\log n}}\right\}$$

to satisfy both the interference and destination bottleneck constraints. Substituting the two values of $r(n)$, we have the following theorem.

Theorem 4. *The multicast capacity of $\Omega(\frac{m\log\log n}{ck\log n})$ can be achieved under multichannel scheduling scheme without redundancy.*

Proof. Consider the choice of $r(n)$. When $k = O(nm\log\log n/c\log n)$, we have $r(n) = \sqrt{\frac{c}{nm}}$. Substituting it into the capacity lower bound derived in Lemma 5, we obtain $\lambda = \Omega(\frac{m\log\log n}{ck\log n})$. When $k = \Omega(nm\log\log n/c\log n)$, we have

$$r(n) = \sqrt{\frac{\log\log n}{k\log n}}.$$

The capacity in this case yields to be

$$\Omega\left(\min\left\{\frac{1}{n}, \frac{m\log\log n}{ck\log n}\right\}\right) = \Omega\left(\frac{m\log\log n}{ck\log n}\right).$$

Therefore, it can be seen that λ is always dominated by the term $\frac{m\log\log n}{ck\log n}$ and thus achieves a lower bound of $\Omega(\frac{m\log\log n}{ck\log n})$. □

Lemma 6. *Multichannel scheduling algorithms that do not use redundancy cannot achieve an average delay of $D = \Theta(\frac{\log k}{r^2(n)})$.*

Proof. It has been shown in [26] that using 2-hop relay algorithm without redundancy can achieve a delay of $\Theta(\frac{n\log k}{d})$ for multicast, where d represents the number of nodes per cell. Notice that we partition the network into cells with each having an area of $r^2(n)$. Thus, a delay of $\Theta(\frac{\log k}{r^2(n)})$ can be achieved in single channel scenario. In multichannel scenario, the bandwidth of each channel is reduced to W/c bits/s. Since we assume that the packet size is scaled with respect to the throughput obtained for each end-to-end flow, each packet arriving at a node in the cell departs within a constant time. Hence, there may be an increase in the end-to-end latency by a constant factor independent of n and c. We, therefore, obtain the average delay of $D = \Theta(\frac{\log k}{r^2(n)})$ for multicast in multi-channel network. □

Note that $r(n)$ is set to be $\min\{\sqrt{\frac{c}{nm}}, \sqrt{\frac{\log\log n}{k\log n}}\}$, the delay can be further rewritten as follows:

When $k = O(\frac{nm\log\log n}{c\log n})$, $r(n) = \sqrt{\frac{c}{nm}}$, we have

$$D = \Theta\left(\frac{nm\log k}{c}\right).$$

When $k = \Omega(\frac{nm\log\log n}{c\log n})$, $r(n) = \sqrt{\frac{\log\log n}{k\log n}}$, we have

$$D = \Theta\left(\frac{k\log n\log k}{\log\log n}\right).$$

Comparing with the results under the scheme of 2-hop relay without redundancy in [26], we find the delay is largely reduced. This is because multiple pairs can be scheduled in our work for simultaneous transmission as long as they are assigned with different channels whereas only one pair is allowed to transmit at one time under a single channel. More detailed comparison and discussion can be referred to Section 7.2.

7 DISCUSSION

In this section, we will give some discussion based on the results obtained in previous sections. Particularly, we will take a deeper look into the results and disclose the impact of multicast destination number k, the ratio of channels to interfaces c/m as well as node mobility on scaling performance.

7.1 Joint Impact of k and c/m on Multicast Capacity

We take a look into the static network first. An interesting finding is that the results are quite delicate, depending on both the ratio of channels to interfaces c/m and the destination number k. That is, multicast capacity exhibits three distinctive regions when c/m falls into the range of $O(\log n)$, $[\Omega(\log n), O(n\frac{\log\log n}{\log n})]$ and $\Omega(n\frac{\log\log n}{\log n})$, respectively. And for each range of c/m, the corresponding capacity region is further partitioned due to different ranges of k. This differs our results from that obtained under unicast traffic where capacity region is solely determined by c/m. A remarkable phenomenon is that multicast is likely to incur more capacity loss. A more clear picture of the capacity loss region with regard to both k and c/m is
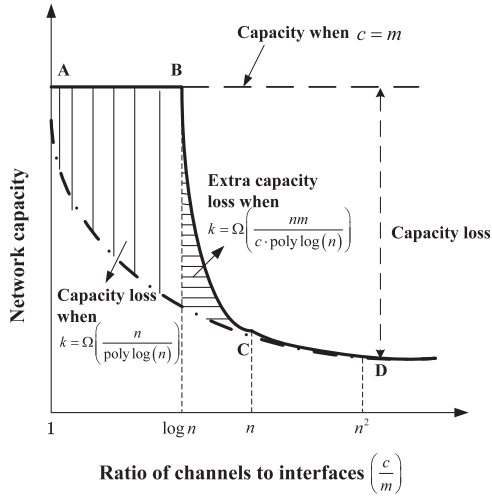
Fig. 3. Capacity loss region incurred by both the ratio of channels to interfaces and number of destinations k.

Fig. 4. Multicast delay scaling with regard to c/m and k. The comparison between our results and that in [26] is also present in the figure.

illustrated in Fig. 3. As can be seen from the figure, when $c/m = O(\log n)$, multicast capacity of

$$\Theta\left(\frac{1}{\sqrt{n \log k}}\right)$$

is achievable at the same level as single channel scenario, given that k should be restricted to $k = O(n/\text{polylog}n)$. However, as k reaches to a value close to the total number of nodes n in the network, capacity is reduced to $\widetilde{\Theta}(m/ck)$. The reason behind is that a sufficiently large k will enhance the effect of destination bottleneck, overwhelming the connectivity constraint which is turned out to be the dominant constraint under unicast traffic when $c/m = O(\log n)$. Analogous phenomenon also occurs at c/m falling into the range of $[\Omega(\log n), O(n\frac{\log \log n}{\log n})]$, where destination bottleneck is the strongest and incurs further capacity loss after k exceeds the threshold $\widetilde{\Theta}(\frac{nm}{c})$. In contrast, interference constraint plays a more significant role in capacity bottleneck when k is restricted to be less than $\widetilde{\Theta}(\frac{nm}{c})$.

7.2 The Impact of c/m in Mobile Networks

Now, we turn our attention to the mobile case for discussion about the impact of c/m on multicast scaling. In particular, we will divide the discussion into two parts, based on capacity and delay performance, respectively.

7.2.1 The Impact on Capacity Scaling

Theorems 3 and 4 both indicate that with i.i.d. mobility introduced into the network, it is possible to unify the different capacity regions shown in static case and improve the capacity to $\widetilde{\Theta}(m/ck)$ at all ranges of c/m. This is attributed to the fact that mobile network is free of connectivity constraint and mobility can be appropriately utilized to break the bottleneck from interference. An observation from the proof of Theorem 4 is interference constraint is stronger than destination constraint when k is no more than $\widetilde{\Theta}(nm/c)$. As a counterpart, the effect of destination constraint becomes more apparent after k exceeds $\widetilde{\Theta}(nm/c)$. However, it is delightful that interference constraint can be effectively controlled for not further
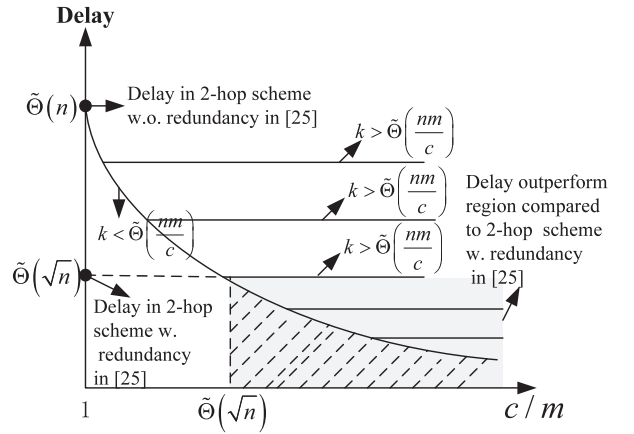
degrading capacity even if it is the dominant constraint. This is because routing tree cannot be established due to the movement of nodes and the corresponding traffic flows assigned to each node per cell is, therefore, solely determined by $F(n)$.

7.2.2 The Impact on Delay Scaling

A notable phenomenon in mobile network is that there is an interplay between the capacity and delay scaling. The improvement of one metric is achieved at the cost of sacrificing the other one. Note that, in this paper, we focus on 2-hop relay mode without redundancy since our main goal is to achieve the maximum capacity bound. Therefore, even in multichannel scenario, the delay also increases to the maximum value when capacity upper bound is achieved. Nevertheless, we can still have an exciting delay result when comparing with [26], which is based on single-channel framework. Fig. 4 illustrates the delay performance obtained in both [26] and the present work. It is proved in [26] that multicast delay is $\Theta(n \log k)$ under 2-hop algorithm without redundancy. Executing under the same algorithm in multichannel scenario, we achieve a delay of $\Theta(nm \log k/c)$ when k is no more than $\widetilde{\Theta}(nm/c)$ (see the curve in Fig. 4) and a delay of $\Theta(k \log k \log n / \log \log n)$ when k exceeds $\widetilde{\Theta}(nm/c)$ (see the straight lines in Fig. 4). Obviously, delay is reduced by c times in the former case. And a delay smaller than $\Theta(n \log k)$ is still achievable in the latter case as long as k is up to n except for a logarithmic factor, as is shown by the straight lines in the figure. It surprisingly suggests that delay can be greatly reduced in multichannel case even without redundancy relays introduced! The reason behind is that the existence of multichannel allows multiple pairs communicating simultaneously without causing interference to each other, which effectively contracts the time consumption on transmission from a source to all its destinations.

Rather than simply stick to 2-hop relay algorithm without redundancy between the two works, we take a further look into our results and that under 2-hop relay algorithm with redundancy in [26]. It turns outs that our results still outperform the delay scaling of [26] in such situations. As is demonstrated in [26], no delay smaller than $\Theta(\sqrt{n \log k})$ is achievable even with more than $\Theta(\sqrt{n \log k})$

redundancies introduced in the 2-hop relay scheme. However, in our work, a delay smaller than $\Theta(\sqrt{n \log k})$ is achievable when k is no more than $\widetilde{\Theta}(nm/c)$, as long as $c/m = \Omega(\sqrt{n \log k})$. And it still works when k is in the range of $[\widetilde{\Theta}(nm/c), \widetilde{\Theta}(\sqrt{n})]$, also provided that $c/m = \Omega(\sqrt{n \log k})$.[1] All those gains are summarized into delay outperform region shown in Fig. 4. This effectively fills in the blank in single-channel where the relay hop count is strictly constrained at 2. We, therefore, obtain a useful insight that delay improvement is possible through frequency reuse other than seeking for complicated techniques or conventionally relying on relays.

7.3 Future Directions

Although being the first attempt to study multicast scaling in MC-MR networks, we claim that there are still several directions in which this work could be extended in the future. One natural direction is to explore the heterogeneity in MC-MR networks, where nodes may have nonuniform distribution. It is expectable that heterogeneous distribution is likely to bring out the gain in scaling laws. Another case we anticipate to investigate is that nodes move according to some more realistic mobility models, such as random walk, random way point and Brownian motion, and so on. We believe that multicast scaling will exhibit different performance in such situations. The extension to social networks is also of interest, where some prominent features such as small-world and human interactions are considered. This is also a potential way to improve multicast performance in MC-MR networks.

8   CONCLUSION

We analyze multicast scaling performance in MC-MR networks in this paper. A fixed bandwidth W is equally split into c channels with $0 < m \le c$ interfaces equipped on each node for channel switching. Totally n nodes are distributed throughout the network, each acting as a source with k randomly and uniformly selected destinations. We investigate capacity and delay in both the static and mobile networks and obtain three remarkable findings from our results. First, we find capacity loss exists even if the ratio $c/m = O(\log n)$ when k exceeds a threshold. Second, we demonstrate mobility helps to improve multicast capacity in MC-MR networks. Third, it turns out that delay can be largely reduced through frequency reuse without introducing redundancies and even outperforms the results in [26] in certain cases. To our best knowledge, we are the first to study multicast scaling law in MC-MR networks from a general perspective.

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1. Notice that this range does not exist when $c/m = O(\sqrt{n \log k})$ since $\widetilde{\Theta}(nm/c) > \widetilde{\Theta}(\sqrt{n})$.

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