Capacity of Wireless Networks with Social Characteristics

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***Abstract*—This paper studies the throughput capacity of wireless networks with social characteristics. We propose a simple model to reflect both the social relations between nodes and power-law node degree distribution, and then examine their impact on capacity. We show the fact that two features above lead to traffic locality and improve capacity. Moreover, multicasting may be employed to further enhance performance when information is desired to be published from the source to all its contacts, of which the number follows power-law distribution. In addition, we propose the corresponding capacity-achieving communication schemes which optimally exploit the underlying structure. Our study is an attempt to understand how social relations may impact on network capacity from a theoretical perspective, and provides fundamental insight on the design and analysis of real wireless networks.**

***Index Terms*—Capacity, Wireless, Social Characteristics**

I. Introduction

The structure of large scale wireless networks are remarkably transformed by a myriaid of newly emerged and rapidly penetrating applications. Typical examples include online social networks such as MySpace, Facebook, Orku, LiveJournal, Cyworld and Flickr, which have attracted tens of millions of users integrating these sites into their daily lives. These online social services have offered an unprecedented opportunity for measurement-based studies on human social networks at massive scale. It is observed that numerous social networks including Youtube (over 190 million users), Orkut (over 62 million), LiveJournal (over 5.5 million) [2], Cyworld (over 12 million), Myspace (over 130 million) [3], and Flickr (over 1 million) [4], [5] exhibit the characteristics such as the way people select friends and the number of such friends.

However, most of the existing research with the two characteristics aforementioned mainly focuses on measurement-based analysis of structural and topological characteristics, and little is known about their impact on network performance metrics such as throughput, delay, etc. That motivates us to present a look into the throughput capacity of large scale wireless social networks from a theoretical perspective. Previous work about mobile social service [6], [7], [8], geosocial networking [9], [10], [11], [12] and ad hoc social networks [13], [14], [15] have already pointed out that the social relations bring new challenges as well as opportunities in system design, and new communication protocols may be conceived to exploit the underlying social structure for better performance. As some initial attempts, Azimdoost *et al.* [16] present the modeling framework for the capacity of a wireless network in which nodes communicate in the context of social groups. They assumed that in a wireless network with n nodes, each node is socially connected to its neighbors and also to q other long-range contacts. Under the assumption that the probability of selecting destination follows power-law distribution with parameter α, the order capacity is derived as a function of the number of nodes n, the social group concentration α, and the size of social groups q. The model is slightly modified by Kiskani *et al.* [17], who further assume that selection of destination within a social group also follows another power law distribution with respect to distance. Different capacity regions are then computed as a function of the social network size for each node. By separating the highly popular nodes with those who are less popular, Azimdoost and Sadjadpour [18] show that nodes with different social status impact the capacity differently. However, all those works only provide the capacity upper bound, and the optimal communication schemes are still not investigated.

In this paper, we bridge the theoretical analysis of fundamental scaling laws of wireless networks with the insights already gained through practical protocol development. By doing so, we provide a theoretical foundation to the design of intelligent scheduling and routing schemes that exploit social relations, analytically demonstrating the benefits of such schemes in terms of throughput capacity.

In particular, to address the aforementioned two major features of such large scale networks, we deploy the rank-based model, where the probability of befriending a particular node is inversely proportional to the αth power of the number of closer nodes. The network is assumed to be comprised by $n+1$ uniformly distributed wireless nodes. We choose the rank-based model over the distance-based one since the latter one underestimates the friendship probability of the distant nodes in the low-density region, when the geographical distribution of users is inhomogeneous in common occurrence. In contrast, the rank-based model states that the friendship probability depends on both the geographic distance and node density, as is pointed out by both Liben-Nowell *et al.* [1] and Li *et al.* [19]. For each node, a random number of friends that follows power-law distribution with parameter β will be selected independently.

We study two kinds of common traffic patterns, i.e., *unicast* and *multicast*. The first type represents messaging service between two friends while the latter represents pattern information broadcasted to all the friends of the source, such as tweets in Twitter and posts in Facebook. This differs our work from the previous ones [16], [17], [18], where only unicast is taken into consideration. We show that the per-node unicast capacity is $\tilde{\Theta}(1/\sqrt{n})$[1] when $\alpha \le 1$, $\tilde{\Theta}(n^{\alpha-3/2})$ when $1 < \alpha < 3/2$, and $\tilde{\Theta}(1)$ when $\alpha > 3/2$. When $\alpha = 0$, the per-node multicast capacity is $\tilde{\Theta}(1/n)$ when $\beta \le 1$, $\tilde{\Theta}(n^{\beta-2})$ when $1 < \beta < 3/2$, and $\tilde{\Theta}(1/\sqrt{n})$ when $\beta > 3/2$. In the more general but intricate case that α is arbitrary, we conjecture the multicast capacity to be $\tilde{\Theta}(n^{\alpha+\beta-3})$ when $\alpha, \beta \in [1, 3/2]$. The results above are significantly better than the capacity of networks with classic uniform traffic, which is $\tilde{\Theta}(n^{-\frac{1}{2}})$ in the unicast case and $\tilde{\Theta}(n^{\beta-5/2})$ in the multiple unicast case, thanks to the traffic locality and multicast gains resulted from the underlying network structure. The corresponding capacity-achieving communication schemes are also discussed.

It is worth noting that both the rank-based model and the power law node degrees are heavy-tailed distributions. Heavy-tailed distributions are useful modeling tools in realistic settings, but are often difficult for analysis because they imply a great degree of variations in the system, i.e., some of the source-destination pairs are close neighbors while some may be very far away. In addition, some nodes have extremely large number of followers (such as celebrities) while some others may only have a few. However, our results show that despite the great heterogeneities in the network, a uniform optimal performance can be guaranteed.

A. *Related Works of Capacity Scaling*

The asymptotic capacity of traditional wireless networks is first studied in [20], where Gupta and Kumar show that the maximal unicast throughput achievable by each node for a uniformly distributed destination is $\Theta(1/\sqrt{n \log n})$[2]. Grossglauser and Tse [21] later introduce mobility to the nodes and show that by employing a store-carry-forward paradigm, capacity can be improved to $\Theta(1)$, at the expense of increased delay. A series of works [22], [23], [24], [25], [26] have then been focusing on the analysis of optimal throughput-delay tradeoff under different mobility models.

Among numerous papers following Gupta and Kumar's framework to investigate the capacity of various kinds of specific wireless networks, the most related ones consider heterogeneous networks. Garetto et al. [25], [26] study the capacity scaling in ad hoc networks with heterogeneous nodes mobility. Alfano et al. [27], [28] consider the case that nodes are distributed heterogeneously according to a shot-noise Cox process, such that clusters may be formed. Huang and Wang [29] analyze a network consisting of nodes with heterogeneous priority. Both data collection and coverage in sensor networks are also investigated in terms of their performance [30], [31]. However, none of the works study the impact of heterogenous traffic (service) pattern, which is an important characteristic of real world social networks.

Finally, multicast in traditional wireless ad hoc network is investigated by Li [32], who shows that per-node multicast capacity is $\Theta(1/\sqrt{nk \log n})$ when $k = O(n/\log n)$ and $\Theta(1/k\sqrt{\log n})$ when $k = \omega(n/\log n)$, where k is the number of destinations per multicast session. Wang et al. [33] generalize the result to anycast traffic pattern and Mao et al. [34] study multicast networks with infrastructure support.

The rest of the paper is organized as follows. We introduce the system model in Section 2, and derive the unicast capacity results in Section 3. Section 4 discusses multicast capacity results and we conclude the paper in Section 5.

II. SYSTEM MODEL

In this paper, we denote the probability of an event E as $\Pr(E)$ and say E happens with high probability ($w.h.p.$) if $\lim_{n\to\infty} \Pr(E) = 1$. By convention, we use $\{c_i\}$ to denote some positive constants independent of n.

A. *Network Topology*

We define the network extension $\mathcal{O}$ to be a unit torus, i.e., the side length of the network is 1. The size normalization and wrap-around conditions are common technical assumptions adopted in previous works to avoid tedious technicalities. These assumptions will not change the main results of this paper. There are $n+1$ nodes with wireless communication capability in the network and exchange information in an ad hoc manner. Their locations are $\{X\}_{i=1}^{n+1}$, which are a series of independent random variables uniformly distributed in $\mathcal{O}$. At a given time t, nodes may be denoted by their positions, i.e., we refer to node i as $X_i(t)$.

B. *Communication Model*

We assume all nodes share a wireless channel with bandwidth W (bps). We base our analysis on the following classic wireless interference model that governs direct wireless transmissions between nodes.

Definition 1: Protocol Model [20]: All nodes use a common transmission range R_T for all their wireless communication. A wireless transmission from node i to j is successful only if : 1) $\|X_i(t) - X_j(t)\| \le R_T$; and 2) For every other node k that is simultaneously transmitting, follows, $\|X_k(t) - X_j(t)\| \ge (1+\Delta)R_T$, where constant Δ defines the area of guard zone.

C. *Node Relationship and Traffic Pattern*

It is worth noting that modeling node relationship and network topology is rather challenging arguably due to the complicated nature of nodes' behaviors, the diverse structure characteristics observed from real data sets and the massive scale. Sala et al. [35] show that among the existing models for synthetic graphs, only very few of them may capture

[1]The order notation $\tilde{\Theta}(\cdot)$ hides polylogarithmic factors for better readability. Refined results are available in Section III and IV.

[2]We use the following notation throughout our paper:
$f(n) = o(g(n)) \Leftrightarrow \lim\limits_{n\to\infty} \frac{f(n)}{g(n)} = 0$,
$f(n) = \omega(g(n)) \Leftrightarrow \lim\limits_{n\to\infty} \frac{g(n)}{f(n)} = 0$,
$f(n) = O(g(n)) \Leftrightarrow \limsup\limits_{n\to\infty} \frac{f(n)}{g(n)} < \infty$,
$f(n) = \Omega(g(n)) \Leftrightarrow \liminf\limits_{n\to\infty} \frac{f(n)}{g(n)} < \infty$,
$f(n) = \Theta(g(n)) \Leftrightarrow f(n) = O(g(n))$ and $g(n) = O(f(n))$.

the full structural characteristics or produce results with high fidelity. Furthermore, most of these models are based on numerical methods and will incur high computational and memory complexity.

We approach the modeling of node relationship in a totally new and novel way that is motivated by a geographical perspective. Both daily experience and real data traces from online social networks [1], [5], [36] have indicated that friendships forming and communication patterns are closely related to geography and are usually highly localized. We adopt the following rank-based model in [1] to characterize the relation between friendship and node location. Consider two nodes i and j, define the rank of j with respect to i as:

$$Rank_i(j) = |\{k : D(k, i) < D(i, j)\}|,$$

where $D(\cdot, \cdot)$ is the distance between two nodes and k is any of the $n+1$ nodes in the network. Then we model the probability that j is a friend of i as

$$\Pr\{i \to j\} \propto \frac{1}{Rank_i^{\alpha}(j)},$$

where $\alpha \geq 0$. Denoting for short $G_1 = \sum_{j=1}^{n} 1/j^{\alpha}$, the distribution law is

$$\Pr\{i \to j\} = \frac{1}{G_1 Rank_i^{\alpha}(j)}. \tag{1}$$

Liben-Nowell et al. [1] show that this model accounts for the majorities of the friendships in the LiveJournal online community. Further, the work in [37] suggests that the model indeed guarantees small-world properties, such that with geographical information only, a friendship chain with at most $\Theta(\log^3 n)$ hops can be established between an arbitrary source node and a target node chosen uniformly at random from the whole population. This clear-cut property remarkably coincides with the fact that shortcuts can be found between two arbitrary nodes in LiveJournal with only geographical information.

Another important feature is the power-law degree distribution [2], [3], [4], [5]. We assume K_i, the number of friends of a particular node i, is drawn according to Zipf distribution.

$$\Pr\{K_i = k\} = \frac{1}{G_2 k^{\beta}},$$

where $G_2 = \sum_{j=1}^{n} 1/j^{\beta}$ is the normalizing factor and $\beta \geq 0$ is the power-law parameter. K_i friends are chosen independently according to the rank-based model (1). We focus on the case that $\beta > \alpha$ such that no node will not be repetitively chosen in K_i trials $w.h.p.$[3]

We study two kinds of major traffic patterns in such networks, i.e., *unicast* and *multicast*. The traffic type represents (private) messaging service between two friends, i.e., sources will select their destinations with the rank-based model (1). The latter traffic type models information broadcasted to all the nodes that have relationship with the source, such as tweets in Twitter and posts in Facebook. Therefore, multiple friends will be chosen according to the above power-law model.

[3]Otherwise, if some friends are repetitively selected, our results provide achievable lower bounds of capacity.

We note that our model may not be a perfect characterization of general graphs. For example, some of the friendships in LiveJournal appear to be geography independent and may be better correlated with other dimensions such as occupations, age, etc. However, a complete reproduction of all the features in a realistic network is too difficult, if at all possible, and we believe it is beneficial to make the proper simplifications towards a tractable model and a meaningful look into the throughput capacity in networks with heterogeneity.

D. Capacity Definition

Definition 2: Feasible unicast(multicast) throughput: Per-node throughput $g(n)$ is said to be feasible if there is a spatial and temporal scheme for scheduling transmissions, such that by operating the network in a multi-hop fashion and buffering at intermediate nodes when awaiting transmission opportunities, every source can send g(n) bits/sec to its $1(K_i)$ chosen destination nodes. That is, there is a $T < \infty$ such that in every time interval $[(i-1) \cdot T, i \cdot T]$, every source can send $T \cdot g(n)$ bits to each of its $1(K_i)$ destinations.

Definition 3: Asymptotic per-node multicast capacity $\lambda_m(n)$ of the network is said to be of order $\Theta(g(n))$ if there exist two positive constants c_1 and c_2 such that:

$$\begin{cases} \lim_{n\to\infty} \Pr\{\lambda_m(n) = c_1 g(n) \text{ is feasible}\} = 1 \\ \lim_{n\to\infty} \Pr\{\lambda_m(n) = c_2 g(n) \text{ is feasible}\} < 1 \end{cases}$$

Similarly we define the asymptotic per-node unicast capacity $\lambda_u(n)$.

E. Notations

In table I, we list all the parameters that will be used in later analysis, proofs and discussions.

TABLE I
NOTATIONS

Notation	Definition
$n+1$	The total number of nodes in the network.
W	The total transmission bandwidth available.
R_T	transmission range
α	Power law parameter indicating the strength of the relation between two nodes.
β	Power law parameter indicating the degree of a node.
$\lambda_u(n)$	Asmptotic per-node unicast capacity.
$\lambda_m(n)$	Asmptotic per-node multicast capacity.

III. Main Results

A graphical representation of our results is reported in Figures 1 and 2, respectively. We adopt the order notation $\widetilde{\Theta}(\cdot)$ to hide poly logarithmic factors for better readability. Refined results are available in Section V.

Figures 1 plots the per-node unicast capacity $\lambda_u(n)$ achieved versus different values of parameter α. As a counterpart, Figure 2 represents the per-node multicast capacity $\lambda_m(n)$, which exhibits different scaling behaviors with different values of parameter β. A common phenomenon from both figures is that the capacity increases as α and β increase. And the maximum capacity can be achieved in the range

$\alpha > 3/2$ for unicast and $\beta > 3/2$ for multicast. In unicast, larger α means that the destination selected locates more closer to the source. This leads to a stronger traffic locality, which greatly reduces the transmission length between source and destination. As α becomes smaller, such transmission length gradually increases. Specifically, when $\alpha < 1$, a length of 1, i.e., the network size is needed in order to transmit a packet from the source to its destination. Under such circumstance, there is no capacity gain since the capacity yields the same result as the one achieved in traditional ad hoc wireless networks. For multicast, a larger β means a smaller number of friends in each multicast session. Specifically, when $\beta > 3/2$, the capacity result is close to the unicast one in traditional wireless networks, since the number of friends in each multicast session is $\Theta(1)$ in such case. In contrast, when $\beta < 1$, almost all the nodes in the network are selected as friends in each multicast session and traffic pattern therefore yields to broadcast, which brings about the capacity of $1/n$.

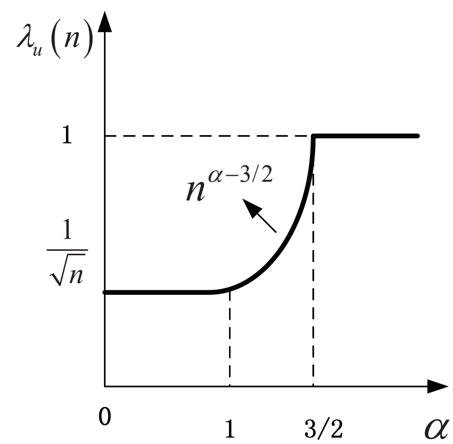

Fig. 1. The per-node unicast capacity $\lambda_u(n)$ versus α.

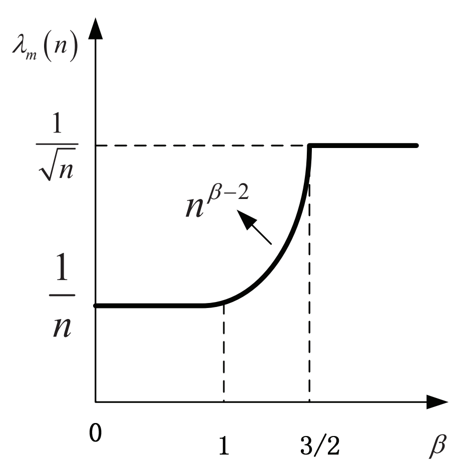

Fig. 2. The per-node multicast capacity $\lambda_m(n)$ versus β.

IV. Unicast

A. Traffic Locality

Comparing with classic unicast networks, the traffic pattern in our model is significantly different because the destinations are selected according to the rank-based model, which will result in a certain degree of traffic locality. Intuitively, as parameter α increases, sources will be more likely to befriend a node located in closer proximity, and therefore less distance or hops are needed to be covered in the packet delivery process. This amounts to a smaller interference per traffic flow, and in terms imply a larger degree of transmission concurrency can be achieved. As a result, the unicast capacity is increased.

However, the non-uniformity of the traffic pattern will cause significant difficulty in analysis. In order to proceed, we first need to establish some important properties and implications of the ranked-based model. Denote Y_i as the destination selected by X_i, the following lemma shows the distribution of the distance between Y_i and X_i conditioning on rank.

Lemma 1: Consider a generic node i, conditioning on the event that $Rank_i(Y_i) = r$, the probability density function (PDF) of random variable $D(X_i, Y_i)$ is:

$$f_{r:n}^{(D)}(d) = \frac{n!}{(r-1)!(n-r)!} 2\pi^r d^{2r-1}(1-\pi d^2)^{n-r} \qquad (2)$$

for $0 \le d \le \frac{1}{\sqrt{\pi}}$.

Proof: Let X_j, $j \neq i$ be an arbitrary node in the system. According to our model, X_j is uniformly distributed, and the cumulative distribution function (CDF) of $D(X_i, X_j)$ follows[4]:

$$\begin{aligned} F^{(D)}(d) = \Pr\{D(i,j) \le d\} &= \int\int_{x^2+y^2\le d^2} 1 \mathrm{d}x\mathrm{d}y \\ &= \pi d^2, \quad 0 \le d \le \frac{1}{\sqrt{\pi}}. \end{aligned}$$

And the corresponding PDF is :

$$f^{(D)}(x) = 2\pi d, \quad 0 \le d \le \frac{1}{\sqrt{\pi}}.$$

Now, consider the mechanism of the rank-based model, conditioning on the event of $Rank_i(Y_i) = r$, it is clear that $D(X_i, Y_i)$ is the rth order statistics [38] of n independent and identically distributed (i.i.d.) $\{D(i,j)\}_{j\neq i}$. By convention we denote $D(X_i, Y_i)$ as $D_{r:n}$.

[4]More rigorously, taking the four corners of the square extension into account, the CDF should be:

$$\Pr\{D \le d\} = \begin{cases} \pi d^2 & 0 \le d \le \frac{1}{2} \\ \pi d^2 (\frac{\pi - 4\arccos\frac{1}{2d}}{\pi}) + \sqrt{4d^2-1} & \frac{1}{2} < d \le \frac{\sqrt{2}}{2} \end{cases}$$

This piecewise function is awkward for presentation and we therefore simplify the extension to be a disk in the lemma derivation. However, it can be easily shown that this slight modification does not have any significance in order sense.

The CDF of $D_{r:n}$ may be obtained by standard technique,

$$\begin{aligned}
F_{r:n}^{(D)}(d) &= \Pr\{D_{r:n} \le d\} \\
&= \Pr\{\text{at least } r \text{ of } D(i,j)\text{'s are at most } d\} \\
&= \sum_{k=r}^{n} \Pr\{\text{exactly } k \text{ of } D(i,j)\text{'s are at most } d\} \\
&= \sum_{k=r}^{n} \binom{n}{k} \{F^{(D)}(d)\}^k \{1 - F^{(D)}(d)\}^{n-k} \\
&= \sum_{k=r}^{n} \binom{n}{k} (\pi d^2)^k (1 - \pi d^2)^{n-k} \\
&= \int_0^{\pi d^2} \frac{n!}{(r-1)!(n-r)!} t^{r-1}(1-t)^{n-r}\mathrm{d}t \qquad (3) \\
&= \int_0^{d} \frac{n!}{(r-1)!(n-r)!} 2\pi^r t^{2r-1}(1-\pi t^2)^{n-r}\mathrm{d}t, \qquad (4)
\end{aligned}$$

where Equation (3) holds because of the fact that,

$$\sum_{k=r}^{n} \binom{n}{k} p^r (1-p)^{n-r} = \int_0^{p} \frac{n!}{(r-1)!(n-r)!} t^{r-1}(1-r)^{n-r}\mathrm{d}t, \quad 0 < p < 1,$$

which may be proved by repeated integration by parts. From Equation (4), we observe that the density function of $D_{r:n}$ is

$$f_{r:n}^{(D)}(d) = \frac{n!}{(r-1)!(n-r)!} 2\pi^r d^{2r-1}(1-\pi d^2)^{n-r}$$

for $0 \le d \le 1/\sqrt{\pi}$. ■

In the next step we characterize the conditional expectation of $D(X_i, Y_i)$.

Lemma 2: Conditioning on the event that $Rank_i(Y_i) = r$ and denote $\mathbb{E}$ as expectation, then

$$\begin{aligned}
\mathbb{E}\{D(X_i, Y_i) | r:n\} &= \sqrt{\pi} \frac{\Gamma(n+1)}{\Gamma(n+3/2)} \frac{\Gamma(r+1/2)}{\Gamma(r)} \\
&\sim \sqrt{r/n}.
\end{aligned}$$

Proof: By definition,

$$\begin{aligned}
\mathbb{E}\{D_{r:n}\} &= \int x f_{r:n}^{(D)} \mathrm{d}x \\
&= \int_0^{\frac{1}{\sqrt{\pi}}} \frac{n!}{(r-1)!(n-r)!} 2\pi^r x^{2r}(1-\pi x^2)^{n-r}\mathrm{d}x \\
&= \int_0^{1} \frac{n!}{(r-1)!(n-r)!} \frac{2}{\sqrt{\pi}} t^{2r}(1-t^2)^{n-r}\mathrm{d}t \\
&= \frac{B(r+1/2, n-r+1)}{\sqrt{\pi}B(r, n-r+1)} \\
&= \frac{1}{\sqrt{\pi}} \frac{\Gamma(n+1)}{\Gamma(n+3/2)} \frac{\Gamma(r+1/2)}{\Gamma(r)},
\end{aligned}$$

where $B(\cdot,\cdot)$ is the complete beta function:

$$B(p,q) = \int_0^1 t^{p-1}(1-t)^{q-1}\mathrm{d}t, \quad p, q > 0$$

and $\Gamma(\cdot)$ is the complete gamma function:

$$\Gamma(p) = \int_0^\infty e^{-t} t^{p-1}\mathrm{d}t, \quad p > 0.$$

With Stirling's approximation of gamma function [43],

$$\Gamma(p) = \sqrt{2\pi p}\left(\frac{p}{e}\right)^p \left(1 + O\left(\frac{1}{p}\right)\right)$$

holds,

$$\begin{aligned}
\mathbb{E}\{D_{r:n}\} &\sim \frac{\sqrt{n+1}(\frac{n+1}{e})^{n+1}}{\sqrt{n+\frac{3}{2}}(\frac{n+\frac{3}{2}}{e})^{n+\frac{3}{2}}} \frac{\sqrt{r+\frac{1}{2}}(\frac{r+\frac{1}{2}}{e})^{r+\frac{1}{2}}}{\sqrt{r}(\frac{r}{e})^r} \\
&\sim \left(\frac{n+1}{n+\frac{3}{2}}\right)^{n+1} \frac{1}{(n+\frac{3}{2})^{\frac{1}{2}}} \left(\frac{r+\frac{1}{2}}{r}\right)^r \left(r+\frac{1}{2}\right)^{\frac{1}{2}} \\
&\sim \left(1 - \frac{1}{2(n+\frac{3}{2})}\right)^{n+1} \cdot \left(1 + \frac{1}{2r}\right)^r \sqrt{\frac{r}{n}} \\
&\sim \sqrt{r/n}.
\end{aligned}$$

■

Before proceeding to the capacity results, we introduce a useful lemma on estimating the partial sum of p-series by the integral test inequality.

Lemma 3: Suppose $g(x)$ is a continuous decreasing function and $g(x) > 0$ for all $x \ge 1$, then

$$\int_1^n g(x)\mathrm{d}x \le \sum_{m=1}^{n-1} g(m) \le g(1) + \int_1^{n-1} g(x)\mathrm{d}x.$$

We conclude this subsection with the expectation of $D(X_i, Y_i)$ in general.

$$\begin{aligned}
\mathbb{E}\{D(X_i, Y_i)\} &= \mathbb{E}_r\{\mathbb{E}\{D(X_i, Y_i) | r:n\}\} \\
&\sim \frac{1}{G_1\sqrt{n}} \sum_{r=1}^{n} \frac{\sqrt{r}}{r^\alpha}
\end{aligned}$$

Recall that G_1 is the normalizing factor in (1). Setting $g(x) = 1/x^\alpha$, by Lemma 3 it is clear that,

$$G_1 = \begin{cases} \Theta(1) & \alpha > 1 \\ \Theta(\log n) & \alpha = 1 \\ \Theta(n^{1-\alpha}) & 0 \le \alpha < 1. \end{cases}$$

Similarly, by setting $g(r) = \sqrt{r}/r^\alpha$,

$$\sum_{r=1}^{n} \frac{1}{r^{\alpha - \frac{1}{2}}} = \begin{cases} \Theta(1) & \alpha > 3/2 \\ \Theta(\log n) & \alpha = 3/2 \\ \Theta(n^{3/2-\alpha}) & 0 \le \alpha < 3/2. \end{cases}$$

Combing the above formulas we have

$$\mathbb{E}\{D(X_i, Y_i)\} \sim \begin{cases} 1/\sqrt{n} & \alpha > 3/2 \\ \log n/\sqrt{n} & \alpha = 3/2 \\ n^{1-\alpha} & 1 < \alpha < 3/2 \\ 1/\log n & \alpha = 1 \\ 1 & 0 \le \alpha < 1. \end{cases} \qquad (5)$$

B. Upper Bound of Capacity

Based on the spatial characteristics of the traffic pattern, an upper bound of capacity can be derived by establishing the relation between throughput and the average distance that the packets need to be relayed.

Lemma 4: Suppose that on average a packet is relayed over a total distance not less than $\mathfrak{D}$, then $\lambda_u(n) = O(1/\mathfrak{D}\sqrt{n})$.

Proof: The proof follows that in [20], [23]. Consider a time interval T which is large enough, then the total number of packets transmitted end-to-end between all source-destination pairs during T is $c_P\lambda_u(n+1)T$, where the positive constant $1/c_P$ is the average number of bits per packet. By the law of large numbers, with high probability, the total distance traveled by these packets is at least $c_P\lambda_u(n+1)T\mathfrak{D}$. Denote h_p as the number of hops packet p is relayed, and let l_p^h, $h = 1, \ldots, h_p$, be the transmission range of the hth hop. Denote for short $N_p = c_P\lambda_u(n+1)T$, clearly it holds,

$$\sum_{p=1}^{N_p}\sum_{h=1}^{h_p} l_p^h \geq c_P\lambda_u(n+1)T\mathfrak{D}. \tag{6}$$

Consider that at time t generic nodes i, j are transmitting directly to nodes k and l, respectively. According to the protocol interference model, the following conditions must hold in order for successful reception:

$$\begin{aligned} d(X_j, X_k) &\leq (1+\Delta)d(X_i, X_k) \\ d(X_i, X_l) &\leq (1+\Delta)d(X_j, X_l). \end{aligned}$$

Therefore,

$$\begin{aligned} d(X_j, X_i) &\geq d(X_j, X_k) - d(X_i, X_k) \\ &\geq \Delta d(X_i, X_k). \end{aligned}$$

Likewise, we have

$$d(X_i, X_j) \geq \Delta d(X_j, X_l).$$

Hence,

$$d(X_i, X_j) \geq \frac{\Delta}{2}\left(d(X_i, X_k) + d(X_j, X_l)\right).$$

The above inequality states that as a consequence of the protocol model[5], disks of radius $\Delta/2$ times the transmission range centered at the transmitter are disjoint from each other. This " transmission consumes area " argument serves as one cornerstone of the upper bounds on achievable throughput. Notice that 1) $\mathcal{O}$ has unit area; 2) for each of these disjoint disks, at least 1/4 of it must lie within $\mathcal{O}$ and 3) transmitting a packet requires $1/c_P W$ duration of time, therefore,

$$\frac{1}{4}\sum_{p=1}^{N_p}\sum_{h=1}^{h_p}\pi\left[\frac{\Delta}{2}l_p^h\right]^2 \leq c_p WT. \tag{7}$$

By the Cauchy-Schwarz Inequality,

$$\left[\sum_{p=1}^{N_p}\sum_{h=1}^{h_p} l_p^h\right]^2 \leq \left[\sum_{p=1}^{N_p}\sum_{h=1}^{h_p}(l_p^h)^2\right]\left[\sum_{p=1}^{N_p}\sum_{h=1}^{h_p} 1\right], \tag{8}$$

where observing the fact that at any time there are at most $n+1$ transmissions in the network, the last factor can be reduced to,

$$\sum_{p=1}^{N_p}\sum_{h=1}^{h_p} 1 = \sum_{p=1}^{N_p} h_p \leq c_p WT(n+1), \tag{9}$$

Substituting (6)-(9) we have,

$$\begin{aligned} \frac{16c_P WT}{\pi\Delta^2} &\geq \sum_{p=1}^{N_p}\sum_{h=1}^{h_p}(l_p^h)^2 \\ &\geq \frac{\left[\sum_{p=1}^{N_p}\sum_{h=1}^{h_p} l_p^h\right]^2}{\sum_{p=1}^{N_p} h_p} \\ &\geq \frac{(c_p\lambda_u(n+1)T\mathfrak{D})^2}{c_p WT(n+1)}, \end{aligned}$$

Thus,

$$\lambda \leq \frac{4W}{\Delta\mathfrak{D}}\frac{1}{\sqrt{\pi(n+1)}} \sim \frac{1}{\mathfrak{D}\sqrt{n}}.$$

■

With Lemma 4 and (5), we have

Theorem 1: Under the ranked-based model, an upper bound of per-node unicast capacity is

$$\lambda_u(n) \sim \begin{cases} O(1) & \alpha > 3/2 \\ O(1/\log n) & \alpha = 3/2 \\ O(n^{\alpha - 3/2}) & 1 < \alpha < 3/2 \\ O(\log n/\sqrt{n}) & \alpha = 1 \\ O(1/\sqrt{n}) & 0 \leq \alpha < 1, \end{cases}$$

C. Capacity Achieving Scheme

In this subsection, we will show that a straightforward cell tessellation multi-hop relaying scheme suffices to achieve the capacity upper bound in Theorem 1.

Optimal Scheme for Unicast:

1. Tessellate $\mathcal{O}$ into squarelets (cells) with area $a(n)$.
2. Employ a cellular time-division multi-access (TDMA) transmission scheme such that each cell is scheduled to be active regularly according to cell time-slots. When a cell is activated, nodes within it are allowed to transmit to nodes inside the same cell or neighboring cells.
3. Denote a straight line segment connecting the source X_i and the destination Y_i as S-D line. Sources send their packets to destinations hop by hop along the cells which the S-D lines intersect.
4. When a cell is scheduled to be active, it transmit a single packet for each of the passing through S-D lines. This is again accomplished by adopting a TDMA scheme such that the cell time-slot is further divided into sub packet time-slots.

The simple scheme above is similar to the one used in [22]. To establish the optimality of the scheme, we need to 1) choose a proper $a(n)$ which guarantees network connectivity; 2) show that the TDMA scheme allows high spatial reuse and concurrent transmissions, and 3) analyze the service load of the cells. We begin with a well-known lemma in [39] which is a standard application of Chernoff bounds [40].

[5] It is also possible to establish similar observations under the generalized physical model [39].

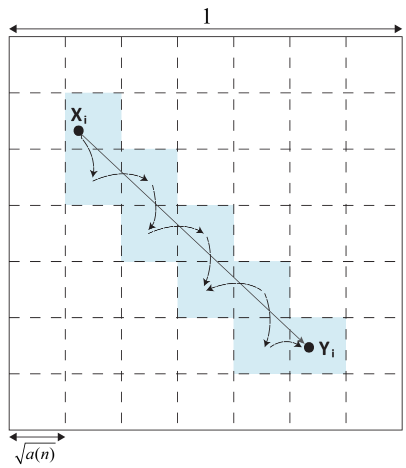

Fig. 3. Multi-hop scheme from X_i to Y_i.

Lemma 5: Let $a(n) = K \log n/n$, for any $K > 1$, uniformly over $\mathcal{O}$ it holds that each cell contains at least one nodes but no more than $Ke \log n$ nodes w.h.p.

Lemma 5 ensures connectivity and as a result, we choose $a(n) = \Theta(\log n/n)$ for minimal interference. The next lemma is a simple consequence of the protocol model and the well-known fact about vertex coloring of graphs of bounded degree, see [20] for a proof.

Lemma 6: With the TDMA scheme described above, each cell has a constant fraction of time to be active (See illustration 4.).

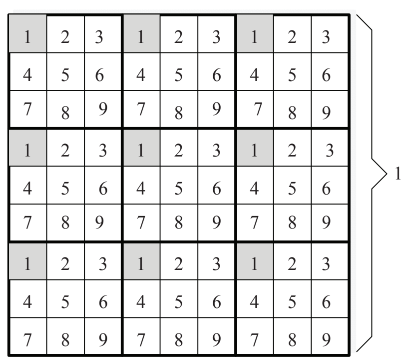

Fig. 4. 9-TDMA scheme where the whole network is divided into clusters with equal area. Each 9 groups are categorized as a group. All the grey cells in each group (numbered with 1) can transmit simultaneously in a time slot. In the next time slot all the cells numbered with 2 transmit and so on.

Lemma 6 equivalently states that each cell has a constant throughput capability. In the following, we calculate how many traffic flows shall share this throughput.

Lemma 7: The number of S-D lines passing through any cells is $O\left(n\mathbb{E}[D(X_i, Y_i)]\sqrt{a(n)} + na(n)\right)$ w.h.p. uniformly over $\mathcal{O}$.

Proof: Consider $n+1$ S-D pairs in the network and denote d_i as the distance between S-D pair (X_i, Y_i), i.e., $d_i = D(X_i, Y_i)$. Denote h_i as the number of hops per packet for S-D pair i, then

$$h_i = \frac{d_i}{\sqrt{a(n)}} + 1$$

where the additional one hop is to guarantee that at least one hop is required for transmission even if $d_i = o(\sqrt{a(n)})$. Define $H = \sum_{i=1}^{n+1} h_i$, i.e., the sum of hops required for each source in the network to send a single packet to its destination.

Now, consider a generic cell V and define the Bernoulli random variable I_k^i for S-D pairs $1 \le i \le n+1$ and $1 \le k \le h_i$, such that $I_k^i = 1$ if the kth hop originated from S-D pair i intersects cell V. Therefore, the total number of S-D lines passing through the cell is $I = \sum_{i=1}^{n+1} \sum_{k=1}^{h_i} I_k^i$. Notice that all I_k^i are identically distributed; further, I_k^i and I_l^j are pairwise independent if $i \neq j$. However, I_k^i and I_l^i are dependent since an S-D line can intersect a given cell by at most once.

Now, I is the major quantity of interest because it characterizes the relaying load of a cell. In the following we shall first determine its expectation, and then bound the tail probability:

$$\begin{aligned}
\mathbb{E}[I] &= \mathbb{E}_H[\mathbb{E}[I|H]] \\
&= \mathbb{E}_H[\sum_{i=1}^{n+1} \sum_{k=1}^{h_i} \mathbb{E}[I_1^1]] \\
&= \mathbb{E}_H[H\mathbb{E}[I_1^1]] = \mathbb{E}_H[Ha(n)] \\
&= a(n)\mathbb{E}\left[\sum_{i=1}^{n+1} \frac{d_i}{\sqrt{a(n)}} + 1\right] \\
&= (n+1)(\mathbb{E}[d_i]\sqrt{a(n)} + a(n)),
\end{aligned}$$

where in the third equation, $\mathbb{E}[I_1^1]$ equals $a(n)$ since by the symmetry of the torus, any hop is equally likely to originate from any of the $1/a(n)$ cells.

The remaining part of the proof essentially needs to show that I will not deviate too much from $\mathbb{E}[I]$ w.h.p. A common technique is to apply Chernoff bounds to obtain the tail probability of I, which is a sum of random variables (RVs). However, two major challenges are: 1) I is a sum of *random number* of RVs and 2) these RVs are not independent. Traditional Chernoff bounds cannot be used under these conditions, whereas we are going to prove that a similar bound still holds for our probability structure.

First, it is helpful to bound the range of H. We claim that w.h.p. $(1-\epsilon)\mathbb{E}[H] < H < (1+\epsilon)\mathbb{E}[H]$, for any constant $\epsilon > 0$. Indeed, define by short $P(H, \epsilon) = \Pr\{|H - \mu_H| < \epsilon\mu_H\}$, with Chebyshev's inequality,

$$\begin{aligned}
P(H, \epsilon) &= \Pr\left\{\left|\frac{H}{n+1} - \mathbb{E}[h_i]\right| < \epsilon\mathbb{E}[h_i]\right\} \\
&\ge 1 - \frac{\mathrm{Var}(h_i)}{(n+1)\epsilon^2\mathbb{E}^2[h_i]},
\end{aligned}$$

where $\mathrm{Var}(h_i)$ is the variance of h_i (for all i). And,

$$\begin{aligned}
\mathrm{Var}(h_i) &= \mathbb{E}[h_i^2] - \mathbb{E}^2[h_i] \\
&= (\mathbb{E}[d_i^2] - \mathbb{E}^2[d_i])/a(n).
\end{aligned}$$

Since $d_i < 2/\sqrt{2}$, $\text{Var}(h_i) = O(1/a(n)) = O(n/\log n)$. Therefore, for some constant c, it holds

$$P(H,\epsilon) \geq 1 - \frac{1}{\epsilon^2\mathbb{E}^2[h_i]c\log n} \to 1, \quad \text{as } n \to \infty.$$

Let $\bar{\mathcal{H}}$ be the event that H is bounded by $(1 \pm \epsilon)\mu_H$.

Now we construct random variable $\tilde{I} = \sum_{i=1}^{H} \tilde{I}_i$, where $\tilde{I}_i$ are i.i.d. Bernoulli random variables with the same distribution as I_k^i. Because of the dependency between I_l^i and I_k^i (i.e., the event that both I_l^i and I_k^i equals 1 is not possible), $\tilde{I}$ is stochastically larger than I. By the property of stochastic ordering, for any increasing function $\phi(\cdot)$ such that the expectation[6] exists, we have

$$\mathbb{E}[\phi(I)] \leq \mathbb{E}[\phi(\tilde{I})]. \tag{10}$$

Let t be an arbitrary positive constant, define for short $P^+(I,\delta) = \Pr\{I \geq (1+\delta)\mathbb{E}[I]\}$, proceed with the main steps in the proof of Chernoff bounds:

$$\begin{aligned} P^+(I,\delta) &= \Pr\{I \geq (1+\delta)\mathbb{E}[I]\} \\ &= \Pr\{\exp(tI) \geq \exp(t(1+\delta))\mathbb{E}[I]|\} \\ &\leq \frac{\mathbb{E}[\exp(tI)]}{\exp(t(1+\delta)\mathbb{E}[I])} \qquad (11) \\ &\leq \frac{\mathbb{E}[\exp(t\tilde{I})]}{\exp(t(1+\delta)\mathbb{E}[\tilde{I}])}, \qquad (12) \end{aligned}$$

where (11) is the consequence of Markov inequality and (12) holds from (10). Exploiting the independence between $\tilde{I}_i$, yields

$$\mathbb{E}[\exp(t\tilde{I})] = \mathbb{E}[\prod_{i=1}^{H}\exp(t\tilde{I}_i)] = \mathbb{E}_H[\prod_{i=1}^{H}\mathbb{E}[\exp(t\tilde{I}_i)|H]].$$

Let $p = a(n) = \Pr\{I_i = 1\}$ be the success probability,

$$\begin{aligned} P^+(I,\delta) &< \frac{\mathbb{E}_H[\prod_{i=1}^{H}(pe^t + 1 - p)]}{\exp(t(1+\delta)\mathbb{E}[\tilde{I}])} \\ &< \frac{\mathbb{E}_H[\prod_{i=1}^{H}\exp(p(e^t-1))]}{\exp(t(1+\delta)\mathbb{E}[\tilde{I}])} \qquad (13) \\ &= \frac{\mathbb{E}_H[\exp(\sum_{i=1}^{H}p(e^t-1))]}{\exp(t(1+\delta)\mathbb{E}[\tilde{I}])} \\ &< \frac{\exp(\sum_{i=1}^{(1+\epsilon)\mathbb{E}[H]}p(e^t-1))}{\exp(t(1+\delta)\mathbb{E}[\tilde{I}])}\Pr\{\bar{\mathcal{H}}\} + \Pr\{\bar{\mathcal{H}}^c\} \qquad (14) \\ &= \frac{\exp((e^t-1)(1+\epsilon)\mathbb{E}[\tilde{I}])}{\exp(t(1+\delta)\mathbb{E}[\tilde{I}])}\Pr\{\bar{\mathcal{H}}\} + \Pr\{\bar{\mathcal{H}}^c\}, \qquad (15) \end{aligned}$$

where (13) holds due to inequality $1 + x < e^x$; (14) holds due to the law of total probability and the monotonicity of exp and; (15) holds because of the fact that $\mathbb{E}[\tilde{I}] = \mathbb{E}[H]\mathbb{E}[\tilde{I}]$. Lastly, by setting $t = \log(1+\delta)$ and noticing that $\mathbb{E}[I] = \mathbb{E}[\tilde{I}]$, we recover a bound which is arbitrarily closed to the classic Chernoff bound,

$$P^+(I,\delta) < \left[\frac{e^{(1+\epsilon)\delta}}{(1+\delta)^{(1+\delta)}}\right]^{\mathbb{E}[I]}\Pr\{\bar{\mathcal{H}}\} + \Pr\{\bar{\mathcal{H}}^c\}.$$

Now we may choose δ as[7],

$$\delta = \left(\frac{4\log n^2}{\mathbb{E}[I]}\right)^{\frac{1}{2}-\epsilon} < \left(\frac{8\log n}{na(n)}\right)^{\frac{1}{2}-\epsilon} = O(1).$$

Thus,

$$\begin{aligned} P^+(I,\delta) &= \Pr\{I \geq (1+O(1))\mathbb{E}[I]\} \\ &< \frac{1}{n^2}\Pr\{\bar{\mathcal{H}}\} + \Pr\{\bar{\mathcal{H}}^c\}. \end{aligned}$$

Therefore, for a general cell V, the number of S-D lines passing through it is upper bounded by $\Theta(\mathbb{E}[I]) = O\left(n\mathbb{E}[D(X_i,Y_i)]\sqrt{a(n)} + na(n)\right)$ with probability $1-1/n^2$ conditioning on event $\bar{\mathcal{H}}$. Since $\bar{\mathcal{H}}$ happens w.h.p. and with the uniform bound the above bound holds uniformly for all cells in $\mathcal{O}$ with probability $1 - 1/n$, this completes the proof. ∎

Remark 1: Lemma 7 has an interesting implication: notice that the rank-based model follows power law and is in fact a heavy-tailed distribution. Therefore the tail of $D(X_i, Y_i)$ plays an important role and cannot be ignored. Hence the number of hops h_i is likely to deviate much from its expectation and its variance might not be bounded. These observations lead to concerns that whether the load of the cells are disproportional so bottlenecks may be formed in the network and capacity is reduced. The answer is no, according to Lemma 7, which shows that we still have nice convergence in probability uniformly over all cells in the network. This not only enables succinct capacity results to be obtained, but also implies that we can combat network heterogeneities and achieve load balancing using simple scheduling schemes.

Combining Lemma 5, 6 and 7, the following theorem is straightforward.

Theorem 2: The per-node throughput of the scheme for unicast is $\Omega\left(1/n(\mathbb{E}[D(X_i,Y_i)]\sqrt{a(n)} + a(n))\right) = \Omega\left((\log n + \sqrt{n\log n}\mathbb{E}[D(X_i,Y_i)])^{-1}\right)$. That is,

$$\lambda_u(n) \sim \begin{cases} \Omega(1/\log n) & \alpha > 3/2 \\ \Omega(1/\log^{\frac{3}{2}} n) & \alpha = 3/2 \\ \Omega(n^{\alpha-\frac{3}{2}}/\sqrt{\log n}) & 1 < \alpha < 3/2 \\ \Omega(\sqrt{\log n}/\sqrt{n}) & \alpha = 1 \\ \Omega(1/\sqrt{n\log n}) & 0 \leq \alpha < 1. \end{cases}$$

Comparing with the results in Theorem 1,

Corollary 1: The lower bounds in Theorem 2 is tight up to a logarithmic factor.

Remark 2: In fact except for the case that $\alpha > 3/2$, the lower bounds in Theorem 2 differ from the upper bounds in Theorem 1 by only a factor of $1/\sqrt{\log n}$. This well-known difference is due to the simplicity of the cell tessellation scheme that employs an almost uniform transmission range of $\Theta(\sqrt{\log n/n})$. However, such slight performance drawback

[6]It is easy to show that the same result holds for conditional expectation in our case. Refer to [41] for more about stochastic ordering.

[7]See [40] for details on the choosing technique.

can be eliminated by adopting a more sophisticated tessellation scheme and applying percolation theory in routing [42]. Though it is not our main focus, we remark that it is not difficult to extend percolation theory based schemes to our framework and achieve a throughput that strictly meets the upper bounds.

V. Multicast

As a major kind of traffic in many online networks, the information from a source is often desired to be disseminated to all its corresponding friends, such as tweets in Twitter and posts in Facebook. In this section we discuss the network throughput capacity under such traffic pattern, i.e., *multicast*.

Multicasting in traditional wireless network has been investigated in [32], [33]. Since relaying links may be shared by different destinations in a multicast session, multicast is more efficient and is able to achieve a better throughput than multiple unicast. A common approach for multicasting is to establish a spanning tree structure for routing.

However, comparing with traditional multicast, a major challenge that we face in studying multicast is that the number of destinations (friends) in each multicast session is assumed to be a random variable following a power-law distribution, while in previous related works it is assumed to be a fixed quantity. Intuitively, this implies that the multicast tree in our case is more random in size.

The problem is further complicated by the rank-based destination selection mechanism. In previous works, destinations in a multicast session are assumed to be selected independently and uniformly from the population, whereas under our framework they are selected in a much more complicated way. In fact, the rank-based selection mechanism implies that the locations of the destinations are subtly dependent, which cause significant difficulties in the analysis of multicast trees. In order to proceed, we have to therefore limit our analysis to the special case that $\alpha = 0$, such that the destinations are selected independently and uniformly over the whole network. We note that this degenerated rank-based model is equivalent to the uniform model widely adopted in related works, and more importantly, the simplification enable us to focus on the impact of power-law distributed destination numbers without entangling with the delicate multicast tree generated by the rank-based model. Our conjecture on the more general case of arbitrary α is proposed at the end of the section.

A. Upper Bound of Capacity

Consider a generic source X_i and denote $Y_i^1, Y_i^2 ... Y_i^{K_i}$ as its K_i friends (destinations). Denote EMST(U) as the Euclidean minimum spanning tree of set U, and $|\text{EMST}(U)|$ represents its total Euclidean edge lengths. The following lemma is a famous result on the asymptotic length of the Euclidean minimum spanning tree generated by i.i.d. point processes.

Lemma 8: Let Y_i, $1 \leq i < \infty$ be independent and identically distributed random variables in $\mathbb{R}^d$, $d \geq 2$, denote $M_k = |EMST(\{Y_1, ..., Y_k\})|$, then with probability 1,

$$\lim_{k\to\infty} M_k = c(d) n^{(d-1)/d} \int_{\mathbb{R}^d} f(x)^{(d-1)/d} \mathrm{d}x,$$

where f denotes the density of the distribution of Y_i and $c(d) > 0$ is a constant independent of k.

With $d = 2$ and $f(x) = 1$ in our case, it is clear that $M_k \sim \sqrt{k}$ as $k \to \infty$. Then we may compute the average length of the Euclidean minimum spanning tree covering the source X_i and its K_i destinations, where K_i follows power-law distribution with parameter β.

$$\begin{aligned}
&\mathbb{E}[\text{EMST}(X_i, Y_1, ... Y_{K_i})] \\
&\qquad = \mathbb{E}_{K_i}[\text{EMST}(X_i, Y_1, ... Y_k) | K_i = k] \\
&\qquad \sim \frac{1}{G_2} \sum_{k=1}^{n} \frac{\sqrt{k}}{k^\beta} \\
&\qquad \sim \begin{cases} 1 & \beta > 3/2 \\ \log n & \beta = 3/2 \\ n^{3/2-\beta} & 1 < \beta < 3/2 \\ \sqrt{n}/\log n & \beta = 1 \\ \sqrt{n} & 0 \leq \beta < 1. \end{cases}
\end{aligned}$$

Then with the minimum spanning tree we can establish a upper bound for the multicast capacity.

Lemma 9: Let $U_i = \{X_i, Y_i^1 ... Y_i^{K_i}\}$, if on average $|\text{EMST}(U)|$ is at least $\mathfrak{D}$, then $\lambda_m(n) = O(1/\mathfrak{D}\sqrt{n})$.

Proof: We define a multicast session as the duration from a packet arrives at the source till the packet is delivered to all destinations. Note that by the definition of Euclidean minimum spanning tree, in a multicast session the packet must be relayed over a distance of at least $|\text{EMST}(U)|$. Again consider a time interval T which is large enough such that the total number of packets transmitted between all multicast sessions is $c_P \lambda_m (n+1) T$. Denote h_p as the number of hops packet p is relayed, l_p^h as the transmission range of the hth hop, and $N_p = c_P \lambda_m (n+1) T$, it follows,

$$\begin{aligned}
\sum_{p=1}^{N_p} \sum_{h=1}^{h_p} l_p^h &\geq c_P \lambda_m T \sum_{i=1}^{n+1} |\text{EMST}(U_i)| \\
&\geq c_P \lambda_m T (n+1) \mathfrak{D}, \qquad (16)
\end{aligned}$$

where (16) follows from the strong law of large numbers because $|\text{EMST}(U_i)|$ are i.i.d. distributed and $\mathbb{E}[|\text{EMST}|] < \infty$. The rest part of the proof is clear by applying the same logic as Lemma 4. ■

Theorem 3: If the number of destinations per multicast session follows power-law distribution with parameter β, an upper bound of the per-node multicast capacity is

$$\lambda_m(n) \sim \begin{cases} O(1/\sqrt{n}) & \beta > 3/2 \\ O(1/\log n\sqrt{n}) & \beta = 3/2 \\ O(n^{\beta-2}) & 1 < \beta < 3/2 \\ O(\log n/n) & \beta = 1 \\ O(1/n) & 0 \leq \beta < 1. \end{cases}$$

B. Capacity Achieving Scheme

For multicasting, the cell partition TDMA scheme that we employ in Section III.C is still highly efficient for scheduling active transmissions in the network. However, routing becomes a major issue in multicast since an optimal routing tree needs to be constructed. Our main idea is to first construct a Euclidean spanning tree using Prim's algorithm, and then

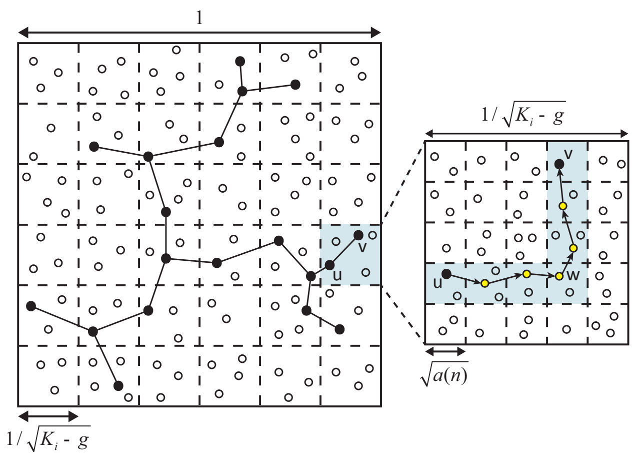

Fig. 5. Multicast routing tree and multi-hop scheme in step 2.

convert it to a multicast routing tree.

Optimal Routing Tree for Multicast Session U_i:

1. Construct a spanning tree using Prim's algorithm:
(1)Initially, nodes in U_i form K_i components. Set $g = 1$.
(2)Partition the network into at most $K_i - g$ squares, such that their side length is $1/\sqrt{K_i - g}$[8].
(3)Find a square that contains two nodes from two different connected components. Merge the two components by adding a edge between the two nodes.
(4)Return the constructed tree ST(U_i) if $g = K_i - 1$, otherwise $g := g + 1$ and goto step (2).
2. Tessellate the network extension into squarelets (cells) with area $a(n)$. For each edge uv in ST(U_i), arbitrarily select a point w in the cell that lies in the same row as u and the same column as v, select a node in each of the cells that uw and wv crosses and connect these nodes to form a path from u to v.
3. Combine the paths and remove cycles, if any. Return the obtained multicast routing tree MRT(U_i). Notice that in Step 1-(3), the square exists due to Pigeonhole principle, and in step 2, the node exists as long as the connectivity criterion $a(n) = \Omega(\log n/n)$ is satisfied.

Intuitively, we can use these cells (with area $a(n)$) as scheduling units and employ the TDMA scheduling scheme proposed in Section III.C, and route the packets along tree MRT(U_i). In order to analyze throughput, it is important to study the "load" of each cell under these schemes.

Lemma 10: Given an arbitrary cell $\mathbf{s}$, the probability that a multicast session U_i is routed throughput $\mathbf{s}$ is upper bounded by $c_3\overline{|\text{EMST}(U_i)|}\sqrt{a(n)}$.

Proof: Notice that the construction of MRT(U_i) consists of $K_i - 1$ steps, and $\mathbf{s}$ may be invoked in any of these steps. Denote I_g as the indicator that whether $\mathbf{s}$ is invoked in step g, it follows,

$$\Pr\{I_g = 1|K_i\} = \frac{1}{K_i - g} \cdot p_{\mathbf{s}}(g),$$

[8]More strictly it should be $\left\lceil 1/\sqrt{\lfloor K_i - g\rfloor}\right\rceil$, but we assume it to be an integer for the ease of presentation.

where $1/(K_i - g)$ is the probability that the square (with side length $1/\sqrt{K_i - g}$) containing $\mathbf{s}$ is selected in the gth iteration of Prim's algorithm, and $p_{\mathbf{s}}$ is the probability that $\mathbf{s}$ is selected in this square. Within this square which is further tessellated into cells with area $a(n)$, assume that $\mathbf{s}$ is in the pth row and qth column, it follows,

$$\begin{aligned} p_{\mathbf{s}}(g) = &(p-1)a^{\frac{3}{2}}(K_i - g)^{\frac{3}{2}} \cdot \left(\frac{1}{\sqrt{a(K_i - g)}} - p + 1\right) \\ &+ (q-1)a^{\frac{3}{2}}(K_i - g)^{\frac{3}{2}} \cdot \left(\frac{1}{\sqrt{a(K_i - g)}} - q + 1\right) \\ \leq &\, 2\sqrt{a(n)(K_i - g)}, \end{aligned} \tag{17}$$

where the first (resp. second) term in (17) is the probability that $\mathbf{s}$ lies in the same row (resp. column) as u (resp. v). Therefore,

$$\begin{aligned} \Pr\{\mathbf{s} \text{ selected by } U_i\} &\leq \sum_{g=1}^{K_i - 1} \Pr\{I_g = 1|K_i\} \\ &\leq \sum_{k=1}^{n+1}\sum_{g=1}^{k-1} \frac{2\sqrt{a(n)(k-g)}}{k-g} \Pr\{K_i = k\} \\ &\leq \sum_{k=1}^{n+1} 4\sqrt{2ka(n)} \Pr\{K_i = k\} \\ &= \frac{4\sqrt{2a(n)}}{c(d)} \overline{|\text{EMST}(U_i)|}. \end{aligned}$$

The lemma holds by setting $c_3 = 4\sqrt{2a(n)}/c(d)$. ■

Theorem 4: Denote $N(\mathbf{s})$ as the number of multicast sessions that invoke $\mathbf{s}$ for routing, then uniformly over all squarelets, it follows,

$$\lim_{n\to\infty} \Pr\left\{\cap_{\mathbf{s}}\left\{N(\mathbf{s}) \leq c_4 n \overline{|\text{EMST(U)}|}\sqrt{a(n)}\right\}\right\} = 1,$$

where c_4 is a positive constant.

Proof: Given an squarelet $\mathbf{s}$, by definition:

$$N(\mathbf{s}) = \sum_{i=1}^{n+1} \mathbf{1}_{\{\mathbf{s} \text{ invoked by } U_i\}},$$

where $\mathbf{1}_{\{\mathbf{s} \text{ invoked by } U_i\}}$ are i.i.d. Bernoullian random variables with mean $p_1 \leq c_3\overline{|\text{EMST}(U_i)|}\sqrt{a(n)} = p_2$. Denote $N^*(\mathbf{s})$ as the corresponding sum of i.i.d. Bernoullian random variables with mean p_2, then clearly $N^*(\mathbf{s})$ is statistically larger than $N(\mathbf{s})$. By applying Chernoff bounds we get,

$$\begin{aligned} \Pr\{N(\mathbf{S}) > 2\mathbb{E}[N^*(\mathbf{S})]\} &< \Pr\{N^*(\mathbf{S}) > 2\mathbb{E}[N^*(\mathbf{S})]\} \\ &< (e/4)^{np_2} < e^{-np_2/8}. \end{aligned}$$

Since $\overline{|\text{EMST}(U_i)|} \geq \Theta(1)$, $a(n) > \Theta(\log n/n)$,

$$\begin{aligned} &\Pr\left\{\cap_{\mathbf{s}}\left\{N(\mathbf{s}) \leq 2c_3 n \overline{|\text{EMST(U)}|}\sqrt{a(n)}\right\}\right\} \\ &\geq 1 - \sum_{\mathbf{s}} \Pr\{N(\mathbf{S}) > 2\mathbb{E}[N^*(\mathbf{S})]\} \\ &\geq 1 - ne^{-\sqrt{n\log n}/8} \to 1 \quad \text{as } n \to \infty. \end{aligned}$$

Setting $c_4 = 2c_3$ finishes the proof. ■

Lastly, by employing a TDMA scheme as Section 3.3 such that every squarelet has a constant fraction of time to transmit, and further dividing a time slot into minislots such that a squarelet can deliver the traffic for every multicast session that invokes it, we have,

Theorem 5: The per-node throughput of the scheme for multicast is $\Omega\left(1/n\overline{|\text{EMST}(U)|}\sqrt{a(n)}\right) = \Omega\left(1/\sqrt{n\log n}\overline{|\text{EMST}(U)|}\right)$, i.e.,

$$\lambda_m(n) \sim \begin{cases} \Omega(1/\sqrt{n\log n}) & \beta > 3/2 \\ \Omega(1/\log^{3/2} n\sqrt{n}) & \beta = 3/2 \\ \Omega(n^{\beta-2}/\sqrt{\log n}) & 1 < \beta < 3/2 \\ \Omega(\sqrt{\log n}/n) & \beta = 1 \\ \Omega(1/n\sqrt{\log n}) & 0 \leq \beta < 1. \end{cases}$$

Corollary 2: The lower bound in Theorem 5 is tight up to a logarithmic factor.

Remark 3: Again, notice that there is a small gap of $\sqrt{\log n}$ between the upper and lower bounds. This gap can be eliminated by modifying our communication scheme using percolation theory.

Remark 4: The results in Theorem 5 match well with Theorem 2 when $\alpha < 1$ and $\beta > 3/2$. For more sophisticated cases that α takes an arbitrary value, the multicast capacity is difficult to be obtained since calculating $\text{EMST}(U_i)$ appears intimidating. However, the nice consistency of the results in Theorem 5 with that in Theorem 2 helps us to conjecture a more general result for arbitrary a. Interestingly, the results in Theorem 5 match well with Theorem 2 when $\alpha < 1$ and $\beta > 3/2$.

Conjecture 1: If the number of destinations per multicast session follows a power-law distribution with parameter β, and each destination is selected according to the rank-based model with parameter α, then the multicast capacity is $\Theta(1/\overline{|D(X_i, Y_i)|}\ \overline{|\text{EMST}(U_i)|}\sqrt{n})$.

VI. Conclusion And Future Works

This paper studies the throughput capacity of wireless networks with social characteristics. We propose a simple model which captures the two key characteristics observed in real large scale networks, i.e., the way people selects friends and the number of friends, and examine their impact on capacity. We show the fact that social relations leads to traffic locality and improves capacity in wireless networks. In addition, in the common traffic pattern where information is desired to be disseminated from the source to all its contacts (e.g., friends, fans or followers) whose number follows power-law distribution, multicast may be employed to further enhance performance.

There are still many interesting directions for us to explore in the future. As is mentioned in previous sections, The network performance may be quite different from that obtained under static model. Furthermore, it is also interesting to take energy consumption and delay performance in mobile network into consideration. Because energy-efficiency and latency minimization are both hot topic in recent study of wireless networks.

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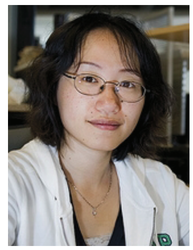

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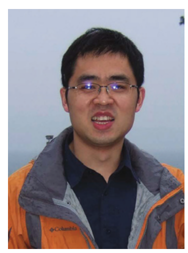

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