HW-5

Exercises for Fundamentals in Data Science, 2017 Summer Semester

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1. **EX5.5**

Solution. If the parity function $f(x)$ on $d \geq 2$ Boolean variables $x = (x_1, x_2, ..., x_d)$, can be represented by a linear threshold function. Then we can get

$$f(x) = W^T x = w_1 x_1 + w_2 x_2 + ... + w_d x_d.$$

Now consider $x = (1, 0, 0, ..., 0)$, by conditions we have $f(x) = w_1 x_1 = w_1 = 1$. By using same method, we can get $w_1 = w_2 = ... = w_d = 1$. That means $f(x) = x_1 + x_2 + ... + x_d$. But then we choose $x = (1, 1, 0, 0, ..., 0)$, we have the contradiction that $1 + 1 = 2 = f(x) = 0$. Therefore, the assumption is wrong.

□

2. **EX5.9**

Solution. (a)

$$K_3(x, y) = f(x)f(y)K_1(x, y) = f(x)f(y)\phi_1^T(x)\phi_1(y) = (f(x)\phi_1(x))^T(f(y)\phi_1(y)) = \phi_3^T(x)\phi_3(y)$$

(b)

$$\phi_1^T(x)\phi_1(y) + \phi_2^T(x)\phi_2(y) = [\phi_1(x), \phi_2(x)]^T[\phi_1(y), \phi_2(y)]$$

Let $\phi_3(x) = [\phi_1(x), \phi_2(x)]$, we have $K_1 + K_2 = \phi_3^T(x)\phi_3(y)$.

(c)

$$K_1K_2 = \phi_1^T(x)\phi_1(y)\phi_2^T(x)\phi_2(y) = [\phi_2(x)\phi_1(x)]^T[\phi_2(y)\phi_1(y)]$$

Let $\phi_4(x) = \phi_1(x)\phi_2(x)$, we can get $K_1K_2 = \phi_4^T(x)\phi_4(y)$.

□

3. **EX5.10**

Solution. For a set of 2 points, there exists a way to shatter them. But for any sets larger than 2, it can't be shattered by H. Because when we have at least three points named x_1, x_2, x_3, we can assume that x_3 have the coordinates equals to $max(x_1, x_2)$'s coordinates. Then we can not shatter these three points because we can not get a subset only contain x_3. Hence $H[n] = O(n^2)$. □

4. **EX5.11**

Solution. (a) $H[n] = 2d$ Let the $2d$ points be unit vectors in both directions of d coordinates. Let S denote this set. Then for any subset of S one can easily create an axis-parallel rectangle by examining each coordinate and choose a_i, b_i satisfying $a_i \leq min\{S_i\}$, $b_i \geq max\{S_i\}$ and $a_i \geq max\{\bar{S}_i\}$, $b_i \leq min\{\bar{S}_i\}$.

(b) If there are $2d + 1$ points, consider the minimum box that contains all the points. Since there can be at most $2d$ points on the box's surface, at least one point named A must in the box. Then $S/\{A\}$ can not be included by a box, since any box enclose $S/\{A\}$ must enclose A.

□