Homework 2 Solution Reference

1. (2.15).
   The surface area in terms of $V(d)$ and $A(d)$:
   $area = A(d-1) \cdot r^{d-1} + hA(d-2) \cdot r^{d-2}$
   $volume = r^d V(d) + hA(d-1) \cdot r^{d-1}$

2. (2.16).(The answer is just as a reference. It may be incorrect.)
   Volume=$V(d) \times 2^d = \frac{2\pi^{\frac{d}{2}}}{d\Gamma(\frac{d}{2})} = \frac{2\pi^{\frac{d}{2}}}{(\frac{d}{2})!}2^d \approx \frac{\pi^{\frac{d}{2}}}{(\frac{d}{2e})^{\frac{d}{2}}}2^d = (\frac{8\pi e}{d})^{\frac{d}{2}}$
   when radius is r, the volume should be:
   $volume = (\frac{4\pi re}{d})^{\frac{d}{2}} \Rightarrow ln(volume) = \frac{d}{2}(ln(4\pi re) - ln(d))$
   only when $ln(4\pi re) - ln(d) = \frac{1}{d}c$, that is $4\pi re = de^{\frac{c}{d}}$
   So only when $r \to +\infty$, the volume is a constant.

3. (2.21).

   (a) Give $f(x) = e^x - x - 1$, So $f'(x) = e^x - 1 \begin{cases} > 0 & (x > 0) \\ = 0 & (x = 0) \\ < 0 & (x < 0) \end{cases}$
   $\because f(x)$ has minimal value at point $x = 0$, as $(x) = 0$,
   $\therefore e^x \geq x + 1$ is always achieved.

   (b) Taylor expression of e^x: $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots + \frac{x^k}{k!} + \cdots$
   $\therefore | e^x - (x+1) | \approx \frac{x^2}{2} \leq 0.01 \Rightarrow x \in [-\frac{\sqrt{2}}{10}, \frac{\sqrt{2}}{10}]$

4. (2.27).
   Extract the features of every web pages and transform them into 0,1 vectors;
   Calculate the distance of each two web pages;
   If the distance > the setting threshold value, then there are similar. Otherwise, they are not similar.

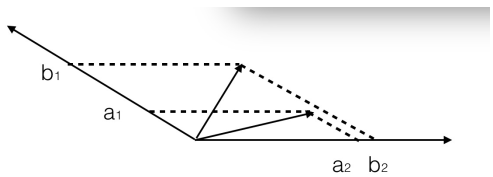
5. (2.38).

   (a) Prove that the representation of any vector in this basis is unique:
   Suppose $\vec{a} = a_1\vec{e_1} + a_2\vec{e_2} + \cdots + a_d\vec{e_d} = \vec{b} = b_1\vec{e_1} + b_2\vec{e_2} + \cdots + b_d\vec{e_d}$
   $a_1, a_2, \cdots, a_d$ and $b_1, b_2, \cdots, b_d$ are not same.
   $\therefore \vec{0} = (a_1 - b_1)\vec{e_1} + (a_2 - b_2)\vec{e_2} + \cdots + (a_d - b_d)\vec{e_d}$
   $\because \vec{e_1}, \vec{e_2}, \cdots \vec{e_d}$ are linear independent
   $\therefore a_1 - b_1 = 0, a_2 - b_2 = 0, \cdots, a_d - b_d = 0$, it is a contradiction.
   So the assumption is not achieved, the representation of any vector in this basis is unique.

   (b) $\vec{Z} = (\frac{\sqrt{2}}{2}, 1)\binom{\vec{e_1}}{\vec{e_2}} = \frac{\sqrt{2}}{2}(1,0) + (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) = (0, \frac{\sqrt{2}}{2})$
   $\left|\vec{Z}\right|^2 = \frac{1}{2}$

   (c) It is Not necessary.
   as shown in the figure, $a_1 < b_1$ and $a_2 < b_2$, but $|Z| < |Y|$

   (d) suppose $(0,1) = a_1\vec{e_1} + a_2\vec{e_2}, then \Rightarrow a_1 = 1, a_2 = \sqrt{2} \Rightarrow (0,1) = (1, \sqrt{2})_e$
   suppose $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) = a_1\vec{e_1} + a_2\vec{e_2}, then \Rightarrow a_1 = \sqrt{2, a_2 = 1} \Rightarrow (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) = (\sqrt{2}, 1)_e$
   suppose $(1,2) = a_1\vec{e_1} + a_2\vec{e_2}, then \Rightarrow a_1 = 3, a_2 = 2\sqrt{2} \Rightarrow (1,2) = (3, 3\sqrt{2})_e$


b1
a1
a2 b2