

HW1

2.1

$$1) E(x)=1/2, \quad E(x^2)=1/3, \quad E(x-y)=0, \quad E(xy)=1/4, \quad E(x-y)^2=1/6$$

$$2) E(x)=0, \quad E(x^2)=1/12, \quad E(x-y)=0, \quad E(xy)=0, \quad E(x-y)^2=1/6$$

3) 设 $A(a_1, \dots, a_d), B(b_1, \dots, b_d)$ 为两个随机的点, 其中 $a_i \sim U(0,1), b_i \sim U(0,1)$

$$|AB|^2 = \sum_{i=1}^d (a_i - b_i)^2$$

$$E((a_i - b_i)^2) = E(a_i^2) + E(b_i^2) - 2E(E(a_i b_i)) = 1/6$$

$$E(|AB|^2) = d/6$$

2.5

$$1) P(x \geq 3) \leq \frac{E(x)}{3} = \frac{2}{3}$$

$$2) P(x \geq 3) = P(x^2 \geq 9) \leq \frac{E(x^2)}{9} = \frac{16}{27}$$

$$3) \frac{4^r}{3^{r(r+1)}}$$

2.11

$$1) 2^{d-k} \binom{d}{k}$$

$$2) \sum_{k=0}^d 2^{d-k} \binom{d}{k} = 3^d$$

$$3) A(d) = 2d * V(d-1) = 2d * 1^d = 2d$$

$$4) A(d) = 2d * V(d-1) = 2d * 2^d = 2^d * d$$

$$5) V(d) = 1^d = 1,$$

令所有的边长度缩短为 $1 - \varepsilon$, 则体积变成: $V'(d) = (1 - \varepsilon)^d$

$$\frac{V'(d)}{V(d)} = (1 - \varepsilon)^d \rightarrow 0, \text{ 当 } d \rightarrow \infty$$

2.12

$$dS = r \sin \theta * r d\theta * d\varphi = r^2 \sin \theta d\theta d\varphi$$

$$S = \int_0^{2\pi} d\varphi \int_0^\theta r^2 \sin \theta d\theta = 2\pi r^2 (1 - \cos \theta)$$

$$\text{当 } \theta = 36^\circ, r = 1, S \approx 1.2$$

2.14

$$\text{horizontal plane: } A(3) = 4\pi$$

$$\text{half-circular ball: } \frac{A(4)}{2} = \frac{2\pi^2}{2\Gamma(2)} = \frac{\pi^2}{2}$$