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A General Privacy Preserving Auction Mechanism for Secondary Spectrum Markets

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***Abstract*—Auctions are among the best-known market-based tools to solve the problem of dynamic spectrum redistribution. In recent years, a good number of strategy-proof auction mechanisms have been proposed to improve spectrum utilization and to prevent market manipulation. However, the issue of privacy preservation in spectrum auctions remains open. On the one hand, truthful bidding reveals bidders' private valuations of the spectrum. On the other hand, coverage/interference areas of the bidders may be revealed to determine conflicts.**

In this paper, we present PISA, which is a PrIvacy preserving and Strategy-proof Auction mechanism for spectrum allocation. PISA provides protection for both bid privacy and coverage/interference area privacy leveraging a privacy preserving integer comparison protocol, which is well applicable in other contexts. We not only theoretically prove the privacy preserving properties of PISA, but also extensively evaluate its performance. Evaluation results show that PISA achieves good spectrum allocation efficiency with light computation and communication overheads.

I. Introduction

INDUSTRY experts indicate that the fast growing wireless technology is being stalled by the scarcity of radio spectrum [10]. This scarcity is often considered to be a result of the static and rigid spectrum allocation by the government. The spectrum may be idle when the primary users are not engaged in data transmission, while at the same time many unlicensed users are starving for radio spectrum. Such a static allocation mechanism can not fully utilize the limited spectrum. In order to improve spectrum utilization, secondary spectrum markets have emerged, where auctions are used to dynamically redistribute channels (*e.g.*, [2, 8, 9, 12, 13, 33, 36, 37, 46]). Different from the auctions held by the government, auctions in secondary spectrum markets occur dynamically and more frequently. The auctioneers may be primary users who tend to lease their channels in order to receive proper payoff during their idle time. The bidders may be secondary wireless service providers that need spectrum to serve their subscribers, or a mobile device that needs spectrum to transmit data.

In spectrum auctions, strategy-proofness (defined in Section II) is the topic of major research efforts, which stimulates bidders to bid their true valuations of the spectrum. It eliminates the overhead of gaming over each other and the auctioneer can allocate the channels to who value it the most. However, different from the primary spectrum auctions where all bids are open and auction results are posted on FCC web pages, there are privacy concerns in secondary spectrum auctions. Truthful bidding divulges the bidder's true valuations towards the spectrum, which are closely related to the profits of winning the spectrum. Bidders are not willing to share such information with other bidders or the auctioneer. Let us consider that the auctioneer is a cellular network provider, named A, and another cellular network provider B participates in the auction as a bidder to request for channels to transmit data. The bid may imply B's economic situation, which is highly sensitive information. B is reluctant to disclose it to the auctioneer (*i.e.* A), who is a competitor in some sense. Furthermore, corrupt auctioneers may exploit such knowledge to their advantage. For instance, the auctioneers/sellers may set the reserve price accordingly in future auctions to increase their own revenue. Unfortunately, most existing works fail to protect bid privacy in auction design.

Moreover, spectrum allocation may disclose the bidders' coverage/interference areas. Spectrum is spatially reusable. Two bidders distanced by space can simultaneously use the same channel for transmission. In auctions, bidders may be required to reveal their coverage/interference areas to the auctioneer to determine conflict. However, coverage/interference areas may divulge the location information of the bidders, especially when the bidders are mobile devices. It may also disclose other sensitive information, such as their business models and subscriber distribution. Thus, bidders are reluctant to share their coverage/interference information with the auctioneer.

Therefore, privacy preservation and strategy-proofness are both important factors in designing spectrum auctions. However, there are several challenges. First, due to the spatial reusability of spectrum, well-separated bidders can share the same channel. Existing privacy preserving auctions (*e.g.*, [4, 28]) are designed for traditional goods (*e.g.*, paintings, jewelry), where each commodity can only be allocated to one

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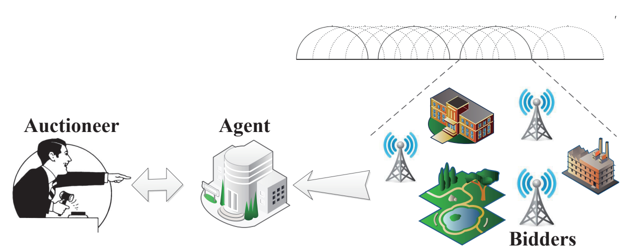

Fig. 1. System Architecture

bidder. When it comes to spectrum auctions, they may either fail or lead to significant degradation of spectrum utilization. Second, strategy-proofness and bid privacy are somewhat contradictory objectives. Strategy-proofness encourages bidders to reveal their true valuations of the spectrum, while bid privacy tends to prevent the auctioneer and other participants from learning the bidders' true valuations. Third, different from conventional auctions, spectrum allocation is constrained by geographic conditions. The allocation process should satisfy the geographic constraints while preserving bidders' coverage/interference area privacy.

In this paper, we consider the problem of privacy preservation in spectrum auctions, and propose PISA, which is a PrIvacy preserving and Strategy-proof Auction mechanism for secondary spectrum markets. As shown in Figure 1, we introduce a third party (*e.g.*, [21, 22, 32]), namely an agent, who acts as an intermediary between the bidders and the auctioneer. The agent should be non-profit and we require the agent to be a well-established organization. Therefore, some trustworthy non-profit organizations are suitable to play the role of the agent, such as Spectrum Bridge [26]. Although the agent may be a well-established party, bidders are still reluctant to share private information with any party, the agent being no exception. Thus, in PISA, the agent and the auctioneer cooperate to perform the auction, but neither of them can infer any sensitive information about the bidders without collusion. The essence of PISA lies in our privacy preserving bid comparison protocol, and we further extend the protocol to privately determine geographic conflict.

We summarize our contributions as follows:

1) To the best of our knowledge, PISA is the first strategy-proof spectrum auction mechanism that protects both bid privacy and coverage/interference area privacy without sacrificing social welfare.
2) We present a protocol to perform efficient comparison between integers without revealing their actual values. Our protocol can compare arbitrary large integers and is well applicable in other contexts.
3) We implement PISA and extensively evaluate its performance. The evaluation results show that PISA achieves good channel utilization, with low computation and communication overhead.

The rest of the paper is organized as follows. In Section II, we review technical preliminaries. In Section III, we present the detailed design of basic PISA, which preserves bid privacy of the winners. In Section IV, we enhance PISA to provide stronger privacy protection (*i.e.*, coverage/interference area privacy and k-anonymous bid privacy for all bidders). We present our evaluation results in Section V. In Section VI, we briefly review related works. Finally, we conclude and point out future research directions in Section VII.

II. Preliminaries

In this section, we first briefly review some solution concepts and present our auction model. Then, we define a generic strategy-proof spectrum auction mechanism. Finally, we introduce a useful homomorphic cryptosystem.

A. Solution Concepts

We recall the solution concepts used in our study. Let s_i denote player i's preference strategy and s_{-i} denote the strategy profile of all the players except for player i. $u_i(s_i, s_{-i})$ is the utility of player i when the strategy of player i is s_i and the strategies of all other players are s_{-i}.

Definition 1 (Incentive Compatible [23]). *A mechanism is incentive compatible if for any strategy $s_i' \neq s_i$ and any other players' strategy profile s_{-i}, the utility u_i of the player i always satisfies the following condition:*

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}).$$

Intuitively, in an incentive compatible mechanism, players can maximize their utilities by reporting truthful preference information, regardless of other players' strategy profiles. Next, we introduce another related concept.

Definition 2 (Individual Rational [23]). *For any strategy s_i and any other players' strategy profile s_{-i}, the utility u_i of the player i always satisfies the following condition:*

$$u_i(s_i, s_{-i}) \geq 0.$$

A mechanism is individually rational if each player always gets a non-negative utility, which means that each player can gain no less utility from faithful participation than non-participation. We now give the formal definition of *strategy-proof mechanism*.

Definition 3 (Strategy-Proof Mechanism [19][30]). *A mechanism is strategy-proof when it satisfies both incentive-compatibility and individual-rationality.*

In a strategy-proof mechanism, misbehavior cannot result in any extra profit. Each player can maximize her utility by truthful participation.

In the field of privacy preservation, *k-anonymity* [29] is widely used to quantify the degree of privacy preservation (*e.g.*, [39]). A scheme provides k-anonymous protection when a person cannot be distinguished from at least $k-1$ other individuals.

Definition 4 (k-anonymity [29]). *A privacy preserving scheme satisfies k-anonymity, if a participant cannot be identified by the sensitive information with probability higher than $1/k$.*

B. Auction Model

We model the procedure of spectrum allocation as a sealed-bid auction, involving an auctioneer, an agent, and a number of bidders. For clarity, we assume that there is a single channel to be shared among the bidders[1].

The auctioneer may be a primary user who tends to lease her idle channel to a group of conflict-free secondary users, in order to receive proper payoff during her idle time. The auctioneer may also be a specialized third-party platform for spectrum management, such as Spectrum Bridge [26]. Bidders simultaneously submit their encrypted bidding tuples via the agent to the auctioneer. The auctioneer decides the winners and their charges.

We consider that there is a set $\mathbb{B} = \{1, 2, \cdots, z\}$ of bidders. Let $\vec{v} = (v_1, v_2, \cdots, v_z)$ denote the valuation profiles of the bidders, which is their private information. Accordingly, the bidders' bidding profile is denoted by $\vec{b} = (b_1, b_2, \cdots, b_z)$. Let $\vec{x} = (x_1, x_2, \cdots, x_z)$ and $\vec{y} = (y_1, y_2, \cdots, y_z)$ denote the vector of bidders' latitudes and longitudes, respectively. Bidder i can share the channel with bidder j, if their coverage/interference areas do not overlap.

The auctioneer determines the charging profile $\vec{p} = (p_1, p_2, \cdots, p_z)$ and the allocation profile $\vec{a} = (a_1, a_2, \cdots, a_z)$, where $a_i = 1$ indicates that bidder i is allocated the channel, while $a_i = 0$ indicates not. The utility of bidder i can be defined as

$$u_i = (v_i - p_i)a_i.$$

The goal of all bidders is to maximize their own utilities. Here, we assume that the bidders do not collude with each other.

In contrast to the bidders, the overall objective of the auction is to guarantee strategy-proofness, and to maximize channel allocation efficiency subject to the conflicting conditions.

C. A Generic Spectrum Auction Scheme

As pointed out in [40], the spatial interference constraints make the problem of finding the optimal allocation in the above auction model NP-complete. A practical solution is to resort to monotonic allocation in order to improve computation efficiency, and to apply critical charging to guarantee strategy-proofness. In this subsection, we present a representative auction scheme that achieves both strategy-proofness and computational efficiency.

We model the geographic conflicts of the bidders as a conflict graph $\mathbb{G}$. On the conflict graph, each bidder is represented by a vertex. Two bidders (vertices) are connected if their coverage/interference areas overlap. Two important concepts are *critical neighbor* and *critical value*.

Definition 5 (Critical Neighbor [40]). *Given $\mathbb{G} \setminus \{i\}$, the critical neighbor $CN(i)$ of bidder i is a neighbor of i where if i bids lower than $CN(i)$, i will not be allocated, and if i bids higher than $CN(i)$, i will be allocated.*

Definition 6 (Critical Value [40]). *The critical value of bidder i is defined as the bid of $CN(i)$; if $CN(i)$ does not exist, the critical value of i is 0.*

Algorithm 1 describes the monotonic allocation procedure, where $\mathbb{A}$ denotes the set of available bidders and $N(i)$ denotes the set of neighbors of bidder i in $\mathbb{G}$. Lines 2-6 iteratively allocate the channel to the highest bidder i in $\mathbb{A}$, and eliminate i and her neighbors from further consideration. The algorithm stops when there is no bidder left in $\mathbb{A}$.

Algorithm 1 Monotonic Allocation Algorithm

Input: Conflict graph $\mathbb{G}$, bidder set $\mathbb{B}$, and bidding profile $\vec{b}$.
Output: Allocation profile $\vec{a}$.
1: $\mathbb{A} \leftarrow \mathbb{B}$; $\vec{a} \leftarrow 0^z$.
2: **while** $\mathbb{A} \neq \emptyset$ **do**
3: $\quad i \leftarrow \underset{j \in \mathbb{A}}{\operatorname{argmax}}(b_j)$.
4: $\quad a_i \leftarrow 1$.
5: $\quad \mathbb{A} \leftarrow \mathbb{A} \setminus (N(i) \cup \{i\})$.
6: **end while**
Return $\vec{a}$.

Algorithm 2 shows the critical charging procedure. Lines 5-11 determine the critical value of bidder i. Each winner is charged with his/her critical value. In the case of a tie, we may break the tie either randomly or by the bidders' identifiers.

Algorithm 2 Critical Charging Algorithm

Input: Conflict graph $\mathbb{G}$, bidder set $\mathbb{B}$, bidding profile $\vec{b}$, allocation profile $\vec{a}$, and bidder i.
Output: Payment p_i.
1: **if** $a_i = 0$ **then**
2: $\quad$ **Return** 0.
3: **end if**
4: $\mathbb{A} \leftarrow \mathbb{B} \setminus \{i\}$.
5: **while** $\mathbb{A} \neq \emptyset$ **do**
6: $\quad k \leftarrow \underset{j \in \mathbb{A}}{\operatorname{argmax}}(b_j)$.
7: $\quad$ **if** $k \in N(i)$ **then** $\quad \triangleright\ CN(i) = k$.
8: $\quad\quad$ **Return** b_k.
9: $\quad$ **end if**
10: $\quad \mathbb{A} \leftarrow \mathbb{A} \setminus (N(k) \cup \{k\})$.
11: **end while**
Return 0.

Here we have the first theorem. Please refer to [40] for the proof.

Theorem 1. *The generic spectrum auction mechanism is strategy-proof.*

We note that the generic spectrum auction mechanism may reveal the bidders' private information to the auctioneer. We present our approaches to protect bidders' privacy in Section III and Section IV.

D. Boneh-Goh-Nissim (BGN) cryptosystem

Homomorphic encryption is a form of encryption that enables specific types of computations to be carried out

[1]For multi-channel case, we can change the monotonic allocation algorithm and the critical charging algorithm according to [40], and apply our privacy preserving bid comparison protocol.

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on ciphertexts, and obtain a new ciphertext, which can be decrypted to match the result of computations applied directly to the original plaintexts.

In this work, we adopt Boneh-Goh-Nissim (BGN) cryptosystem [3]. It supports computations of unlimited number of additions with at most one multiplication. Thus it can evaluate quadratic multi-variate polynomials on ciphertexts.

Before introducing the BGN cryptosystem, we recall *bilinear map* and *bilinear group*, which are the bases of the BGN cryptosystem.

Definition 7 (Bilinear Map [3]). *Let $\mathbb{G}_1$ and $\mathbb{G}_2$ be two cyclic groups of order n, for some large n. A map $e : \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_2$ is said to be bilinear if*

$$e(P^a, Q^b) = e(P, Q)^{ab},$$

for all $P, Q \in \mathbb{G}_1$ and $a, b \in \mathbb{Z}$.

Definition 8 (Bilinear Group [3]). *$\mathbb{G}_1$ is a bilinear group if there exists a group $\mathbb{G}_2$ and a bilinear map e, s.t.,*

1) *$\mathbb{G}_1$ and $\mathbb{G}_2$ are two multiplicative cyclic groups of finite order n.*
2) *g is a generator of $\mathbb{G}_1$.*
3) *$e : \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_2$ is a bilinear map and $e(g, g)$ is a generator of $\mathbb{G}_2$.*

Given n, [3] presents an approach to constructing a bilinear group of order n. Due to limitations in space, we do not elaborate on it here. Next, we describe the three algorithms making up the BGN cryptosystem.

KeyGen(τ): Given a security parameter $\tau \in \mathbb{Z}^+$, generate two random τ-bit primes q_1, q_2, and set $n = q_1 q_2 \in \mathbb{Z}$. Generate a bilinear group $\mathbb{G}_1$ of order n as described in [3]. Let $e : \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_2$ be the bilinear map.

Randomly pick two generators g, u from $\mathbb{G}_1$ and set $h = u^{q_2}$. Then h is a random generator of a q_1-order subgroup of $\mathbb{G}_1$. The public key is $\mathcal{PK} = (n, \mathbb{G}_1, \mathbb{G}_2, e, g, h)$. The private key is $\mathcal{SK} = q_1$.

Encrypt($\mathcal{PK}, m$): We assume that the message space consists of integers from $\{0, 1, \cdots, T\}$ with $T < q_2$. To encrypt a message m with the public key $\mathcal{PK}$, pick a random integer r from $\mathbb{Z}_n$, and compute

$$C = g^m h^r \in \mathbb{G}_1.$$

Here, C is the ciphertext of m.

Decrypt($\mathcal{SK}, C$): To decrypt a ciphertext C using the private key $\mathcal{SK} = q_1$, note that

$$C^{q_1} = (g^m h^r)^{q_1} = (g^{q_1})^m.$$

To recover m, it suffices to compute the discrete log of C^{q_1} in base g^{q_1}. Although it appears inefficient to do decryption, BGN cryptosystem is well suited to our scenario. In our application, we only need to decide whether a ciphertext is an encryption of 0 or not.

Next, we show the homomorphic properties of BGN cryptosystem. Given ciphertexts $C_1 = g^{m_1} h^{r_1} \in \mathbb{G}_1$ and $C_2 = g^{m_2} h^{r_2} \in \mathbb{G}_1$,

$$C_1 \oplus C_2 = C_1 C_2 h^r = g^{m_1} h^{r_1} g^{m_2} h^{r_2} \cdot h^r = g^{m_1+m_2} h^{r_1+r_2+r}$$

is the ciphertext of $m_1 + m_2$ for a random $r \in \mathbb{Z}_n$.

Furthermore, BGN cryptosystem allows one multiplication using the bilinear map. For h is of order q_1, we rewrite $h = g^{\alpha q_2}$ for some unknown $\alpha \in \mathbb{Z}$. Set $\hat{g} = e(g, g)$ and $\hat{h} = e(g, h) = \hat{g}^{\alpha q_2}$. Hence, $\hat{g}$ is of order n and $\hat{h}$ is of order q_1. Pick a random $r' \in \mathbb{Z}_n$, then

$$\begin{aligned} C_1 \otimes C_2 &= e(C_1, C_2)\hat{h}^{r'} = e(g^{m_1} h^{r_1}, g^{m_2} h^{r_2})\hat{h}^{r'} \\ &= e(g^{m_1+\alpha q_2 r_1}, g^{m_2+\alpha q_2 r_2})\hat{h}^{r'} \\ &= e(g, g)^{(m_1+\alpha q_2 r_1)(m_2+\alpha q_2 r_2)}\hat{h}^{r'} \\ &= e(g, g)^{m_1 m_2} e(g, g)^{\alpha q_2 (m_1 r_2 + m_2 r_1 + \alpha q_2 r_1 r_2)}\hat{h}^{r'} \\ &= e(g, g)^{m_1 m_2} e(g, h)^{m_1 r_2 + m_2 r_1 + \alpha q_2 r_1 r_2}\hat{h}^{r'} \\ &= \hat{g}^{m_1 m_2}\hat{h}^{\bar{r}} \in \mathbb{G}_2 \end{aligned}$$

is the ciphertext of $m_1 \times m_2$, where $\bar{r} = m_1 r_2 + m_2 r_1 + \alpha q_2 r_1 r_2 + r'$. We note that the system is still additively homomorphic in $\mathbb{G}_2$.

III. Basic PISA

In this section, we first introduce our privacy preserving bid comparison protocol. Then we introduce our spectrum auction mechanism, namely PISA, which preserves the winners' bid privacy and achieves strategy-proofness. Here we assume that the auctioneer has full knowledge of bidders' coverage/interference areas to construct the conflict graph. In Section IV, we enhance our design to provide stronger privacy protection, *that is*, coverage/interference area privacy, k-anonymous bid privacy for both winners and losers.

A. Privacy Preserving Bid Comparison

To determine the auction winners, it suffices to let the auctioneer know whether b_i is higher than b_j, for any i and j from $\mathbb{B}$, without revealing the exact values of b_i and b_j. This problem is a variant of secure comparison. A generic solution is based on Yao's garbled circuits [38], which have a predefined number of inputs. However, in spectrum auctions, it is difficult to determine the number of bidders before the bidding phase. Hence Yao's approach is not practical here. Therefore, we design a more flexible approach. Our protocol is based on bit-wise comparison, allowing us to compare arbitrary large integers. We describe it separately for clarity, and it is well applicable in other contexts.

We consider two l-bit binary bids $b_i = (b_i^l b_i^{l-1} \cdots b_i^1)_2$ and $b_j = (b_j^l b_j^{l-1} \cdots b_j^1)_2$, where b_i^1 and b_j^1 denote the least significant bits, while b_i^l and b_j^l denote the most significant bits. For each integer $k \in [1, l]$, we define

$$\omega_{ij}^k = (b_i^k - b_j^k)^2 = (b_i^k)^2 + (b_j^k)^2 - 2b_i^k b_j^k, \qquad (1)$$

$$\lambda_{ij}^k = \zeta_{ij}^k[(b_i^k)^2 - (b_j^k)^2 + 1 + \sum_{r=k+1}^{l} \omega_{ij}^r], \qquad (2)$$

where $\zeta_{ij}^k \in_R \mathbb{Z}^+$ is a random positive number. In this subsection, l, k and t are all integers. Then, we get the following lemma.

Lemma 1. *For any $i, j \in \mathbb{B}$, we have $b_i < b_j$, if and only if there exists exactly one $k \in [1, l]$, where $\lambda_{ij}^k = 0$.*

Proof. For any $t \in [1, l]$, we have $\zeta_{ij}^t > 0$ and

$$\omega_{ij}^t = (b_i^t - b_j^t)^2 \geq 0,$$

$$(b_i^t)^2 - (b_j^t)^2 + 1 \geq 0 - 1 + 1 = 0,$$

hence $\lambda_{ij}^t \geq 0$. Next, we prove the necessary and sufficient conditions.

- Given $b_i < b_j$, there exists $k \in [1, l]$, such that $b_i^k = 0 < b_j^k = 1$ and $b_i^t = b_j^t$ for any $t \in [k+1, l]$. Consequently,

  $$\omega_{ij}^k = (b_i^k - b_j^k)^2 = 1,$$

  and

  $$\omega_{ij}^t = (b_i^t - b_j^t)^2 = 0, \forall t \in [k+1, l].$$

  Hence,

  $$\lambda_{ij}^k = \zeta_{ij}^k [0^2 - 1^2 + 1 + 0] = 0.$$

  We further distinguish two cases.
  - For $t \in [1, k-1]$, since $\omega_{ij}^k = 1$, we have

    $$\sum_{r=t+1}^{l} \omega_{ij}^r > 0.$$

    Hence, $\lambda_{ij}^t > 0$.
  - For $t \in [k+1, l]$,

    $$(b_i^t)^2 - (b_j^t)^2 + 1 = 1 > 0.$$

    Hence, $\lambda_{ij}^t > 0$.

  Therefore, there exists exactly one $k \in [1, l]$, where $\lambda_{ij}^k = 0$.
- Given $\lambda_{ij}^k = 0$, for $\zeta_{ij}^k > 0$, we can infer that

  $$(b_i^k)^2 - (b_j^k)^2 + 1 = 0 \wedge \sum_{r=k+1}^{l} \omega_{ij}^r = 0.$$

  Hence, $b_i^k = 0 < b_j^k = 1$ and $b_i^t = b_j^t$ for any $t \in [k+1, l]$. Therefore, $b_i < b_j$.

This completes our proof. □

We note that both Equation (1) and (2) are quadratic polynomials in b_i^k and b_j^k. Consequently, we can evaluate them using the BGN cryptosystem. With Lemma 1, we can compare two bids without knowing their exact values.

B. Design Details

In basic PISA, bidders submit their encrypted bidding tuples to the agent who preprocesses them before transferring them to the auctioneer. The auctioneer can decrypt the encrypted tuples and find only the necessary information to run the auction, without inferring any nonessential information. In this subsection, we present the design details of basic PISA, which comprises four phases shown as follows.

Phase 1: Initialization

Before the auction, the auctioneer sets up the parameters for BGN cryptosystem and runs KeyGen(τ) (as shown in Section II-D). Then, the public key $\mathcal{PK} = (n, \mathbb{G}_1, \mathbb{G}_2, e, g, h)$ is announced, while the private key $\mathcal{SK} = q_1$ is not revealed. The auctioneer also sets the possible bidding range of integers $R = [b_{min}, b_{max}]$, where b_{min} and b_{max} are two l-bit binaries.

Phase 2: Bidding

In the bidding phase, each bidder i decides her bid $b_i \in R$ according to her valuation v_i. The bidder i encrypts every bit b_i^k from her bid b_i with the auctioneer's public key $\mathcal{PK}$:

$$E(b_i^k) = Encrypt(\mathcal{PK}, b_i^k), k \in [1, l],$$

where $Encrypt()$ is the encryption function defined in Section II-D. For ease of expression, we denote the series of encrypted bits of b_i as

$$E(\tilde{b_i}) = (E(b_i^k))_{k \in [1,l]}.$$

Then, the bidder i sends $[i, E(\tilde{b_i})]$ as her bidding tuple to the agent.

Phase 3: Preprocessing

After receiving all the encrypted bidding tuples from the bidders, the agent preprocesses the ciphertexts.

For each bidder i, the agent appends $E(\tilde{i})$ to the least significant end of $E(\tilde{b_i})$. Here, similar with $E(\tilde{b_i})$, $E(\tilde{i})$ is the series of encrypted bits of bidder i's binary ID number. Now the bidding tuple turns out to be:

$$[i, E(\tilde{b_i})||E(\tilde{i})],$$

where $||$ is the concatenation operator. The suffix does not affect the comparison result of the two bids, except the case of tie. With the suffix, the tie can be broken according to the bidders' ID number.

Then, for any pair of bidders i and j, the agent computes:

$$\begin{aligned} E(\omega_{ij}^k) &= E((b_i^k)^2 + (b_j^k)^2 - 2b_i^k b_j^k) \\ &= e(E(b_i^k), E(b_i^k)) \times e(E(b_j^k), E(b_j^k)) \\ &\quad \times e(E(b_i^k), E(b_j^k))^{-2}, \\ E(\lambda_{ij}^k) &= E((b_i^k)^2 - (b_j^k)^2 + 1 + \sum_{r=k+1}^{l+\lceil \log_2 z \rceil} \omega_{ij}^r)^{\zeta_{ij}^k} \\ &= e(E(b_i^k), E(b_i^k))^{\zeta_{ij}^k} \times e(E(b_j^k), E(b_j^k))^{-\zeta_{ij}^k} \\ &\quad \times e(g, g)^{\zeta_{ij}^k} \times \prod_{r=k+1}^{l+\lceil \log_2 z \rceil} E(\omega_{ij}^r)^{\zeta_{ij}^k}, \end{aligned} \quad (3)$$

for each $k \in [1, l + \lceil \log_2 z \rceil]$ and $\zeta_{ij}^k \in_R \mathbb{Z}^+$. Here, $\lceil \log_2 z \rceil$ is the length of bidders' binary ID number, since there are z bidders in total.

Finally, the bidder sends the following tuples to the auctioneer:

$$[i, j, E(\tilde{\lambda}_{ij})], \forall i, j \in \mathbb{B} \wedge i \neq j,$$

where $E(\tilde{\lambda}_{ij})$ is the list of $E(\lambda_{ij}^k)$ for $k \in [1, l + \lceil \log_2 z \rceil]$. We note that the elements in $E(\tilde{\lambda}_{ij})$ can be randomly permuted.

Phase 4: Opening

(a) Conflict graph construction: The auctioneer can construct the conflict graph $\mathbb{G} = (\mathbb{V}, \mathbb{E})$ according to bidders' geographic distribution, where each bidder is represented by a vertex. Two vertices are connected if their coverage/interference areas overlap. Here, $N(f) = \{h \in \mathbb{V} | (f, h) \in \mathbb{E}\}$.

(b) Monotonic allocation: For each edge $(f, h) \in \mathbb{E}$, the auctioneer can decrypt $E(\tilde{\lambda}_{fh})$, and check whether it contains a λ^k_{fh} that is equal to 0 for $k \in [1, l+\lceil\log_2 z\rceil]$. If so, $b_f < b_h$; otherwise, $b_f > b_h$.

We can update Algorithm 1 to protect the bidders' bidding values. Algorithm 3 shows the privacy preserving winner allocation procedure. We define matrix $\mathbb{S} = [\tilde{\lambda}_{fh}]_{f,h\in\mathbb{V}}$. In lines 2-6, we iteratively pick bidder f, who does not have 0 in $\tilde{\lambda}_{fh}$, for any $h \in \mathbb{A} \setminus \{f\}$ (*i.e.*, bidder f has the highest bid in $\mathbb{A}$); and eliminate f and her neighbors from $\mathbb{A}$. When the set $\mathbb{A}$ becomes empty, the algorithm outputs the allocation profile $\vec{a}$.

Algorithm 3 Privacy Preserving Allocation Algorithm

Input: Conflict graph $(\mathbb{V}, \mathbb{E})$ and matrix $\mathbb{S} = [\tilde{\lambda}_{fh}]_{f,h\in\mathbb{V}}$.
Output: Allocation profile $\vec{a}$.
1: $\mathbb{A} \leftarrow \mathbb{V}$; $\vec{a} \leftarrow 0^z$.
2: **while** $\mathbb{A} \neq \emptyset$ **do**
3: Pick $f \in \mathbb{A}$, *s.t.*, $\nexists h \in \mathbb{A}\setminus\{f\}$ and $k \in [1, l+\lceil\log_2 z\rceil]$, such that $\lambda^k_{fh} = 0$.
4: $a_f \leftarrow 1$.
5: $\mathbb{A} \leftarrow \mathbb{A} \setminus (N(f) \cup \{f\})$.
6: **end while**
Return $\vec{a}$.

(c) Critical charging: Since the auctioneer is not given the encrypted bids from the agent, the charges to the winners cannot be computed directly according to Algorithm 2. However, the auctioneer can determine the critical neighbor of each winner.

Algorithm 4 Privacy Preserving Critical Neighbor Determination Algorithm— $CN(f)$

Input: Conflict graph $(\mathbb{V}, \mathbb{E})$, matrix $\mathbb{S} = [\tilde{\lambda}_{fh}]_{f,h\in\mathbb{V}}$, and bidder f.
Output: Critical neighbor $CN(f)$.
1: $\mathbb{A} \leftarrow \mathbb{V} \setminus \{f\}$.
2: **while** $\mathbb{A} \neq \emptyset$ **do**
3: Pick $f' \in \mathbb{A}$, *s.t.*, $\nexists h \in \mathbb{A} \setminus \{f'\}$ and $k \in [1, l+\lceil\log_2 z\rceil]$, such that $\lambda^k_{f'h} = 0$.
4: **if** $f \in N(f')$ **then**
5: **Return** f'.
6: **end if**
7: $\mathbb{A} \leftarrow \mathbb{A} \setminus (N(f') \cup \{f'\})$.
8: **end while**
Return $NULL$.

Algorithm 4 shows our privacy preserving critical neighbor determination procedure. In lines 2-8, we determine bidder f's neighbor f', who would be allocated the channel if f is absent from the auction. Then, the algorithm outputs f' as the critical neighbor of f. If no such f' exists, the algorithm returns $NULL$. We note that Algorithm 4 differs from Algorithm 2, because it outputs the bidder f's critical neighbor $CN(f)$, instead of the critical value.

(d) Outcome announcement: We denote the vector of critical neighbors by $\overrightarrow{CN} = (CN(f))_{f\in\mathbb{V}:a_f=1}$. The auctioneer needs to resort to the agent for the encrypted bids of the critical neighbors. Next, the agent replies with a vector of encrypted bids of the critical neighbors

$$\overrightarrow{C} = (E(\tilde{b}_i))_{i\in\overrightarrow{CN}}.$$

Finally, the auctioneer can decrypt the encrypted critical bids in $\overrightarrow{C}$, and announce the winners together with their charges.

C. Analysis

We consider the computational complexity for the bidders, the agent and the auctioneer, respectively: bidders have to carry out BGN encryption for each bit in their bids. Thus, each bidder has to carry out encryption l times, where l is the number of bits in the bid; the agent has to carry out pre-processing for each pair of bidders, so the computational complexity of the agent is $\mathrm{O}(z^2 l)$. Here, z is the number of bidders; both the allocation algorithm and charging algorithm run at $\mathrm{O}(z^2 l)$, thus the computational complexity of the auctioneer is $\mathrm{O}(z^2 l)$.

The strategy-proofness of PISA is inherited from the generic auction mechanism. We omit the proof here, and directly draw the following conclusion.

Theorem 2. *PISA satisfies strategy-proofness.*

Besides privacy preservation and strategy-proofness, PISA also achieves the following nice properties.

1) Compared with the generic spectrum auction scheme, PISA protects bidders' privacy without sacrificing spectrum allocation efficiency.
2) In PISA, bidders are allowed to choose their bids from a contiguous integer range. Compared with existing mechanisms (*e.g.*, [15]), in which bids are limited to a small set of predefined values, PISA provides bidders with more bidding flexibility.
3) PISA preserves the communication pattern of an auction protocol. In PISA, bidders are not required to communicate with each other. After submitting the bidding tuples to the agent, they are free from burdensome computation and communication.

Basic PISA is based on our privacy preserving bid comparison protocol, hence the auctioneer can compare two bids without knowing their values. As an intermediary, the agent can not decrypt the encrypted bidding tuples to learn the bids. Therefore, we protect the bid privacy of winners against both the auctioneer and the agent. However, the bids of critical neighbors are revealed as the charges of the winners. In the next section, we enhance our design to thwart such privacy breaches and provide protection for coverage/interference area privacy.

IV. Extended PISA

In this section, we intend to provide k-anonymous bid privacy for both winners and losers, together with protection for coverage/interference area privacy. To preserve coverage/interference area privacy, we generalize each bidder's coverage/interference area to a square with side length $2r_i$.

As shown in Figure 2, bidder i and j are conflicting bidders in case (a), while they can share the same channel in case (b). Compare with the commonly used circular conflict areas, this assumption may overestimate interference. To evaluate the impact of this assumption on channel allocation, in Section V, we compare our square conflict model with a conflict graph obtained from a real measurement.

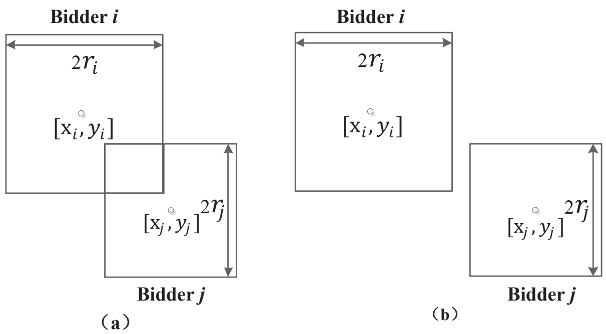

Fig. 2. Square coverage/interference area examples. In case (a), bidder i and j have conflict. In case (b), the two bidders do not have conflict.

We define

$$x_i^{\triangleright} = x_i + r_i, \qquad x_i^{\triangleleft} = x_i - r_i,$$
$$y_i^{\vartriangle} = y_i + r_i, \qquad y_i^{\triangledown} = y_i - r_i,$$

where $x_i^{\triangleright}$, $x_i^{\triangleleft}$, $y_i^{\vartriangle}$, and $y_i^{\triangledown}$ are t-bit binaries. Bidder i and j are out of the coverage/interference range of each other, if the following condition holds:

$$(x_i^{\triangleright} < x_j^{\triangleleft} \vee x_i^{\triangleleft} > x_j^{\triangleright}) \vee (y_i^{\vartriangle} < y_j^{\triangledown} \vee y_i^{\triangledown} > y_j^{\vartriangle}) = TRUE. \quad (4)$$

Given Lemma 1, we can evaluate $x_i^{\triangleright} < x_j^{\triangleleft}$, $x_i^{\triangleleft} > x_j^{\triangleright}$, $y_i^{\vartriangle} < y_j^{\triangledown}$, and $y_i^{\triangledown} > y_j^{\vartriangle}$ privately. Due to limitations of space, we only focus on the differences in phases of bidding, preprocessing, and opening.

A. Design Details

Besides the processing of coverage/interference areas, the key difference lies in the preprocessing phase, where the agent performs a secret permutation to anonymize the bidders.

Phase 1: Initialization

It is similar to the basic PISA in Section III-B.

Phase 2: Bidding

In addition to $E(\tilde{b}_i)$, each bidder i also calculates $E(\tilde{x}_i^{\triangleright})$, $E(\tilde{x}_i^{\triangleleft})$, $E(\tilde{y}_i^{\vartriangle})$, and $E(\tilde{y}_i^{\triangledown})$. Then, the bidder sends

$$[i, E(\tilde{b}_i), E(\tilde{x}_i^{\triangleright}), E(\tilde{x}_i^{\triangleleft}), E(\tilde{y}_i^{\vartriangle}), E(\tilde{y}_i^{\triangledown})]$$

to the agent as the bidding tuple.

Phase 3: Preprocessing

For any pair of bidders i and j, the agent computes $E(\tilde{\lambda}_{ij})$ as specified in Section III-B. In addition, the agent calculates

$$E(\alpha_{ij}^k) = E((x_i^{\triangleright k})^2 - (x_j^{\triangleleft k})^2 + 1 + \sum_{r=k+1}^{t} (x_i^{\triangleright r} - x_j^{\triangleleft r})^2)^{\xi_{ij}^k},$$

for each $k \in [1, t]$ and $\xi_{ij}^k \in_R \mathbb{Z}^+$. Same as before, we let $E(\tilde{\alpha}_{ij}) = (E(\alpha_{ij}^k))_{k \in [1,t]}$. The agent also carries out the calculations on $(x_i^{\triangleleft}, x_j^{\triangleright})$, $(y_i^{\vartriangle}, y_j^{\triangledown})$ and $(y_i^{\triangledown}, y_j^{\vartriangle})$, and results in $E(\tilde{\beta}_{ij})$, $E(\tilde{\gamma}_{ij})$, and $E(\tilde{\delta}_{ij})$, respectively.

After finishing the above calculations, the agent carries out a secret permutation π on the results to make them anonymous to the auctioneer. Then, the agent sends the following anonymous tuples to the auctioneer:

$$[\pi(i), \pi(j), E(\tilde{\lambda}_{ij}), E(\tilde{\alpha}_{ij}), E(\tilde{\beta}_{ij}), E(\tilde{\gamma}_{ij}), E(\tilde{\delta}_{ij})],$$

for each $i, j \in \mathbb{B}$ and $i \neq j$.

Phase 4: Opening

The auctioneer decrypts $E(\tilde{\alpha}_{fh})$, $E(\tilde{\beta}_{fh})$, $E(\tilde{\gamma}_{fh})$, and $E(\tilde{\delta}_{fh})$, to get $\tilde{\alpha}_{fh}$, $\tilde{\beta}_{fh}$, $\tilde{\gamma}_{fh}$, and $\tilde{\delta}_{fh}$, respectively. We note that bidder f and h cannot share the channel, if the following condition holds:

$$\Omega_{fh} = \left(0 \notin \tilde{\alpha}_{fh} \wedge 0 \in \tilde{\beta}_{fh} \wedge 0 \notin \tilde{\gamma}_{fh} \wedge 0 \in \tilde{\delta}_{fh}\right) = TRUE. \quad (5)$$

The auctioneer can construct the conflict graph $\mathbb{G} = (\mathbb{V}, \mathbb{E})$, where

$$\mathbb{V} = \{\pi(i) | i \in \mathbb{B}\},$$
$$\mathbb{E} = \{(f, h) | f, h \in \mathbb{V} \wedge \Omega_{fh} = TRUE\}.$$

Then the auctioneer carries out Algorithm 3 and Algorithm 4 on permuted bidders. Algorithm 3 outputs allocation profile $\vec{a}'$ on permuted bidders, while Algorithm 4 returns the permuted critical neighbors. Since the auctioneer is unaware of the agent's perturbation $\pi : \mathbb{B} \to \mathbb{V}$, the original identifiers of the bidders remain anonymous. Let $\overrightarrow{W} = (f)_{f \in \mathbb{V}: a'_f = 1}$ denote the vector of permuted winners, and $\overrightarrow{CN} = (CN(f))_{f \in \mathbb{V}: a'_f = 1}$ denote the corresponding vector of permuted critical neighbors. To prevent the agent from finding one-to-one correspondence between the winners and their critical neighbors, the auctioneer permutates $\overrightarrow{CN}$ to get $\overrightarrow{CN}'$. The auctioneer needs to resort to the agent for the winners' original identifiers and the encrypted bids of the critical neighbors, by sending $\overrightarrow{W}$ and $\overrightarrow{CN}'$ to the agent. Next, the agent replies with a vector of winner identifiers

$$\overrightarrow{W}' = (i)_{\pi(i) \in \overrightarrow{W}},$$

and a vector of encrypted bids of the critical neighbors

$$\overrightarrow{C}' = (E(\tilde{b}_i))_{\pi(i) \in \overrightarrow{CN}'}.$$

Finally, the auctioneer can decrypt the encrypted critical bids in $\overrightarrow{C}'$, map the critical bids to the winners by reversing the permutation done on $\overrightarrow{CN}$, and announce the winners together with their charges.

B. Illustrative Example

The following example may help to illustrate how extended PISA works. Suppose there are five bidders 1, 2, 3, 4 and 5, located at $(6, 10)$, $(10, 14)$, $(10, 6)$, $(14, 10)$ and $(10, 16)$ respectively. They are supposed to bid higher than 0, but lower than 16. Each of them has a valuation of the channel in binary form: $b_1 = 0010_2, b_2 = 0100_2, b_3 = 1010_2, b_4 = 0101_2, b_5 =$

0110_2. For simplicity, we assume $r_1 = r_2 = r_3 = r_4 = r_5 = 5$.

In the auction, each bidder submits an encrypted bidding tuple, which contains his/her encrypted bid and coverage/interference boundaries. The bidding tuple follows the format of $[i, E(\tilde{b}_i), E(\tilde{x}_i^{\triangleright}), E(\tilde{x}_i^{\triangleleft}), E(\tilde{y}_i^{\vartriangle}), E(\tilde{y}_i^{\triangledown})]$. For instance, bidder 1 submits

$$[1, E(\widetilde{0010}), E(\widetilde{1011}), E(\widetilde{0001}), E(\widetilde{1111}), E(\widetilde{0101})]$$

to the agent. Here 1 is the identity,

$$b_1 = 0010_2,$$

$$x_1^{\triangleright} = 6 + r_1 = 1011_2, x_1^{\triangleleft} = 6 - r_1 = 0001_2,$$

$$y_1^{\vartriangle} = 10 + r_1 = 1111_2, y_1^{\triangledown} = 10 - r_1 = 0101_2.$$

Similarly, bidder 3 submits

$$[3, E(\widetilde{1010}), E(\widetilde{1111}), E(\widetilde{0101}), E(\widetilde{1011}), E(\widetilde{0001})].$$

After collecting all the bids, the agent carries out preprocessing as specified in Phase 3. For simplicity, we omit the suffix. Then, the agent anonymizes all the tuples, and sends the permuted ones to the auctioneer. For example, the agent sends the following information about bidders 1 and 3 to the auctioneer:

$$[4'(3), 5'(1), E(\mathbf{2} \times 2, \mathbf{4} \times 2, \mathbf{1} \times 2, \mathbf{3} \times 2),$$

$$E(\mathbf{4} \times 2, \mathbf{5} \times 3, \mathbf{3} \times 4, \mathbf{1} \times 4), E(3 \times 0, \mathbf{5} \times 3, \mathbf{3} \times 2, \mathbf{2} \times 4),$$

$$E(\mathbf{7} \times 2, \mathbf{3} \times 1, \mathbf{2} \times 4, \mathbf{2} \times 4), E(\mathbf{3} \times 0, \mathbf{3} \times 1, \mathbf{2} \times 2, \mathbf{2} \times 4)].$$

Here, the numbers inside parentheses in the first two terms are the bidders' true identifiers, which are hidden from the auctioneer. Furthermore, $\pi(1) = 5'$ and $\pi(3) = 4'$ are the anonymous identification after the secret permutation π. The numbers in bold are the random numbers generated by the agent (*e.g.*, 2, 4, 1, 3 in the third term correspond to the random number ζ_{ij} in Equation (3)).

The auctioneer can decrypt the ciphertexts and do the comparison. In this example,

$$\begin{aligned}
\Omega_{4'5'} &= [0 \notin (\mathbf{4} \times 2, \mathbf{5} \times 3, \mathbf{3} \times 4, \mathbf{1} \times 4)] \\
&\wedge [0 \in (\mathbf{3} \times 0, \mathbf{5} \times 3, \mathbf{3} \times 2, \mathbf{2} \times 4)] \\
&\wedge [0 \notin (\mathbf{7} \times 2, \mathbf{3} \times 1, \mathbf{2} \times 4, \mathbf{2} \times 4)] \\
&\wedge [0 \in (\mathbf{3} \times 0, \mathbf{3} \times 1, \mathbf{2} \times 2, \mathbf{2} \times 4)] \\
&= TRUE.
\end{aligned}$$

Thus bidder $4'$ and $5'$ cannot share the channel. The auctioneer constructs the conflict graph based on the comparison results, as shown in Figure 3. The term by each edge $(f, h) \in \mathbb{E}$ denotes $[f, h, E(\tilde{\lambda}_{fh})]$ (elements in $E(\tilde{\lambda}_{fh})$ are multiplied with random positive numbers and are randomly perturbed).

From Figure 3, the auctioneer can learn that

$$b_{1'} > b_{2'}, \{b_{1'}, b_{3'}\} > b_{2'} > b_{5'},$$

$$b_{4'} > b_{3'} > b_{2'}, b_{4'} > \{b_{3'}, b_{5'}\}, \{b_{2'}, b_{4'}\} > b_{5'}.$$

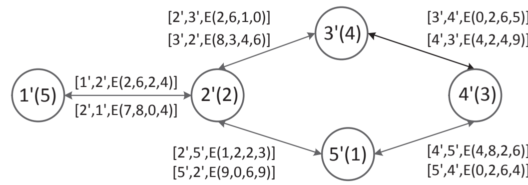

Fig. 3. Conflict graph constructed by the auctioneer, without revealing actual coverage/interference areas of the bidders.

Then the auctioneer can determine winners and their critical neighbors using Algorithm 3 and Algorithm 4, respectively. The winners turn out to be bidder $1'$ and $4'$, and the corresponding critical neighbors are bidder $2'$ and $3'$, respectively. The auctioneer permutes $\overrightarrow{CN} = \{2', 3'\}$ to get $\overrightarrow{CN}' = \{3', 2'\}$.

Next the auctioneer consults the agent with $\overrightarrow{W} = \{1', 4'\}$ and $\overrightarrow{CN}' = \{3', 2'\}$, and the agent replies with answer 5, 3, $E(\tilde{b}_4)$, and $E(\tilde{b}_2)$. Finally, the auctioneer announces bidder 3 and 5 as winners, with charges 5 and 4, respectively.

C. Analysis

It is evident that extended PISA inherits the nice properties from basic PISA, including strategy-proofness and the three properties listed in Section III-C. To avoid redundancy, we do not elaborate on them again here. In this subsection, we demonstrate some other nice properties of our auction mechanism.

Theorem 3. *Extended PISA guarantees k-anonymity for bid privacy, where $k = |\overrightarrow{CN}|$.*

Proof. We distinguish the following two cases:

Case 1: Bidder i is a critical neighbor, *i.e.*, $\pi(i) \in \overrightarrow{CN}$. We consider from the perspectives of both the agent and the auctioneer.

On the one hand, the agent cannot decrypt bidder i's bidding tuple without the secret key $\mathcal{SK}$, hence the agent cannot find out what the bid is. When the auctioneer consults the agent with $\overrightarrow{CN}'$ in the opening phase, the agent learns that bidder i is one of the critical neighbors. However, the order of bidders in $\overrightarrow{CN}'$ has been permutated by the auctioneer. The agent does not know which bidder is which winner's critical neighbor. When the auctioneer announces the charges for winners, bidder i's bid is hidden among $|\overrightarrow{CN}|$ charges. Thus, the agent cannot identify bidder i's bid with probability higher than $1/|\overrightarrow{CN}|$.

On the other hand, although the auctioneer can decrypt the bidding tuples, they cannot be linked with the bidders, because all the bidding tuples are anonymized by the agent. The true identifier of a critical neighbor is hidden among $z - |\overrightarrow{W}|$ losers. Thus, the auctioneer cannot identify bidder i with probability higher than $1/(z - |\overrightarrow{W}|)$.

For the set of critical neighbors is a subset of losers ($z - |\overrightarrow{W}| \geq |\overrightarrow{CN}|$), both the agent and the auctioneer cannot identify bidder i's bid with probability higher than $1/|\overrightarrow{CN}|$.

Case 2: Bidder i is not a critical neighbor, *i.e.*, $\pi(i) \notin \overrightarrow{CN}$.

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For bidder $\pi(i) \notin \overrightarrow{CN}$, the agent cannot decrypt the bidding tuple, and the bid is never revealed to the auctioneer. We note that with the suffix, all bids have different encrypted values. There is no possibility that the auctioneer can infer a bid as it happens to be equal to one of the critical values.

This completes our proof. $\square$

As for coverage/interference areas, the agent can not decrypt bidders' encrypted bidding tuples and can only perform homomorphic operations on the ciphertexts. Although the auctioneer can decrypt the preprocessed tuples received from the agent, the bidders' coverage/interference areas remain unknown. The auctioneer constructs the conflict graph based on Lemma 1, without knowledge of the bidders' exact coverage/interference boundaries. Hence bidders' coverage/interference areas are revealed to neither the auctioneer nor the agent. Therefore, we have the following theorem.

Theorem 4. *Extended PISA prevents the agent and the auctioneer from learning the bidders' coverage/interference areas.*

We also analyze the computational complexity: in extended PISA, besides bids, we have to process the four coverage/interference area boundaries. Each bidder has to carry out BGN encryption for $(l + 4t)$ times. Here, t is the number of bits in the four boundaries; similarly, the agent has to carry out pre-processing for each pair of bidders, so the computational complexity of the agent is $\mathrm{O}\big(z^2(l + 4t)\big)$; for each pair of bidders, the auctioneer has to judge whether their conflict squares overlap. Thus, the auctioneer has to carry out $(4z^2t)$ times of decryption to construct the conflict graph. The allocation algorithm and charging algorithm still run at $\mathrm{O}\big(z^2 l\big)$, thus the computational complexity of the auctioneer is $\mathrm{O}\big(z^2(l + 4t)\big)$.

V. Evaluation

We have implemented PISA and evaluated its efficiency and overhead through simulations. In this section, we show our evaluation results.

A. Methodology

In our evaluations, we implement the BGN cryptosystem with security parameter $\tau = 80$, using the Stanford pairing-based cryptography library (PBC), which is a C library built on the GMP library to perform the mathematic operations underlying pairing-based cryptosystems.

In each set of evaluations, we vary a factor among bidder number, the size of the terrain, and the number of digits in a bid, while fixing the other two. The number of bidders varies from 20 to 200, and the bidders are randomly distributed in the terrain. The size of the terrain ranges from 256 meters $\times$ 256 meters to 2048 meters $\times$ 2048 meters, such that the coordinates can be represented by 8 to 11 bits. The coverage/interference range is randomly selected from 50 to 150 meters, and hence the mean value is 100 meters, which is the transmission range of 802.11b. The bid of each bidder ranges from 2 to 1023, which can be represented by at most 10 bits. The default values for bidder number, the size of terrain, and the number of digits in a bid is 200, 2048 meters $\times$ 2048 meters, 10, respectively.

We measure the following metrics in our evaluation:

- Channel utilization: The average number of bidders allocated to the channel.
- Computational overhead: The processing time required by each party to run the auction.
- Communication overhead: The size of data that must be sent to convey information.

We run a series of evaluations on a PC with Intel® Core(TM) i5 3.1GHz processer and 4GB memory under Ubuntu 10. All the results on performance are averaged over 100 runs.

B. Allocation Efficiency

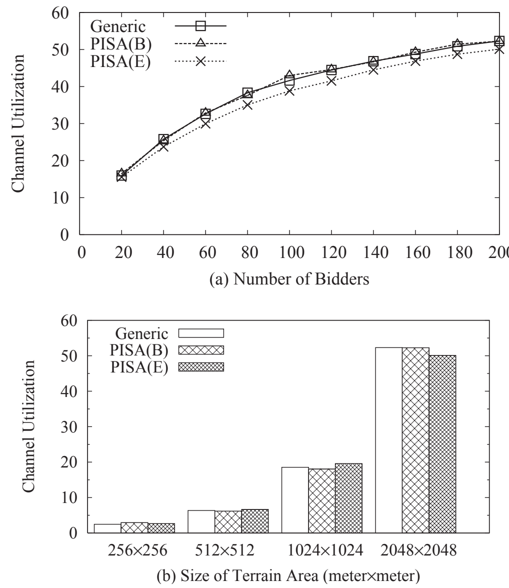

Fig. 4. Channel utilizations of the generic spectrum auction scheme, basic PISA (Section III) and extended PISA (Section IV).

In this section, we compare PISA with VERITAS [40], *i.e.*, the generic spectrum auction without privacy preservation.

Figure 4(a) shows the channel utilizations achieved by the generic spectrum auction scheme, basic PISA and extended PISA as a function of the number of bidders, when the terrain area is 2048 meters $\times$ 2048 meters. We can see that the channel utilizations achieved by all the mechanisms are non-decreasing concave functions of the number of bidders.

Figure 4(b) shows the case, in which we vary the size of the terrain and fix the number of bidders at 200. Again, we can see that the generic auction scheme, basic PISA and extended PISA all have increasing channel utilization. Bidders are randomly distributed over the terrain, a larger terrain results in less conflicts, and hence higher channel utilization.

Figure 4 validates our claim that, compared with the generic spectrum auction scheme, PISA protects bidders' privacy without sacrificing channel utilization.

To show the impact of square conflict area assumption, we compare it with a conflict graph obtained from real measurements. We utilize the data collected by Zhou *et al.* [41]. This

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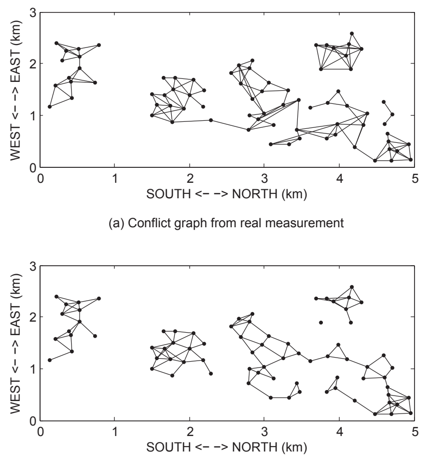
dataset contains 78 APs of the Google WiFi network, covering a $7km^2$ residential area in Mountain View, California.

We obtain two conflict graphs: one from the real measurement [41], and the other one constructed by the square conflict model. Here, we set the interference range to be 150 meters. The two conflict graphs are shown as follows. Figure 5(a) is from [41] and Figure 5(b) is constructed by the square conflict model. In Figure 5(a), there are 151 edges, whereas is Figure 5(b), there are 171 edges.



(a) Conflict graph from real measurement

(b) Conflict graph from square conflict model

Fig. 5. Conflict graphs by real measurement and square conflict model

We use these two different conflict graphs as input and run the spectrum allocation algorithm. Bids are randomly distributed over $[50, 100]$. We repeat the experiments for 1000 runs. The average channel utilization for Figure 5(a) is 26.2, whereas the number for Figure 5(b) is 26.1. The two numbers are quite close. This is reasonable as there are only minor differences between the two conflict graphs. Thus, we can conclude that the assumption of the square conflict areas has a minor impact on channel utilization.

C. Overhead

PISA integrates cryptographic tools to protect bidders' privacy. An efficient privacy preserving mechanism should have a low overhead. We evaluate the computation and communication overheads, by varying the number of bidders, the size of the terrain, and the number of bits in the bid.

Figure 6 shows the computational overhead of the agent and the auctioneer. We do not plot the bidders' computational overhead, because each bidder just encrypts several bits of information, and the computational overhead is only about 25 milliseconds. We can see that the computational overhead is mainly from the agent, because the agent is responsible for a large number of encryption operations. Furthermore, we can find that the agent spends far more time in extended PISA than in basic PISA. This is because in basic PISA, the agent spends most of the time processing bids. However, in extended PISA, the agent also needs to preprocess the four coverage/interference boundaries between bidders. Similarly, the auctioneer in extended PISA has to decrypt more ciphertexts to construct the conflict graph, hence the higher computational overhead than in basic PISA.

Specifically, Figure 6(a) shows the run time against the number of bidders with 10-bit bids and a 2048 meters $\times$ 2048 meters terrain area. We find that the computational overhead of the agent grows as a quadratic polynomial of bidders' number. This is reasonable, because in Phase 3: Preprocessing, the agent has to carry out preprocessing for each pair of bidders. As shown in Figure 6(b), the computational overhead of the agent in basic PISA changes slightly as the size of terrain area grows. However, in extended PISA, the agent has to process the ciphertexts of coordinates for the auctioneer to build the conflict graph, thus her computational overhead increases with the size of the terrain. Figure 6(c) shows that the run time of the agent increases almost linearly with the number of bits in a bid. This is reasonable, as for each pair of bidders, the agent has to compute Equation (3) k times, where k is the number of bits in the bids. Generally, in our evaluations, the time required by the agent in basic PISA is less than 50 seconds while in extended PISA, the agent requires about a few minutes' processing time (less than 5 minutes). To speed up computation, we can use parallel computing to save computation time. Since bidders are not involved in burdensome computation after submitting bids, they are not required to stay connected with the agent nor the auctioneer. They can simply wait for the auctioneer to broadcast the results, hence, we believe this time gap is acceptable.

Figure 7 plots the communication overhead induced by basic PISA and extended PISA. The communication overhead of each bidder is about 96 bytes in basic PISA and 546 bytes in extended PISA. It is trivial compared with the total communication overhead, hence we do not show them on the figures. As shown in Figure 7, the communication overhead of extended PISA is much higher than basic PISA. This is because, in addition to transmitting the preprocessed results of bids, the agent in extended PISA has to transmit the preprocessed results of four coverage/interference boundaries to the auctioneer.

Similar to Figure 6(a), the communication overhead in Figure 7(a) grows as a quadratic polynomial of bidders' numbers. As shown in Figure 7(b), the communication overhead of extended PISA grows almost linearly with the size of the terrain. With the increases in terrains, we need more bits to represent bidders' coverage/interference areas. Thus, communication overheads increase linearly with the number of bits needed to represent the terrain. Similar to the computational overhead, the communication overhead increases almost linearly with the number of bits in a bid, which is shown in Figure 7(c). Generally, the communication overhead of basic PISA is less than 4 MB, while the communication overhead of extended PISA is less than 25 MB.

From Figure 6 and Figure 7, we can conclude that PISA protects bidders' privacy with tolerable computation and communication overheads. Since bidders are not engaged in burdensome computation and communication, both the computation and communication overheads for bidders are negligible, which is an appealing property in auction design.

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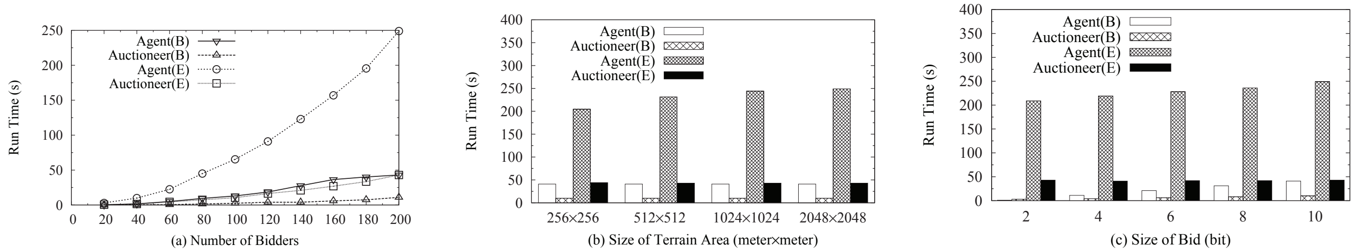

Fig. 6. Computational overheads induced by basic PISA (Section III) and extended PISA (Section IV).

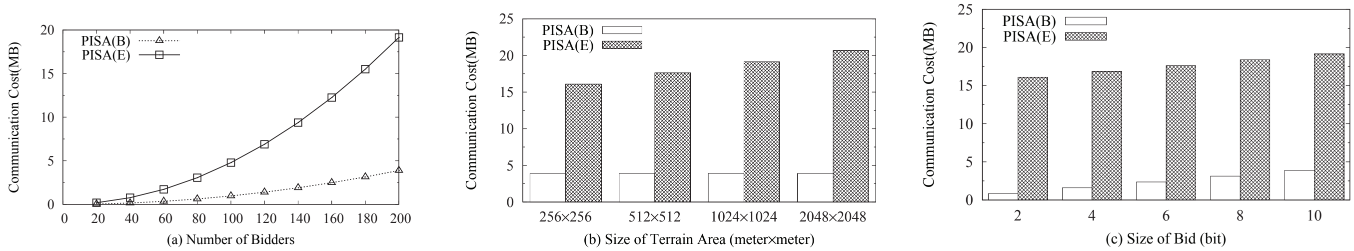

Fig. 7. Communication overheads induced by basic PISA (Section III) and extended PISA (Section IV).

VI. Related Work

We briefly review related works in this section.

A. Privacy Preserving Mechanism Design

Some works have been devoted to privacy preserving mechanism design. Wang *et al.* [31] incentized SUs to contribute their sensing data for collaborative sensing by providing differential privacy protection in the presence of malicious service providers and SUs. Naor *et al.* [22] proposed Yao's garbled circuits for use in auctions. However, the number of bidders in spectrum auctions cannot be known before the bidding phase, hence Yao's garbled circuits are not applicable here. Sui and Boutilier [27] studied efficiency and privacy tradeoffs in mechanism design. Their results show that sacrifices in efficiency can provide gains in privacy. Similarly, Feigenbaum *et al.* [11] proposed a general framework to analyze the tradeoff between communication cost and privacy. In [7], the authors present a protocol based on homomorphic encryption for secure comparison of integers, which is well applicable for auctions. There are a great number of existing works on privacy preserving auctions (*e.g.*, [1, 5, 17, 25, 28]), which are designated for traditional goods (*e.g.*, paintings, jewelry), where each commodity can only be allocated to one bidder. When it comes to spectrum auctions, they may either fail or lead to significant degradation of spectrum utilization. Assume that we directly apply one of the existing privacy preserving auction schemes to spectrum auctions, each channel will be allocated to only one bidder, which cannot fully exploit the spatial reusability of spectrum, resulting in extremely low channel utilization.

B. Dynamic Spectrum Auction

Auctions are widely used to handle spectrum allocation, and researchers have proposed various spectrum auction mechanisms (*e.g.*, [35, 36, 40, 43]). TAHES [12] and TRUST [42] are both truthful double spectrum auctions. Dong *et al.* [9] and Zhu *et al.* [46] applied combinatorial auctions to allocate spectrum. Deek *et al.* [8] and Xu *et al.* [37] investigated various forms of cheating in online auctions. Al-Ayyoub and Gupta [2] and Jia *et al.* [16] aimed at maximizing the revenue of primary users. Most of the existing literature mainly focuses on the economic aspects of the auction.

Pan *et al.* [24] proposed a secure spectrum auction leveraging paillier cryptosystem. Their design requires multiple auctioneers, which is normally considered to be impractical. Liu *et al.* [18] studied location privacy in spectrum auctions, however, their auctions are not strategy-proof. In a closely related study, Huang *et al.* [15] proposed a novel spectrum auction mechanism to preserve bid privacy. However, SPRING [15] is based on bid-independent bidder grouping, and thus may result in terribly poor spectrum utilization in extreme cases. Furthermore, in [15], bidders are only allowed to choose bids from a small set of predefined values. PISA differs significantly from [15] as PISA is based on monotonic allocation and critical charging instead of bidder grouping. Moreover, PISA allows bidders to choose bids from a continuous integer range, which is more flexible. Furthermore, PISA protects both bid privacy and coverage/interference area privacy. Recently, Zhu *et al.* [44, 45] extended the exponential mechanism in [20] and proposed the first differentially privacy preserving spectrum auction with approximate revenue maximization, under the assumption that the auctioneer is trustworthy. However, as mentioned before, bidders are reluctant to share their bidding information with anyone else, including the auctioneer. Thus, this assumption is not always true. In other related works [6, 14, 34], the authors adopted the similar system architecture. They mainly focused on protecting bid privacy, but did not consider bidders' coverage/interference area privacy.

VII. Conclusion and Future Work

In this paper, we have presented PISA, which is the first privacy preserving and strategy-proof spectrum auction mechanism that can protect both bid privacy and coverage/interference area privacy, without sacrificing social welfare. PISA is based on our novel and efficient privacy preserving integer comparison protocol, which can compare arbitrary large integers and is well applicable in other contexts. Analytical results have demonstrated PISA's privacy preserving properties and evaluation results have shown that PISA achieves good spectrum allocation efficiency, with light computation and communication overheads.

As for future work, it will be interesting to study potential attacks against our model. Yet another possible direction is to provide privacy protection for both buyers and sellers in double spectrum auctions.

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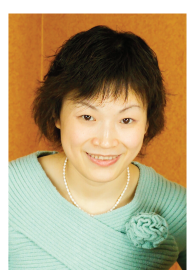

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