K Nearest Neighbor Queries and KNN-Joins in Large Relational Databases (Almost) for Free

Bin Yao, Feifei Li, Piyush Kumar

Florida State University

Introduction

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  spatial databases, pattern recognition, DNA sequencing.

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- Our goal: design *relational algorithms* for KNN and KNN-Joins.
  - Readily applied on relational databases without updating the engine.
  - Augmented with ad-hoc query conditions and optimized by the query optimizer.
  - Do it in SQL!

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- kNN-Join solution:
  - the iJoin algorithm
  - the Gorder algorithm

Problem formulation

- Data set P stored in table R_P: $\{pid, Y_1, \cdots, Y_d, A_1, \cdots, A_h\}$.
  Query set Q stored in table R_Q: $\{qid, X_1, \cdots, X_d, B_1, \cdots, B_g\}$.

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- KNN queries: let $A = kNN(q, \mathsf{R}_P)$,

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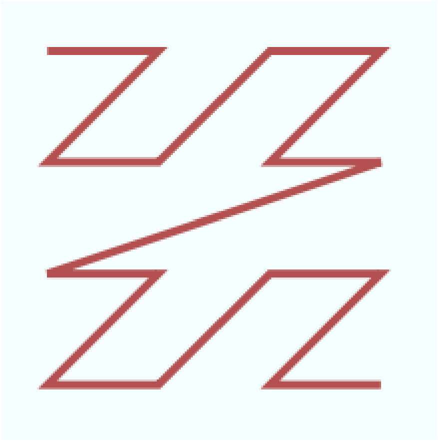
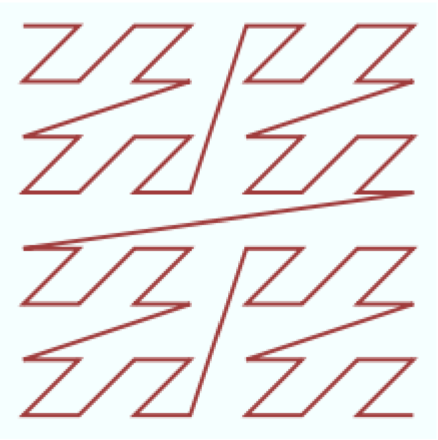
- Approximate k nearest neighbors:
  Suppose q's kth nn from P is p^* and $r^* = |q, p^*|$,
  p be the kth NN of q for some kNN algorithm A and $r^p = |q, p|$,
  $(p, r^p) \in \mathbb{R}^d \times \mathbb{R}$ is $(1 + \epsilon)$-approximate solution of kNN if
  $r^* \leq r^p \leq (1 + \epsilon) r^*$.

Z-value and Z-order curve

- z-value of a point:
  For point $(2, 6)$, binary representation is $(010, 110)$, z-value is $011100 = 28$.

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 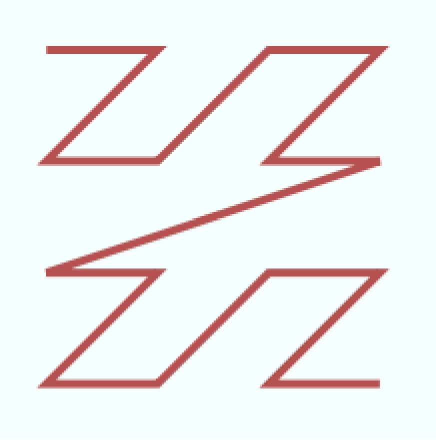 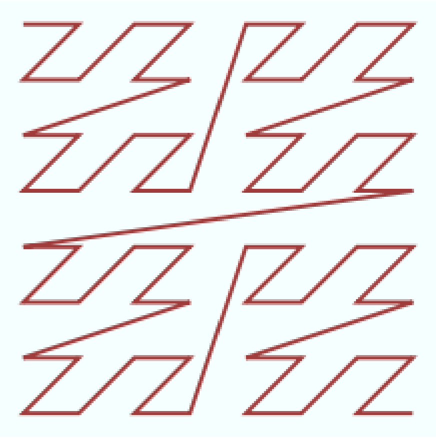

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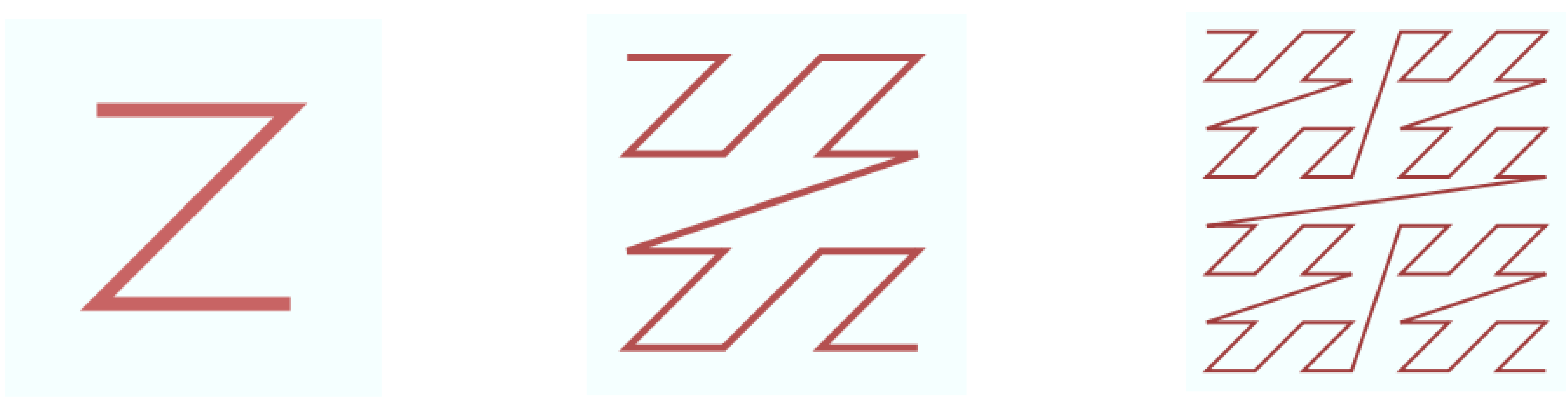

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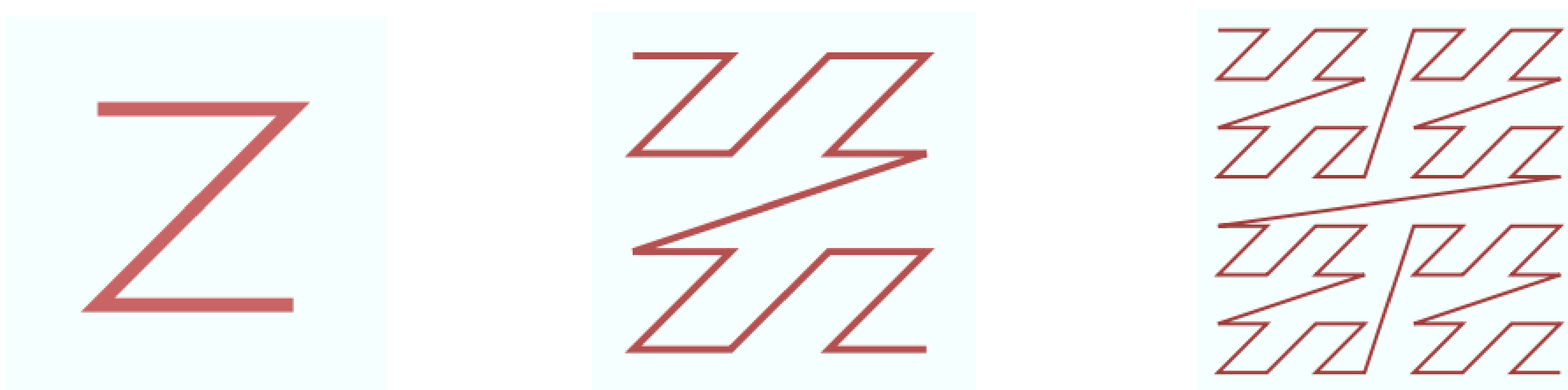

- A well-known approach:
  - Map points in a multi-dimensional space into one dimension by using z-values.
  - Translate the kNN search into one dimensional range search on the z-values.

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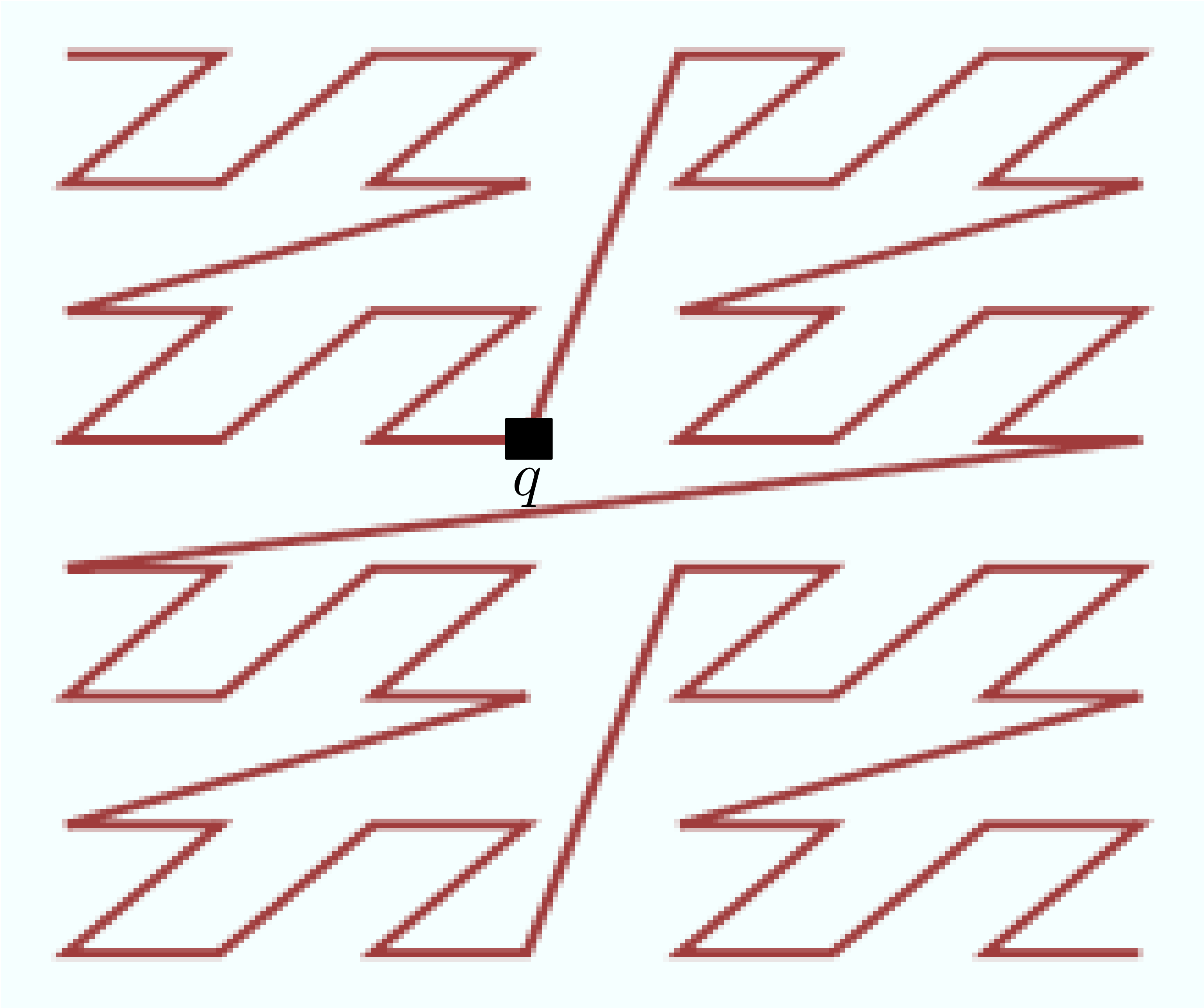

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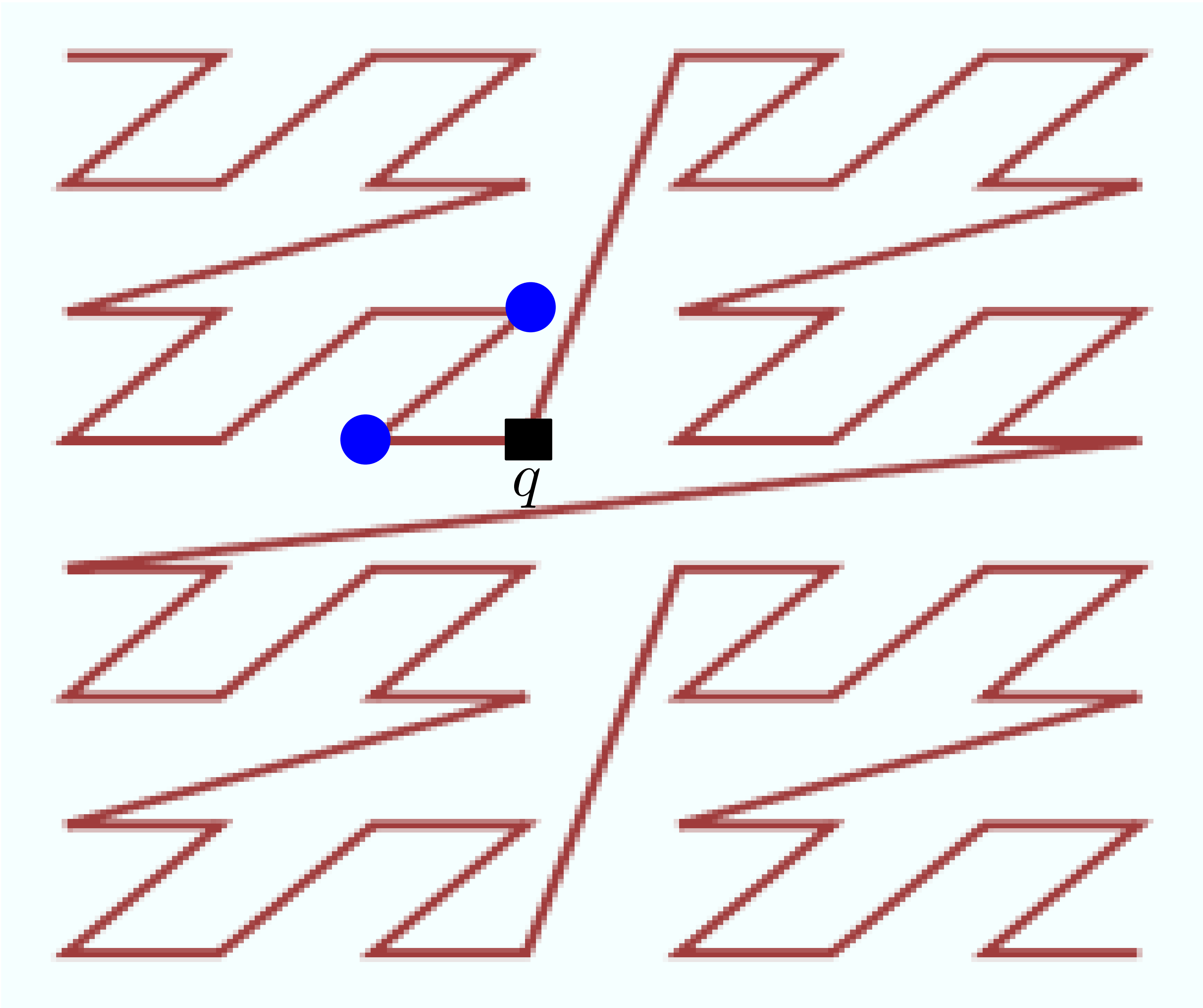

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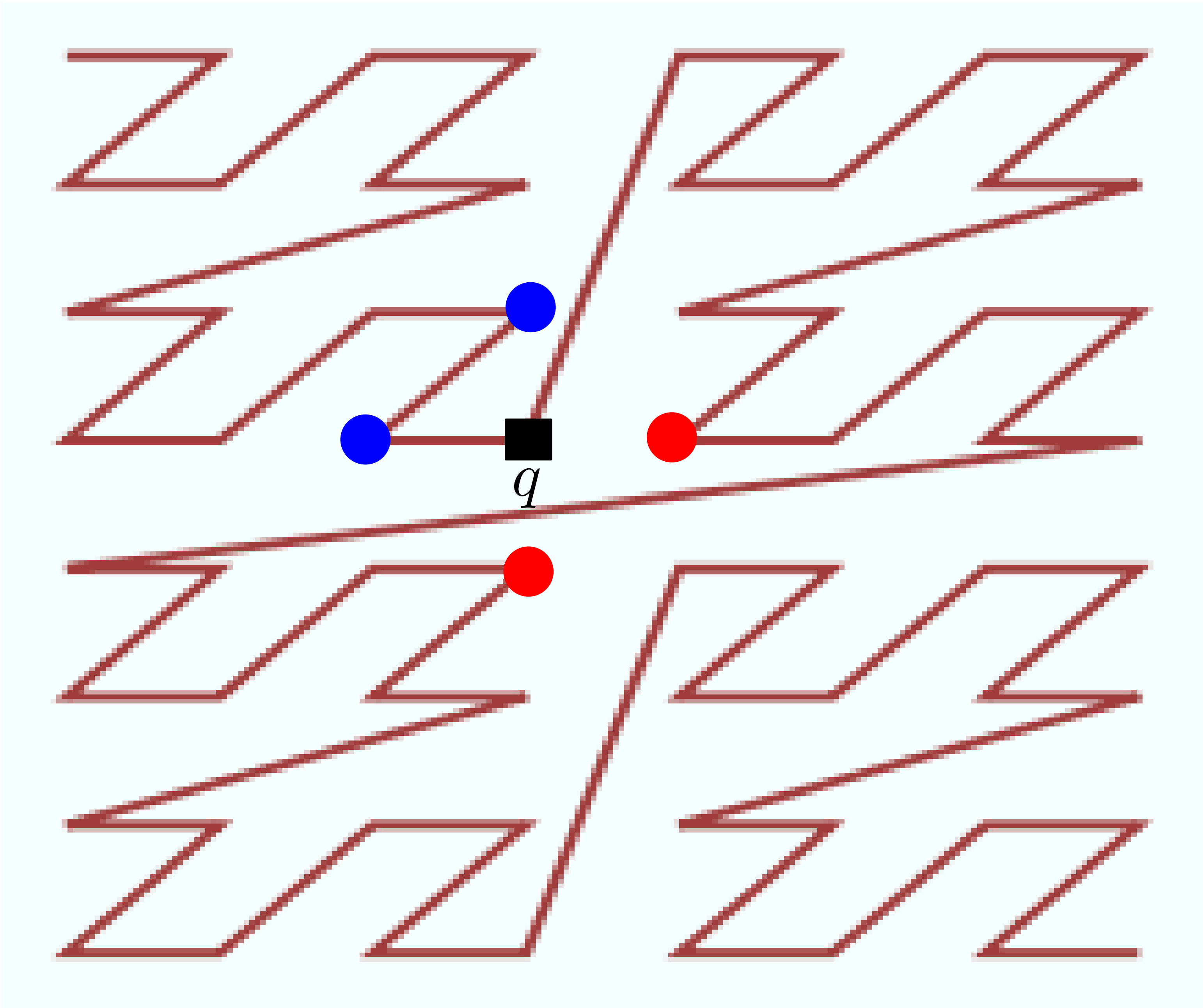

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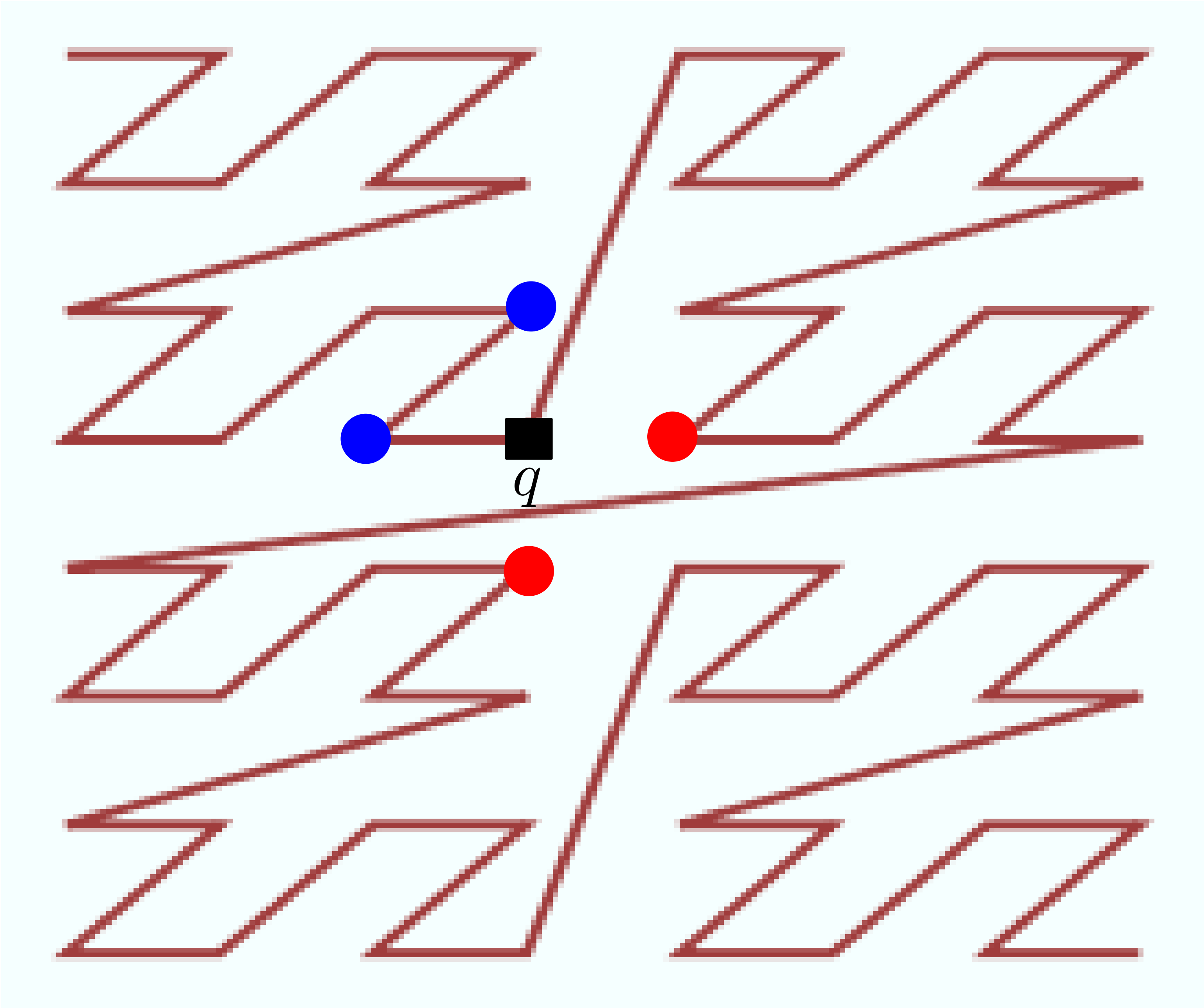

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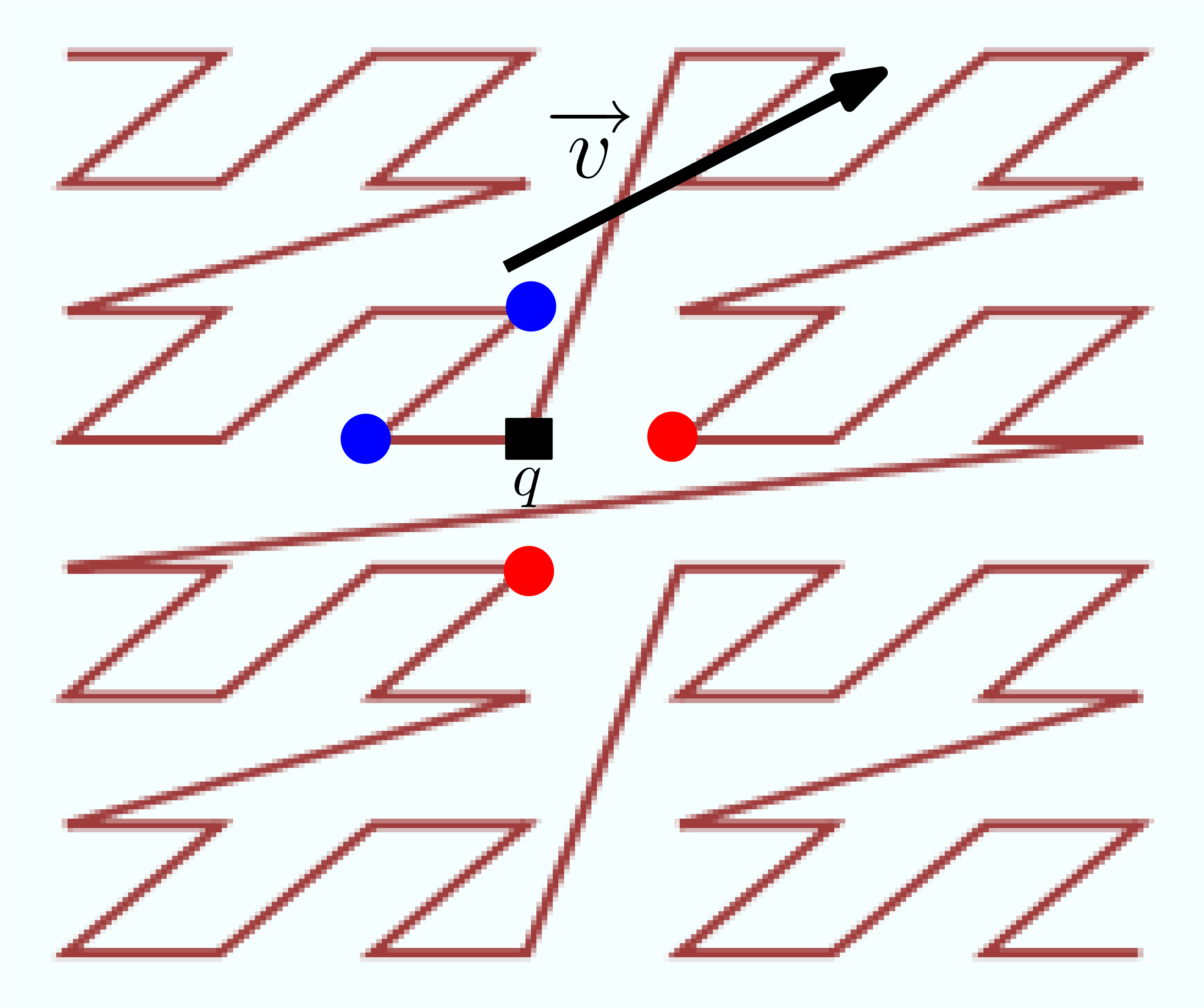

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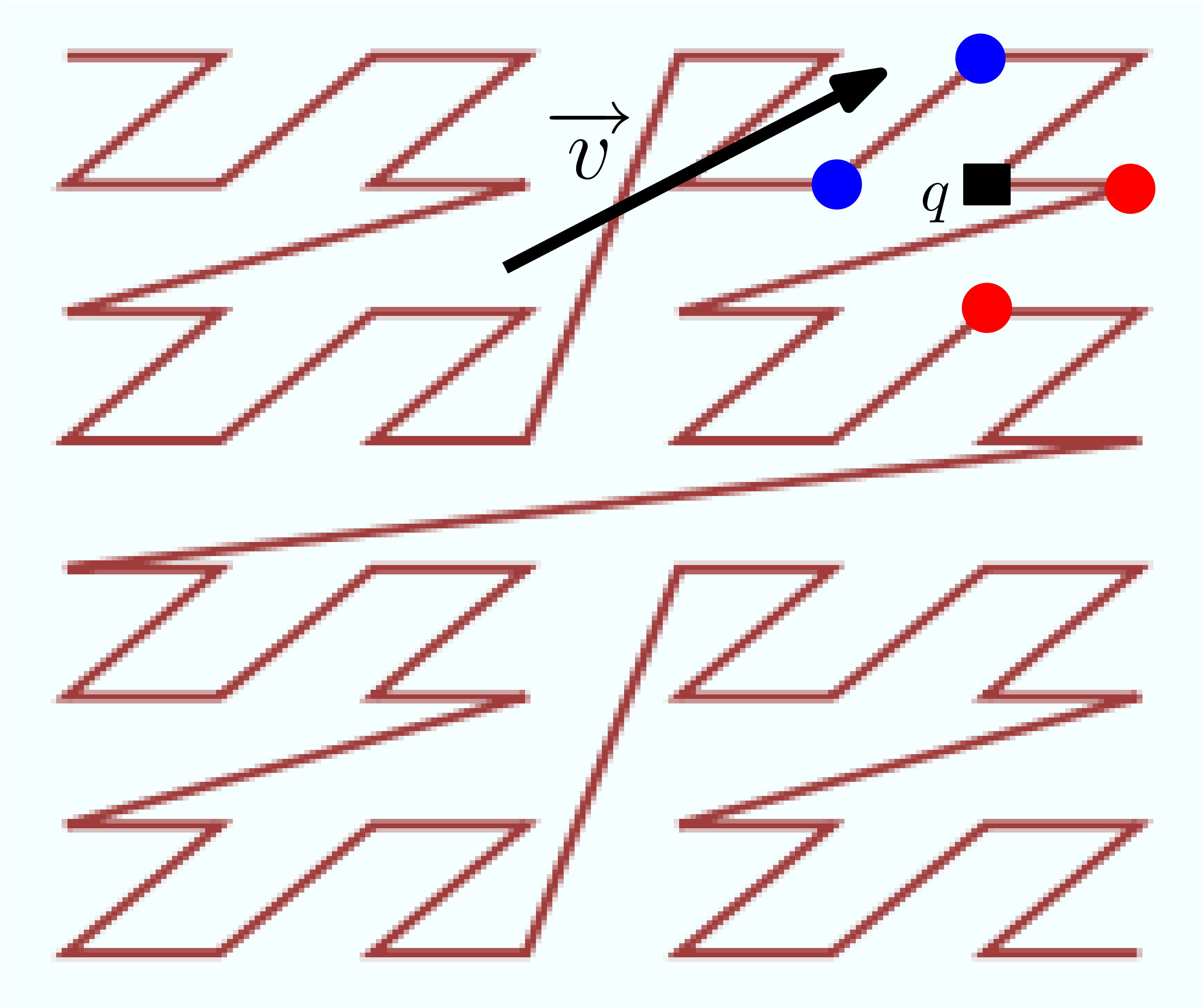

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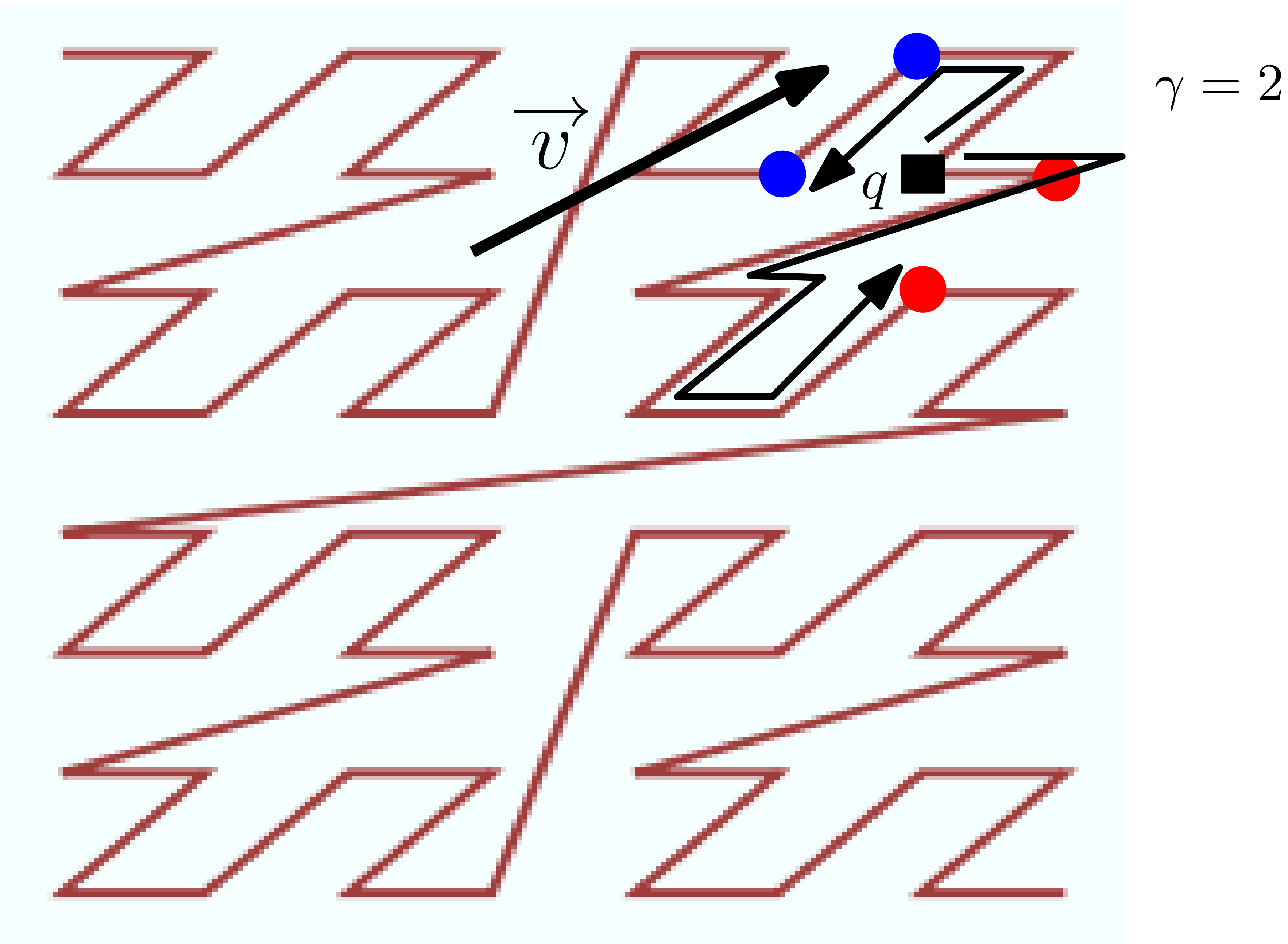

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- Theorem 1:
  Using $\alpha = O(1)$ and $\gamma = O(k)$, z$^{\chi}$-kNN guarantees an expected constant factor approximate kNN result with $O(\log_f \frac{N}{B} + k/B)$ number of page accesses (clustered index on z-values).

Approximation algorithm

z$^\chi$-kNN (point q, point sets $\{P^0, \ldots, P^\alpha\}$)

Candidates $C = \emptyset$;
For $i = 0, \ldots, \alpha$ $\{$
Find z_p^i as the successor of z_{q+v_i} in P^i;
Let C^i be γ points up and down next to z_p^i in P^i;
For each point p in C^i, let $p = p - v_i$;
$C = C \bigcup C^i$;
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Let $A^\chi = k$NN(q, C) and output A^χ.

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SQL statement for approximation algorithm

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         WHERE R_P.zval≥T.zval
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ORDER BY Euclidean(q.X_1,q.X_2,C.Y_1,C.Y_2)
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- Can we do better?

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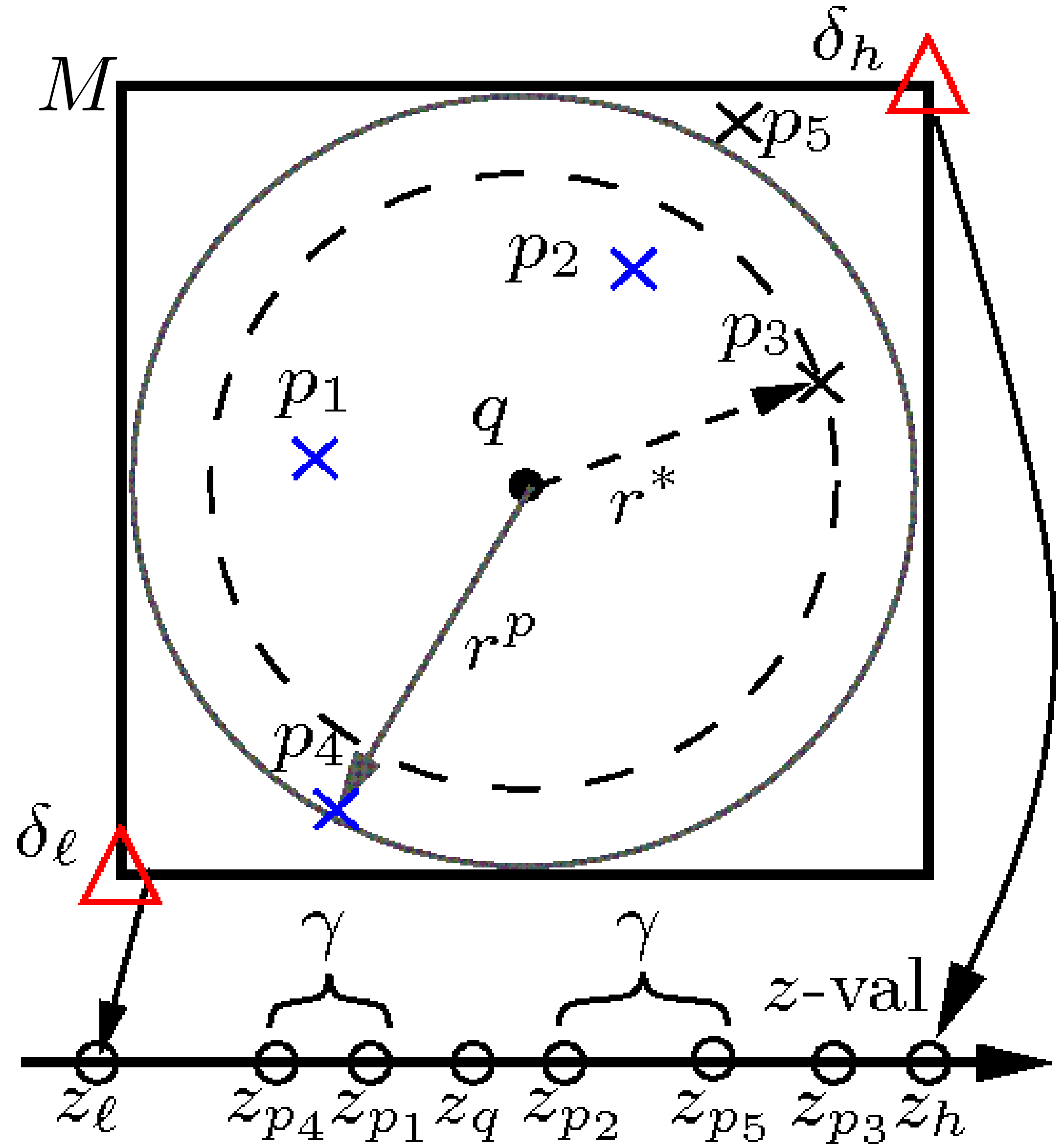

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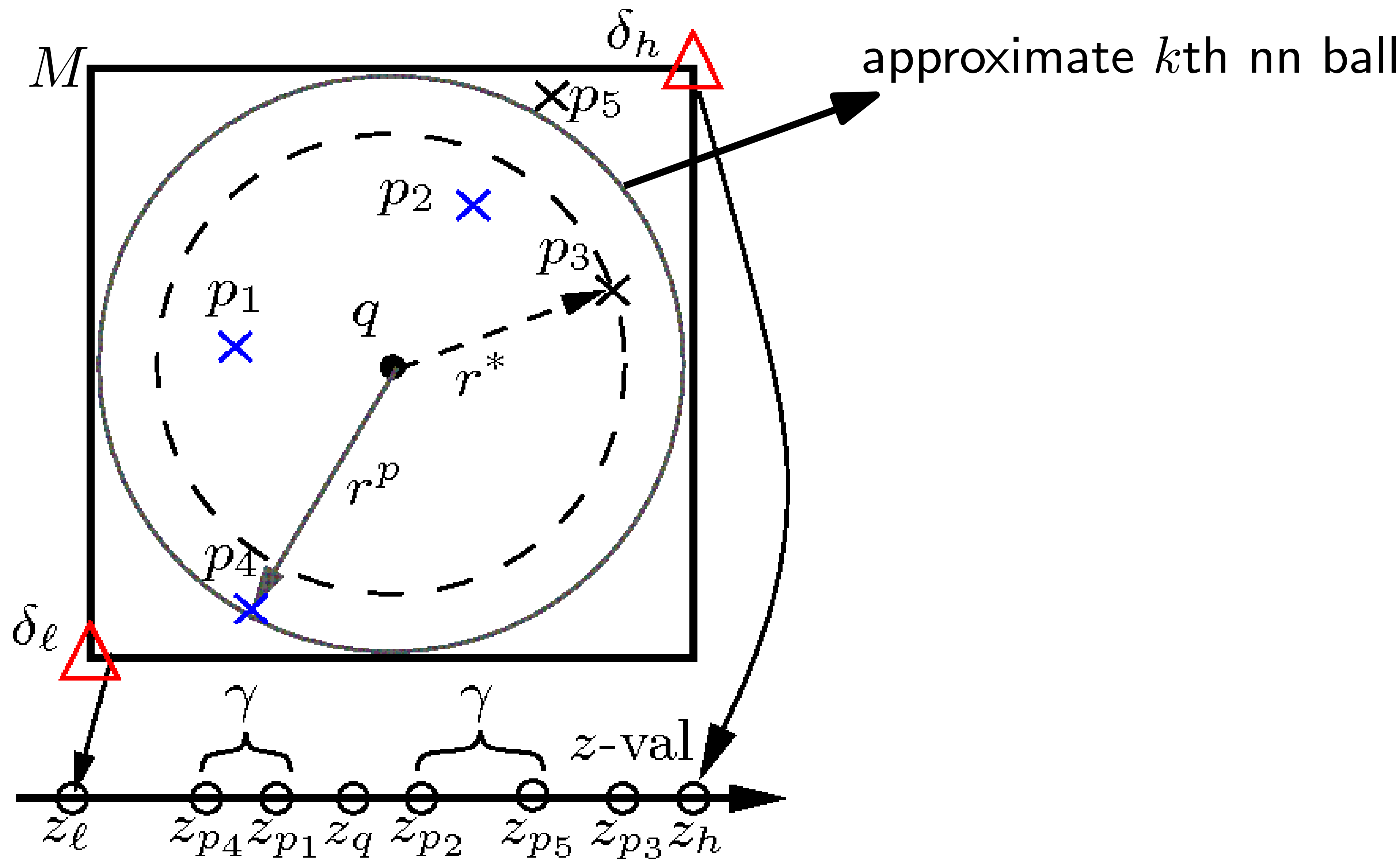

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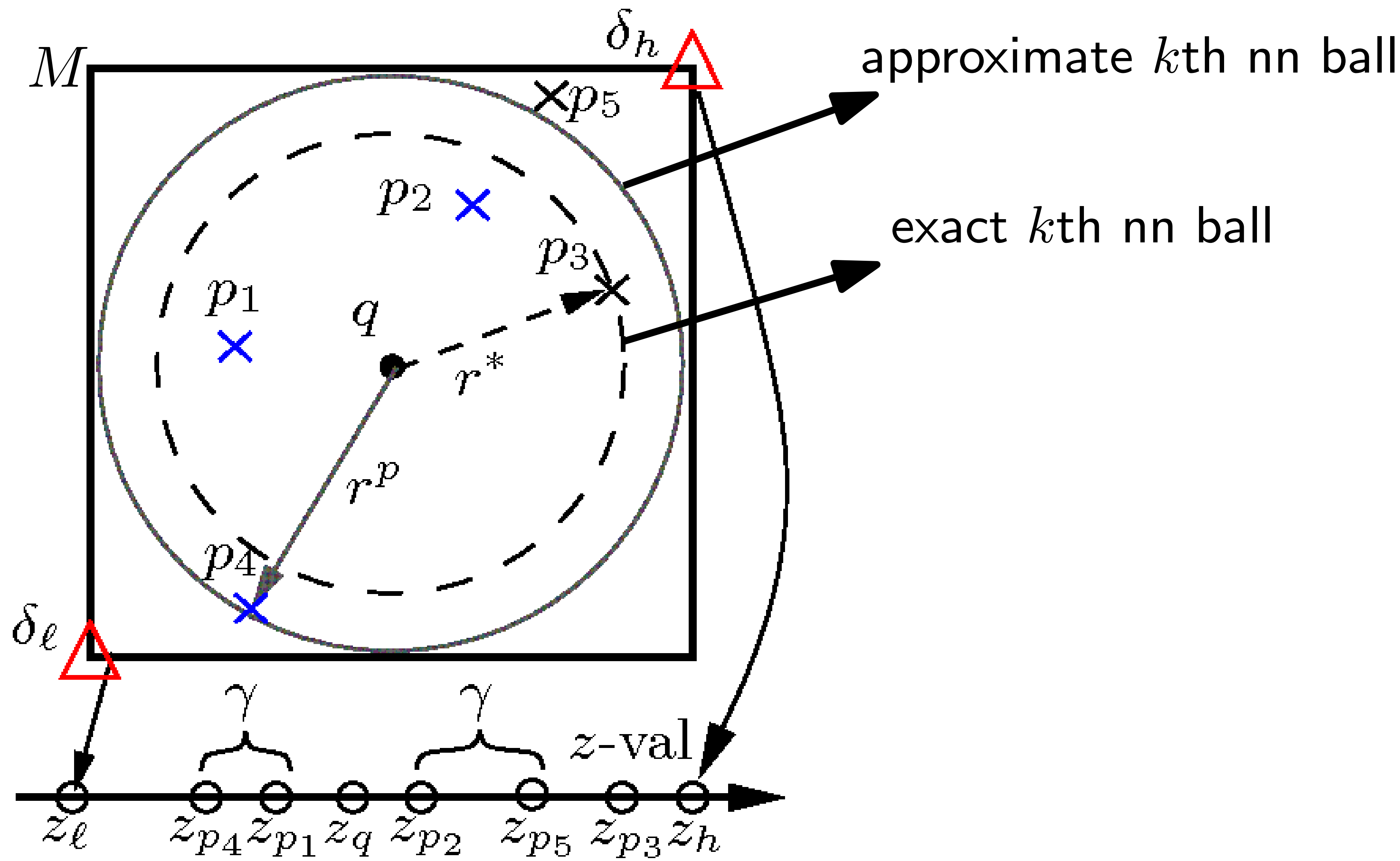

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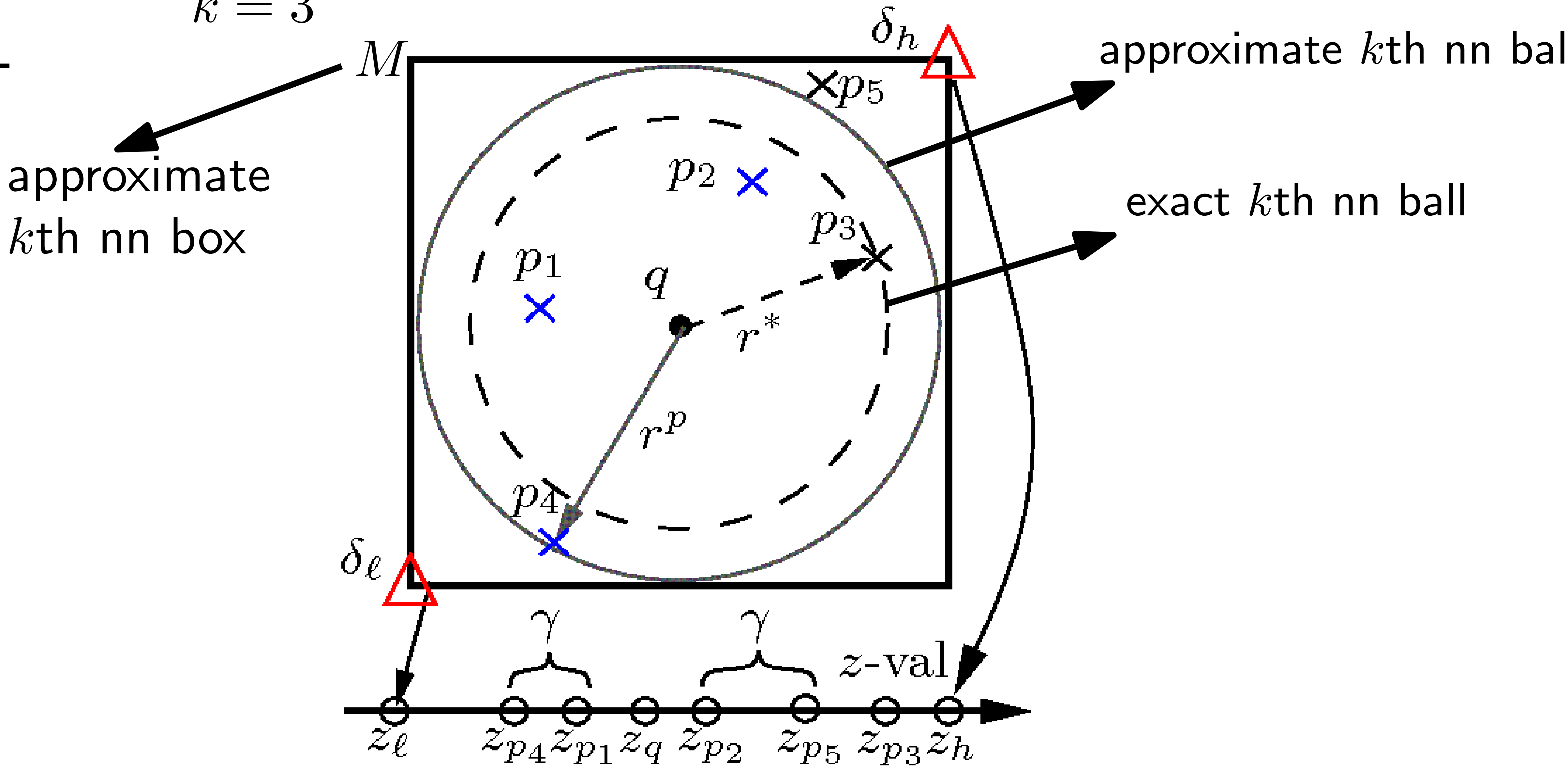

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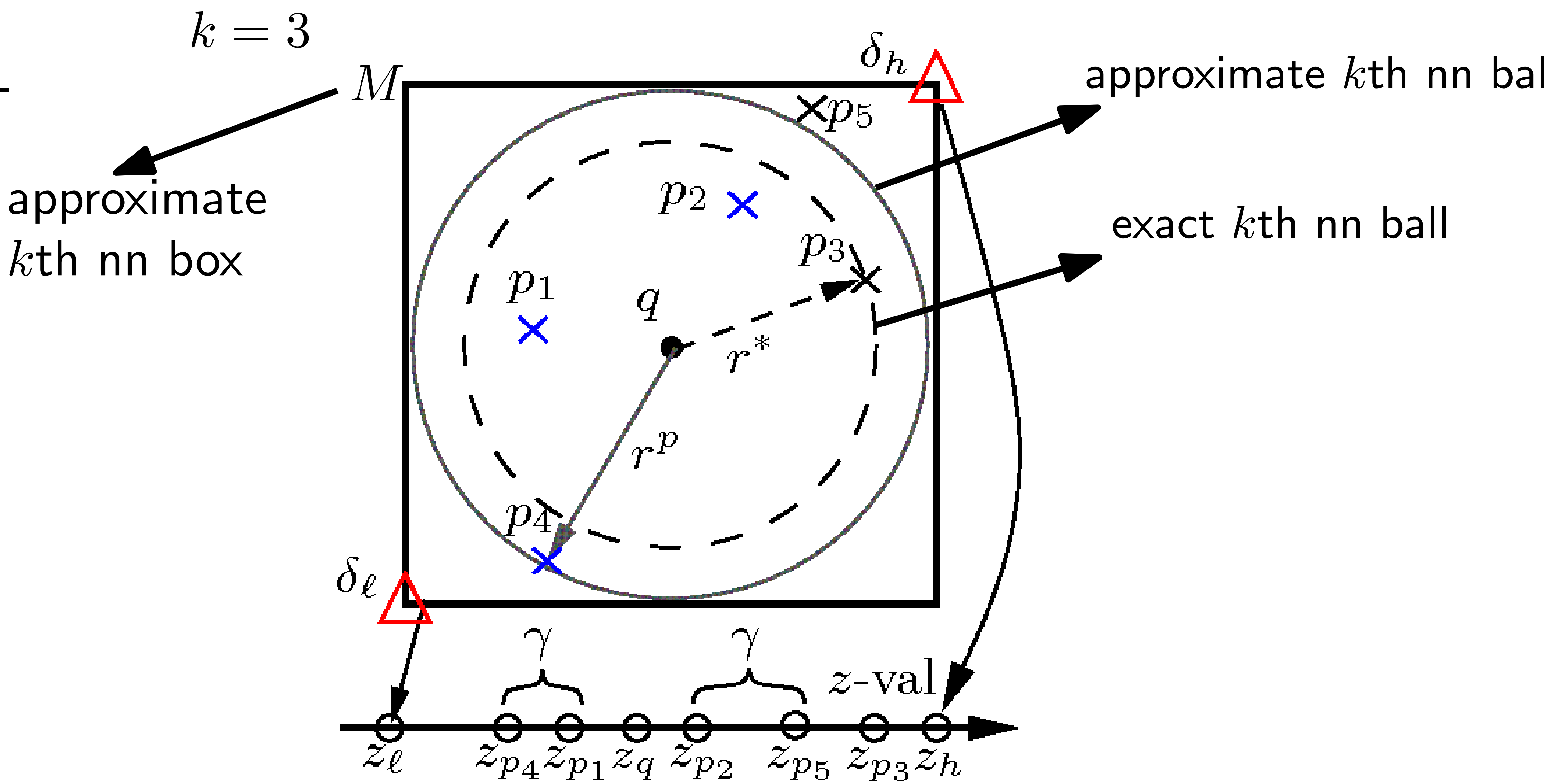

Lemma 4: For a rectangular box M and its lower-left and upper-right corner points δ_ℓ, δ_h, $\forall p \in M$, $z_p \in [z_\ell, z_h]$, where z_p stands for the z-value of a point p and z_ℓ, z_h correspond to the z-values of δ_ℓ and δ_h respectively.

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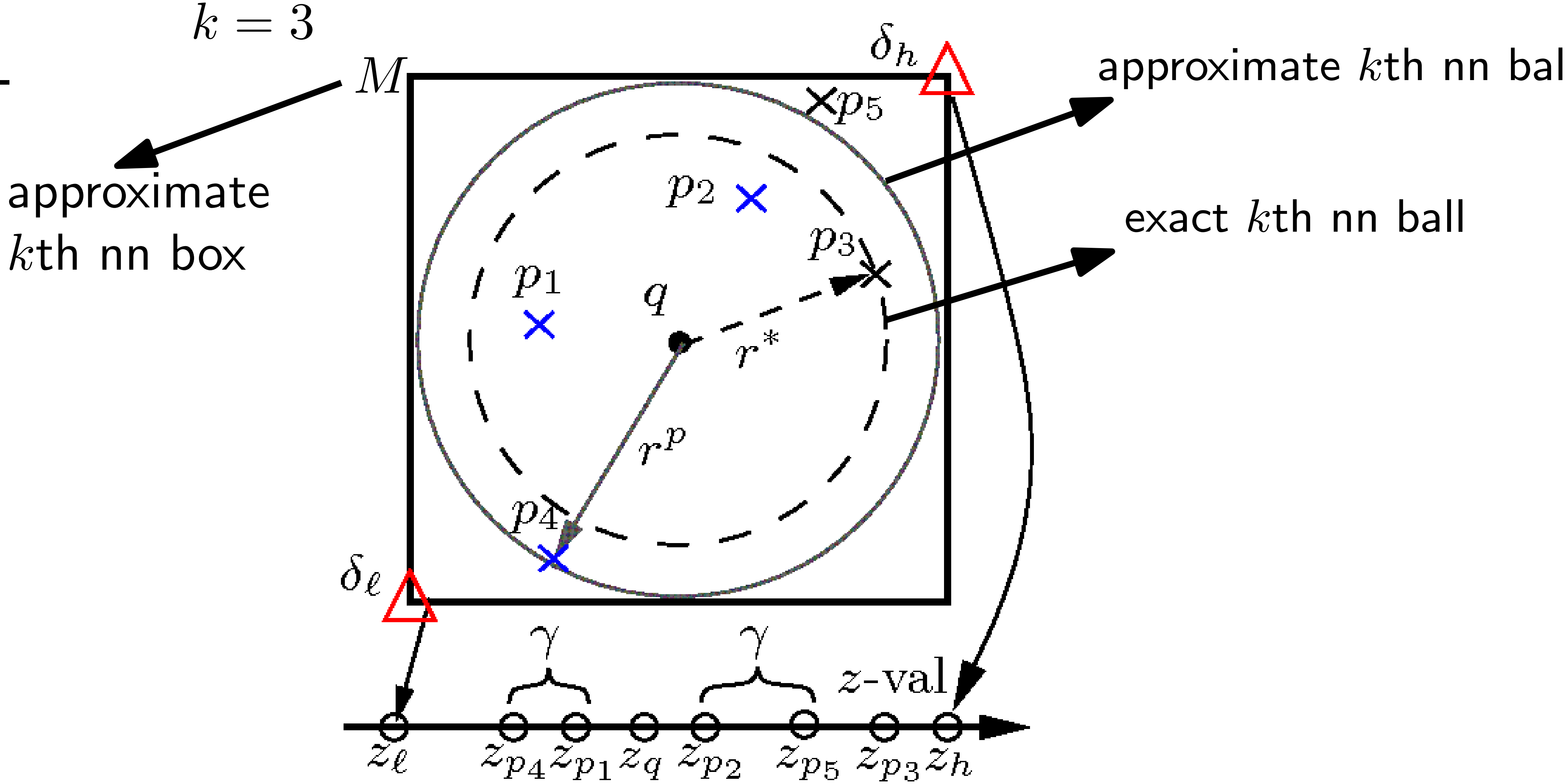

Corollary 1: Let z_ℓ and z_h be the z-values of δ_ℓ and δ_h points of $M(A^\chi)$. For all $p \in A$, $z_p \in [z_\ell, z_h]$.

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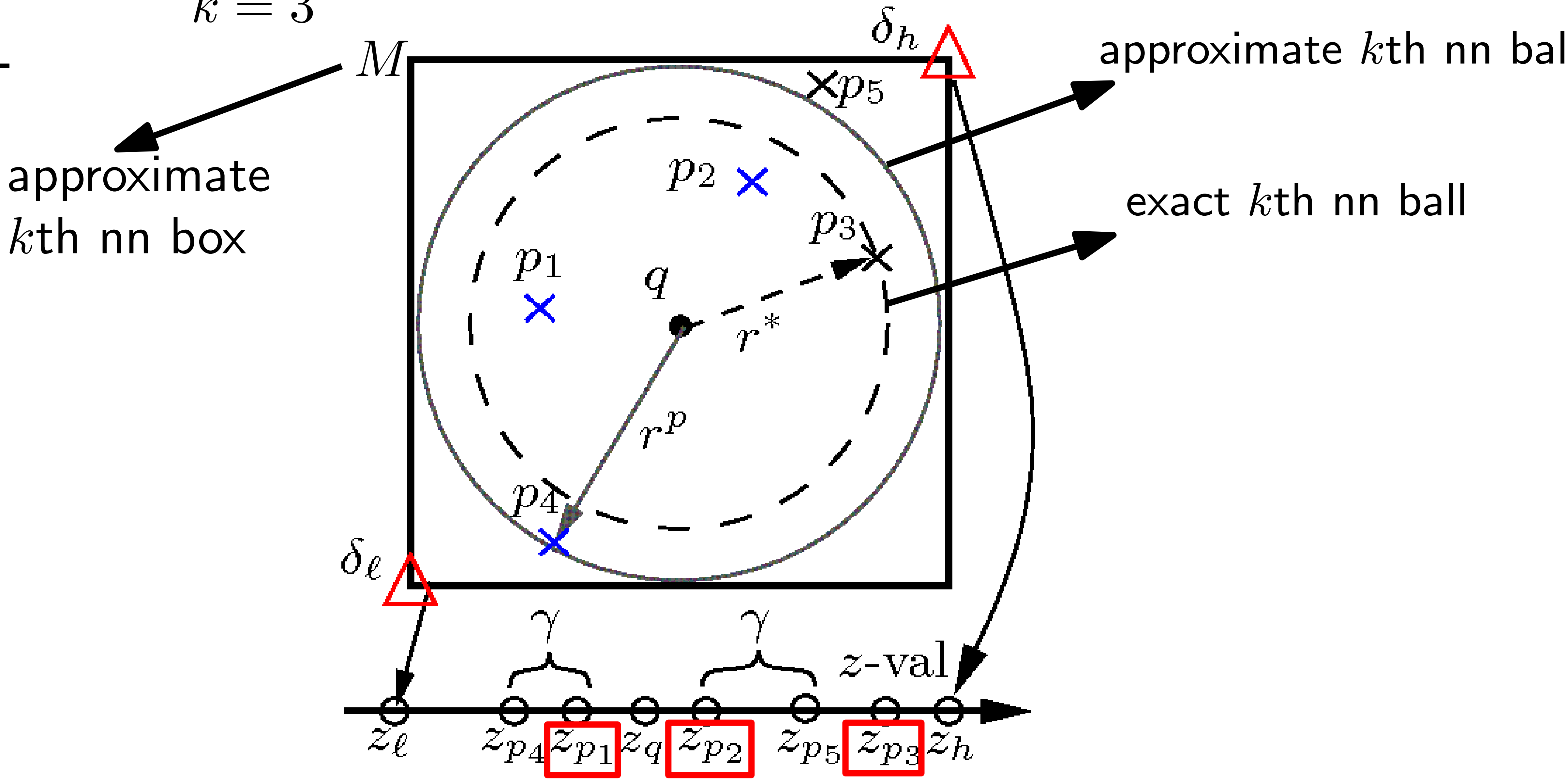

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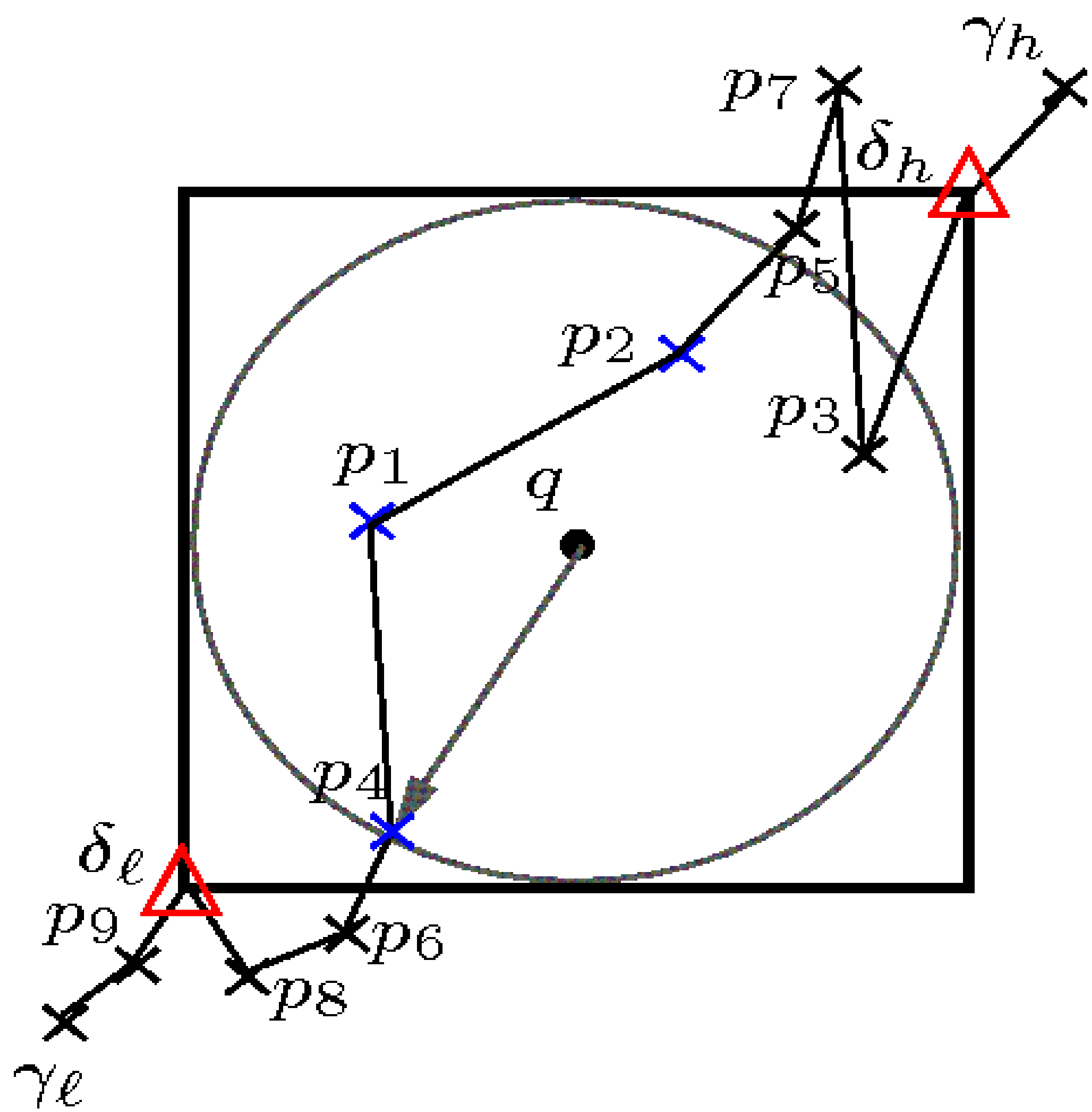

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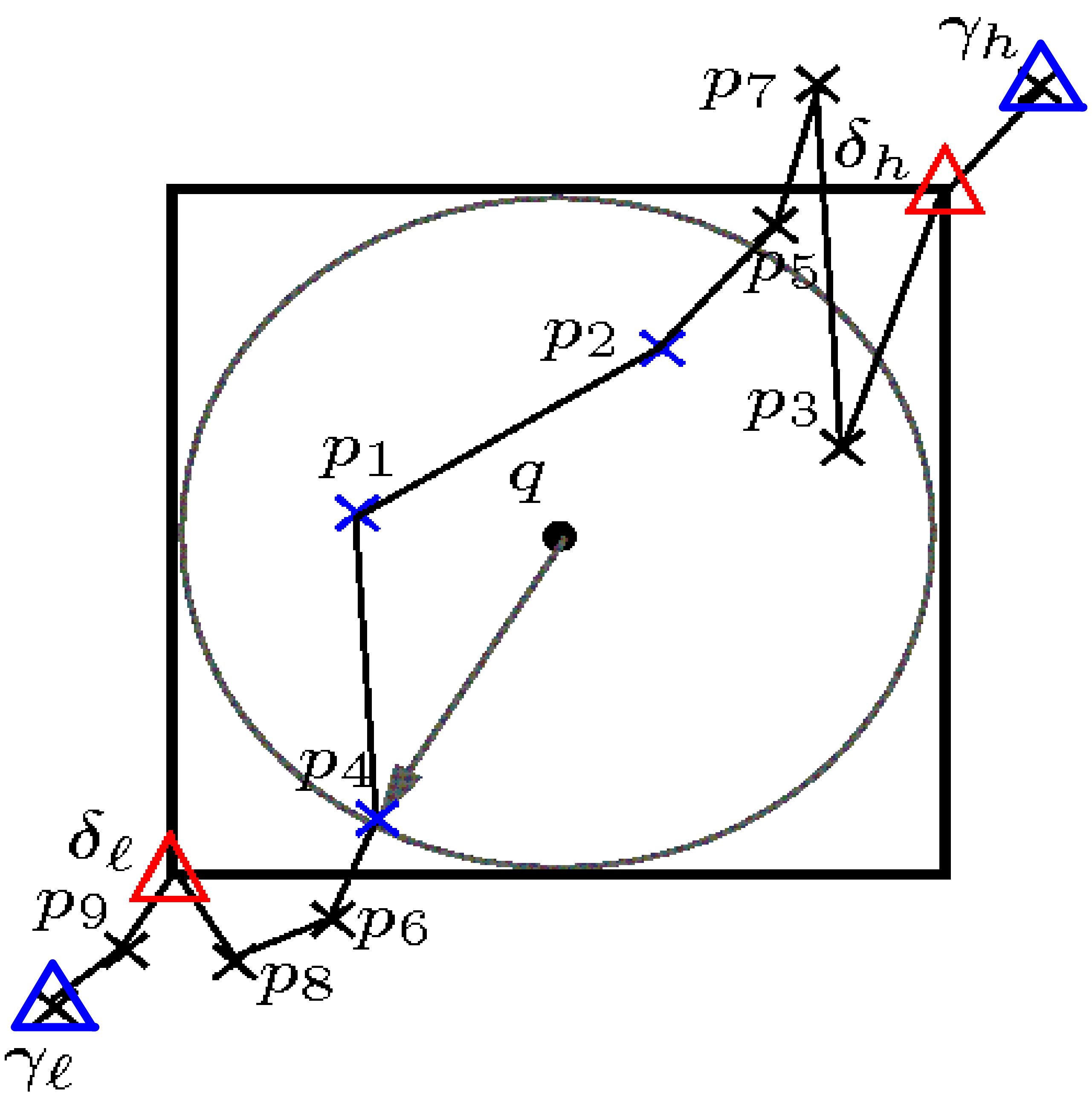

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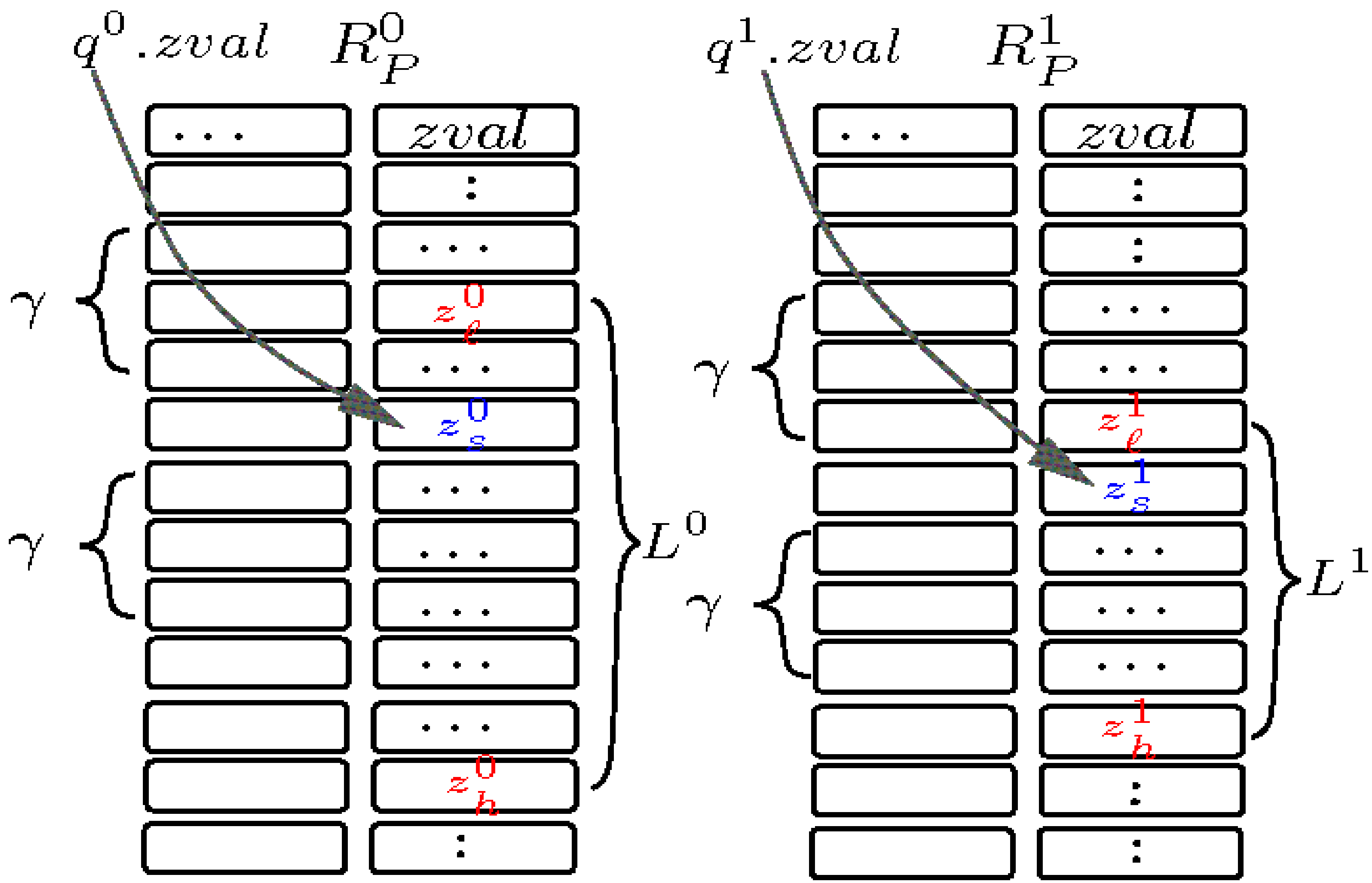
If not, we can find A by doing a range query with $[z_\ell^j, z_h^j]$ on any of the α tables. Ideally, we use the table with smallest $[z_\ell^j, z_h^j]$.



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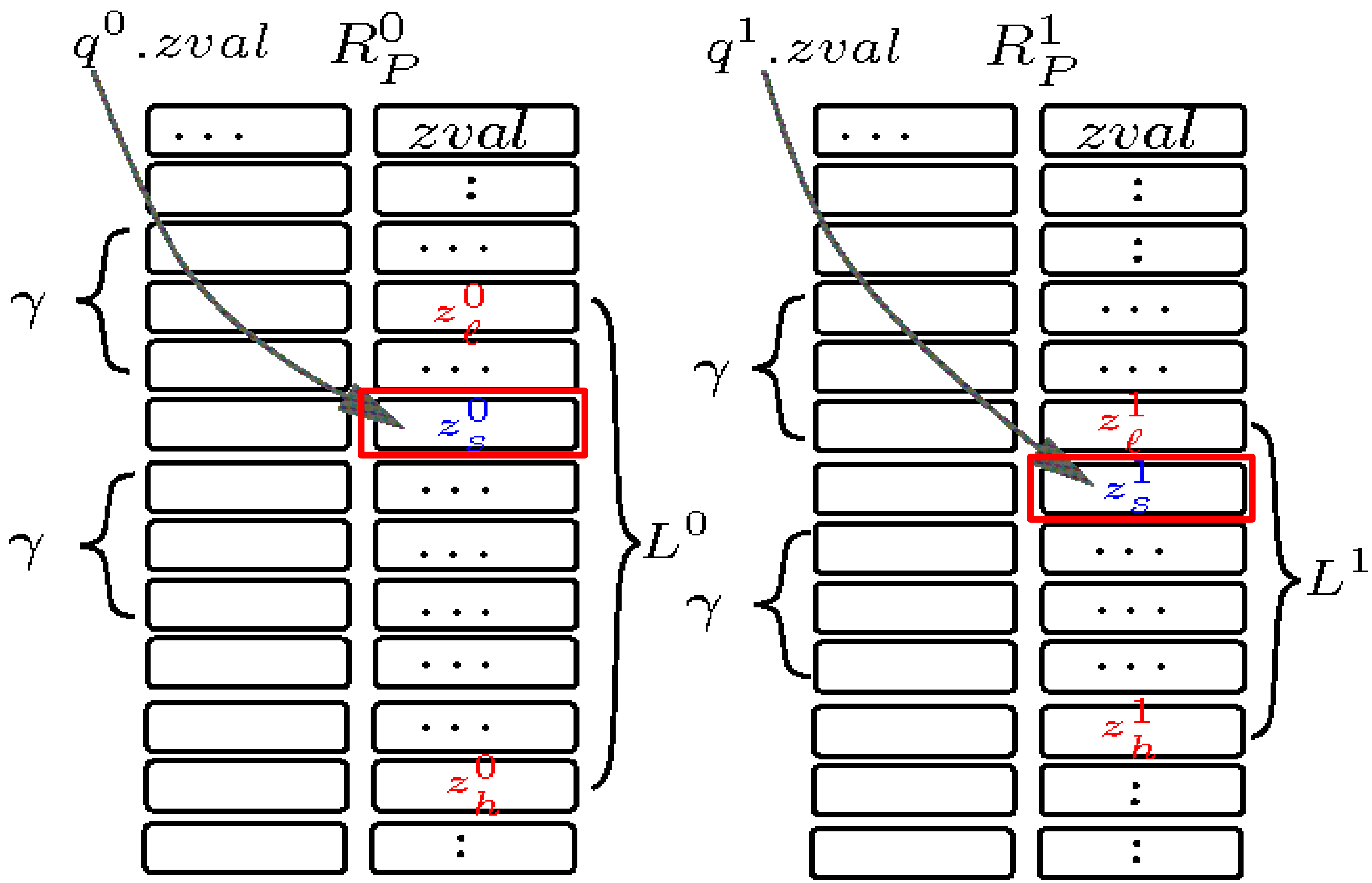

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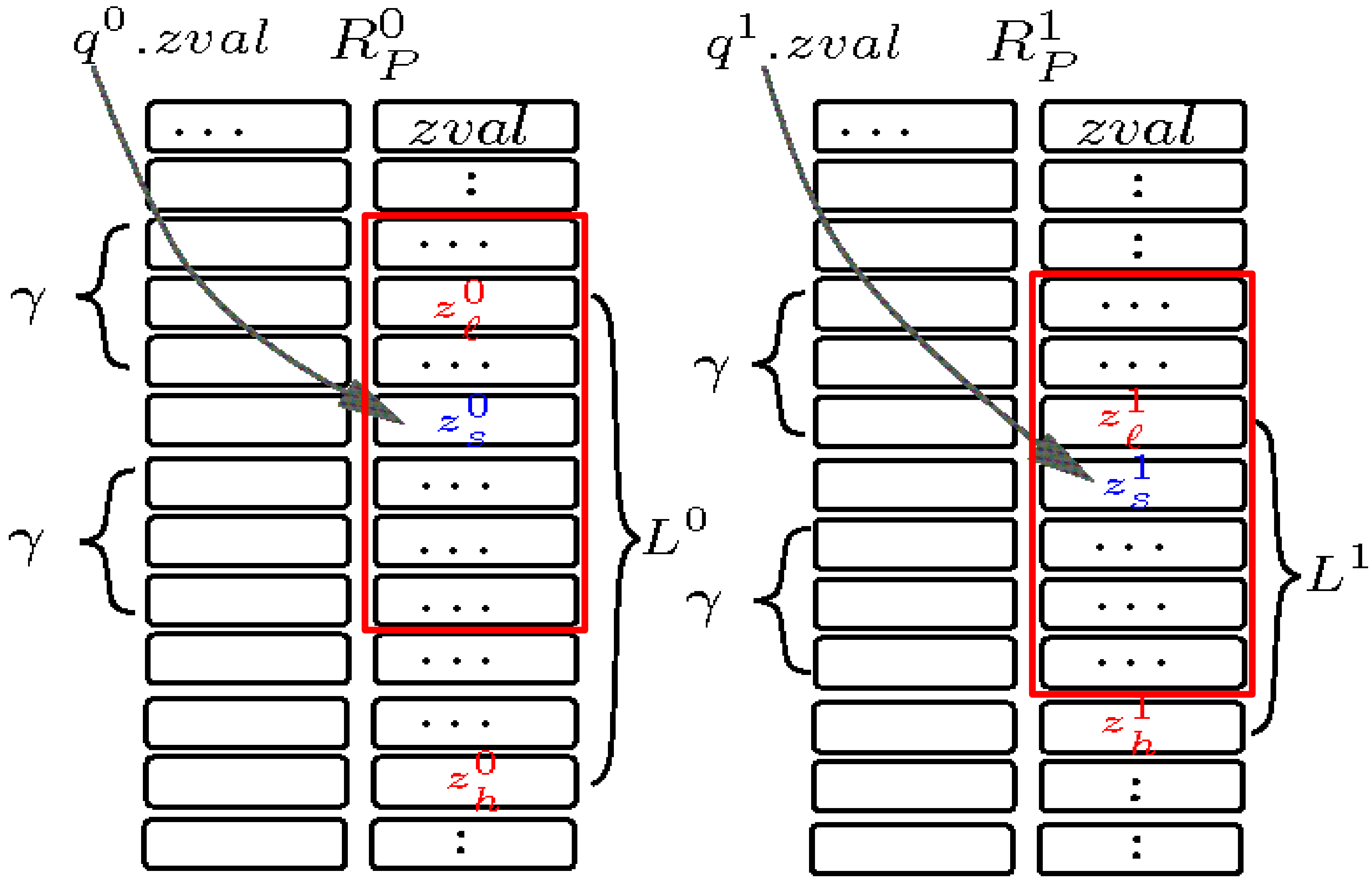

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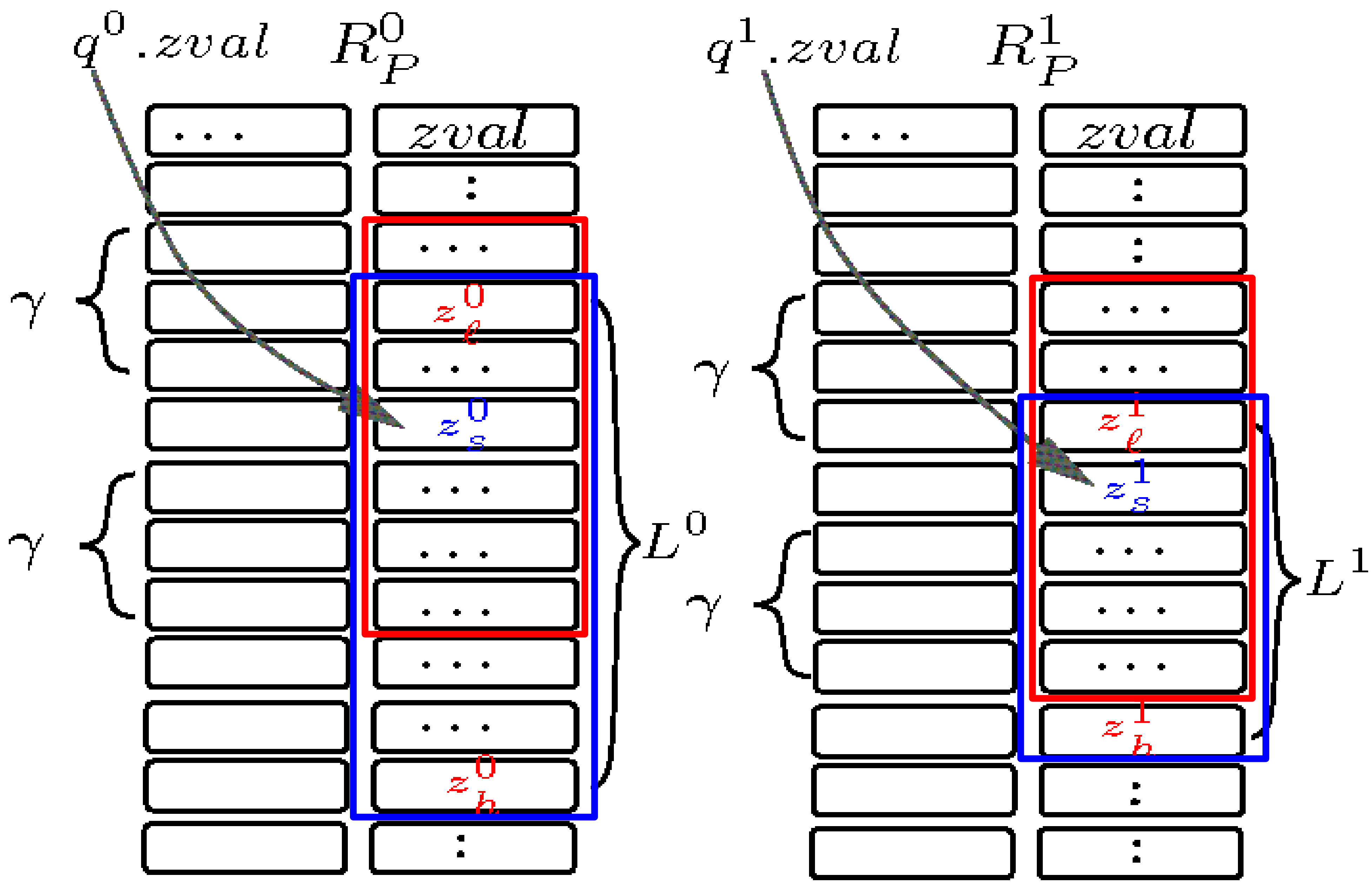

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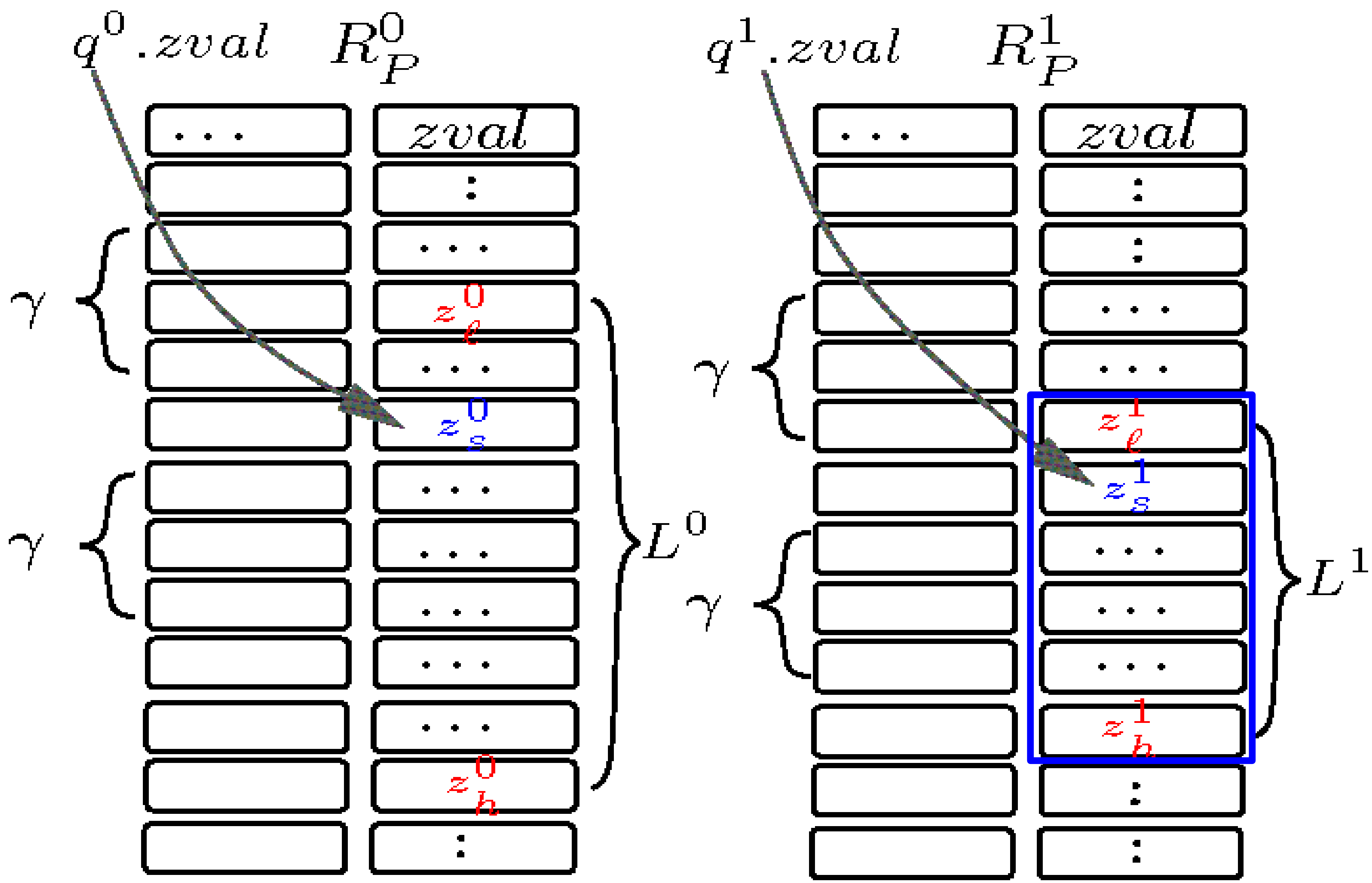

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- Deal with data in any dimension: without changing the framework; for large dimensionality (say $d > 20$), using LSH-based method.
- Updates: for deletion, delete record r based on its pid from all talbes $\mathsf{R}^0, \ldots, \mathsf{R}^\alpha$; for insertion, calculate the z-values of the point for all randomly shifted versions, insert them into corresponding tables.

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- All algorithms are implemented in Microsoft SQL Server 2005. Experiments are conducted on an Intel Xeon CPU @ 2.33GHz. The memory of the SQL Server is set to 1.5GB.

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- Two synthetic data sets: uniform points and random clustered points.

Experiment Setup

- All algorithms are implemented in Microsoft SQL Server 2005. Experiments are conducted on an Intel Xeon CPU @ 2.33GHz. The memory of the SQL Server is set to 1.5GB.

- Real data sets: points representing the road-networks for states in USA

- Two synthetic data sets: uniform points and random clustered points.

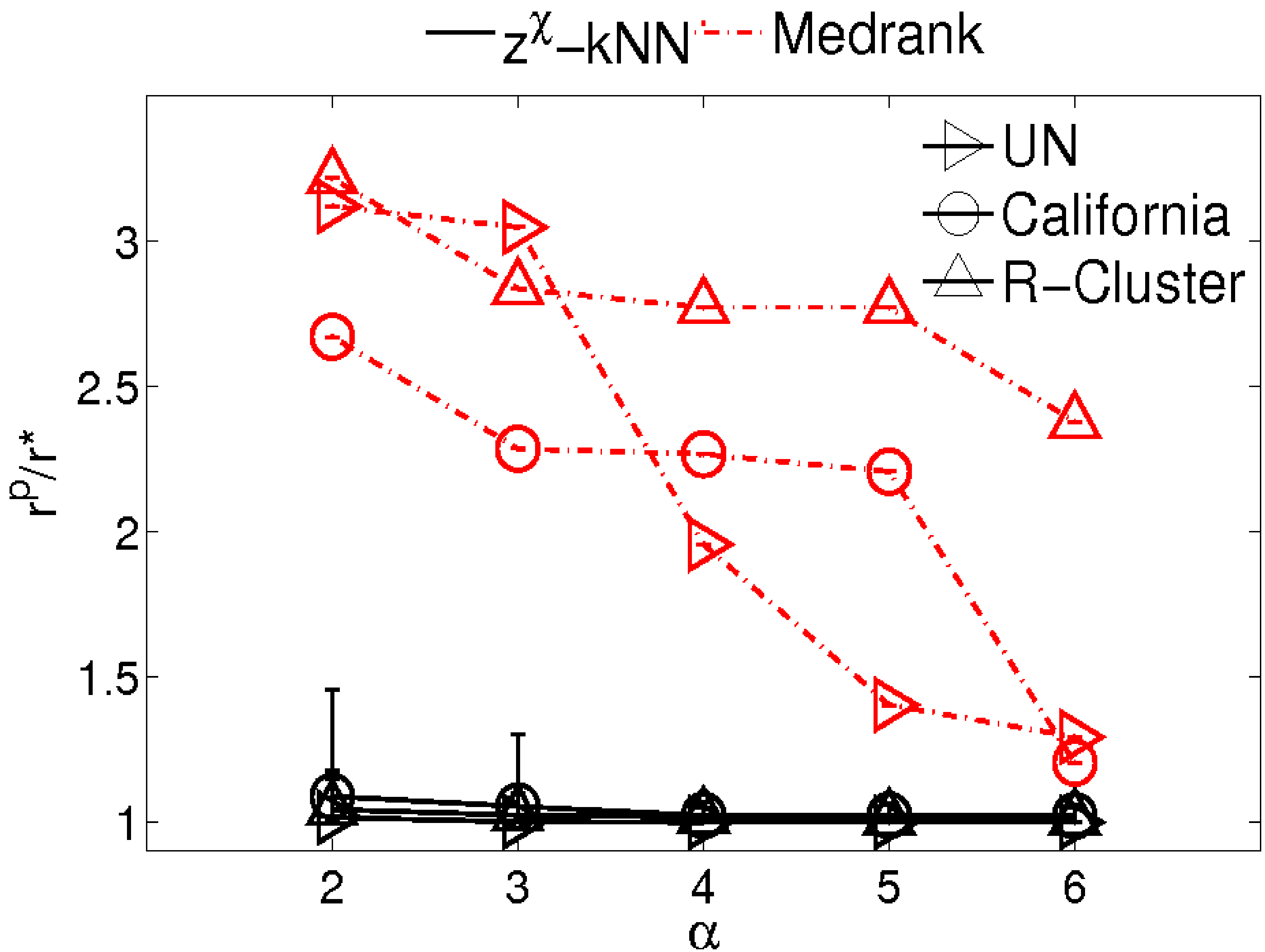
- Compare against the Medrank and iDistance algorithms (implemented by SQL statement and store precedure).

Experiment Setup

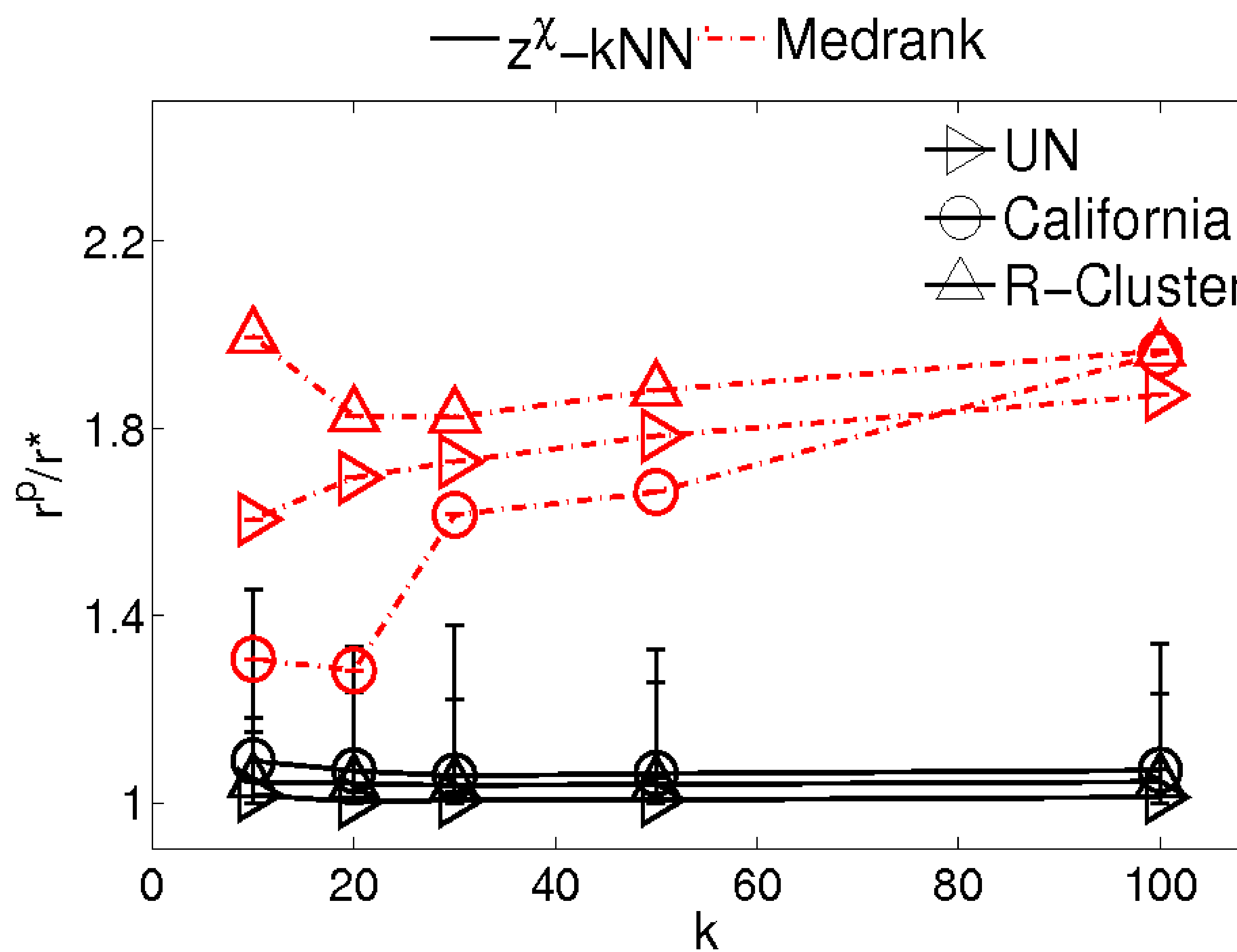
- The default experimental parameters are summarized below

Symbol	Definition	Default Value
k	number of neighbors	10
N	size of points set	1,000,000
α	randomly shifted copies	2
γ	number of points up and down	$2k$
d	dimensionality	2

Results for the kNN query: approximation quality



Results for the kNN query: approximation quality



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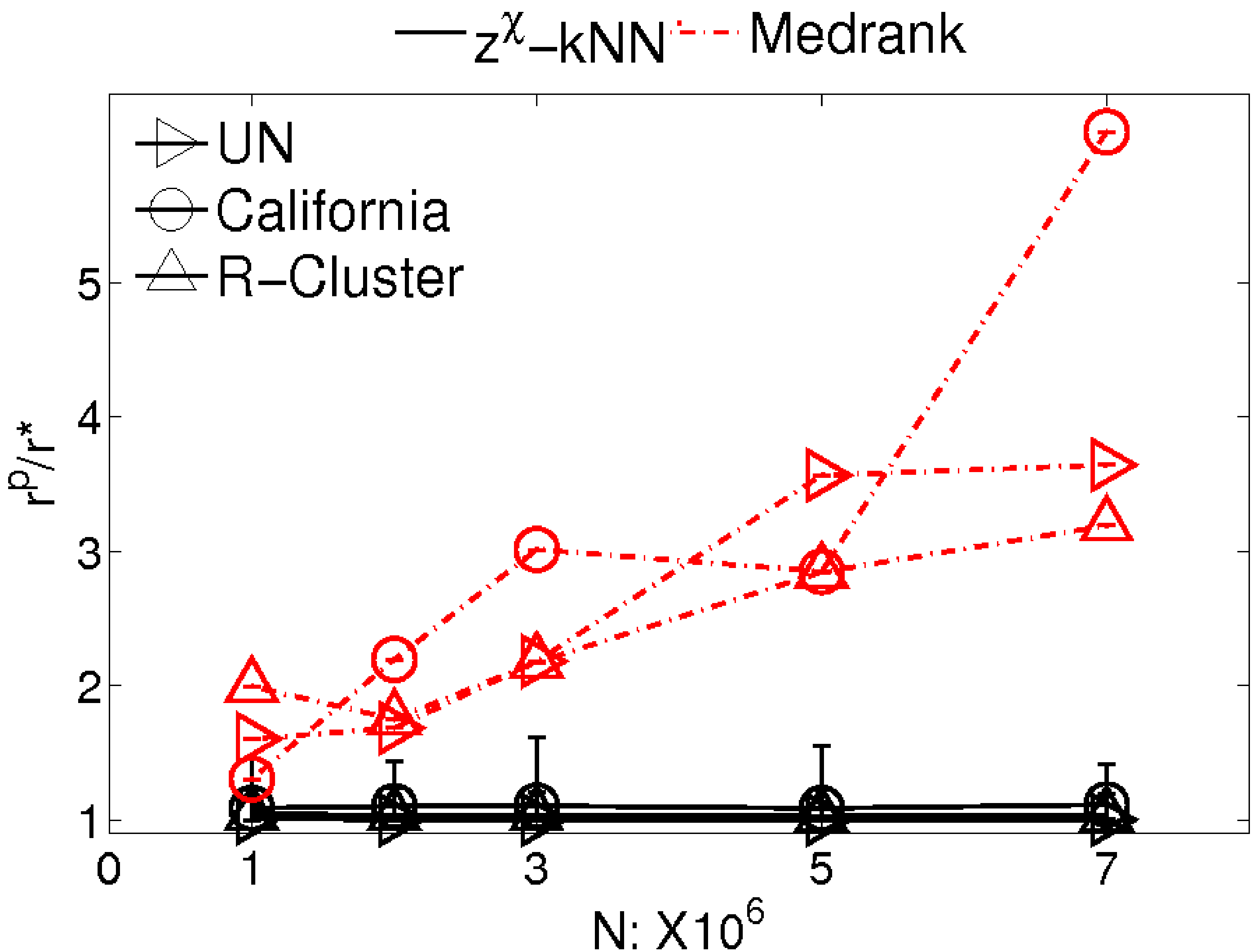

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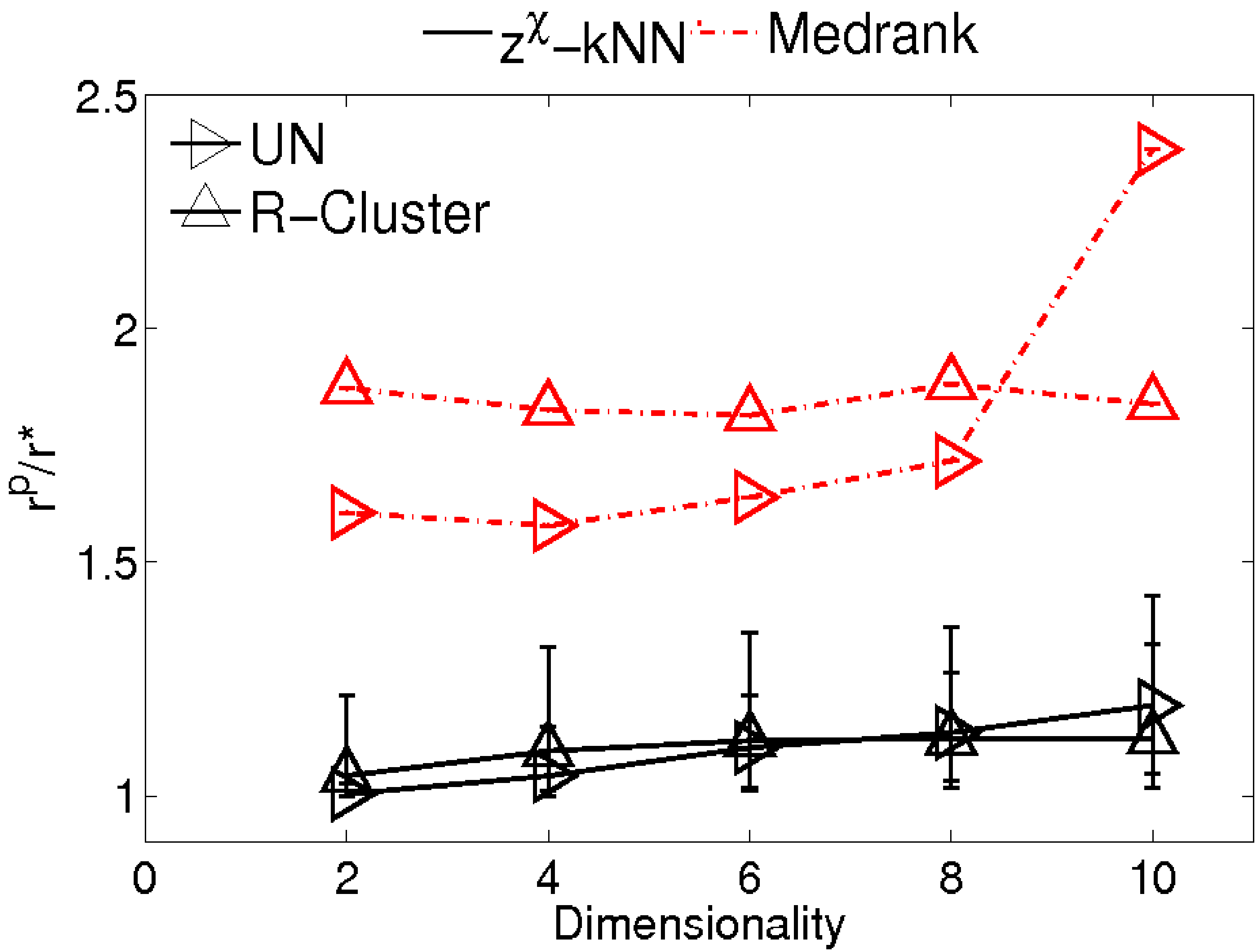
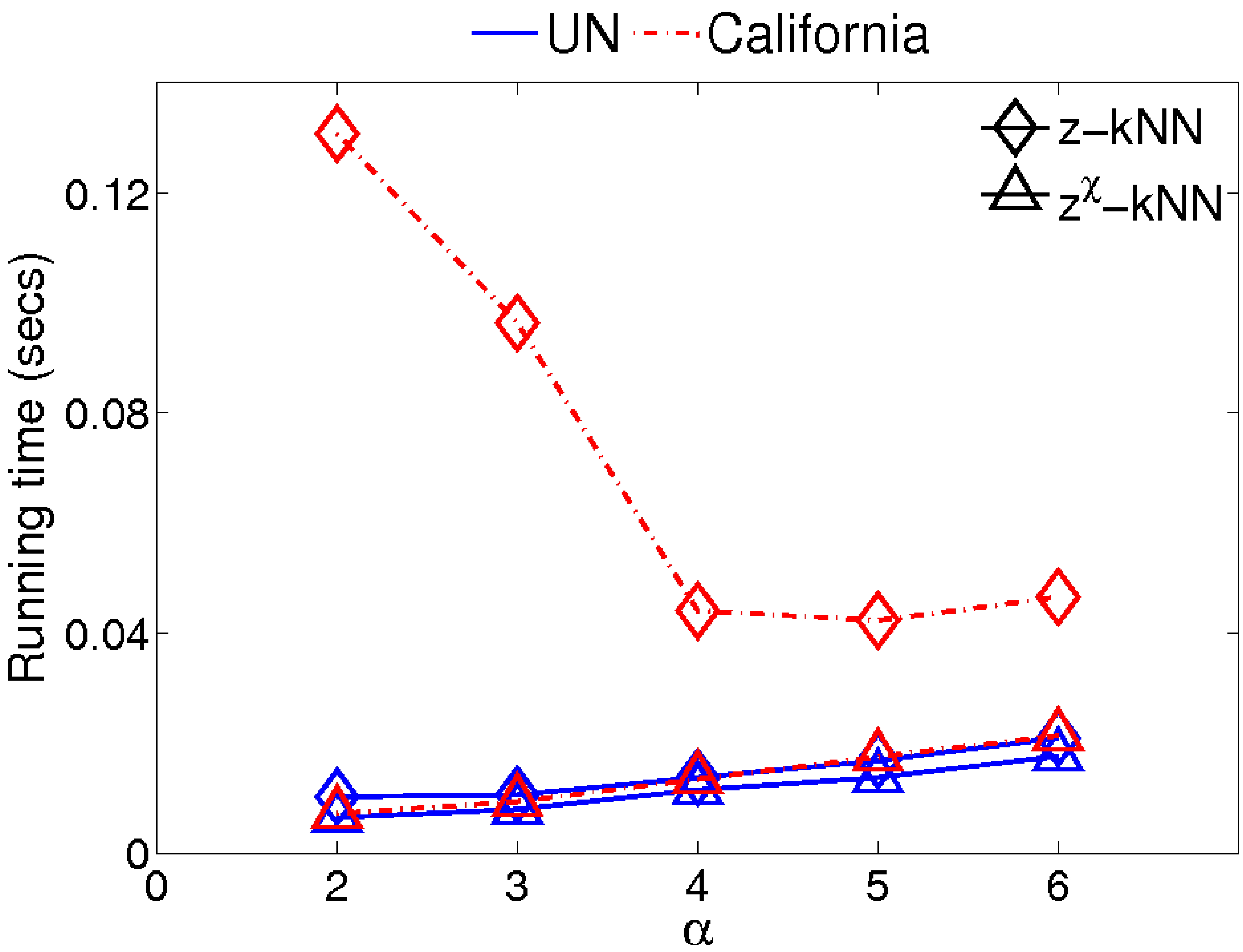


Results for the kNN query: running time



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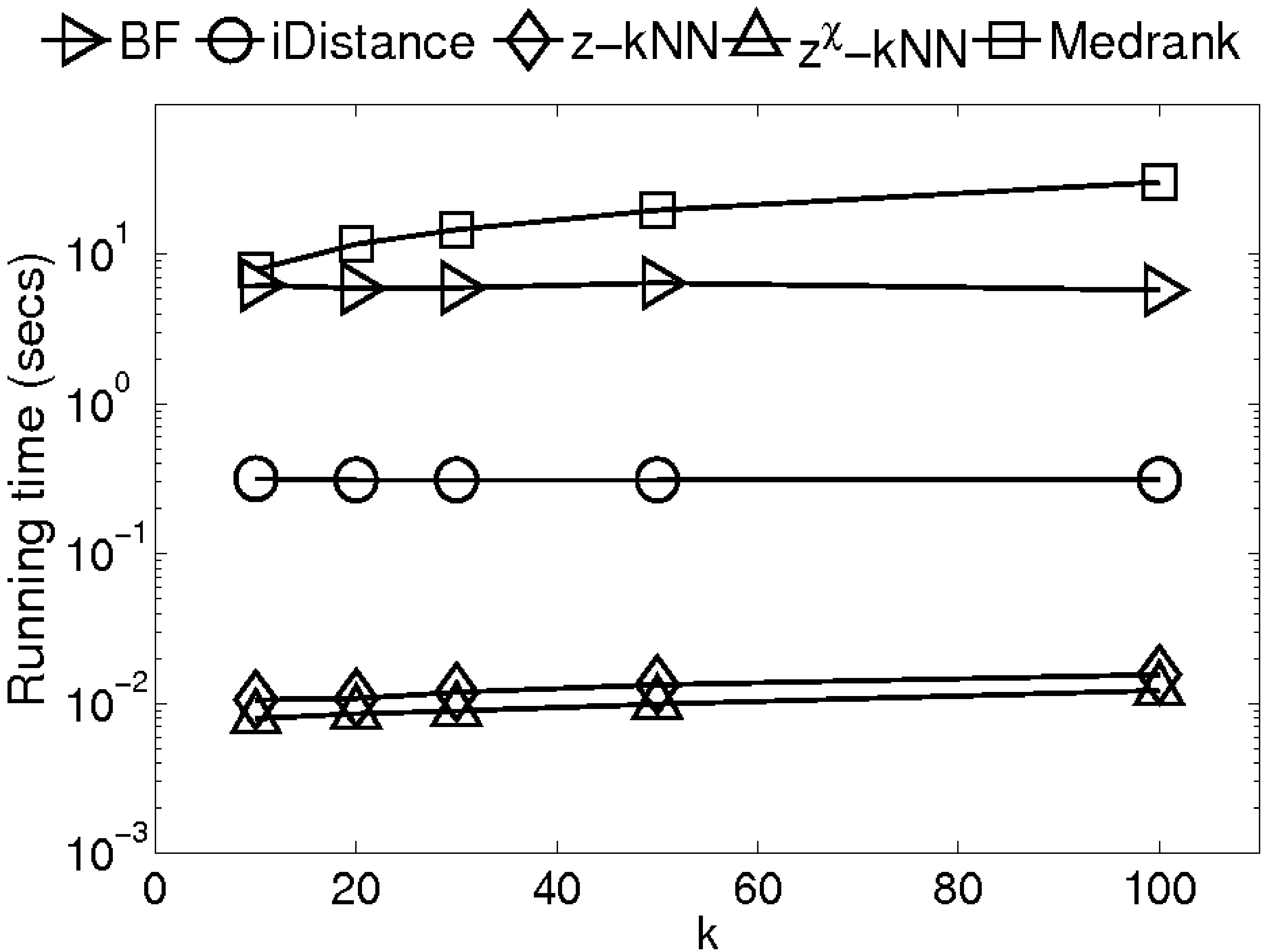
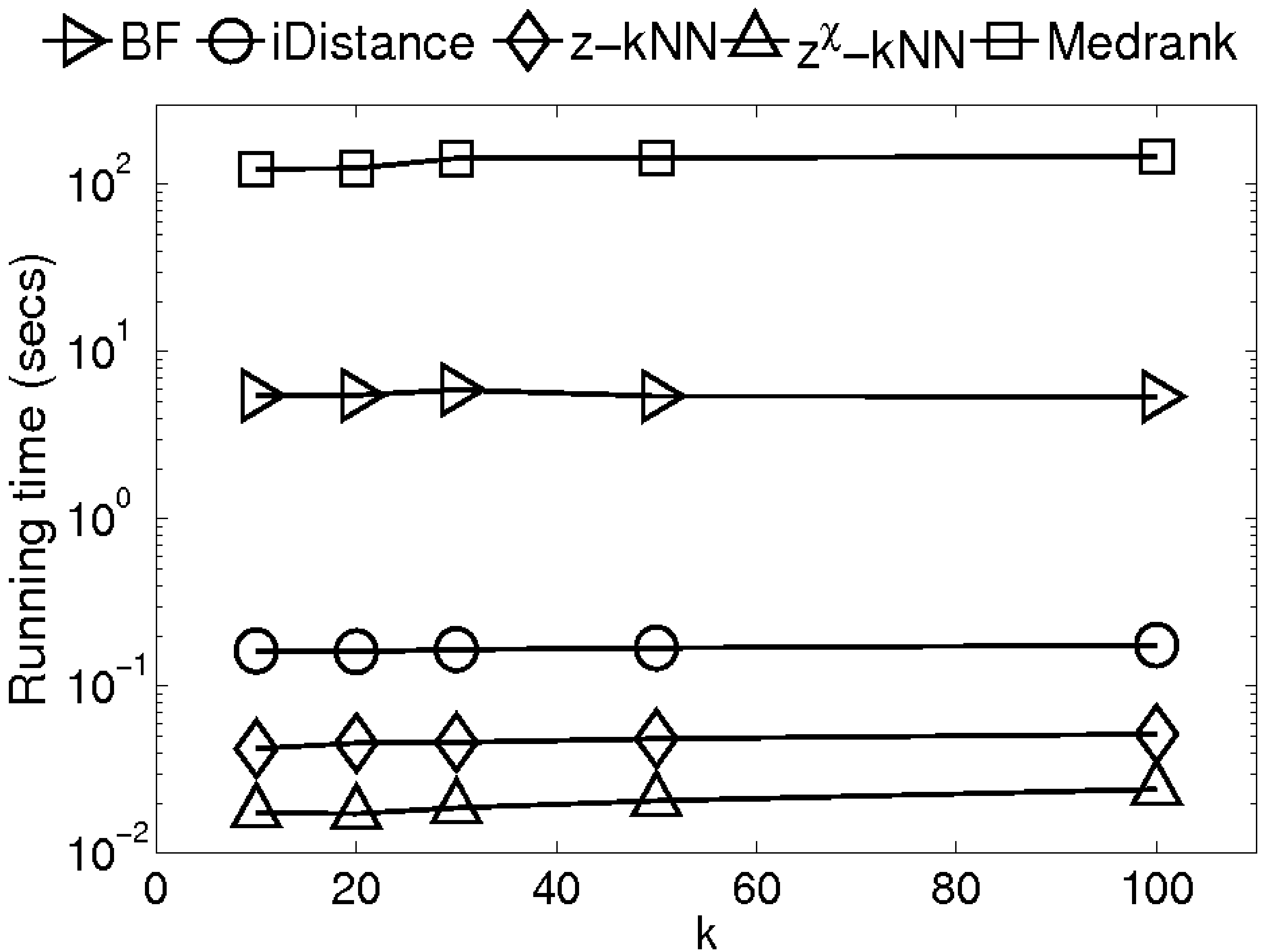


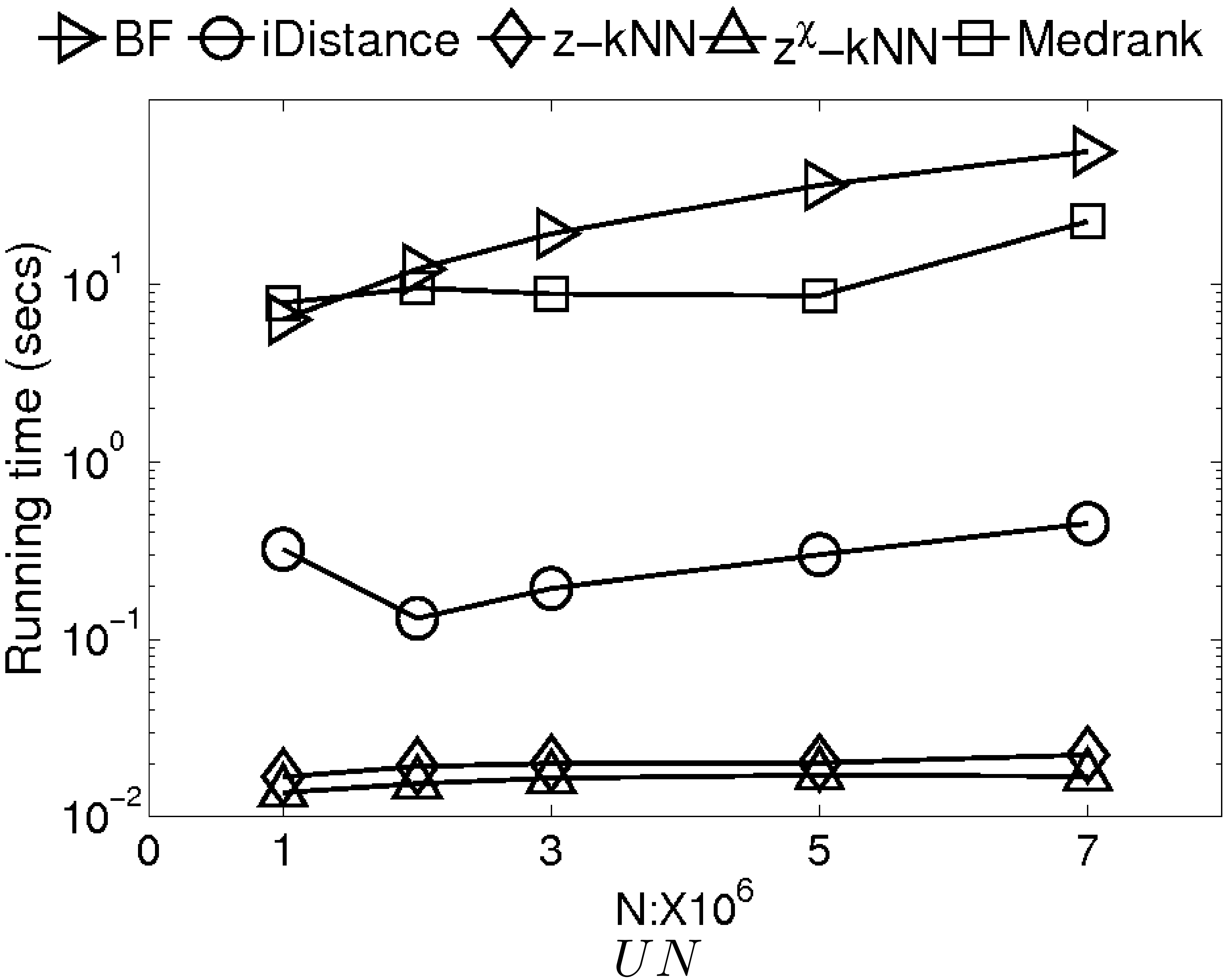
UN

Results for the kNN query: running time

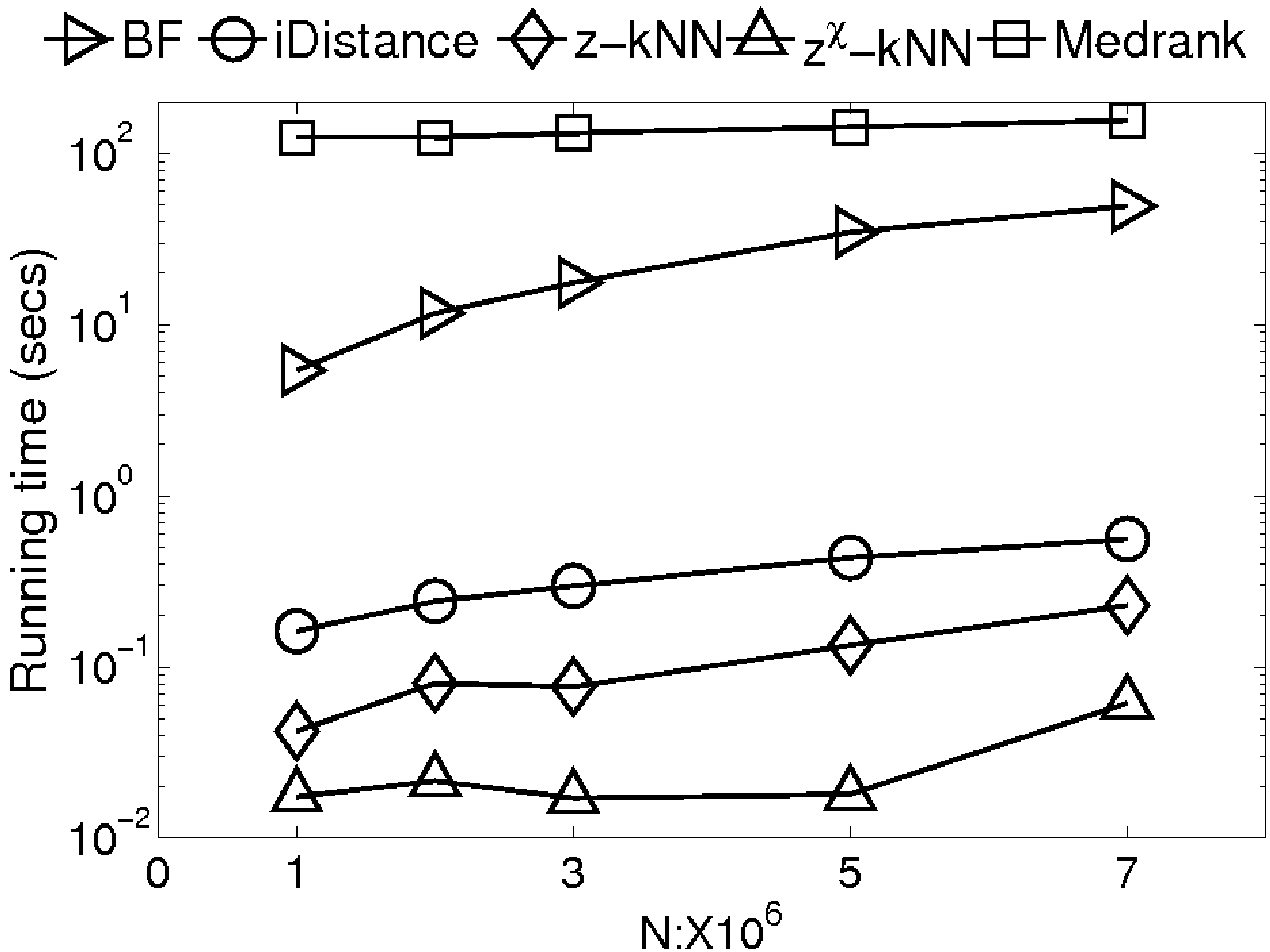


$California$

Results for the kNN query: running time

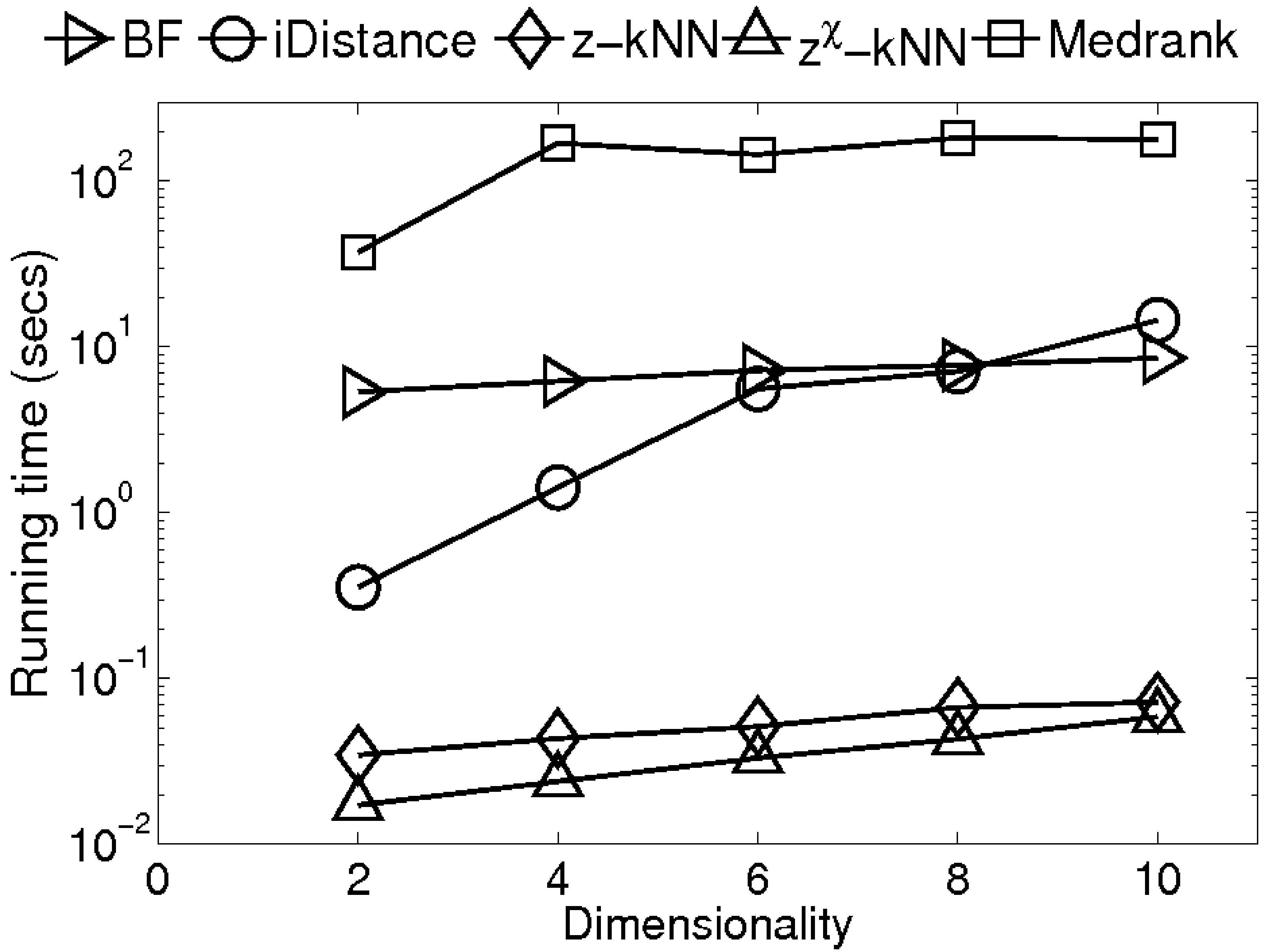


Results for the kNN query: running time



California

Results for the kNN query: running time



Conclusions

- Presented a constant approximation for the kNN query, with logarithmic page accesses in any fixed dimension and extended it to the exact solution, both using just $O(1)$ random shifts.

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- All the algorithms can be implemented by SQL operators in relational databases.
- Our approach naturally supports kNN-Joins.
- No changes are required for different dimensions, and the update is trivial.
- Future research:
  - Study other related, interesting queries in this framework, e.g., the reverse nearest neighbor queries.
  - Examine the relational algorithms to the data space other than the L_p-norms, such as the road networks.

The End

$$THANK \quad YOU$$

Q and A