Capacity and Delay Analysis for Multicast

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***Abstract*—In this paper,we consider capacity and delay for multicast under multi-hop.Assuming the network is a unit which has a cell partitioned structure and users move according to a independent and identically distribution (i.i.d) mobility model,we achieved the bounder of capacity and delay.The algorithm is a new one named after multi-hop relay algorithm for multicast.The capacity we get is $O(\frac{1}{log(N*logK)})$ and the delay we attain is $\Theta(logN)$. Compared to multicast under two-hop - delay ($\Theta(\sqrt{NlogK})$) ,capacity $\Theta(\frac{1}{K})$, achieved by Xinbing Wang *[IEEE Trans VOl.19 NO.5]* ,the delay becomes better but capacity becomes worse.**

I. Introduction

Michael J and Eytan Modianop[1] focus on the unicast under two-hop algorithm ,demonstrating that it can achieve the delay of O(N) without redundancy and the delay of $O(\sqrt{N})$ with redundancy.Xinbing Wang[2] and his cooperators extend the unicast to multicast(Motioncast) and find out that we can achieve the delay $\Theta(NlogK)$, $\Theta(\sqrt{NlogK})$ respectively without and with redundancy.However, it is also limited in two-hop.In this paper,we analyse the capacity and delay for multicast under multi-hop.

A mobile ad hoc network (MANET) consists of a collection of wireless mobile nodes dynamically forming a temporary network without the support of any network infrastructure or centralized control. In these networks, nodes often operate not only as sources, but also as relays, forwarding packets for other mobile nodes.We use the following cell partitioned network model: The network is partitioned into nonoverlapping cells of equal size (see Fig. 1). There are mobile users independently roaming from cell to cell over the network, and time is slotted so that users remain in their current cells for a timeslot, and potentially move to a new cell at the end of the slot. If two users are within the same cell during a timeslot, one can transfer a single packet to the other. Each cell can support exactly one packet transfer per timeslot, and users within different cells cannot communicate during the slot. Multihop packet transfer proceeds as users change cells and exchange data. The cell partitioning reduces scheduling complexity and facilitates analysis.

We consider the following simplified mobility model: Every timeslot, users choose a new cell location independently and identically distributed over all cells in the network. Such a mobility model is, of course, an oversimplification. Indeed, actual mobility is better described by Markovian dynamics, where users choose new locations every timeslot from the set of cells adjacent to their current cell. However, analysis under the simplified independent and identically distributed (i.i.d.) mobility model provides a meaningful bound on performance in the limit of infinite mobility.With this assumption, the network topology dramatically changes every timeslot, so that network behavior cannot be predicted and fixed routing algorithms cannot be used. Rather, because information about the current and future locations of users is unknown, one must rely on robust scheduling algorithms.[1]

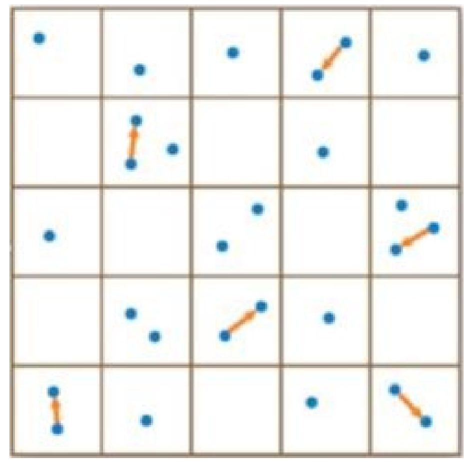


Fig. 1. Network model

Multicast is a fundamental service for supporting information communication and collaborative task completion among a group of users and enabling cluster-based system design in a distributed environment [3].Compared to unicast,multicast considers the cooperation and is more suitable to some real cases.Multi-hop increase the probability of node with packet meeting destinations,so the delay becomes better.We find the bound of delay is $\Theta(logN)$ and the capacity is $O(\frac{1}{log(N*logK)})$.We compare our result to the multicast under two-hop .Through the comparison,we find out the inherent trade-off between capacity and delay.

II. NETWORK MODEL AND DEFINITION

A. NETWORK MODEL

Cell partitioned Network: The network is a unit square and there are N mobile nodes in it. Divide it into $C = \Theta(N)$ non overlapping cells with equal size.We assume nodes can

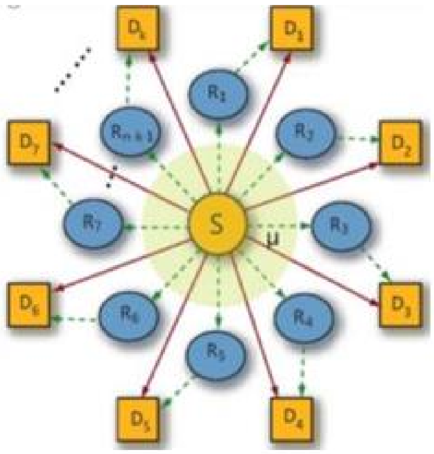


Fig. 2. The source-destination relationships

communicate with each other only when they are within a same cell ,and at each timeslot there is at most one transmission.

Mobility Model: Dividing time into constant duration slots and all nodes following the following ideal i.i.d. mobility . The initial position of each node is equally likely to be any of the C cells independent of others. And at the beginning of each time slot, nodes randomly choose and move to a new cell i.i.d. over all cells in the network. This model captures the characteristic of the infinite mobility. With the help of mobility, packets can be carried by the nodes until they reach the destinations The cell-partitioned network model restricts communication to one transmission per cell per timeslot.

The source-destination relationships: we assume the number of users is divisible by k+1 and number all the nodes from 1 to N . We uniformly and randomly divide the network into different groups with each of them having k+1 nodes. Assume packets from each node in a specific group must be delivered to all the other nodes within the group. Nodes not belonging to the group can serve as relays. The relationships do not change as nodes move around.As shown in fig.2.

B. DEFINITION

Stability: For a fixed λ_i, the network is stable if there exists a scheduling algorithm so that the queue in each node does not grow to infinity as time goes to infinity.

Capacity: The per-node capacity of the network is the maximum rate λ_i that it can stably support.

Delay: The delay for a packet is defined as the time it takes the packet to reach all its k destinations after it arrives at the source.

III. CAPACITY,DELAY,AND MULTI-HOP RELAY ALGORITHM

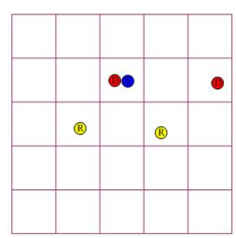
Multi-hop Relay Algorithm for Multicast: Every timeslot and for each cell containing at least two users:

1) If there exists a source-destination pair within the cell,randomly choose such a pair (uniformly over all such pairs in the cell). If the source contains a new packet intended for that destination, transmit. But when the receiver has received it before, ignore it . Else remain idle.

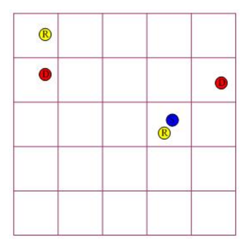
2) If there is no sourcedestination pair in the cell, designate a random user within the cell as sender. Independently choose another user as receiver among the remaining users within the cell. With equal probability, randomly choose one of the two options.

- Send a Relay packet to its Destination: If the designated transmitter has a packet destined for the designated receiver, send that packet to the receiver. But when the receiver has received it before, ignore it .Else remain idle.
- Send a New Relay Packet: If the designated transmitter has a packet, relay that packet to the designated receiver. But when the receiver has received it before, ignore it .Else remain idle.

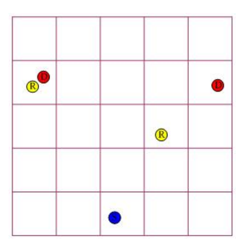
When the packet has transmitted to all of the k destinations, delete it.



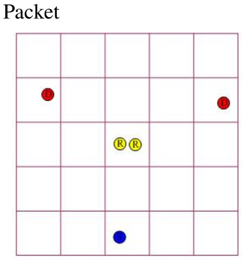
(a) Send a packet to its Destination



(b) Send a New Relay Packet



(c) Send a Relay packet to its Destination



(d) Send a New Relay Packet

Fig. 3. Multihop Relay Algorithm for Multicast

Theorem 1: Algorithms permitting at most one transmission in a cell at each time slot cannot achieve an average delay better than$\Omega(\frac{N*logK}{d})$ .In particular, If $d=\Theta(1)$,the min delay is$\Omega(N*logK)$.

*Proof:*In order to achieve the best delay,consider an ideal situation where the network is empty and only node 1 sends a single packet to k destinations. At timeslot T ,there are K_t nodes which is no more than2^T have the packet.Let p represent the probability of one destination having not received the packet.
Observe that during slots$\{1,2,3,...T\}$, there are at most2^T nodes have the packet.Hence,

$$p=(1-\frac{1}{C})^{K_t*T} \quad (1)$$
$$\geq(1-\frac{1}{C})^{2^T*T} \quad (2)$$
$$\geq(1-\frac{1}{C})^{2^{2T}} \quad (3)$$

LetT_N represent the time required to reach destinations under this optimal policy for sending a single packet.

$$P_r(T_N>T)=1-C_K^0(1-p)^K \quad (4)$$
$$\geq 1-[1-(1-\frac{1}{C})^{2^{2T}}]^K \quad (5)$$

Choosing T=$\frac{1}{2}log(\frac{N*logK}{d})$ and letting N$\rightarrow\infty$ and K$\rightarrow\infty$,it yields that

$$P_r(T_N>T)\geq 1-C_K^0(1-p)^K \quad (6)$$
$$=1-[1-(1-\frac{1}{C})^{\frac{N*logK}{d}}]^K \quad (7)$$
$$=1=(1-e^{-logK})^K \quad (8)$$
$$=1-(1-\frac{1}{K})^K \quad (9)$$
$$=1-e^{-1} \quad (10)$$

Thus,

$$E\{T_N\}\geq E\{T_N|T_N>\frac{1}{2}log(\frac{N*logK}{d}\}* \quad (11)$$
$$P_r(T_N>\frac{1}{2}log(\frac{N*logK}{d})) \quad (12)$$
$$\geq\frac{1-e^{-1}}{2}log(\frac{N*logK}{d}) \quad (13)$$

From (12) ,we prove the theorem.

Lemma 1: Under the Multihop Relay Algorithm ,for any network size $N\geq 2$, the expected time$E\{T_N\}$ for the packet to reach all destinations satisfy :

$$E\{T_N\}\leq E\{S_1\}+E\{S_2\}$$

where

$$E\{S_1\}\leq\frac{log(N)(2+d)}{log(2)(1-e^{-d/2})}$$

$$E\{S_2\}\leq 1+\frac{2}{d}(1+log(N/2))$$

$E\{T_N\}$:the total time to reach all destinations
S_1:represent the time required to send the packet to at least N/2 users
S_2 :the time required to deliver the packet to the destination users given that at least N/2 users initially hold the packet.

Proof: This Lemma derives from Lemma 3 in [1], we just modify it according to our model.First,we consider the boundary of $E\{\alpha_i\}$.Let$\mu_1,\mu_2,\mu_3,...\mu_{K_t}$ represent the users containing the packet at time t. Each of these users μ_i delivers the packet to α_i new users on the next timeslot, where is a binary random variable taking a value of either 0 or 1 . Whenever there are at least N/2 users which do not currently hold the packet, we have that$E\{S_2\}\leq\theta_1\theta_2\theta_3$, where$\theta_1$ represents a lower bound on the probability that at least one of the new users enters the cell of user , θ_2 represents a lower bound on the probability that user is selected to transmit its replica among all other packet-holding users within the cell,and θ_3 represents the probability of transmitting to a relay packet.Through calculate,we can get $E\{\alpha_i\}\geq\frac{1-e^{-d/2}}{2+d}$ and according to Appendix H in [1],we can get the boundary of $\{S_1\}$.The boundary of $E\{S_2\}$ has been proved in [1].

Theorem 2: Under the multi-hop relay algorithms for multicast,we can achieve the delay $\Theta(logN)$ and achieve the capacity $O(\frac{1}{log(N*logK)})$.

Proof: Theorem 1 and Lemma 1 is the special case where there is only one packet transmitting in the network.If we consider each source node as a Geo/Geo/1 queue, the delay we achieved in Th1 and Le1 can be seen as service time. According to the property of Geo/Geo/1 queue,we can achieve the above delay and capacity.

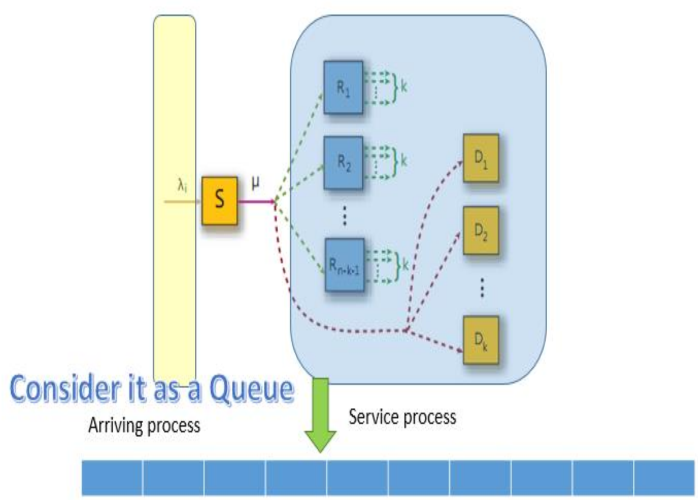


Fig. 4. Decoupled queue model

Based on Theorem 1 and Lemma 1,the lower bound of service time is $\Omega(log(N*logk))=\Omega(logN+log(logK))=\Omega(logN)$ and the upper bound of service is $O(logN)$,i.e. the boundary of service time is $\Theta(logN)$. According to the property of Geo/Geo/1 queue ,the delay D_n equals service time S_n plus waiting time W_n.And also:

$$E\{D_n\} = E\{S_n\} + E\{W_n\} \tag{14}$$

$$E\{S_n\} = \frac{1}{\mu} \tag{15}$$

$$E\{W_n\} = \frac{\alpha}{\mu(1-\alpha)} \tag{16}$$

$$\alpha = \frac{\lambda(1-\mu)}{\mu(1-\lambda)} \tag{17}$$

$$\tag{18}$$

Thus,

$$E\{D_n\} = \frac{1}{\mu(1-\alpha)} \tag{19}$$

And,we can easily find that $\Theta(\frac{1}{1-\alpha}) = \Theta(1)$,so $E\{D_n\}$ is as the same order as $E\{S_n\}$.In other word,

$$E\{D_n\} = \Theta(logN) \tag{20}$$

And the capacity $\mu = \frac{1}{E\{S_n\}}$=$O(\frac{1}{logNlogK})$.

Compared to the multicast under two-hop with and without redundancy,we can easily find that the multi-hop improves the delay at the expanse of capacity.In other word,the capacity and delay cannot both better than those of two-hop.Thus, there must be a tradeoff between the delay and capacity .The comparison is demonstrated in Table 1.

TABLE I
DELAY AND CAPACITY UNDER DIFFERENT ALGORITHM FOR MULTICAST

scheme	capacity	delay
Two-hop relay w.o redund	$\Theta(\frac{1}{K})$	$\Theta(NlogK)$
Two-hop relay w. redund	$\Omega(\frac{1}{K\sqrt{NlogK}})$	$\Theta(\sqrt{NlogK})$
multi-hop	$O(\frac{1}{logNlogK})$	$\Theta(logN)$

IV. CONCLUSION AND FUTURE WORK

In this paper,we analyze the delay and capacity for multicast under multi-hop.We take the cell-partitioned network model and assume the mobility of node following i.i.d model.We get the capacity $O(\frac{1}{log(NlogK)})$ and $\Theta(logN)$ .Compared to two-hop,the delay becomes better while the capacity aggravates.Therefore,there must be a inherent trade-off between them.

Due to time limited,the trade-off between capacity and delay is not proved in theoretically.And we assume the mobility of node is infinite which is not satisfied in real case.Solving these problem will be our future work.

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