The capacity of mobile network in energy-constrained model

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***Abstract*—We calculate the throughput of an energy-constrained mobile network, we have two charging models.One of which using base station to recharge the battery of each mobile node,another one using base station and vehicles to recharge each mobile node. We find that in the first model the throughput is closely related to the ratio between the number of nodes n and the number of base station W. And in the second model,the throughput is closely related to the number of nodes n,the number of base station W,and the number of vehicles m.Through analytic and numerical results, we show that in these two models,the scaling law of the throughput can reach to $\Theta(min(1,\frac{W}{n})c^{min(1,\frac{W}{n})})$ and $\Theta(min(1,\frac{Wm}{n^2})c^{min(1,\frac{Wm}{n^2})})$.**

***Keywords*—Capacity, base station, vehicle,energy-constrained, Markov chains.**

I. Introduction

A fundamental characteristic of mobile wireless networks is the time variation of the channel strength of the underlying communication links. Such time variation can be due to multipath fading, path loss via distance attenuation, shadowing by obstacles and interference from other users. The impact of such time variation on the design of wireless networks permeates throughout the layers, ranging from coding and power control at the physical layer to cellular handoff and coverage planning at the networking layer.

In their seminal work [1], Gupta and Kumar claimed that the per node throughput for each S-D pair follows$\Theta(1/\sqrt{n\log n})$,where n is the number of nodes in the network. Later, Glossglauser and Tse showed that by exploiting mobility of each node and using relay, the per node throughput becomes constant regardless of the node density [2]. And various scheduling and routing strategies exploiting mobility have been proposed [3–5]. These works assume that the available transmission energy of each node is unlimited, which is not the case in practice. After the battery runs out, the throughput falls to zero.

The capacity of ad-hoc wireless networks is constrained by the various cause.Among the work above,these work did not concern the effect of the energy constrained model.while in [6],Seung-Woo Ko,Seung MinYu,and Seong-Lyun Kim analyzed the capacity of the mobile network in energy-constrained model.And in his model,Seung-Woo Ko,Seung MinYu,and Seong-Lyun Kim using vehicle-charging model to analyzed the capacity of the mobile network.And also in in [7],the author incorporate WPT into a wireless random network and claimed that WPT can prolong the lifetime of wireless sensor networks.

And the main purpose of this paper is analyzing the capacity of the ad-hoc wireless network in two models:Base station charging model and Base station and vehicle charging model.

The rest of this paper is organized as follows. Section II introduces the Base station charging network model of this paper. Section III presents the analysis of the Base station charging model. Section IV introduces the Base station and vehicle charging network model. Section V details the analysis of the Base station and vehicle charging model.Section VI gives simulation of this paper and finally inVII,we concludes this paper.

II. Network Model

In this section we introduces the network model and we have four parts:

- The network charging model.
- Two-phase scheduling policy.
- interference solution.
- Charging model.

A. *The network charging model*

The Base Station charging network model is presented in Figure.1.The black triangle represents Base station,which can charge the node(either the receiver node which is represented by red circle or the transmitter node,which is represented by red square)if and only if the node is covered by Base Station with a charging range R.The node which becomes transmitter can transmits packets to the receiver node if and only if the receiver node is within the transmission range r.

And also,Base station is distributed in the network statically and without overlapping with each other.In each time slots,node are randomly distributed in the network according to i.i.d model.

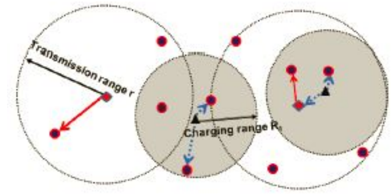

Fig. 1: network charging model,referenced in [6]

B. Two-phase scheduling policy.

We can now apply this scheduling policy π,which is applied in paper [2],to our basic problem. The overall algorithm is divided into two phases (presented in Figure.2): (1) scheduling of packet transmissions from sources to relays (or the final destination), and (2) scheduling of packet transmissions from relays (or the source) to final destinations. These two phases are interleaved: in the even timeslots, phase 1 is run; in the odd time-slots, phase 2 is run.

In phase 1, we can apply the scheduling policy π to transmit packets from sources to relays or destinations. In phase 2, we again apply the policy π, but this time to transmit packets from relays to final destinations (or, as in phase 1, from a source directly to the destination). More specifically, when a receiver is identfied for a sender under π, the sender checks if it has any packets for which the receiver is the destination; if so, it will transmit it. It should be noted that every packet goes through at most two hops: it is transmitted once in phase 1 from its source to an intermediate relay, and once in phase 2 from a relay to the final destination. We allow for packets to be directly transmitted from their source to their destinations in both phases, if a sender-receiver pair happens to be a source-destination pair as well.

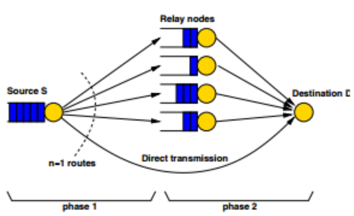

Fig. 2: graph of two-phase scheduling policy,referenced in [2]

C. interference solution

For the interference model, we adopt the protocol model from [1]. Transmitter i successfully delivers a packet to receiver j when the following conditions are satisfied:

- The distance between them is no more than r.
- The distances between node j and the other transmitting nodes are no less than r.

D. Charging model.

The maximum battery capacity of each node is set to L units of energy. A node cannot transmit a packet when its battery is exhausted. In order to recharge the battery,W Base stations are employed in the network. Base stations exploit magnetic resonance coupling to recharge nodes. Base stations distributed in the whole network without overlapping with each other. The maximum energy transfer rate from Base stations to a node is E units of energy per time slot.

The recharged units of energy v(d) is:

$$v(d) = \begin{cases} 0 & \text{if } R_1 < d \\ k & \text{if } R_{k+1} < d < R_k, k = 1, 2, \cdots, E-1 \\ E & \text{if } 0 < d < R_E, \end{cases} \quad (1)$$

where $R_x = \{d : E \cdot \tau(d) = x\}$ and $\tau(d)$ denotes the energy transfer efficiency that is a non-increasing function of $d(0 \leq \tau(d) \geq 1)$.

And also,we have charge protocol below:

- When there are less than X nodes within R1, Base station selects all of them.
- When there are more than X nodes within R1, Base station randomly selects X nodes among them.

III. ANALYSIS OF THE BASE STATION CHARGING MODEL

Let us denote by p_c and p_t the conditional probabilities that a Base station charges a node given the battery of the node is not full, and a node transmits a packet given the node has at least one unit of energy in its battery, respectively. The probabilities p_c and p_t are expressed as a function of the number of nodes n, the number of Base stations W, the maximum number of nodes charged by one Base station at a time X, the transmission range r and the charging range R.

Theorem 1. *The probability that a Base station charges a node given the battery of the node is not full p_c can expressed as:*

$$p_c = W\pi R^2 \sum_{i=0}^{n-1} min[1, \frac{X}{l+1}][C_{n-1}i(\pi R^2)^i(1-\pi R^2)^{n-1-i}] \quad (2)$$

Proof: The probability that there are number of i nodes in the range of the Base station and m-i nodes out of the range:

$$p_i = C_{n-1}i(\pi R^2)^i(1-\pi R^2)^{n-1-i} \quad (3)$$

So from p_i,we can calculate p_c equals:

$$p_c = W\pi R^2 \sum_{i=0}^{n-1} min[1, \frac{X}{l+1}][C_{n-1}i(\pi R^2)^i(1-\pi R^2)^{n-1-i}] \quad (4)$$

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Theorem 2. *The probability that a node transmits a packet given the node has at least one unit of energy in its battery p_t can expressed as:*

$$p_t = q[1 - \{1-(1-q)\pi r^2\}^{n-1}] \quad (5)$$

Theorem 3. *we define the probability that a node has at least one unit of energy is p_{on}*

we can calculate p_{on} equals:

$$p_{on} = \begin{cases} \Theta(W/n) & \text{if } W < \Theta(n) \\ \Theta(1) & \text{otherwise} \end{cases} \tag{6}$$

Proof: We can show the transition status of the energy of a node using Markov Chain model showed in Figure3.

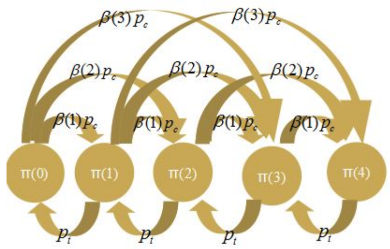

Fig. 3: status transition graph of Markov Chain

Our goals is solving this Markov Chain and to calculate p_{on},and we have this equilibrium equation according to the Markov Chain mentioned above:

$$(p_c + p_k)p_k = p_t p_{k+1} + \sum_{i=0}^{k-1} p_i p_c \beta_{k-i} \tag{7}$$

and to solve this equation above,we use G conversion,and also we have:

$$P(z) = \frac{p_t(1-\rho)(1-z)}{p_t(1-z) - p_c[1-G(z)]} \tag{8}$$

Notice that ρ equals $1 - p_0$,which equals p_{on}.And also we have:

$$1 - p_0 = \rho = \frac{p_c G'(1)}{p_t} \tag{9}$$

,which is demonstrated in [8].

Notice that $G'(1)$ is not related with W and n.So we have:

$$p_{on} = \begin{cases} \Theta(W/n) & \text{if } W < \Theta(n) \\ \Theta(1) & \text{otherwise} \end{cases} \tag{10}$$

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Theorem 4. *The throughput of this network model can expressed as:*

$$\Lambda(n) = \Theta(min(1, \frac{W}{n})c^{min(1,\frac{W}{n})}) \tag{11}$$

Proof:

In [2], the authors proved that, from the perspective of a one-node queue, the arrival rate (Phase 1) and service rate (Phase 2)are the same. This ensures the stability of the queue. Therefore, the throughput of an S-D pair is equivalent to the rate at which a source node successfully delivers its packet in Phase 1.

In Phase 1, a node always transmits its own packet. Recall that node i transmits a packet successfully only when node i becomes a transmitter and there is at least one unit of energy, there is at least one receiver within distance r, and there is no co-transmitter within r from the receiver. Given that there are j receivers and v transmitters having enough energy to transmit packets in the network, the probability of delivering its packet successfully, $p_s{'}(j, v)$, is:

$$p_s{'}(j, v) = qp_{on}\{1 - (1 - \pi r^2)^j\} \cdot (1 - \pi r^2)^v \tag{12}$$

And also we can derive the successful transmission probability $p_s{'}$ as follows:

$$\begin{aligned} p_s{'} &= \sum_{j=0}^{n-1} \sum_{v=0}^{n-1-j} f(j; n-1, 1-q) f(v; n-1-j, p_{on}) p_s{'}(j, v) \\ &= qp_{on}(e^{-\pi q p_{on}/4} - e^{\pi(-1+q-qp_{on})/4}) \end{aligned} \tag{13}$$

Using the conclusion of(3),we finally have:

$$\Lambda(n) = \Theta(min(1, \frac{W}{n})c^{min(1,\frac{W}{n})}) \tag{14}$$

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IV. Base Station and vehicle charging model

In the Base station and vehicle charging model,we define that the number of nodes in the whole network is n,the number of the Base station in the whole network is W,and the number of the vehicle in the network is m.

Now we suppose that the **interference solution model** and the **Two-phase scheduling policy** in BS and vehicle model is same with the Base station charging model.The difference is that we use vehicle to charge the node in the whole network,and Base station to charge the vehicle to provide the energy of the vehicle.In other words,vehicle becomes a intermediary in charging between Base stations and nodes.

To simplify,we use only one level charging model rather than multi-level **Charging model** mentioned in the part [2].That is,Base station can only charge one unit of energy each time to the nearby vehicle and also vehicle can only charge one unit of energy to each nodes it covers each time.

And we define p_{BS} is the probabilities that a BS charges a vehicle.p_t is the probabilities that a node transmits a packets given that the node has at least one unit of energy. p_c is the probabilities that a vehicle charges a node.R_{BS} is the BS charging range.R_c is the vehicle charging range.r is the transmission range of nodes. W is the number of Base station.n is the number of nodes.m is the number of vehicles.

V. ANALYSIS OF THE BASE STATION AND VEHICLE CHARGING MODEL

Using the same method of analysis as the previous model,we can presents the process of state transition of a certain vehicle as Figure.4. And we can presents the process of state transition of a certain node as Figure.5.

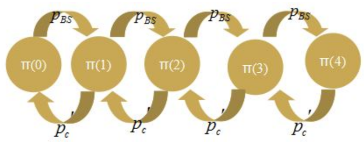

Fig. 4: status transition process of a vehicle

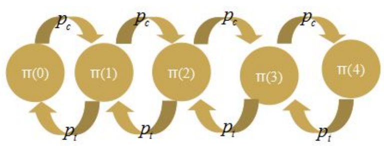

Fig. 5: status transition process of a node

Our purpose is to calculate the probabilities of a vehicle having at least one unit of energy $p_{on-vehicle}$ and the probabilities of a node having at least one unit of energy $p_{on-node}$.

To solve two Markov Chain mentioned in Figure.4 and Figure.5,we use G conversion,the similar method showed in previous model to calculate the probabilities.And we have:

$$p_{on-vehicle} = 1 - p_0 = \gamma \frac{p_{BS}}{p_c'} = \gamma \frac{W{R_B}^2}{1-(1-\pi R^2)^n} \tag{15}$$

$$p_{on-node} = \gamma\beta \frac{p_{on-vehilce} p_c}{p_t} = \begin{cases} \Theta(\frac{Wm}{n^2}) & \text{if } Wm < \Theta(n^2) \\ \Theta(1) & \text{otherwise} \end{cases} \tag{16}$$

The throughput of this model can present as:

$$\Lambda(n) = \frac{1-e^{-\frac{\pi(q-1)}{4}}}{2} \cdot q \cdot e^{-\frac{\pi q p_{on}}{4}} = \begin{cases} \Theta(\frac{Wm}{n^2} con^{\frac{Wm}{n^2}}) & \text{if } W < \Theta(m) \text{ and } m < \Theta(n) \\ \Theta(1) & \text{otherwise} \end{cases} \tag{17}$$

VI. SIMULATION

A. Base Station Charging Model

In the Base station charging model,we suppose that the whole network is a unit circle.we use 3 Base station to charge the whole network,and the charging range of the Base station is 0.1 units.The transmission range of a node is defined as 0.03 units.And we have two circumstance of simulation.

- The number of nodes n increasing and the number of Base Station W remains constantly.
- The number of nodes n increasing and the number of Base Station W is in proportion to the number of nodes n

The Simulation of Base station charging model and Base station and vehicle charging model is presented in Figure.6,7 and Figure.8,9,respectively.

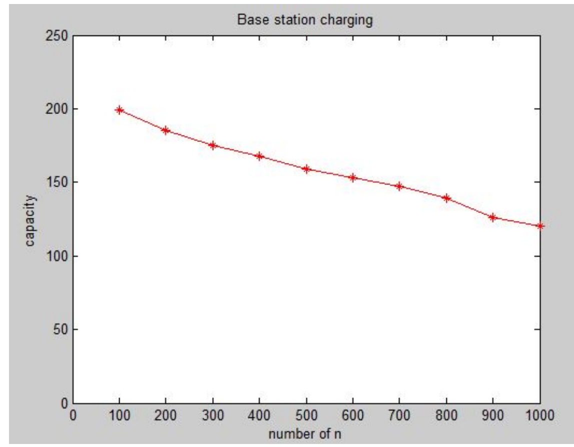

Fig. 6: Base station charging model while m remains constant

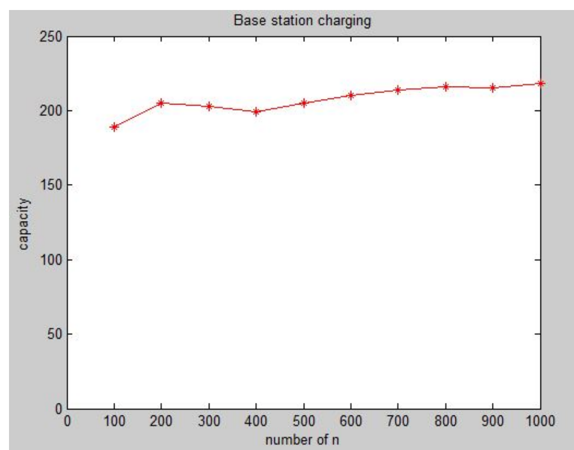

Fig. 7: Base station charging model while m is in proportion to n

We can see that when the number of the Base station W remains constant(shows in Figure.6),the throughput of the network is a decreasing function of n;while the number of the Base station W is in proportion to the number of nodes n(shows in Figure.7),the throughput of the network remains constant.It is apply to the equation $\Lambda(n) = \Theta(min(1, \frac{W}{n})c^{min(1, \frac{W}{n})})$.

B. Base Station and vehicle Charging Model

In the Base station and vehicle charging model,we suppose that the whole network is a unit circle.we use 3 Base station to charge the vehicle in the whole network and 3 vehicle to charge the nodes in the network.The charging range of the Base station is 0.1 units,and the charging range of the vehicle is 0.1 units too.The transmission range of a node is defined as 0.03 units.And we have two circumstance of simulation.

- The number of nodes n increasing and the number of Base Station W and the number of vehicles m remains constantly.
- The number of nodes n increasing and the number of Base Station W is in proportion to the number of nodes m and m is in proportion to the number of nodes n.

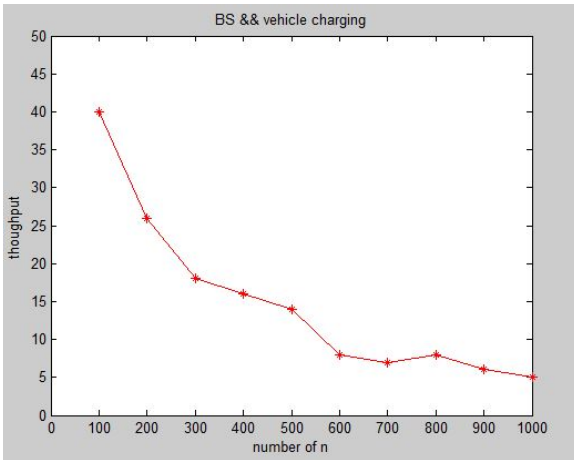

Fig. 8: Base station and vehicle charging model while m and W remains constant

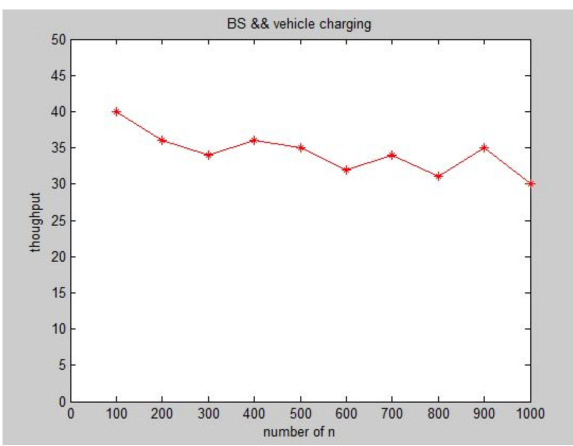

Fig. 9: Base station and vehicle charging model while W is in proportion to m and m is in proportion to n

We can see that when the number of the Base station W and number of vehicle m remains constant(shows in Figure.6),the throughput of the network is a decreasing function of n;while the number of the Base station W is in proportion to the number of vehicles m and m is in proportion to the number of nodes n(shows in Figure.7),the throughput of the network remains constant.It is apply to the equation $\Lambda(n) = \Theta(min(1, \frac{Wm}{n^2})c^{min(1,\frac{Wm}{n^2})})$.

VII. Conclusions

We calculate the throughput of an energy-constrained mobile network, we have two charging models.One of which using base station to recharge the battery of each mobile node,another one using base station and vehicles to recharge each mobile node. We find that in the first model the throughput is closely related to the ratio between the number of nodes n and the number of base station W. And in the second model,the throughput is closely related to the number of nodes n,the number of base station W,and the number of vehicles m.Through analytic and numerical results, we show that in these two models,the scaling law of the throughput can reach to $\Theta(min(1, \frac{W}{n})c^{min(1,\frac{W}{n})})$ and $\Theta(min(1, \frac{Wm}{n^2})c^{min(1,\frac{Wm}{n^2})})$. And to testify our conclusions,we do some simulation job,which shows the validity of our conclusion.

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