Communication Capacity and Delay in Inhomogeneous Social Situation

Guanjie Zheng
School of Electronic Information and Electrical Engineering
Shanghai Jiao Tong University
Shanghai
Email:gjzheng93@163.com

***Abstract*—In this paper, we discuss the communication capacity in social relationships. Here, we concentrate on the inhomogeneous situations, in which home points are randomly distributed and nodes are distributed around their home points by some certain distributions, such as power-law and exponent distribution. Under this assumption, we calculate the capacity and delay. We have proved that,no matter in what inhomogeneous situation, under no-relay situation, the per node capacity convergence to $\lambda(n) = \theta(\frac{1}{n})$, and the delay will be $D = \theta(n)$. Under at-most-one-relay situation, the per node capacity convergence to $\lambda(n) = \theta(1)$, and the delay will still be $D = \theta(n)$. This is very similar to the result of D. Tse in Also, we come up with a new kind of transportation model, in which nodes are attracted by all home points and will change there home points if they are in a larger gravitation than the current one. This may perfectly simulate the people flow in real life.**

I. Introduction

Social relationships are an essential factor influencing people's behaviour. A resident in Suzhou may be attracted by Shanghai and they move to Shanghai. That displays the role social environment plays in people's action. A fan of Kobe Bryant may change their mind if they are surrounded by a group of fans of Lebron James, thus they may have more communication with fans of Lebron James since then. That indicates the impact of companions on the action of human. All of these may have a great influence on the communication between people. It is reasonable that crowd with a higher density will be easier to diffuse the virus, or colleagues may share gossips. In this paper, we will discuss the impact of these factors on the communication capacity and delay.

A lot of work has been done by the previous ones. P.Gupta and P.R.Kumar conducted the pioneer work in [1] by first show the basic capacity and delay of ad hoc networks. They gave the results of $\theta(\frac{1}{\sqrt{nlogn}})$ at an average sense. Then D.Tse in [2] first introduce mobility to this area and show the identification that mobility will increase the capacity. They gave the conclusion that a uniformly distributed ad hoc network can maintain $\lambda(n) = \theta(1)$, $D = \theta(n)$, with at most once relay and i.i.d mobility. Since then, more and more works are published.

Among these works, some of them concentrate on the research of different mobility models, like [3] [4], some of them concentrate on different distributions of the nodes, like [5] [6] [4], and some focus on the combination of hybrid networks, like [3] [7]. Some work, like [8] [9], mainly talk about the power optimization, which may also be involved in this paper. Some work though have little to do with network,like [10] [11] [12] which research on the human nature of mobility, can provide us a lot of help.

In this paper, we discuss the network capacity and delay under a distribution model modified from [4] in order to introduce the influence of nodes scale n. Also, we have come up with a new kind of mobility model which can simulate the people flow in real life very well.

The rest of this paper is recognized as follows. In section II, we introduce our models and basic assumptions. In section III, we analyse the situation without our unique mobility model. In section IV, we analyse the situation considering our jump mobility model. In section V, we display some simulation results. And in section VI and section VII, we give the conclusion and future work correspondingly.

II. Network Model

In this part, we introduce our models. Basically, our work is defined in a wraping-around plate. No matter what shape the plate is, the results will not change. So we choose a circle to simplify the derivation. And we have several home points, every of which will own a number of nodes itself. Usually, we set the number of nodes a home point obsess to be a constant.

First, we assume that these home points satisfy the uniform distribution. So for the home points we have $p = \frac{1}{\pi R^2}$,where R is the equivalent radius of this region. And then every center will displace their own nodes according to some certain requirements.

Next, under some determined mobility model, these center and nodes may move around and transmit the message. We divide the time into slots. At the beginning of every slot, the home points tend to be reshuffled. And then the nodes conduct the jump operation. Finally, the nodes will be reshuffled.

Under this situation, we will derive the per node capacity and delay.

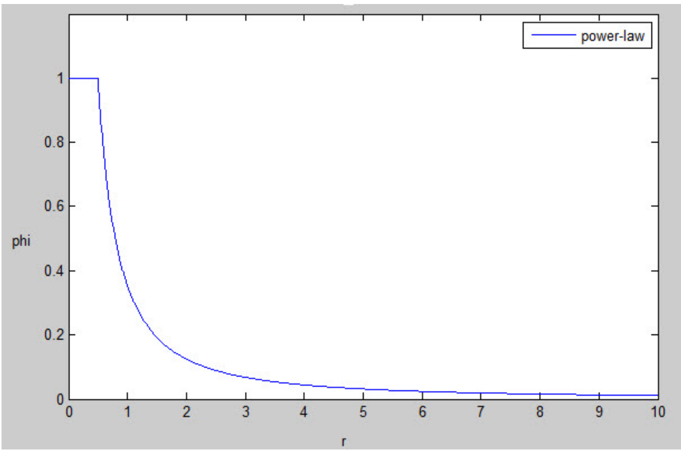

Fig. 1. power-law

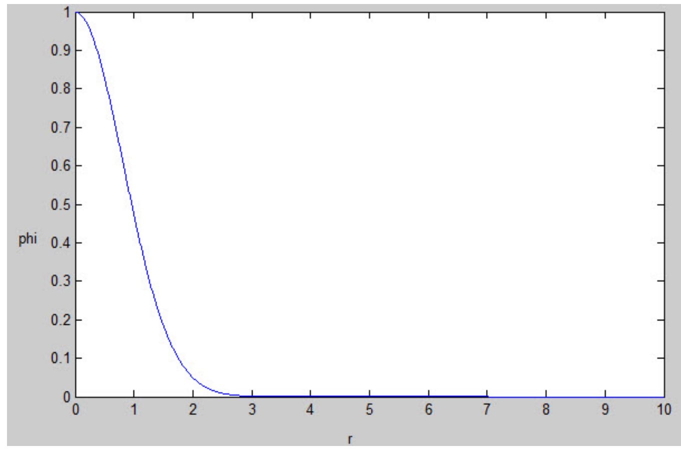

Fig. 2. exponent

A. Parameters

parameters	explanation
R	equivalent radius of the region
r	distance a node from its own home point
x	distance a node from a home point
d	transmission range
n	overall number of nodes
m	number of home points
c	number of nodes per home point

B. Network Model

We introduce two kinds of network model here. One is power-law model, and the other one is exponent model. In the next derivation, we will prove that our results can be set up on any other models, as long as some basic conditions are satisfied.

1) Power-law: This model is modified from the one in [4].

Given the distance r a node from its home point, the density function will be

$$\phi(r) = \frac{1}{A}\begin{cases} r^{-\delta} & r > b \\ b^{-\delta} & 0 \leq r \leq b \end{cases}$$

where

$$A = \int_0^R \int_0^{2\pi} \phi(r) r d\theta dr = \frac{2\pi}{2-\delta}(R^{2-\delta} - \frac{1}{2}\delta b^{2-\delta})$$

A is the normalization constant, and $\delta = n^k$, $b = g(n)$.

Here, we let the parameters to be some typical values. $\delta = 1.5$, $b = 0.5$, $R \to \infty$ then we will obtain Fig. 1.

Here we have that the density will decrease with the increase of r, which fits the fact well. And for the very little r case, we keep the $\phi(r)$ to be a constant. This is for the density cannot be infinity in the real case. See Fig. 1.

To introduce the impact of n on the density, we put the exponent of n on the exponent of r. This is quiet close to the real case, for larger the overall amount of people, more crowded will the area around the home point be. Accordingly, we introduce $g(n)$ to be the boundary of constant density.

Here, a typical value for k is 0.1.

2) Exponent: Given the distance r a node from its home point, the density function will be

$$\phi(r) = \frac{1}{A}e^{-\delta r^2}$$

where

$$A = \frac{\pi}{\delta}(-e^{-\delta R^2} + 1)$$

A is the normalization constant, and $\delta = n^k$.

Here, these parameters have the similar meaning to the previous one. Using a set of typical value, we have Fig. 2.

We can see that, by using exponent model, we avoid the indifferentiability on the boundary point. However, it keeps the similar trend with the power-law one.

C. Mobility Model

For mobility models, we have two mechanisms, i.i.d Mobility, which is prevalent used in previous work, and jump mechanism, which is introduced by us.

1) i.i.d Mobility: For i.i.d mobility, we will have all the home points reshuffled, according to their distribution, at the beginning of each time slot. These means the range of the points could be very large.

2) Jump: For jump mechanism, we can imagine that nodes of a home point may be distributed very far from the home point, though this will be a small-probability incident. Thus, it could be attracted by other home points, and then turn to be the node of others. See Fig. 3.

We can see that a core problem is the pattern how the gravitation perform. In general sense, we have the following formula derived from the fact

$$F(x) = \frac{GMm}{x^2}$$

For we assume all the home points and all the nodes are same correspondingly, the comparison of $\frac{1}{x^2}$ will degenerate into the comparison of x. This means closer home points will perform larger gravitation.

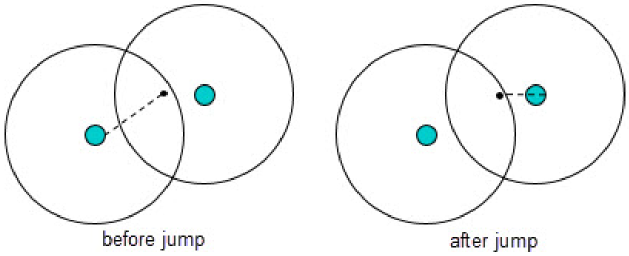

Fig. 3. jump movement

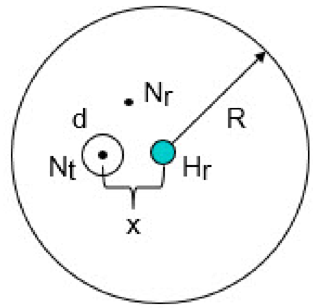

Fig. 4. no jump situation

After defining this, we can decide the time for the nodes choose to conduct a jump. Here, to simulate the judgement of human, we have two kinds of decision policy, conservative policy and jacobinic policy. To illustrate them clearly, we define these symbols. Let H_i, H_k represent the ith home points, and N_j represent the jth node. Then the distance between them will be denoted by x_{ij}, x_{kj}, x_{ik}. And the gravitation will be denoted as F_{ij}, F_{kj}.

Conservative policy permit a node to jump when it have seen that a specific more attractive home points. Then the criterion will be

$$F_{kj} > F_{ij}$$

In current simplified circumstance, we can simplify it to

$$x_{kj} < x_{ij}$$

If this is satisfied, then a node will jump at a probability of p.

Jacobinic policy permit a node to jump when it have seen that the summation of the gravitation of all other home points. Then the criterion will be

$$\sum_{k=1}^{n} \vec{F_{kj}} \geq \vec{F_{ij}}$$

If this is satisfied, then a node will jump at a probability of p.

D. Transmission Pattern

About the transmission pattern, we still use the protocol model that P.Kumar used in [1].

Assume that node N_i is going to transmit to N_j. Then the transmission will succeed iff

$$|N_k - N_j| \geq (1 + \Delta)|N_i - N_j|$$

for $k! = i$. The $\Delta > 0$ actually represents the SNR of the signal received by the receiver.

E. Scheduling Pattern

Here we use the same scheduling pattern in [2].

We assume that each of the nodes is a source node for one session and a destination node for another session. The SD association does not change with time, although the nodes themselves move.

III. Analysis of No-Jump Situation

In this section, we analyse the no-jump situation. Here we analyse the circumstance that a transmitter is going to send a message to a receiver. We denote the transmitter to be N_t and receiver to be N_r. And there home points to be H_t and H_r, correspondingly. Without loss of generality, we can assume H_r to be static, and discuss the impact of the position of N_t. Here we will see, using the protocol model, the only possible situation that a transmission will happen is that N_r is within the transmission range of N_t. So we begin our derivation.

For transmission range d and whole region radius R, we will have a discussion in the following part.

In our discussion, we will mainly focus on the power-law model. However, many of these conclusions may expand to exponent model, or even some other models. We will give the specific explanation in the following part.

We suppose the whole area to be a circle with a radius of R. First, we make H_r to be static. About the receiver, it will follow its own distribution $\phi(r)$. Then for the transmitter N_t, we have assume that its home points H_t is reshuffled at the beginning of every slot, with a distribution of uniform distribution. This will ensure that N_t is also uniformly distributed in the whole area. Therefore, what we need to do is just do the integration of the probability that N_r to be in the circle of N_t, considering different x. See Fig. 4.

For better understanding, we will introduce the theorem D.Tse introduced in [2] again.

Theorem 1 For the current scheduling policy, the expected number $E[N_t]$ of feasible sender-receiver pairs is $\theta(n)$, i.e.

$$\lim_{n\to\infty} = \frac{E[N_t]}{n} = \xi > 0$$

A. Capacity

We define p_l to be the probability that one node to within the transmission range of its source in a single time slot. Then we will have

$$p_l = I = \int_0^R \int_0^{2\pi} \int_0^d r\sqrt{(x - rcos\theta)^2 + (rsin\theta)}^{-n^k} d\theta dr dx$$

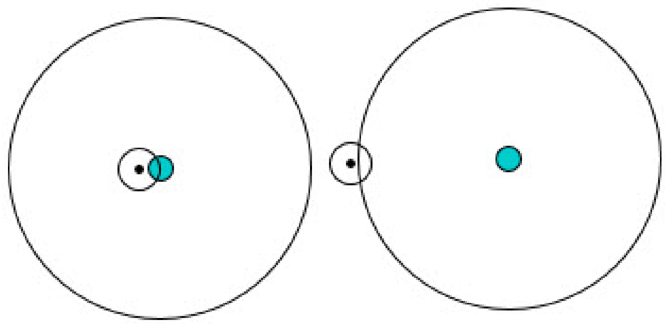

Fig. 5. integration

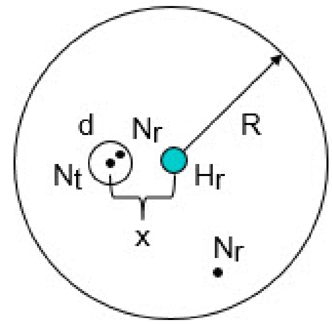

Fig. 6. Consider the Interference

If we want to calculate p strictly, we need to discuss the location of N_t and H_r and then divide it into five subcircumstances, and modify the limit of the integration correspondingly. But the integration will still be hard to resolve. So we apply the Squeeze Theorem.

Here we show the integration process in Fig.7.

1) Upper Bound of p: We notice that our density function $\phi(r)$ is a non-increasing function. Therefore, if we let the density $\phi(r)$ in the transmission range circle to be a constant for a certain x, and always use the density at the most inner point of the circle(reference to the home point H_r), then we will obtain the upper bound of the integration. Let the integration to be I_1.

$$dI_1 = \begin{cases} \frac{1}{\pi R^2}\phi(0)\pi d^2 dx & x \le d \\ \frac{1}{\pi R^2}\phi(x-d)\pi d^2 dx & d < x < R \end{cases}$$

$$p_l = I \le \int_0^R 2\pi x dI_1$$

$$= \int_0^d \frac{1}{\pi R^2}\phi(0)\pi d^2 2\pi x dx + \int_d^R \frac{1}{\pi R^2}\phi(x-d)\pi d^2 2\pi x dx$$

$$= \theta(\frac{d^2}{R^2})$$

2) Lower Bond of p: Similarly, if we let the density $\phi(r)$ in the transmission range circle to be a constant for a certain x, and always use the density at the most outer point of the circle(reference to the home point H_r), then we will obtain the lower bound of the integration. Let the integration to be I_2.

$$dI_2 = \begin{cases} \frac{1}{\pi R^2}\phi(x+d)\pi d^2 dx & x \le R-d \\ \frac{1}{\pi R^2}\phi(R)\pi d^2 dx & R-d < x < R \end{cases}$$

$$p_l = I \ge \int_0^R 2\pi x dI_2$$

$$= \int_0^{R-d} \frac{1}{\pi R^2}\phi(x+d)\pi d^2 2\pi x dx + \int_{R-d}^R \frac{1}{\pi R^2}\phi(R)\pi d^2 2\pi x dx$$

$$= \theta(\frac{d^2}{R^2})$$

B. Capacity Derivation

We can derive the upper thesis,that $p_l = \theta(\frac{d^2}{R^2})$. Actually, d^2 represents the transmission power to some extent.

In general case, we will assume that $d = \emptyset(R)$. This is reasonable, for the scale of d can never be larger than the one of R.

Now we consider the impact of interference. A transmission will succeed if and only if the receiver is within the transmission range of the transmitter and any other node is out of it. So we will have the probability to be

$p = p_l(1-p_l)^{n-1}$

Now we will have this discussion:

- If we let $d = \theta(1)$ and $R = \theta(\sqrt{n})$, which is similar to what Garetto do in [4], we will have $p_l = \theta(\frac{1}{n})$, and then $p = \theta(\frac{1}{n})$. Then we will have $p = \theta(\frac{1}{n})$. In this case, the transmission power will be a constant but the whole region range R will increase with $\sqrt{n}$, which will keep the density of nodes to be constant.
- If we let $d = \theta(1)$ and $R = \theta(1)$, we will have $p_l = \theta(1)$ and $p = \theta(C^n)$. In this case, the transmission power and the whole region range R will be a constant.
- Or if we let $d = \theta(\sqrt{n})$ and $R = \theta(\sqrt{n})$, we will have $p_l = \theta(1)$ and $p = \theta(C^n)$. In this case, both the transmission power and the whole region range R will increase with $\sqrt{n}$.

From now on, we will mainly concentrate on the first kind of circumstance.

Using the derivation similar to the one used in D.Tse, we will have Theorem 2.

Theorem 2 Given that the home points to be uniformly distributed and the nodes to be displaced according to a certain distribution with a non-increasing density function. In no-relay system, we will have

$$\lambda(n) = \theta(\frac{1}{n})$$

for at-most-once-relay system, we will have

$$\lambda(n) = \theta(1)$$

Even, we can expend it to all models with a non-increasing density function.

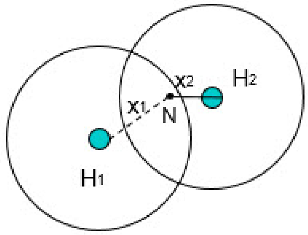

Fig. 7. situation to jump

C. Delay Derivation

Theorem 3 Given a probability for incident A to happen is p_A, then the average times of experiment it will take for A to happen will be $t = \frac{1}{p_A}$.

1) No Relay System: We have $p = \theta(\frac{1}{n})$, so by using Theorem 3, we can derive that $D = \theta(n)$.

2) At-Most-Once-Relay System: By using similar methods in [2], we can let the other $n-2$ nodes to be the relay. Thus, for the whole transmission will be conducted in two hop, so the delay will be

$$D = \frac{1}{n-1}\frac{1}{p} + \frac{n-2}{n-1}(\frac{1}{p} + \frac{1}{p(n-2)}) = \theta(n)$$

Therefore, we have Theorem 4.

Theorem 4 Given that the home points to be uniformly distributed and the nodes to be displaced according to a certain distribution with a non-increasing density function, we will have the average delay for a message to be transmitted from its source to its destination.

In no relay system, it will be

$$D = \theta(n)$$

And in at-most-once-relay system, it will be

$$D = \theta(n)$$

IV. Analysis of Jump Situation

In this section, we analyse the jump situation. First, we will try to derive the probability p_j that a node change its home points in a time slot. And then we will come to the capacity and delay.

A. Probability p_j

For the simplified conservative case, we have

$$p_j = \int_0^{2\pi} d\theta \int_0^R \frac{\pi x^2}{\pi R^2}\phi(x)xdx$$

$$= \frac{2-\delta}{\pi R^2(R^{2-\delta} - \frac{1}{2}\delta b^{2-delta}[\frac{1}{4-\delta} + \frac{-\delta}{4(4-\delta)}b^{4-\delta}])}$$

where $\delta = n^k$, $b = g(n)$.

We can derive that $p_j = \theta(\frac{2-\delta}{4-\delta})$, when $0 < \delta < 2$. The reason why we constrain the δ is that p_j is very close to 0

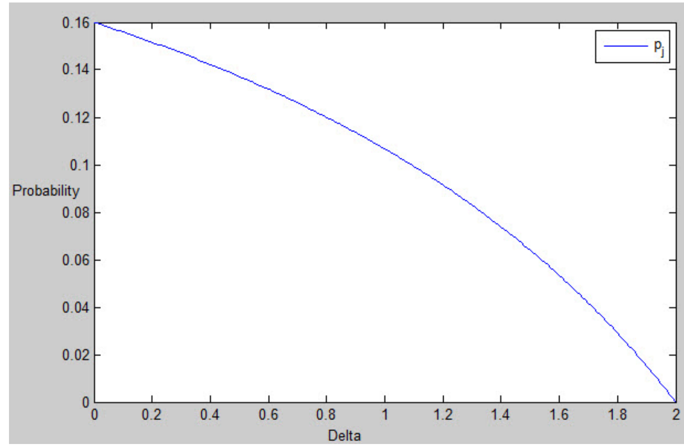

Fig. 8. p_j v.s. δ

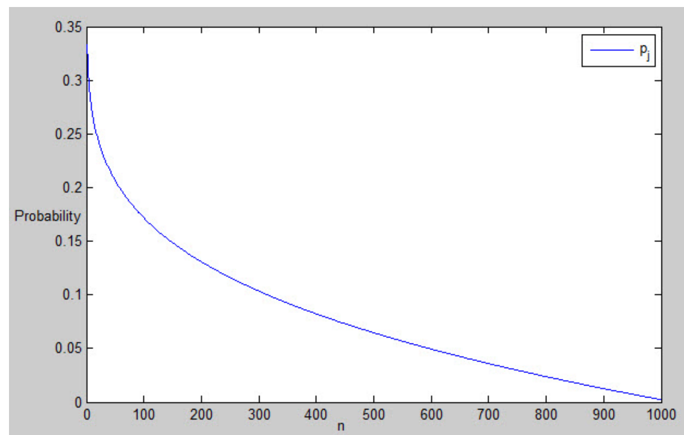

Fig. 9. p_j v.s. n

when $\delta \geq 2$. The results are displayed in Fig. 8. If we let $n^k = \delta = constant$,i.e. $k = 0$, then p_j is a constant regarding n, and in this case, the capacity and delay results will be the same with those in section III.

Actually, we can see that, for the p_j is only the function of δ, that means, if we can properly design δ, then will be able to control the movement of these nodes.

Then we substitute $\delta = n^k$, then we have

$$p = \frac{2-n^k}{4-n^k}$$

And if we use the log-coordinate, we will have the figure in Fig. 10.

We can see, in this case, the figure degenerates back to the δ=constant mode.

For we only consider the case when the numerator is larger than zero, we can simplify it to

$$p_j = C(2-n^k)$$

We can see that, though this movement process may be a dynamic balance, this probability p will definitely have a

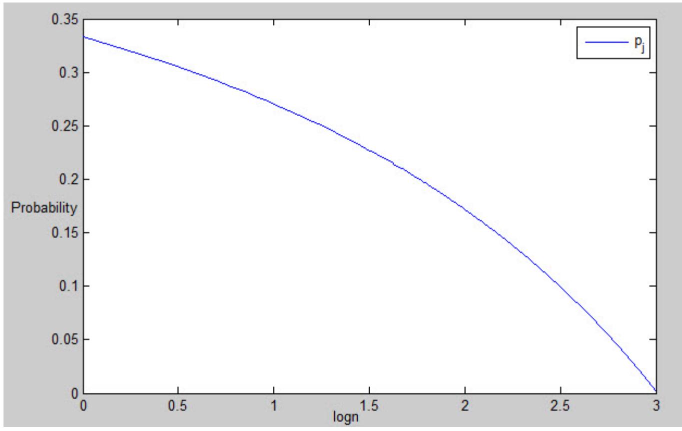

Fig. 10. p_j v.s. $logn$

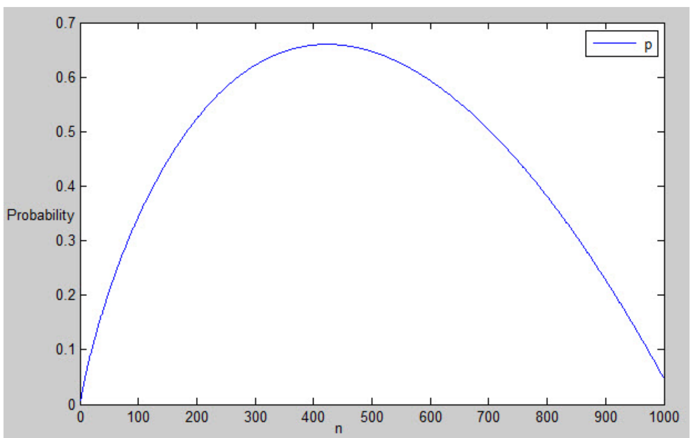

Fig. 11. p v.s. n

influence on the information exchange between different home points.

And if we look at the overall probability for a node to change its home points, that will be

$$p = C^{'}(2 - n^k)n$$

From Fig. 11 we can see that this function will have a maximum at the point. That means we can find the point where a node tends to leave its home points!

V. Simulation

A. Probability p_j

In Fig. 12, here we can see the actual probability p_j is very close to our theoretical deduced value.

Actually, according to the theoretical one, p_j is not equal to 0 but even be negative value. All of this has a boundary limit $\delta = 2$. As for the reason why this will cause such a significant influence, we will try to figure it out.

B. Capacity of No Jump Situation

Here we can see the theoretical one fits the actual one very well in Fig. 13.

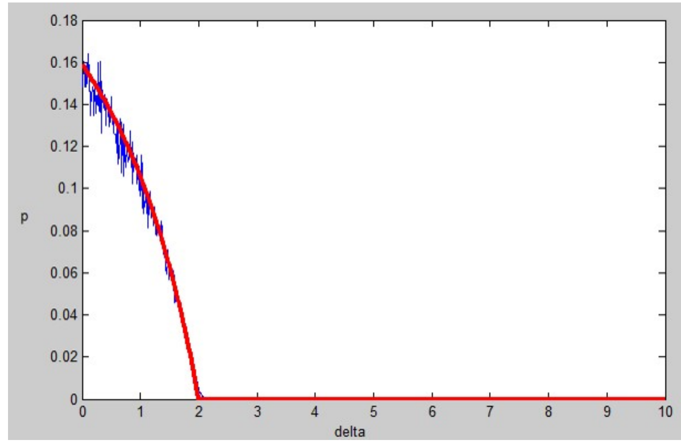

Fig. 12. probability to jump

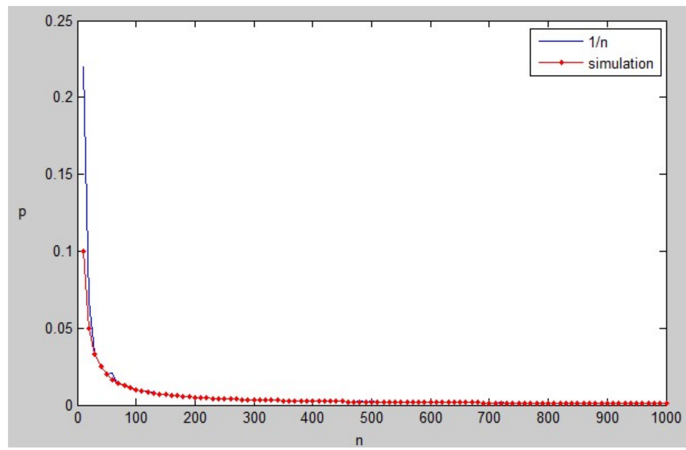

Fig. 13. capacity of no jump situation

VI. Conclusion

In this paper, we discuss the communication capacity and delay in social networks. First, we give out the identification that under any heterogeneous model with a non-increasing node distribution, the no relay capacity will be $\lambda(n) = \theta(\frac{1}{n})$, and delay will be $D = \theta(n)$, and for at-most-one-relay system, the capacity will be $\lambda(n) = \theta(1)$, and delay will be $D = \theta(n)$. In this way, we can eliminate the impact that heterogeneous distribution introduce into the system. This could be useful in some routing mission.

Also, we come up with a new kind of model that will be used to simulate the movement of people. We derived the probability P_J that a node changes its home points in a single time slot. Also, we calculate the capacity and delay under this circumstance. This may be applied to the research on the expansion of gossips, people flow around cities, precaution of disease and so on.

In section III, we have proved that though p will be very small when $R = \theta(1)$, $d = \theta(1)$, but p could be $\theta(1)$, which means the decline of capacity is due to the interference. So this may give us some implies on the derivation of multicast.

As for the capacity and delay with our unique mobility model, now we have some problems figuring them out. But we already can see that, the distribution of nodes will have a significant impact on p_j and hence on the capacity and delay. And we are even able to control these performance by properly design the distribution!

VII. Future Work

In this paper, we have finished the proof of the basic problems, but we have not finished the proof with our own mobility model. Also, due to the finite computational resources, it is impossible for us to do the simulation work with our own model with a very large n. We will try to deal with this problem.

We discuss the single-cast situation based on our model. But in reality, multi-cast may be more prevalent, or the combination of single-cast and multi-cast. We can design a mechanism to decide when to conduct single-cast and when multi-cast, in order to both saving power and realizing maximum capacity.

Also, home points can be heterogeneous in real life. We can add some weight to the home points, which in a way represents the capability of maintaining nodes.

References

[1] P. Gupta and P. Kumar, "The capacity of wireless networks," *Information Theory, IEEE Transactions on*, vol. 46, no. 2, pp. 388–404, Mar 2000.

[2] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad hoc wireless networks," *Networking, IEEE/ACM Transactions on*, vol. 10, no. 4, pp. 477–486, Aug 2002.

[3] X. Chen, W. Huang, X. Wang, and X. Lin, "Multicast capacity in mobile wireless ad hoc network with infrastructure support," in *INFOCOM, 2012 Proceedings IEEE*, March 2012, pp. 271–279.

[4] M. G. E. Leonardi, "Restricted mobility improves delay-throughput trade-offs in mobile ad-hoc networks."

[5] Z. Lu, Y. Wen, and G. Cao, "Community detection in weighted networks: Algorithms and applications," in *Pervasive Computing and Communications (PerCom), 2013 IEEE International Conference on*. IEEE, 2013, pp. 179–184.

[6] S. Yao, X. Wang, X. Tian, and Q. Zhang, "Delay-throughput tradeoff with correlated mobility of ad-hoc networks."

[7] L. K. Law, S. Krishnamurthy, and M. Faloutsos, "Capacity of hybrid cellular-ad hoc data networks," in *INFOCOM 2008. The 27th Conference on Computer Communications. IEEE*, April 2008, pp. –.

[8] J. Zhang, Q. Zhang, B. Li, X. Luo, and W. Zhu, "Energy-efficient routing in mobile ad hoc networks: mobility-assisted case," *Vehicular Technology, IEEE Transactions on*, vol. 55, no. 1, pp. 369–379, Jan 2006.

[9] J. Liu, X. Jiang, H. Nishiyama, and N. Kato, "Exact throughput capacity under power control in mobile ad hoc networks," in *INFOCOM, 2012 Proceedings IEEE*, March 2012, pp. 1–9.

[10] D. Liben-Nowell, J. Novak, R. Kumar, P. Raghavan, and A. Tomkins, "Geographic routing in social networks," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 102, no. 33, pp. 11 623–11 628, 2005.

[11] C. Song, Z. Qu, N. Blumm, and A.-L. Barabási, "Limits of predictability in human mobility," *Science*, vol. 327, no. 5968, pp. 1018–1021, 2010.

[12] M. C. Gonzalez, C. A. Hidalgo, and A.-L. Barabasi, "Understanding individual human mobility patterns," *Nature*, vol. 453, no. 7196, pp. 779–782, 2008.

Acknowledgment

The authors would like to thank Prof. Xinbing Wang, Prof. Xiaohua Tian, Dr. Luoyi Fu.