Wireless Communications and Mobile Networks, Spring 2014, Project Report
MIMO System and Power Allocation

Zengwen Yuan 5110309479

June 20, 2014

Contents

1 Background and Motivation

Since modern wireless communication system is fundamentally limited by random fading channels, we want to tackle this problem to increase the system capacity and enhance the bit error performance.

On the one hand, we want to use some theoretical methodology to exploit the better performance. On the other hand, we want to test the theoretical result in real world, not just do some simulation. So, MIMO technology and power control have sparked our interest, because:

- MIMO technology is suitable to deal with the small scale fading and the multipath propagation.
- Power control can alleviate the distance attenuation and the large scale fading.

However, to implement and test a MIMO system is usually costy and time-consuming. So, we want to build a testbed/platform to quickly implement the system and test the algorithms with *flexibility* and *low cost*. And software defined radio (SDR) is an effective, yet economical solution.

2 Introduction

2.1 MIMO technology

In radio, multiple-input and multiple-output (MIMO), is the use of multiple antennas at both the transmitter and receiver to improve communication performance. It is one of several forms of smart antenna technology.

MIMO technology has attracted attention in wireless communications, because it offers significant increases in data throughput and link range without additional bandwidth or increased transmit power. It achieves this goal by spreading the same total transmit power over the antennas to achieve an array gain that improves the spectral efficiency (more bits per second per hertz of bandwidth) and/or to achieve a diversity gain that improves the link reliability (reduced fading). Because of these properties, MIMO is an important part of modern wireless communication standards such as IEEE 802.11n (Wi-Fi), 4G, 3GPP Long Term Evolution, WiMAX and HSPA+.[1]

2.2 Channel state information in MIMO

In MIMO systems, a transmitter sends multiple streams by multiple transmit antennas. The transmit streams go through a matrix channel which consists of all $N_t N_r$ paths between the N_t transmit antennas at the transmitter and N_r receive antennas at the receiver. Then, the receiver gets the received signal vectors by the multiple receive antennas and decodes the received signal vectors into the original information. A narrowband flat fading MIMO system is modelled as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{y}$ and $\mathbf{x}$ are the receive and transmit vectors, respectively, and $\mathbf{H}$ and $\mathbf{n}$ are the channel matrix and the noise vector, respectively.

Referring to information theory, the ergodic channel capacity of MIMO systems where both the transmitter and the receiver have perfect instantaneous channel state information is

$$C_{\mathrm{perfect-CSI}} = E\left[\max_{\mathbf{Q};\,\mathrm{tr}(\mathbf{Q})\leq 1} \log_2 \det\left(\mathbf{I} + \rho \mathbf{H}\mathbf{Q}\mathbf{H}^H\right)\right] = E\left[\log_2 \det\left(\mathbf{I} + \rho \mathbf{D}\mathbf{S}\mathbf{D}\right)\right] \tag{2}$$

where $()^H$ denotes Hermitian transpose and ρ is the ratio between transmit power and noise power (i.e., transmit SNR). The optimal signal covariance $\mathbf{Q} = \mathbf{V}\mathbf{S}\mathbf{V}^H$ is achieved through singular value decomposition of the channel matrix $\mathbf{U}\mathbf{D}\mathbf{V}^H = \mathbf{H}$ and an optimal diagonal power allocation matrix $\mathbf{S} = \mathrm{diag}(s_1, \ldots, s_{\min(N_t,N_r)}, 0, \ldots, 0)$. The optimal power allocation is achieved through waterfilling,[2] that is

$$s_i = \left(\theta - \frac{1}{\rho d_i^2}\right)^+, \quad \text{for } i = 1, \ldots, \min(N_t, N_r), \tag{3}$$

where $d_1, \ldots, d_{\min(N_t,N_r)}$ are the diagonal elements of $\mathbf{D}$, $(\cdot)^+$ is zero if its argument is negative, and θ is selected such that $s_1 + \ldots + s_{\min(N_t,N_r)} = N_t$.

If the transmitter has only statistical channel state information, then the ergodic channel capacity will decrease as the signal covariance $\mathbf{Q}$ can only be optimized in terms of the average mutual information as [3]

$$C_{\text{statistical-CSI}} = \max_{\mathbf{Q}} E\left[\log_2 \det\left(\mathbf{I} + \rho \mathbf{H}\mathbf{Q}\mathbf{H}^H\right)\right]. \quad (4)$$

The spatial correlation of the channel has a strong impact on the ergodic channel capacity with statistical information.

If the transmitter has no channel state information it can select the signal covariance $\mathbf{Q}$ to maximize channel capacity under worst-case statistics, which means $\mathbf{Q} = 1/N_t \mathbf{I}$ and accordingly

$$C_{\text{no-CSI}} = E\left[\log_2 \det\left(\mathbf{I} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^H\right)\right]. \quad (5)$$

Depending on the statistical properties of the channel, the ergodic capacity is no greater than $\min(N_t, N_r)$ times larger than that of a SISO system.

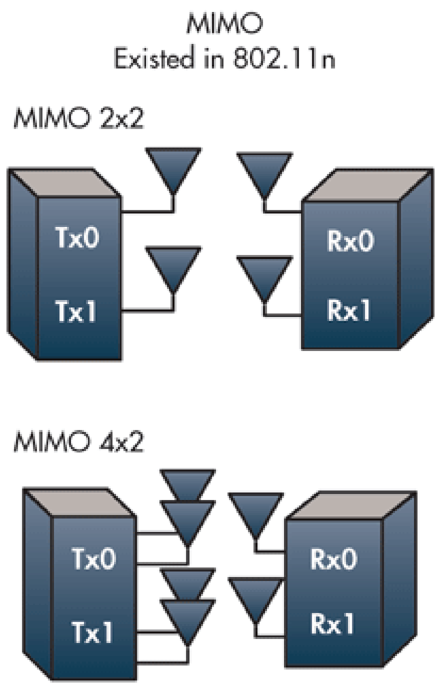

Figure 1: MIMO Existed in 802.11n

2.3 Power Allocation with Large-scale Fading-only Feedback

Now we know that if the channel information $\mathbf{H}$ is known at the transmitter, then we could maximize the SNR and channel capacity. It is usually obtained via feedback (FDD) or via reciprocity (TDD).

However, due to the limit of practical systems, the transmitter usually can only obtain the statistical channel state information. If we want to increase the system capacity and decrease the bit error rate, we have to tune the signal covariance matrix $\mathbf{Q}$, which can be via power allocation. We want optimum power allocation for distributed antenna systems (DAS) in time-varying Rayleigh and Ricean fading channels.

The conventional power allocation schemes for DAS usually use the anntenna with the maximum channel gain, but this is based on the assumption that channel state information (CSI) includes fast small-scale fading and there is no delay in the power allocation control loop. So, it will cost extra bandwidth and processing cost, and suffer from inevitable loop delay.

So, power allocation that considers only large-scale fading could be a proper approach to resolve the problems caused by feedback delay and feedback load.

Large-scale fading arises when the coherence time of the channel is large relative to the delay constraint of the channel. In this regime, the amplitude and phase change imposed by the channel can be considered roughly constant over the period of use, as shown in Fig. 2. The received power change caused by shadowing is often modeled using a log-normal distribution with a standard deviation according to the log-distance path loss model. Large-scale fading can be assumed constant during loop delay in practical cellular environments; also, large-scale fading can be reported to the transmitters with a fairly low frequency compared to the scenario of reporting fast small-scale fading, which significantly reduces the feedback load.

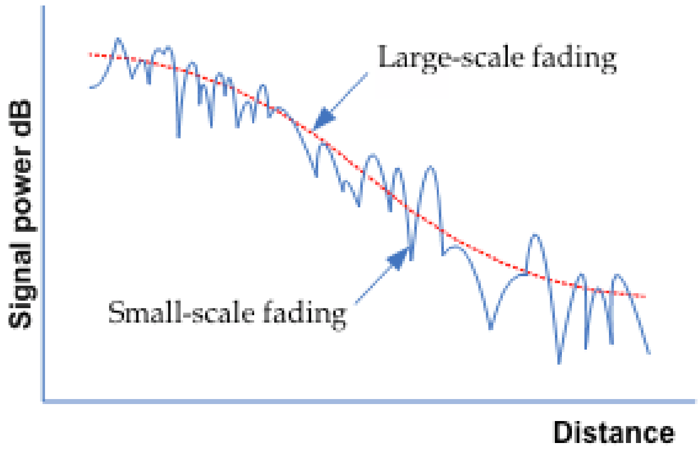

Figure 2: Large-scale fading

2.4 SDR and USRP

Software-defined radio (SDR) is a radio communication system where components that have been typically implemented in hardware (e.g. mixers, filters, amplifiers, modulators/demodulators, detectors, etc.) are instead implemented by means of software on a personal computer or embedded system.[4] While the concept of SDR is not new, the rapidly evolving capabilities of digital electronics render practical many processes which used to be only theoretically possible.

And USRP (Universal Software Radio Peripheral) is an affordable, PC-hosted SDR platform used with softwares to build powerful wireless communications systems. Each USRP device provides an independent transmit and receive channel capable of full duplex operation in some hardware configurations. The USRP (Fig. 3) supports software GNU Radio, LabVIEW, or even MATLAB and Simulink frameworks. The combination of hardware and software offers flexibility and functionality to deliver a rapid prototyping platform for physical layer design, record and playback, signal intelligence, algorithm validation, and more.

The USRP device connects to a host computer via the gigabit ethernet port which requires standard 1 Gigabit Ethernet (GbE) connection. It operates on 6 Volts DC drawing a maximum of 18 Watts of power. Referring to the USRP system block diagram (Fig. 4), the transmit and receive chains operate independently but share a common internal 10-MHz TCXO reference clock from which local oscillator (LO) are derived:[5]

Figure 3: NI USRP-2920

1. **Rx Signal Path**

   On the receiver side incoming analog RF signals enter through either RX 1 or RX 2 connector, selected by a programmable switch passing through an adjustable (0-30dB) gain stage to the mixer for direct-conversion from the LO RF frequency to baseband IQ components. A 2-channel, 100MS/s, 14-bit ADC filters and samples the baseband I and Q analog signals. The digitized I and Q data flow through parallel onboard signal processing (OSP) processes that applies DC offset correction, digital down conversion using a CORDIC to correct minor frequency offsets to achieve the desired RF center frequency, filtering and decimating the 100M S/sec input signal to the user-specified IQ Rate. The downconverted samples are then passed to the host computer at a baseband IQ Rate of up to 25 MS/s in 16-bit mode and 50MS/s in 8-bit mode over the standard GbE connection to the host computer for processing. [9]

2. **Tx Signal Path**

   For transmission, the host computer synthesizes baseband IQ signals and passes the resulting I and Q signal samples to the USRP at up to 25 MS/s in 16-bit mode and 50 MS/s in 8-bit mode over the GbE link. The USRP hardware OSP interpolates and up-converts the synthesized signals to 400MS/s using a digital up conversion process, applies the CORDIC for minor frequency offset corrections based on the requested RF center frequency and then converts the signal to analog with a dual-channel, 16-bit DAC. The resulting analog signal is then filtered and modulated at the specified RF frequency using a direct conversion architecture to mix the LO with the analog baseband IQ signal. An adjustable (0-30dB) gain stage amplifies the signal for transmission through the external TX 1 port.

3 Implementation of the MIMO Testbed

3.1 Features

We setup a SDR-based 8×8 MIMO system testbed in lab, and our MIMO testbed is implemented on NI USRP-2920 hardware. By using up to 16 USRP-2920 devices, we combine them to form a phase-coherent antenna array for transmission and reception, which scales from 2×2 to 8×8 antenna configurations (consisting of up to 8 transmitters and 8 receivers). With our MIMO testbed and NI LabVIEW development environment, we are capable to test and verify a wide range of MIMO and multi-user communications algorithms.

Compared to the state-of-the-art MIMO systems shown in Table 1, our system have several advantages:

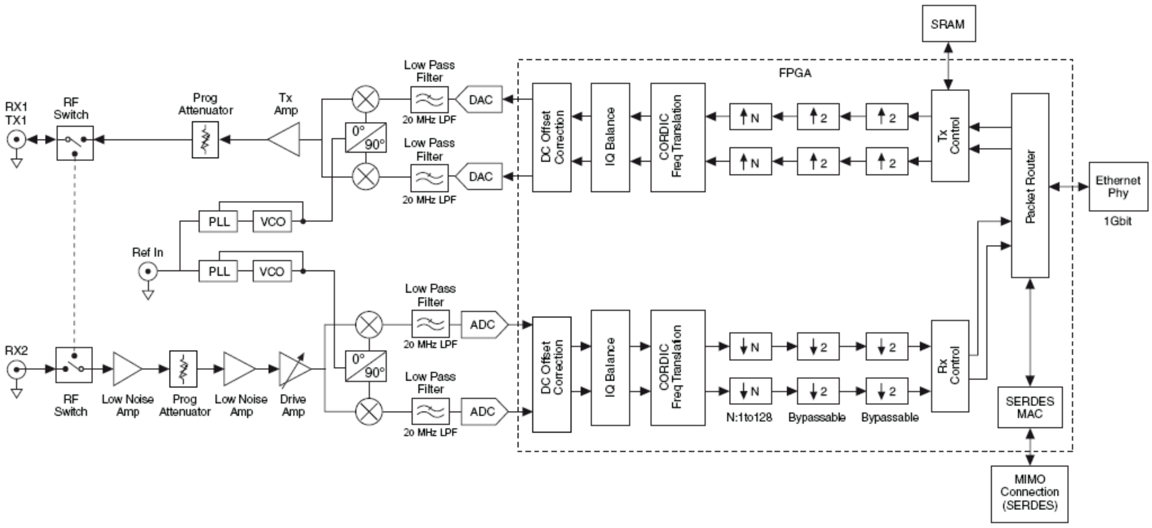

Figure 4: NI USRP-2920 System Block Diagram

Table 1: State-of-the-art MIMO Systems

System	Year	Realization & Feature	Institution
SourceSync	2010	FPGA @ 128 MHz	MIT
AirSync	2012	8 WARPs, Phase Sync	USC
STROBE	2012	WARPLab, ZFBF	Rice
Argos	2013	WARP, 64 antennas	Rice
ADAM	2013	WARP, Adaptive Beamforming	Rice

1. First, our testbed is implemented by NI USRP-2920, which is costs less compare to the WARP Kit. That means we can build a larger scale MIMO testbed under the limited budget.

2. Second, our testbed is scalable and flexible, which can vary from 2×2 to 8×8 antennas. As we know, some wireless communication standards such as LTE Advanced, incorporates up to a 4x4 MIMO uplink and 8x8 MIMO downlink. So a scalable system is very useful.

3. Third, we implemented feedback of large-scale fading channel state information (CSI) for simple power allocation, and we also combined OFDM scheme in our system.

3.2 Time and Frequency Synchronization

For a transceiver to be considered MIMO-capable, the system must meet two basic requirements:

1. The sample clocks must be synchronized and aligned.

2. DSP operations must be performed on samples aligned in time. That is, they must be performed at the same sample clock edge.

For the USRP, the time and frequency can be synchronized between two USRPs using the MIMO expansion port, which also acts as an Ethernet switch, allowing a pair of USRPs to share a single GbE connection. The REF IN (10-MHz reference clock) and PPS (pulse per second) SMA connections on the front of the USRP enable an external frequency reference and time synchronization to supplement the internal TCXO for greater frequency accuracy or to provide synchronization among a larger numbers of devices. And as the scale goes up, this time and frequency synchronization is more important.

As for our testbed, we want to implement the MIMO testbed that all Rx or Tx channels operate as a single receiver or transmitter, so frequency and time synchronization and a phase coherent local oscillator (LO) is needed and we must use the external clock. The MIMO testbed achieves synchronized operation by sharing a 10MHz reference clock and PPS time base among the USRP devices. The hardware we used to generate the clock is Trimble Thunderbolt GPS Disciplined clock which supplies a 10-MHz Ref Clock output and digital PPS output.[6]

3.3 Testbed Configuration

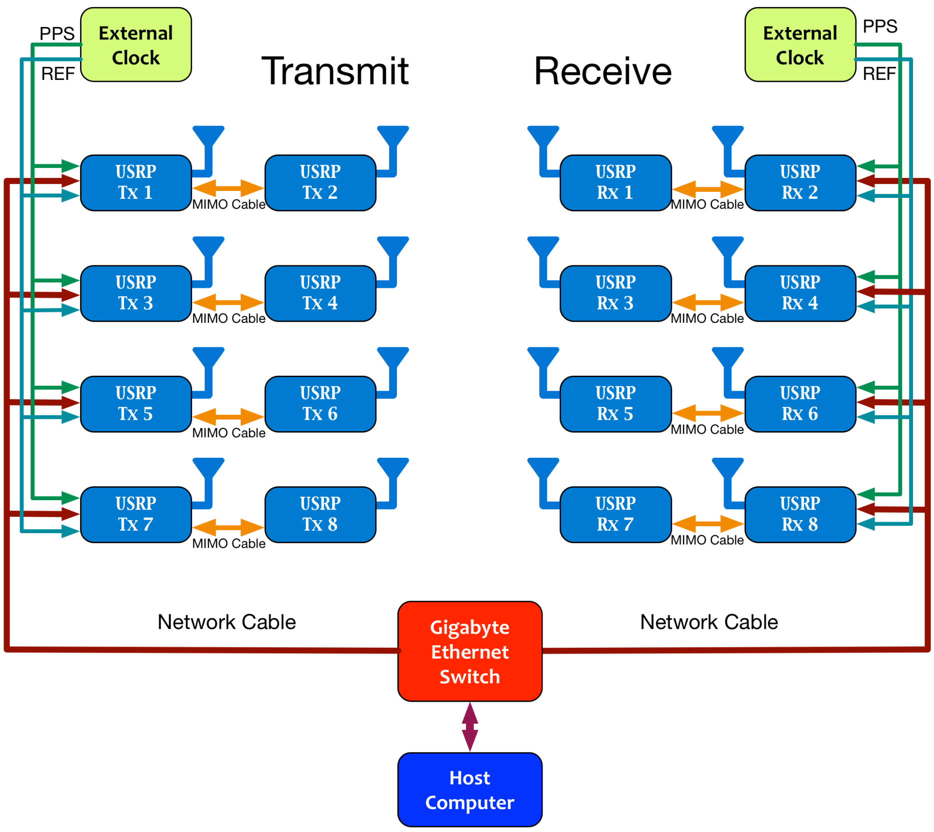

Figure 5: 8×8 MIMO Testbed Configuration

Our testbed is initially configured in a centralized way, as shown in Fig. 5. For the 8×8 configuration, the transmitter consists of 8 USRPs and so does the receiver. Among these 8 USRPs on each side, 4 of them are master USRPs, while the remaining four are slave USRPs. The difference between the master USRP and the slave USRP is that the master USRPs are connected to the external clock and get synchronized, while the slave USRPs rely on the MIMO cable to synchronize with the master USRP. And all master USRPs are connected to a gigabyte ethernet switch, which finally connects to a host computer. This synchronizes all USRP devices in both frequency and time with the coherent LO being derived from a single

10-MHz reference clock and a PPS output.

Note that the power level of both the 10 MHz and PPS reference signals is an important consideration as clock signals are divided among radios. In our application, common coaxial BNC "T" connections divide the signals twice enabling synchronization of the first 4 USRP devices. The Tunderbolt provides a 10MHz reference of approximately 12.5dBm and a 5V digital PPS signals. Fanning the signal out to 4 devices is near the recommended REF IN and PPS input levels for the USRP.

Fig. 6 shows our 8×8 testbed, and Fig. 7 shows the time-domain wave diagram and the channel constellation.

Figure 6: 8×8 MIMO Testbed

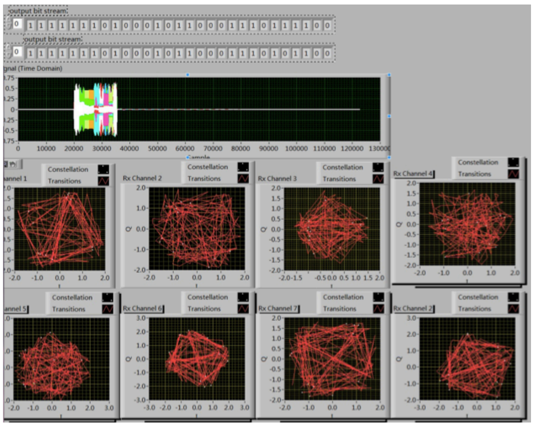

Figure 7: 8×8 MIMO Channel Constellation

4 System Model

4.1 Centralized Structure

Consider a DAS (Distributed Antenna System) given in Fig. 8, N neighboring base stations (transmitters) simulcasts the same information to a mobile user (receiver) within the radius of R, and both transmitter and receiver are equipped with single antenna. And there is a cental controller (a server) connected to all the transmitters. It can receive the feedback of the CSI between transmitters and the receiver and thus derive an optimal power allocation scheme to optimize the system performance.

The received signal at a mobile station is given by

$$\mathbf{y} = \mathbf{h}\mathbf{P}^{\mathbf{1/2}}\mathbf{s} + \mathbf{n} \tag{6}$$

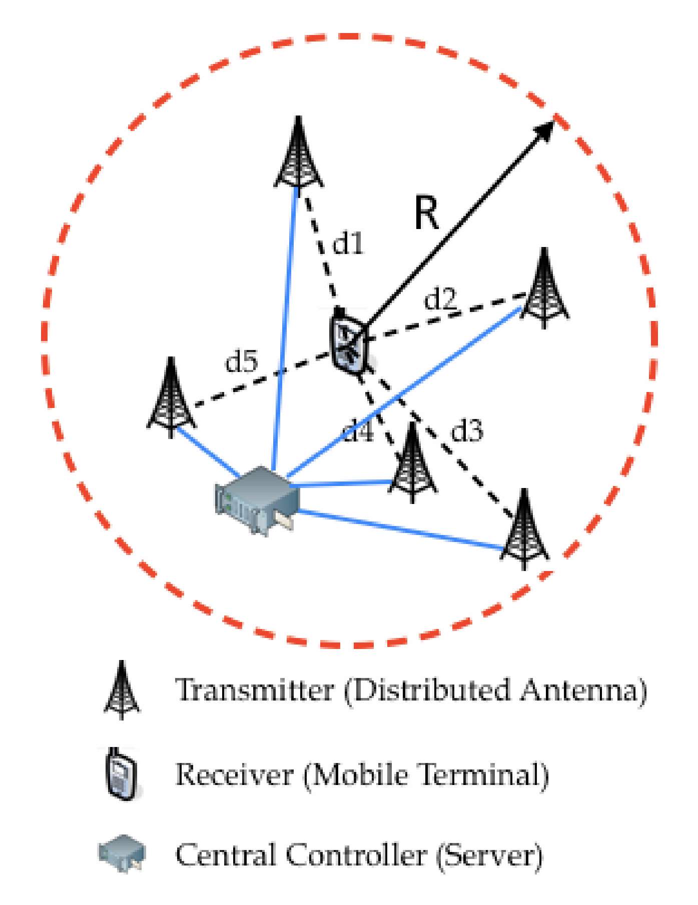

Figure 8: Centralized structure

Where $\mathbf{y}$ is the received signal, $\mathbf{h}$ is the channel gain, $\mathbf{P}$ is the transmission power, $\mathbf{s}$ is the transmitted signal, and $\mathbf{n}$ is noise, $\mathbf{n} \sim \mathcal{CN}(0, \sigma_n^2),\ \sigma_n^2 = N_0/2$.

And we model the channel as the composite fading model, where the channel gain $\mathbf{L}$ consists of large- scale fading component $\mathbf{L}$ and small-scale fading component $\mathbf{L}$ and is expressed as:

$$\mathbf{h} = \mathbf{L^{1/2}f} \tag{7}$$

In Eqs. 7, $\mathbf{L}$ denotes the large-scale fading component from the base station to the mobile, and is given by

$$\mathbf{L} = \mathbf{rG}, \tag{8}$$

where $\mathbf{r} = d^{-\alpha}$ is the path loss, and α is usually between 2 and 4.

The log-normal shadowing $\mathbf{G}$ is given by:

$$\mathbf{G} = 10^{\frac{\xi}{10}}, \tag{9}$$

where $\xi \sim \mathcal{CN}(0, \sigma_\xi^2)$ is the shadowing attenuation, which p.d.f can be written as

$$p_\xi(x) = \frac{1}{\sqrt{2\pi}\sigma_\xi} \exp(-\frac{x^2}{2\sigma_\xi^2}). \tag{10}$$

In Eqs. 7, $\mathbf{f} \sim \mathcal{CN}(0, 1)$ represents the small-scale fading component, and the p.d.f of the magnitude of $\mathbf{f}$ (Rayleigh Distribution) is:

$$p_{|\mathbf{f}|}(x) = \frac{x}{\sigma_{|\mathbf{f}|}^2} \exp(-\frac{x^2}{2\sigma_{|\mathbf{f}|}^2}). \tag{11}$$

So, the total transmission power of all transmitters can be written as

$$P_T = \sum_{i=1}^{N} P_i,\ \ P_i \in \mathbf{P}. \tag{12}$$

At the receiver side, we use MRC (Maximum Ratio Combining) to obtain the combined signal:

$$\mathbf{z} = \mathbf{h}^H \mathbf{P}^{\mathbf{1/2}} \mathbf{y}. \tag{13}$$

Thus, the total received SNR γ (in terms of symbols) is given by

$$\gamma = \sum_{i=1}^{N} \gamma_i = \sum_{i=1}^{N} \frac{|h_i|^2 P_i}{2\sigma_n^2}. \tag{14}$$

4.2 Distributed Structure

The senario of distributed structure is shown in Fig. 9. It is similar to that of centralized structure, the received signal is given by 6, and the rest parameters are same as centralized structure, except that there is no central controller, so centralized power allocation cannot be performed in such structure, we have to model the power allocation between the transmitters as a Stackelberg game.

For mobile user (receiver), we define its revenue as

$$R = 1 - e^{\eta \overline{P_s}}, \tag{15}$$

where $\overline{P_s}$ is the average symbol error rate, it measures the quality of the signal at the receiver side.

The cost of receiver is defined as

$$C = \sum_{i=1}^{N} \mu_i P_i. \tag{16}$$

Thus, the utility function of receiver is given by

$$U_R(\mathbf{P}, \mu) = R - \kappa C, \tag{17}$$

where η and κ are the sensitive factors, which measures the significance of the average sysmbol error rate, and the receiver's sensitivity to the cost.

For base station (transmitter), the utility function of i-th transmitter is similarly given by

$$U_{T,i}(P_i, \mu) = \mu_i P_i - \rho P_i, \tag{18}$$

ρ is the sensitive factor.

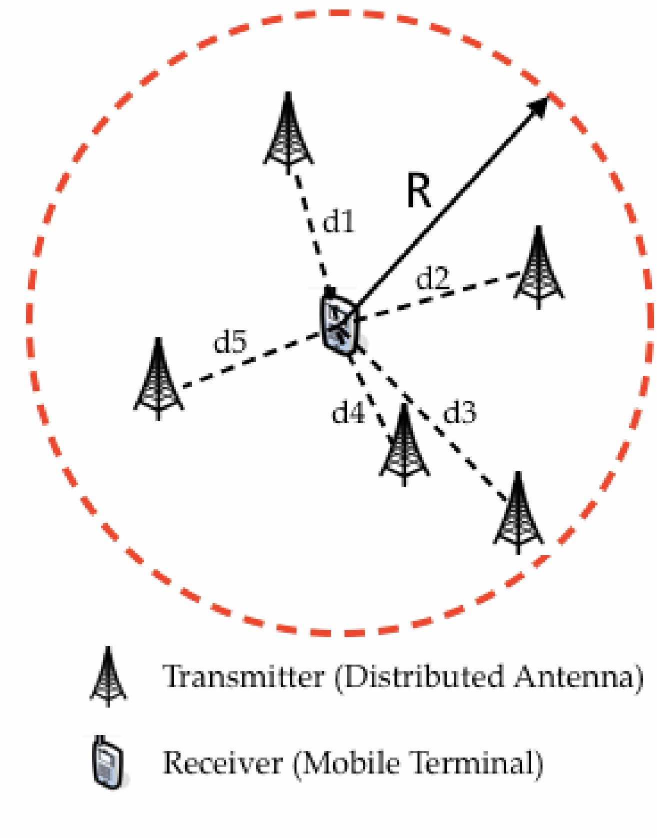

Figure 9: Distributed structure

5 Power Allocation Scheme

5.1 Centralized System Power Allocation

For centralized system with only imperfect CSI (Large-scale only feedback) known at the transmitter, we can view the power allocation as an optimization problem of average symbol error rate under the constraint.

Since the SNR at the receiver side

$$\gamma = \sum_{i=1}^{N} \frac{|h_i|^2 P_i}{2\sigma_n^2}, \tag{19}$$

is a weighted chi-squared distributed random variable, its p.d.f. is given by [7]

$$f_\gamma(\gamma) = \sum_{i=1}^{N} \frac{2\sigma_n^2 \pi_i}{L_i P_i} \exp(-\frac{2\sigma_n^2}{L_i P_i}\gamma), \tag{20}$$

where

$$\pi_i = \prod_{k=1, k\neq i}^{N} \frac{L_i P_i}{L_i P_i - L_k P_k}. \tag{21}$$

Also, the symbol error rate (SER) for a M-modulate scheme is

$$P_s(\gamma) = \alpha_M Q(\sqrt{\beta_M \gamma}), \tag{22}$$

where α_M is the average number of the nearest neighbors in the given constellation, β_M is related to the modulation method.

The error function $Q(x)$ is defined as

$$Q(x) = \frac{1}{2}\text{erfc}(\frac{x}{\sqrt{2}}) \tag{23}$$

Since a pure exponential approximation of the Q-function is given by Chiani, Dardari & Simon (2003) by:[8]

$$Q(x) \approx \frac{1}{12}e^{-\frac{x^2}{2}} + \frac{1}{4}e^{-\frac{2}{3}x^2} \qquad x > 0, \tag{24}$$

the average SER $\overline{P_s}$ can be calculated by

$$\overline{P_s} = \int_0^\infty P_s(\gamma)f_\gamma(\gamma)\mathrm{d}\gamma \tag{25}$$

$$\approx \sum_{i=1}^{N} \frac{\alpha_M \pi_i}{12}\left(\frac{1}{1+\frac{\beta_M}{4\sigma_n^2}L_iP_i} + \frac{3}{1+\frac{\beta_M}{3\sigma_n^2}L_iP_i}\right) \tag{26}$$

$$\approx \sum_{i=1}^{N} \frac{\alpha_M \pi_i}{\frac{3\beta_M}{4\sigma_n^2}L_iP_i + 3} \tag{27}$$

Thus, the optimization problem becomes finding an optimal power allocation scheme $\mathbf{P_{opt}}$ such that

$$\mathbf{P_{opt}} = \arg\min \overline{P_s}, \tag{28}$$

$$\text{s.t.} \sum_{i=1}^{N} P_i = P_T. \tag{29}$$

Since $\overline{P_s}$ is convex, we can use the *Lagrange multiplier* technique to solve this optimization problem.

First we define:

$$g(\mathbf{P}) = \sum_{i=1}^{N} P_i - P_T, \tag{30}$$

Then the following equivalent problem is for us to solve:

$$\begin{cases} \nabla\overline{P_s} = \lambda\nabla g(\mathbf{P}) \\ g(\mathbf{P}) = 0 \end{cases} \tag{31}$$

By solving Eqs. 30 and 31, we can obtain the optimal power allocation scheme for centralized senario, that is

$$P_{i,opt} = \frac{P_T}{N} + \frac{4\sigma_n^2}{3N\beta_M}\left[\sum_{k=1,k\neq i}^{N}\left(\frac{1}{L_k} - \frac{1}{L_i}\right)\right]. \tag{32}$$

5.2 Distributed System Power Allocation

As stated before, we model the distributed power allocation procedure as a Stackelberg game, in which receiver performs as the *leader* of the game, and the transmitters are treated as *followers*. Receiver acts as a leader who takes action first, and the transmitters are followers who observe the leader's action and act accordingly.

So the frame of the Stackelberg game is given in this way:

Receiver (leader) announces it's price $\mu = \{\mu_1, \mu_2, \ldots, \mu_N\}$

$$\Downarrow$$

Transmitter (follower) announces it's power $\mathbf{P} = \{P_1, P_2, \ldots, P_N\}$

The Stackelberg equilibrium can be found using the backward induction method. It first studies the followers' game. For each possible action of the leader, it finds the optimal followers' response that maximizes the followers' payoff. Then given the optimal followers' response strategy, it studies the leader's action and chooses the one that maximizes the leaders utility. The chosen strategy set is the Stackelberg equilibrium.

The backward induction method for the Stackelberg game is given in this way:

Find transmitters' (follower) optimum power allocation $\mathbf{P_{opt}}$ assuming the leader has made action

$$\Downarrow$$

Find receiver's (leader) price set $\mu_{\mathbf{opt}} = \{\mu_1^*, \mu_2^*, \ldots, \mu_N^*\}$ that maximizes the receiver's utility

So, to obtain the optimal power allocation scheme in Stackelberg game for the distributed scenario, we have to solve the sub-optimization problems for both transmitter and receiver.

For i-th transmitter (follower), we have to solve the optimization problem that

$$\max_{P_i \geq 0} U_{T,i}(P_i, \mu) = \mu_i P_i - \rho P_i. \tag{33}$$

Ande we would get the optimal power allocation set $\mathbf{P_{opt}}$:

$$\mathbf{P_{opt}} = \{P_1^*, P_2^*, \ldots, P_N^*\}. \tag{34}$$

For the receiver (leader), the optimization problem is given by

$$\max_{\mu \succeq 0} U_R(\mathbf{P}, \mu) = \kappa R - C, \tag{35}$$

$$\text{s.t.} \sum_{i=1}^{N} P_i^* \leq P_T. \tag{36}$$

By solving this we would get optimal price set $\mu_{\mathbf{opt}} = \{\mu_1^*, \mu_2^*, \ldots, \mu_N^*\}$.

With $\mathbf{P_{opt}}$ and $\mu_{\mathbf{opt}}$, we can assert that a power allocation scheme is derived. However, we still have to check if the derived power allocation scheme satisfies the power constraint in Eqs. 36. If the power constraint is satisfied, then we get the optimal power allocation scheme; otherwise, we need to null power transmitted on the transmitter that consumes maximum power, and solve the problem under that conditon again.

Unfortunately, we haven't derive the result for this distributed senario due to the time limit.

6 Conclusion and Future Work

By now, we have sucessfully setup a scalable MIMO testbed from 2×2 to 8×8 in lab, and we use the imperfect CSI (large-scale fading information) to exploit simple and effective power allocation schemes, considering both the centralized senario and distributed senario. Also, we give the close-form optimum power allocation scheme in centralized senario, and use Stackelberg game to allocate power in distributed senario.

As for future work, we are going to complete this work in the following aspects:

1. Complete the work in distributed senario and derive the numerical results.
2. Expand the senario to N receive antennas.
3. Simulate and test the theoretical results in our MIMO testbed.

References

[1] http://en.wikipedia.org/wiki/Mimo

[2] D. Tse and P. Viswanath, Fundamentals of Wireless Communication, Cambridge University Press, 2005

[3] Love, David; Heath, Robert; n. Lau, Vincent; Gesbert, David; Rao, Bhaskar; Andrews, Matthew (2008). "An overview of limited feedback in wireless communication systems". IEEE Journal on Selected Areas in Communications 26 (8): 13411365. doi:10.1109/JSAC.2008.081002.

[4] Software Defined Radio: Architectures, Systems and Functions (Markus Dillinger, Kambiz Madani, Nancy Alonistioti) Page xxxiii (Wiley & Sons, 2003, ISBN 0-470-85164-3)

[5] Building an Affordable 8x8 MIMO Testbed with NI USRP: http://www.ni.com/white-paper/14311/en/

[6] Trimble Thunderbolt web site: http://www.trimble.com/timing/thunderbolt-e.aspx

[7] Wan Choi and Jeffrey G. Andrews, "Downlink Performance and Capacity of Distributed Antenna Systems in a Multicell Envirnment," IEEE Trans. Wireless Communications, vol. 6, no. 1, pp. 69-73, May. 2007.

[8] Chiani, M.; Dardari, D.; Simon, M. K. (2003), "New Exponential Bounds and Approximations for the Computation of Error Probability in Fading Channels", IEEE Transactions on Wireless Communications 4 (2): 840845, doi:10.1109/TWC.2003.814350.

[9] J. W. Massey, J. Starr, Seogoo Lee, Dongwook Lee, A. Gerstlauer, and R. W. Heath Jr. Implementation of a real-time wireless interference alignment network. Proc. of the Asilomar Conference on Signals, Systems and Computers, 2012.