# Satisfaction Games in Graphical Multi-resource Allocation

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**ABSTRACT:** This report is for the project of course of *wireless communications and networking.* This project is about how to allocate multi-resources efficiently with the concept of game theory. In this report, we mainly introduced the motivation, system model, convergence of NE and the simulation result for our system.

**KEY WORD:** wireless communications, resource allocation, game theory

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# I. Introduction

In communication system, resource allocation is always the fundamental problem – the limited resources and the increasing demand. How to allocate the resources efficiently among so many users to maximize the total utility has always been on focus. Typically there are two different approaches. The first one is the centralized manner while the other depends on the autonomous users or entities, which is called the distributed manner.

- Centralized manner

  In a centralized manner, there is a head operator controls the whole system. The operator collects almost all information about every entities within the system. It will analyze the current situation and comes out a best result. Then the operator will make plans and allocate the resources to each entities to reach his best result.

  In practice, centralized manner is often difficult to realize. Firstly, due to the heterogeneity of users, the operator needs gather massive amounts of information to perform the optimization. Secondly, finding the system-wide optimal solution is usually NP hard. Thus this approach is sometimes not suitable.

- Distributed manner

  In a distributed manner, however, problems can be avoided. In a distributed manner, each user makes the resource allocation decision locally to meet its own demand, while taking the network dynamics and other users' actions into consideration.

  This approach is flexible and particularly suitable. But the distributed manner also has its own problem, that is, the convergence and the price of anarchy. To analyze these issue, models in game theory have been utilized, such as the congestion games, potential games. We have different game models for different situations.

Nowadays everybody has a smartphone. We may use it to listen to music, read e-books, browse the webpages or watch the videos. Our demand for QoS varies according to what we use it for, that is, the QoS for watching a video must be higher than the QoS for browsing a webpage. It is also obvious that we don't need the QoS to be as high as possible. (For example, when listening to music, once the music can be played fluently, the listener will be satisfied and doesn't need a higher QoS.) Thus, the model of satisfaction game is proposed in[1]. In this paper, only single channel allocation is considered. But in practice, cases exist about multi-resource allocation problems. Such as object replication, where each node (entities) can has a storage capacity $K_i$ which it uses to replicate objects locally. Thus we need to extend the original satisfaction games into multi-resource allocation model.

## II. System Model

In this section we formally define the QoS satisfaction game model for spectrum sharing.

A QoS satisfaction game is defined by a tuple

$$(\mathcal{N}, \mathcal{R}, (Q_n^r)_{n\in\mathcal{N}, r\in\mathcal{R}}, (D_n)_{n\in\mathcal{N}}, \mathcal{G}, K, \mathcal{A})$$

where:

- $\mathcal{N}$ is the set of users or entities that compete for resources.
- $\mathcal{R}$ is the set of resources
- Each node has a demand QoS, which is denoted by $(Q_n^r)_{n\in\mathcal{N}, r\in\mathcal{R}}$. $Q(\cdot)$ is a non-increasing function along with the number of interference users choose the same resource. As shown in Figure 1 QoS function. Here we use $I_n^r$ to denote the number of neighbors of user n choose resource r. And the concept of neighbor will be introduced in the next part of interference graph.

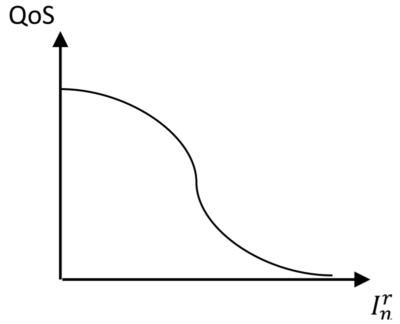


Figure 1 QoS function

- $(D_n)_{n\in\mathcal{N}}$ is the demand of each user n. This demand, as we have mentioned in the Introduction part, varies according to what the user use the resource for. It is only when the user's QoS is larger than or equal to the demand that the user is satisfied.
- $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ represents the interference graph, where $\mathcal{N}$ and $\mathcal{E}$ denotes the set of nodes and set of edges respectively. In reality, users in different qualities may share the same resource without any interference to each other. For example, in spectrum sharing problem, users who are far enough with each other may not cause interference or "congestions" when selecting the same channel. Thus, we need another set called the "neighbor" $(n)$: the neighbor of node $n$, who will interfere with node $i$ if they collect the same resource.
- $K$ is the set of number of resources each node allocates.
- Strategy profile: $\mathcal{A} = \{A_n | n \in \mathcal{N}\}$, where $A_n = \{a_n^r | r \in \mathcal{R}\}$. We define Indicator $a_n^r = \begin{cases} 1, \text{if resource } r \text{ is allocated by node } n \\ 0, \qquad\qquad \text{otherwise} \end{cases}$

To better model our system and make it more concise, we use the threshold

$T_n^r$ to represent both the QoS function and the demand of each user, as shown in Figure 2 QoS with demand and threshold

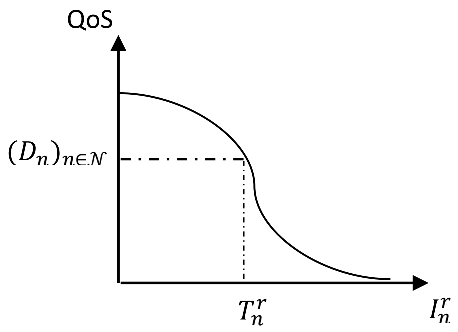


Figure 2 QoS with demand and threshold

After defining the threshold, we can give our utility function for each entity:

$$U_n^r = \begin{cases} 1, & \text{if } I_n^r \geq T_n^r \\ 0, & \text{if } I_n^r < T_n^r \end{cases}$$

Thus the whole utility of the system is:

$$U_n = \sum_r U_n^r$$

In a whole, our system is modeled like this:

$$(\mathcal{N}, \mathcal{R}, T_n^r, \mathcal{G}, K)$$

# III. Convergence of NE

## A. Key concept of Game Theory[2]

- **Definition 1 (Better Reply Update).** The event where a player n changes its choice of strategy from $x\_n$ to r is a better reply update if and only if $U\_n\ (r, \boldsymbol{x}\_(-n)\ ) > U\_n\ (x\_n, \boldsymbol{x}\_(-n)\ )$, where we write the argument of the function as $\boldsymbol{x}=(x\_n,x\_(-n))$ with $\boldsymbol{x}\_(-n)=(x\_1,\ldots,x\_(n-1),x\_(n+1),\ldots,x\_N)$ representing the strategy profile of all users except player n.
- **Definition 2 (Pure Nash Equilibrium).** A strategy profile $\mathbf{x}$ is a pure NE if no users at $\mathbf{x}$ can perform a better reply update, i.e., $\mathrm{U_n}(\mathrm{r},\mathbf{x}_{-\mathrm{n}}) > \mathrm{U_n}(\mathrm{x_n},\mathbf{x}_{-\mathrm{n}})$ for any $\mathrm{r} \in \mathcal{R}$ and $\mathrm{n} \in \mathcal{N}$.
- **Definition 3 (Finite Improvement Property).** A game has the finite improvement property if any asynchronous better reply update process terminates at a pure NE within a finite number of updates.
- **Definition 4 (Potential Function)** A function $\Phi\colon \times_n (\mathcal{A}_n) \to \mathbb{R}$ is a generalized ordinal potential function for the game if the change of $\Phi$ is strictly positive if an arbitrary player $n$ increases his utility by changing his strategy from $A_n$ to $A'_n$. Formally
$$U_n(A'_n, A_{-n}) > U_n(A_n, A_{-n}) \Longrightarrow \Phi(A'_n, A_{-n}) > \Phi(A_n, A_{-n})$$

## B. Convergence Algorithm

We will first introduce the concept of evicted set and inserted set. Since our system is about multi-resource allocation, define these two sets is helpful for our analysis.[3]

The evicted set: $E_n(t) = \{r | a_n^r(0) = 1 \wedge a_n^r(t) = 0\}$

The inserted set: $I_n(t) = \{r | a_n^r(0) = 0 \wedge a_n^r(t) = 1\}$

Then a modified better reply is proposed which can avoid loop-forming in the improvement path and thus reaching NE can be guaranteed.

**Definition 4 (modified better reply)** An modified better reply for multi-resource allocation problem is that for $\forall \mathrm{r} \in \mathrm{E_n}(\mathrm{t})$, $\mathrm{U_n^r}(\mathrm{t}) = 0$; and for $\forall \mathrm{r} \in \mathrm{I_n}(\mathrm{t})$, $\mathrm{U_n^r}(\mathrm{t}) = 1$. $|\mathrm{E_n}(\mathrm{t})| = |\mathrm{I_n}(\mathrm{t})| \le \mathrm{K}(\mathrm{n})$.

This definition can be illustrated with the figure below

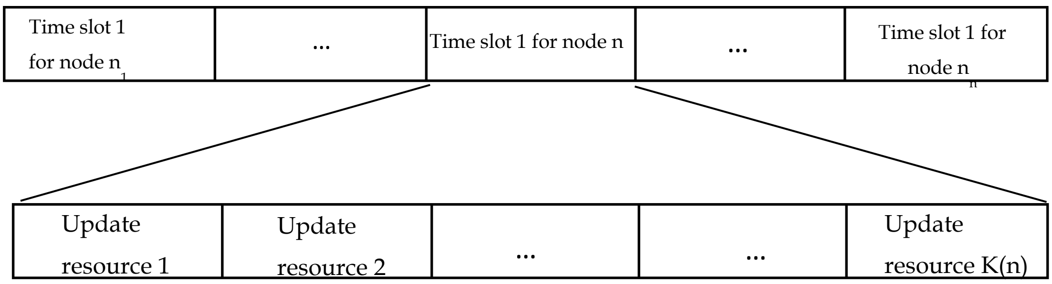


Figure 3 modified better reply

Now we can propose our theorem which indicate the convergence of our system:

**Theorem 1.** In satisfaction game for graphical multi-resource allocation, FIP can be guaranteed under modified better reply.

**Prove:**

We define our potential function as

$$\Phi(\boldsymbol{A}) = \sum_n \sum_r F_n^r(A_n, A_{-n})$$

Where $F_n^r = \begin{cases} 2T_n^r - I_n^r(\boldsymbol{A}), \text{if } a_n^r = 1 \\ 0, \qquad\qquad\quad \text{if } a_n^r = 0 \end{cases}$

We can show that in each modified better reply step, the potential function will increase at least 2, ie.

$$\sum_n F_n^{r'} - \sum_n F_n^r \geq 2$$

When a user is going to do a better reply, he will check if there is any dissatisfactory resources in his allocated set. If there are some, he then will check whether there is any satisfactory resource he hasn't allocated. If there exists, he will evict the dissatisfactory one and allocate the satisfactory resource. Thus, after a user n performing a single-step better reply from r to r′, the potential will change in two aspects: $F_n^r$ and $\sum_n F_n^r(A_n, A_{-n})$, which shows the summation of potential for neighbors in r and r′

$$\Delta F_n^r = 2T_n^{r'} - I_n^{r'}(\boldsymbol{A}') - 2T_n^r + I_n^r(\boldsymbol{A})$$

$$\Delta \sum_n F_n^r(A_n, A_{-n}) = -I_n^{r'}(\boldsymbol{A}') + I_n^r(\boldsymbol{A})$$

Thus $\Delta\Phi(\boldsymbol{A}) = 2(T_n^{r'} - I_n^{r'}(\boldsymbol{A}')) - 2(T_n^r + I_n^r(\boldsymbol{A}))$

Because the user jumps from an dissatisfactory resource into a satisfactory one, we have

$$T_n^{r'} > I_n^{r'}(\boldsymbol{A}') + 1$$

$$T_n^r < I_n^r(\boldsymbol{A})$$

Therefore, for a single-step better reply the increase in potential is at least 2.

Since we have the inequalities: $-N \leq -I_n^r < F_n^r \leq 2T_n^r < 2N + 2$ ,our potential function has a upper bound $\Phi(\boldsymbol{A}) < (3N+2)N\sum_n K(n)$

Therefore, the largest steps one improvement path may need is

$$\frac{1}{2}(3N+2)N\sum_n K(n)$$

# IV. Simulation

Although the maximum step appears to be a non-linear function in terms of the number of entities, this is the worst case. In reality, the convergence rate is much better as shown in our simulation result.

Our simulation is performed in the matlab. We set the threshold value for each node randomly and the interference graph randomly.

Figure 4 below shows the simulation result when we have 6 nodes and 10 resources. We can see that in at most 4 steps our whole system convergences to the NE point.

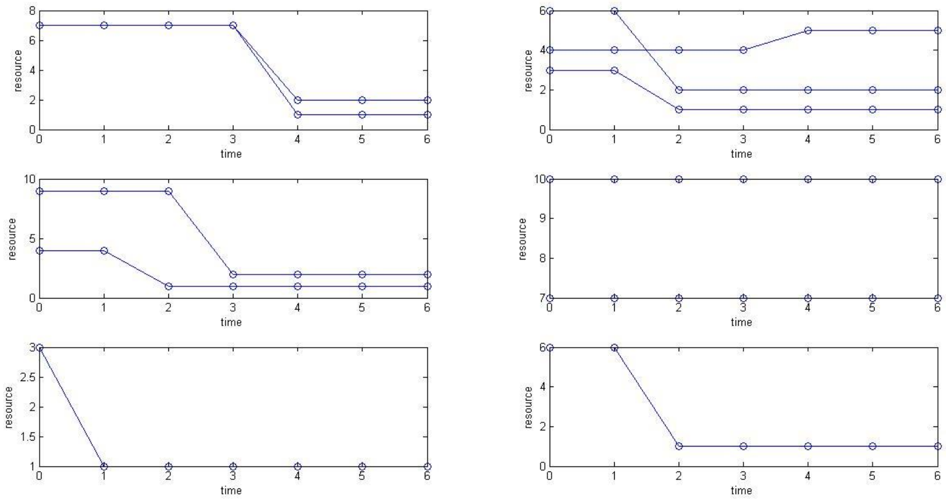


Figure 4 convergence steps with 6 nodes and 10 resources

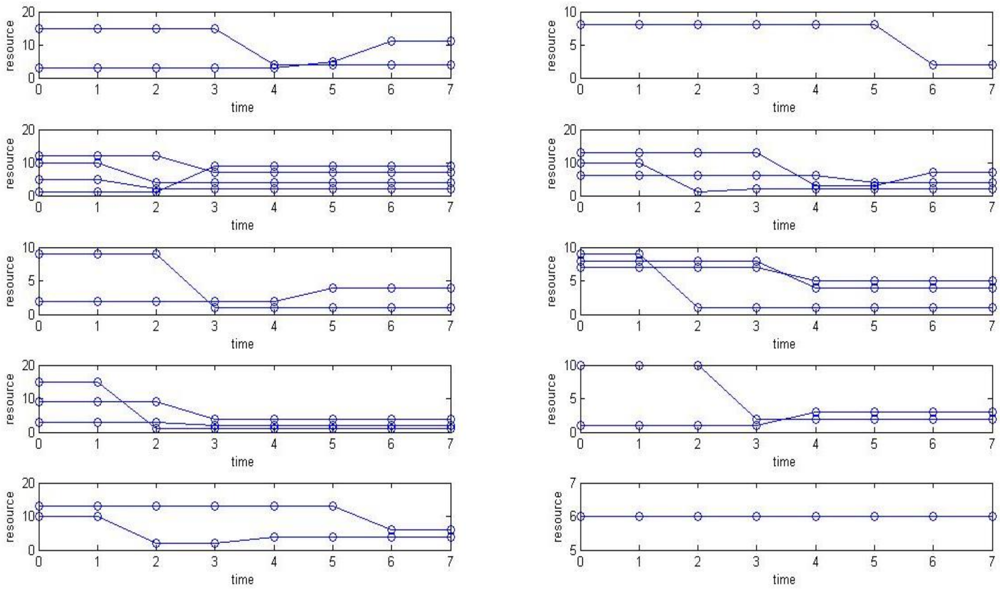


Figure 5 convergence rate with 10 nodes and 15 resource

When the number of nodes and resources become more, our system is still stable and can converge to the NE point quickly. As shown in Figure 5 at most 6 steps are needed to reach NE.

When the nodes and resources' number grow, will our system's converge rate grows in O($N^2$)? The Figure 6 below give us the answer. We set the number of nodes and resources in a form of an array:

R=[7,15,25,35,45,53,65,75,83,92];

N=[10,20,30,40,50,60,70,80,90,100];

The corresponding convergence rate is shown below. Easily seen from the result that in reality, the convergence rate doesn't grows squarely as the maximum step represents.

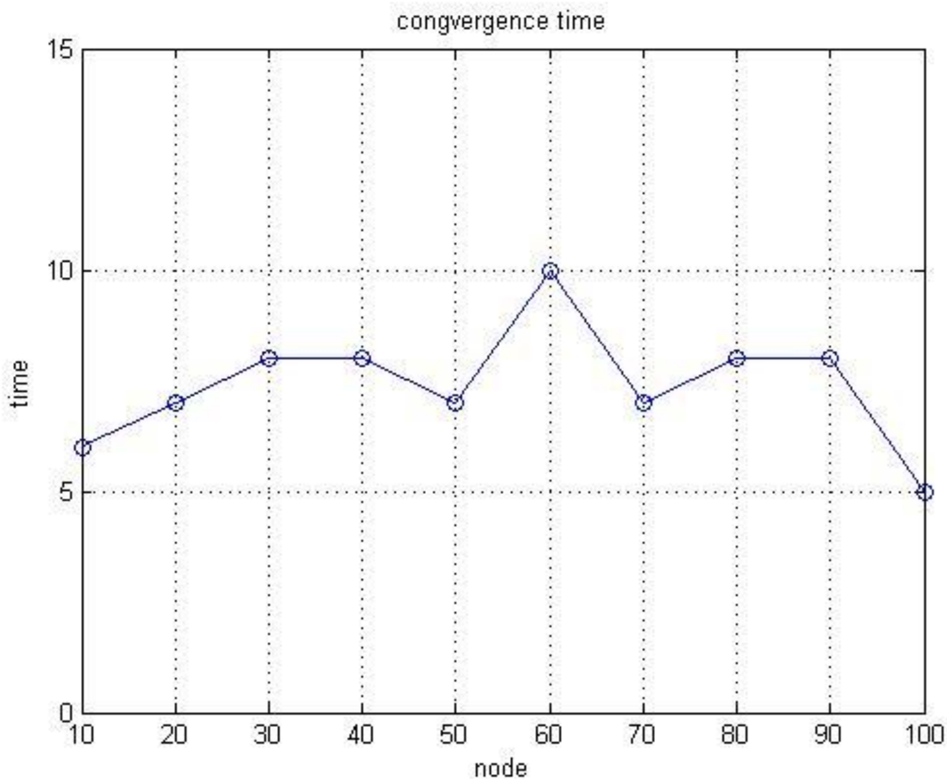


Figure 6 Convergence rate with large number of nodes and resouces

# V. Conclusion

In our project, we finished the following task:

- We use the distributed manner to solve the resource allocation problems
- Instead of single resource allocation, we propose that every node can collect more than one resource
- Utilize the satisfaction game model to make the allocation problem more practical
- Using a modified better reply strategy guarantees that NE can be reached.

In our future work, we want to further perfect our system by:

- Find a faster way than asynchronous update for converge
- To collect more resources (so that we can reduce the demand for each resource) to guarantee that the node is satisfied with every resource he collects.

# VI. Reference

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