# Impact of Mobility and Heterogeneity on Full View Coverage in Camera Sensor Network

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***Abstract*—In this work, we investigate the full view coverage of mobile heterogeneous camera sensor network(CSN). We study the asymptotic coverage under uniform deployment scheme with i.i.d, 1-dimensional random walk, random rotating mobility models and we propose the equivalent sensing radius (ESR) for each case, and derive the critical ESR correspondingly for the first time. We first show that the critical ESR for full view coverage and full coverage are in the same order, as well as the energy consumption, which means we only need to increase the ESR for full coverage by some constant to achieve the full view coverage. From the perspective of critical ESR, we show that 1-dimensional random walk mobility can increase the full view coverage under certain delay tolerance, and thus decreases the sensing energy consumption. Meanwhile, we show that the random rotating scheme should be avoided as it won't improve the performance of full view coverage, considering that it can't decrease the critical ESR by order compared with stationary. And it also can't decrease the energy consumption, yet induce some delay. So for CSNs which aim to achieve full view coverage, they should avoid the random rotating, and other similar movement such as random direction movement.**

## I. Main results

We summarize our main results in this paper as follows.

Under the uniform deployment scheme:

- With i.i.d mobility model, the critical ESR is $R_\star(n) = \sqrt{\frac{\log n + \log(\log n)}{n\theta}}$
- With 1-dimensional random walk mobility model, the critical ESR is $R_\odot = \frac{3\pi(\log n + \log\log n)}{4\theta n}$
- With random steering mobility model, the critical ESR is $R_\diamond = \sqrt{\frac{\log n + \log(\log n)}{n\theta(1-\frac{\phi_y}{4\pi})}}$
- Under the uniform deployment, we demonstrate that 1-dimensional random walk mobility reduces the energy consumption by the order $\Theta(\sqrt{\frac{\log n + \log(\log n)}{n}})$ at the expense of $\Theta(1)$.
- Under the uniform deployment, we demonstrate that random rotating mobility can't decrease the critical ESR by order. And it also can't decrease the energy consumption, yet induce some delay. So for CSNs which aim to achieve full view coverage, they should avoid the random rotating, and other similar movement such as random direction movement.

## II. System Model and Performance Metric

In this section, we mainly talk about the system model regarding sensing, deployment and mobility patterns. Also, we will present several metric to assess the full view coverage performance of the camera sensor network.

### A. Deployment Scheme

In this work, we assume the operational region of the sensor network be an unit square and this square is assumed to be a torus. In this case, we don't need to consider strategy when the sensor reach the edge of the area, and focus on the general cases.

We assume the sensors follow an uniform deployment, which means $n$ sensors are randomly and uniformly deployed in the operational region, independent of each other. This random strategy is favored in the case that the region to be sensed is inimical and hostile, or it will cost much money and can be very difficult to place the senors by human or programmed robots. Under such circumstance, camera sensors might be sprinkled from aircraft, or other methods randomly.

### B. Sensing Strategy

We consider the heterogenous camera sensors similar to [**?**]. A camera sensor S can sense in a sector of radius $r$ and angle $\phi$. The angular bisector of $\phi$ is defined as the *orientation* of S. And the direction a point P faces towards is called its facing direction. The vector $\overrightarrow{PS}$ is called the object's viewed direction, reflecting the viewpoint of senor S. Figure **??** shows these three directions directly.

There are $n$ sensors in the network. We assume that there are $u$ groups $G_1$, $G_2$, ... ,$G_u$ in this heterogenous network considering their difference in radius or angle, and $u$ is a positive number. For each group $G_y(y = 1, 2, ..., u)$ has $n_y = c_y n$ sensors, where $c_y$ is a constant invariant to $n$. Obviously, $c_y$ satisfies $0 < c_y < 1$ and $\sum_{y=1}^{u} c_y = 1$. All sensor in group $G_y$ has identical sensing radius $r_y$ and angle $\phi_y$. We mainly study the asymptotic coverage here, indicating that $n$ is a variable approaching infinity, and $r_y$ and $\phi_y$ are independent variables of $n$, which can also be represented as $r_y(n)$ and $\phi_y(n)$ equivalently.

### C. Mobility Pattern

The sensing process is divided into time slots with unit length and sensors can move according to certain mobility patterns.

- I.I.D Mobility Model: At the beginning of each time slot, each sensor will randomly and uniformly choose a position within the operational region and remains stationary in the rest of the time slot.
- 1-Dimensional Random Walk: Senors in each group can either move horizontally or vertically and for each sensor, they have fixed moving dimension, horizon or vertical direction. At the very beginning of each time slot, each sensor will random

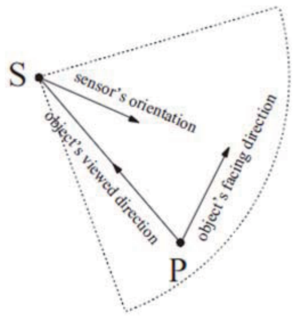


Fig. 1. For sensor S's orientation, and point P's facing direction and viewed direction are shown respectively

choose a direction along its moving dimension, i.e. move right or left, up or down. Then, they will travel in the selected direction for a certain distance $D$, which is a random variable uniformly distributed from 0 to 1. We don't bother about the velocity of the sensor, but they should arrive at the destination within the time slot, and remain stationary until next time slot.

• Random Rotating: The camera sensors can rotate and change their orientations clockwise or counterclockwise. At the very beginning of each time slot, each sensor will random choose a rotating direction, i.e. clockwise or counterclockwise, and then they will move an angle $\Theta$, which is a random variable uniformly distributed between 0 and $2\pi$. Similarly, we don't set requirements on their velocity, but they should reach the destination within the time slot, and remain stationary until the next slot.

•1-Dimensional Random Direction: Senors in each group can either move horizontally or vertically and for each sensor, they have fixed moving dimension, horizon or vertical direction. At the every beginning of each time slot, each sensor will random choose a direction along its moving dimension, then they will chose a rotating direction, horizontally or vertically. After that, they will move a certain distance $D$, which is also a random variable uniformly distributed from 0 to 1, and rotate for a certain angle, a variable evenly distributed from 0 to $2\pi$. Similarly, it will arrive the destination within the time slot.

### D. Performance Metric

• Full View Coverage: We use the definition of full view coverage in [**?**], i.e., if for every point P in the operational area, it's all directions are safe(covered by at least one sensor, and angle between the viewed direction and point's facing direction is less than the effective angle $\theta$). Figure

• $\theta$-Viewed Coverage: We define that orientation k is $\theta$-view covered if this direction is covered by at least one active sensor with effective angle $\theta$.

To access the full view coverage performance of the camera sensor network, we propose the metric, asymptotic coverage.

• Asymptotic Coverage: The ESR of the heterogeneity CSN with i.i.d mobility model is $r_\star = \sqrt{\sum_{y=1}^{u} c_y \frac{\phi_y}{2\pi} r_y^2}$, and the ESR for 1-dimensional random walk mobility model is $r_\circ = \sum_{y=1}^{u} c_y r_y \frac{\phi_y}{2\pi}$, and the ESR for random rotating mobility model is $r_\star = \sqrt{\sum_{y=1}^{u} c_y \frac{\phi_y}{2\pi} r_y^2}$, which is same as the one under i.i.d mobility model.

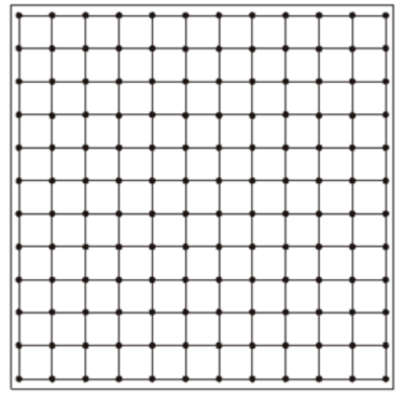

Fig. 2. $\sqrt{m} \times \sqrt{(m)}$ dense grid in an unit square

When we give the definition of ESR in different circumstance, we view $\frac{\phi_y}{2\pi}$ as the weight of each signal sensor's radius. When $\phi = 2\pi$ it is equivalent to a sensor whose sensing range is a circle, and ESR in this case is the same as the ESR defined in [**?**]. By this transformation (i.e. change the meaning of angle $\phi_y$ to the weight each single sensor takt), we covert the sensor to a non-directional one. Then the equivalent sensing area may be denoted as $s_\star(n) = \pi r_\star^2(n)$.

## III. Full View Coverage Under Uniform Deployment Scheme

In this section, we analyze the asymptotic full view coverage under the uniform deployment, with i.i.d, 1-dimensional random walk, and random steering mobility strategies, respectively. Then based on these, we can also get the result for 1-dimensional random direction scheme.

### A. Over View of the Geometric analysis

In ??, the author has proved that we can covert the coverage of the unit square to the coverage of all points of a $\sqrt{m} \times \sqrt{m}$ dense grid $\mathbb{M}$, when $m = n \log n$ based on Theorems in Kumar's work [**?**].

Now, we can also prove that we can covert the full view coverage of a point to the coverage of k orientations, when $k = n \log n$.

**Lemma 1** *Assume $\theta$, $\theta_0$, $k$ are constraints, which satisfies that $\theta_0 = \theta + \frac{2\pi}{k}$. And $\mathbb{K}$ be the set of $k$ orientations, which distributed across the whole circle of a point P uniformly. If these $k$ orientations can all be $\theta$-viewed covered , then the point P can be full view covered by the same network with effective angle $\theta_0$*

*proof*: Let $v$ be an arbitrary direction around point P. Without loss of generality, we may assume it is inside the sector formed by the virtual orientation $a$ and $b$, as shown in Figure **??**. Also without loss of generality, we may assume that it is closest to orientation $a$. By assumption, there exists at least one active sensor that cover orientation $a$, with effective angle $\theta$. Let one of them be located at point $u$, as shown in

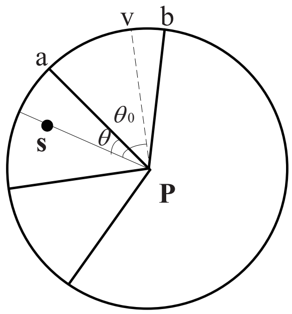


Fig. 3. A set of four nearest orientation include $a$ and $b$ on the direction set

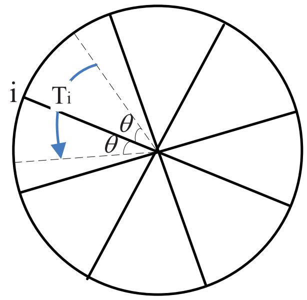


Fig. 4. The proper area $T_i$ for sensor to cover orientation $O_i$

Figure **??**. Then $arg(u, a) < \theta$. The phase shift between $a$ and $v$ is less than $\frac{2\pi}{k}$. Thus we can obtain that

$$arg(u, v) = arg(u, a) + arg(a, v) < \theta + \frac{2\pi}{k} = \theta_0 \quad (1)$$

The same holds for other sensors and directions. And now we know that, $\lim_{k\to\infty} \theta_0 = \theta$, which means, when $k$ is large enough, $\theta_0$ is only slightly larger than $\theta$. Then we can apply the THEOREM 4.1 in [**?**] to build the following theorem, which will be used in subsequent sections .

**Theorem 1.** *For point P, we choose a set of $k$ orientations, $\mathbb{K}$ uniformly distributed along the circle. When $k = n\log n$, we can convert the full view coverage of P with effective angle $\theta$ to the $\theta$-viewed coverage of $k$ different orientations.*

So to identify the full view coverage performance of the operational unit area, we can focus on the M dense gird. And for each point furthermore, we can rely on the $\theta$-viewed coverage of K orientation set.

### *B. Critical ESR Under I.I.D Mobility Model*

We use $H$ to denote the event that the dense grid $\mathbb{M}$ is full view covered, and we drive the critical ESR to guarantee the asymptotic full view coverage of $\mathbb{M}$.

**Definition 1.** *For mobile heterogeneous CSN with i.i.d mobility model, $r_\star$ is the critical sensing radius for $\mathbb{M}$ if*

$$\lim_{n\to\infty} \mathbb{P}(\mathcal{H}) = 1,\ if\ r_\star \geq cr_\star(n) \quad for\ any\ c > 1$$

$$\lim_{n\to\infty} \mathbb{P}(\mathcal{H}) < 1,\ if\ r_\star \leq cr_\star(n) \quad for\ any\ 0 < c < 1$$

**Lemma 2.** *Given $x = x(n)$ and $y = y(n)$, both of which are positive functions of $n$ then $(1-x)^y \sim e^{-xy}$ if $x$ and $x^2 y$ approach 0 as $n \to \infty$*

*proof:* *See Appendix*

**Lemma 3.** *if $r_\star = \sqrt{\frac{\log n + \log\log n + \xi}{\theta n}}$, and $m(n) = n\log n$, $k(n) = n\log n$, for fixed $\gamma < 1$,*

$$mk\prod_{y=1}^{u}(1 - \frac{r_y^2(n)\phi_y\theta}{\pi}) \geq \gamma e^{-\xi} \quad (2)$$

*holds for all sufficient large $n$.*

*proof:* *See Appendix*

*1) Necessary ESR for Full View Coverage of the Dense Grid :* We know that to make sure the $i$ orientation of set $\mathbb{K}$ is $\theta$-viewed, at least one sensor should located in sector $T_i$, whose angular bisector is $i$, with an angle $2\theta$, shown in Figure **??**

We denote the probability of the orientation $O_i$ of the direction set $\mathbb{K}$ of point P is $\theta$-viewed covered by sensor S in group $G_y$ by $\mathbb{P}_{i,P,S}$ Since

$$\begin{aligned}\mathbb{P}_{i,P,S} &= \mathbb{P}(S\ falls\ in\ T_i) \times \mathbb{P}(S\ has\ proper\ orientation)\\ &= \frac{2\theta}{2\pi} \times \pi r_y(n)^2 \times \frac{2\phi_y}{2\pi}\\ &= \frac{r_y(n)^2\phi_y\theta}{\pi}\end{aligned} \quad (3)$$

Let $\widehat{\mathcal{H}}$ denote the event that the dense grid M is not full view covered and we have the following proposition.

**Proposition 1.** *In the mobile heterogeneous CSN with i.i.d. mobility model, if $r_\star = \sqrt{\frac{\log n + \log(\log n) + \xi(n)}{n\theta}}$ and the density of the dense grid M is $m = n\log n$, the density of the orientation set K is $k = n\log n$, then*

$$\liminf_{n\to\infty} \mathbb{P}(\widehat{\mathcal{H}}) \geq e^{-\xi} - e^{-4\xi}$$

*where $\xi = \lim_{n\to\infty} \xi(n)$*

*proof*: To begin with, we study the case where $r_\star = \sqrt{\frac{\log n + \log(\log n) + \xi}{n\theta}}$, for a fix $\xi$ Then referring to Bonferroni inequalities, we have that

$$\begin{aligned}\mathbb{P}(\widehat{\mathcal{H}}) &\geq \sum_{P_i\in\mathbb{M}} \mathbb{P}(\{some\ point\ P_i\ is\ not\ full\ view\ covered\})\\ &\geq \sum_{P_i\in\mathbb{M}} \mathbb{P}(\{P_i\ is\ the\ only\ uncovered\ point\})\\ &\geq \sum_{P_i\in\mathbb{M}}\sum_{O_i\in\mathbb{K}} \mathbb{P}(\{only\ direction\ O_j\ of\ P_i\ is\ uncovered\})\\ &\geq \sum_{P_i\in\mathbb{M}}\sum_{O_j\in\mathbb{K}} \mathbb{P}(\{O_j\ of\ P_i\ is\ uncovered\})\\ &- \sum_{P_i\in\mathbb{M}}\sum_{\substack{O_j\neq O_t\\ O_j,O_t\in\mathbb{K}}} \mathbb{P}(\{O_j\ and\ O_t\ of\ P_i\ are\ uncovered)\end{aligned} \quad (4)$$

Respectively, we can evaluate the two terms on the right hand side of (3). As for the first term, we have

$$\begin{aligned}&\mathbb{P}(\{O_j\ of\ P_i\ is\ uncovered\})\\ &= \prod_{y=1}^{u} \mathbb{P}(\{O_j\ is\ uncovered\ by\ sensors\ in\ G_y\})\\ &= \prod_{y=1}^{u} (1 - \frac{r_y(n)^2\phi_y\theta}{\pi})^{c_y n}\end{aligned} \quad (5)$$

Use Lemma 3, we can bound the first term for any $\gamma < 1$,

$$\sum_{P_i\in\mathbb{M}}\sum_{O_j\in\mathbb{K}} \mathbb{P}(\{O_j\ of\ P_i\ is\ uncovered\}) \geq \gamma e^{-\xi} \quad (6)$$

for all $n > N_\xi$

Now, we assume $s_y = \frac{1}{2} r_y^2 \phi_y$, then $s_\star(n) = \pi r_\star^2(n) = \sum_{y=1} u(n) = \frac{\log n + \log\log n + \xi}{n\theta}$. Hence for all $y = 1, 2, ..., u$, $s_y(n) = \Theta(\frac{\log n + \log\log n + \xi}{n\theta})$, and this indicates that $s_y(n)$ and $s_y^2(n)(c_y n)$ approach 0 as $n \to \infty$. From Lemma 2, we obtain that for arbitrary positive constant $\alpha$

$$(1 - \alpha s_y(n))^{c_y n} \sim e^{-\alpha n(c_y s_y(n))} \tag{7}$$

As when $n \to \infty$, all the sensing areas $s_y$ goes to zero, which implies that the senor who cover $O_j$ can't further cover another direction $O_t$. Hence, the probabilities that direction $O_j$ and $O_t$ is $\theta$-viewed depends on different sets of sensors. So, $\mathbb{P}(O_j is uncovered)$ and $\mathbb{P}(O_t is uncovered)$ are independent. Thus, for $O_j$ and $O_t$ in $\mathbb{K}$, we obtain that

$$\begin{aligned} &\mathbb{P}(\{O_j \ and \ O_t \ are \ uncovered\}) \\ &= \mathbb{P}(\{O_j \ is \ uncovered\})\mathbb{P}(\{O_t \ is \ uncovered\}) \\ &= \prod_{y=1}^{u} (1 - \frac{r_y(n)^2 \phi\theta}{\pi})^{2c_y n} \end{aligned} \tag{8}$$

Then with Lemma 2 and equation (7), we can obtain that

$$\begin{aligned} \sum_{P_i \in \mathbb{M}} \sum_{O_j, O_t \in \mathbb{K}}^{O_j \neq O_t} &\mathbb{P}(\{O_j \ and \ O_t \ of \ P_i \ are \ uncovered) \\ &= m^2 k^2 \prod_{y=1}^{u} (1 - \frac{r_y(n)^2 \phi\theta}{\pi})^{2c_y n} \\ &\sim m^2 k^2 e^{-\frac{2n\theta}{\pi} \sum_{y=1}^{u} c_y r_y(n)^2 \phi_y} \\ &= m^2 k^2 e^{-4n\theta r_\star(n)} \\ &= (n \log n)^4 e^{-4(\log n + \log\log n + \xi)} \\ &= e^{-4\xi} \end{aligned} \tag{9}$$

As for the case that $\xi$ is a function of $n$ with $\xi = \lim_{n\to\infty} \xi(n)$, we know that, $\xi(n) < \xi + \delta$ for any $\delta > 0$, for all $n > N_\delta$. Since $\mathbb{P}(\widehat{\mathcal{H}})$ is monotonously decreasing in $r_\star$ and thus in $\xi$, we have

$$\mathbb{P}(\widehat{\mathcal{H}}) \geq \gamma e^{-(\xi+\delta)} - e^{-4(\xi+\delta)} \tag{10}$$

for all $n > N_{\gamma,\delta}$

From Proposition 1, we know $\mathbb{P}(\widehat{\mathcal{H}})$ is bounded away from zero. Combined with Definition 1, we know that $r_\odot \geq \sqrt{\frac{\log n + \log(\log n)}{n\theta}}$ is necessary to achieve the full view coverage of $\mathcal{M}$.

*2) Sufficient ESR for Full View Coverage of the Dense Grid:* First, we obtain the following proposition.

**Proposition 2.** *In CSN, if $n$ sensors are randomly and uniformly deployed in an unit square, and $r_\star = cr_\star(n)$ where $c > 1$, then*

$$\liminf_{n\to\infty} \mathbb{P}(\widehat{\mathcal{H}}) = 0 \tag{11}$$

*proof:* Let $\mathcal{F}_i$ denote the event that point $P_i$ in $\mathbb{M}$ is not covered, and $\mathcal{F}_{i,j}$ represents the event that orientation $O_j$ of $P_i$ is not $\theta$-viewed covered. Then, we obtain

$$\begin{aligned} \mathbb{P}(\widehat{\mathcal{H}}) &= \mathbb{P}(\bigcup_{i=1}^{m} \mathcal{F}_i) \leq \sum_{i=1}^{m} \mathbb{P}(\mathcal{F}_i) \leq \sum_{i=1}^{m} \sum_{j=1}^{k} \mathbb{P}(\mathcal{F}_{i,j}) \\ &= (n \log n)^2 \prod_{y=1}^{u} (1 - \frac{r_y^2 \phi_y \theta}{\pi})^{c_y n} \\ &\sim (n \log n)^2 e^{-2n\theta(r_\star)^2} \\ &= \frac{1}{(n \log n)^{2c^2-2}} \to 0 \end{aligned} \tag{12}$$

for any $c > 1$.

Then the proof is completed, and from Proposition 2 and Definition 1 we know that $r_\star \geq \sqrt{\frac{\log n + \log(\log n)}{n\theta}}$ is sufficient to achieve the full view coverage of $\mathbb{M}$.

*3) Critical ESR for Full View Coverage of the operational range:* The density of the dense grid $m = n \log n$ and the density of the orientation set $k = n \log n$ are sufficient large to evaluate the full view coverage problem of the whole area. Referring to LEMMA 3.1 in ??, and Lemma 1 and Theorem 1 in our work, use similar approach as THEOREM 4.1, Then we can demonstrate that $r_\star \geq \sqrt{\frac{\log n + \log(\log n)}{n\theta}}$ is sufficient to achieve the full view coverage of the whole range. On the other hand, the necessary condition to full view cover the dense grid $\mathbb{M}$ is surely the necessary condition for the whole operational region.

Hence, we have the following Theorem.

**Theorem 2.** *Under the uniform deployment with i.i.d. mobility model, the critical ESR for mobile heterogenous CSNs to achieve asymptotic full view coverage is* $R_\star(n) = \sqrt{\frac{\log n + \log(\log n)}{n\theta}}$

### *C. Critical ESR Under 1-Dimensional Random Walk Mobility Model*

Under the 1-dimensional random walk mobility model, we study the sensing process and sensor's movement based on slots. We use $\mathcal{H}^\tau$ to denote the event that $\mathbb{M}$ is full view covered in time slot $\tau$, and $\mathbb{P}_\tau(\mathcal{H}^\tau)$ denotes the corresponding probability. Similarly, we define the critical ESR for 1-dimensional random walk model.

**Definition 2.** *For mobile heterogeneous CSN with 1-dimensional random walk mobility model, $r_\odot$ is the critical sensing radius for $\mathbb{M}$ if*

$$\lim_{n\to\infty} \mathbb{P}^\tau(\mathcal{H}^\tau) = 1, \ if \ r_\odot \geq cr_\odot(n) \ \ for \ any \ c > 1$$

$$\lim_{n\to\infty} \mathbb{P}^\tau(\mathcal{H}^\tau) < 1, \ if \ r_\odot \leq cr_\odot(n) \ \ for \ any \ 0 < c < 1$$

*1) Failure Probability of an Orientation in $\mathbb{K}$ :* We use $\mathcal{F}_{i,j}$ to denote the event that orientation $O_j$ of point $P_i$ is not $\theta$-viewed covered, and use $\mathbb{P}(\mathcal{F}_{i,j})$ as the corresponding probability. We denote the probability of the orientation $O_j$ of the direction set $\mathbb{K}$ of point $P_i$ is $\theta$-viewed covered by sensor S in group $G_y$ by $\mathbb{P}_{i,j,S}$.

From Wang ??, we know for 1-dimensional random walk the probability that $S$ falls in the circle around of $P_i$, with radius

$r_y$ is $\mathbb{P}_{i,S} = \frac{4}{3}r_y$. Clearly $\mathbb{P}(S\ falls\ in\ cirle\ around\ P_i) = \mathbb{P}_{i,S}$. Then we obtain

$$\begin{aligned}\mathbb{P}_{i,j,S} &= \mathbb{P}(S\ falls\ in\ T_j) \times \mathbb{P}(S\ has\ proper\ orientation) \\ &= \mathbb{P}(S\ falls\ in\ the\ cirle\ around\ P_i) \times \frac{2\theta}{2\pi} \times \frac{2\phi_y}{2\pi} \\ &= \frac{\phi\theta_y}{\pi^2}\mathbb{P}_{i,S} = \frac{4\theta\phi_y r_y(n)}{3\pi^2}\end{aligned} \tag{13}$$

Then, $\mathbb{P}(\mathcal{F}_{i,j})$ can be easily calculated.

*2) Necessary ESR for Full View Coverage of the Dense Grid:* Here, we use $\widehat{\mathcal{H}^\tau}$ denote the event that the dense grid $\mathbb{M}$ is not fully full view covered in the time slot $\tau$. We have the following technical lemma.

**Lemma 4.** *If $r_\odot = \frac{3\pi(\log n + \log\log n + \xi(n))}{4\theta n}$, and $m(n) = n\log n$, $k(n) = n\log n$, for fixed $\gamma < 1$,*

$$mk\prod_{y=1}^{u}(1 - \frac{4\theta\phi_y r_y(n)}{3\pi^2})^{c_y n} \geq \gamma e^{-\xi} \tag{14}$$

*holds for all sufficient large $n$.*

*proof:* Using the same approaching for Lemma 3.

Now, we present the following proposition regarding the necessary condition.

**Proposition 3.** *In the mobile heterogeneous CSN with 1-dimensional random walk mobility model, if $r_\odot = \frac{3\pi(\log n + \log\log n + \xi(n))}{4\theta n}$ and the density of the dense grid $M$ is $m = n\log n$, the density of the orientation set $K$ is $k = n\log n$, then*

$$\liminf_{n\to\infty}\mathbb{P}_\tau(\widehat{\mathcal{H}^\tau}) \geq e^{-\xi} - e^{-4\xi}$$

*where $\xi = \lim_{n\to\infty}\xi(n)$*

*proof:* The technique used here is similar to that used in the proof of Proposition 1, and we present the main steps here. We first study the case where $r_\star = \sqrt{\frac{\log n + \log(\log n) + \xi}{n\theta}}$, for a fix $\xi$.

$$\begin{aligned}\mathbb{P}_\tau(\widehat{\mathcal{H}^\tau}) \geq &\sum_{P_i\in\mathbb{M}}\sum_{O_j\in\mathbb{K}}\mathbb{P}_\tau(\{O_j\ of\ P_i\ is\ uncovered\}) \\ &- \sum_{P_i\in\mathbb{M}}\sum_{O_j,O_t\in\mathbb{K}}^{O_j\neq O_t}\mathbb{P}_\tau(\{O_j\ and\ O_t\ of\ P_i\ are\ uncovered)\end{aligned} \tag{15}$$

Based on (14), we can bound the first term on the right hand of (15) and have,

$$\mathbb{P}_\tau(\{O_j\ of\ P_i\ is\ uncovered\}) = \prod_{y=1}^{u}(1 - \frac{4\theta\phi_y r_y(n)}{3\pi^2})^{c_y n} \tag{16}$$

Then use Lemma 4, we have

$$\sum_{P_i\in\mathbb{M}}\sum_{O_j\in\mathbb{K}}\mathbb{P}_\tau(\{O_j\ of\ P_i\ is\ uncovered\}) \geq \gamma e^{-\xi} \tag{17}$$

for any $\gamma > 1$ and all $n > N_\xi$.

Then, we can also bound the second term on the right use similar techniques as for (9), and we have

$$\begin{aligned}&\sum_{P_i\in\mathbb{M}}\sum_{O_j,O_t\in\mathbb{K}}^{O_j\neq O_t}\mathbb{P}_\tau(\{O_j\ and\ O_t\ of\ P_i\ are\ uncovered) \\ &= m^2k^2\prod_{y=1}^{u}(1 - \frac{4\theta\phi_y r_y(n)}{3\pi^2})^{c_y n} \\ &\sim m^2k^2 e^{-\frac{8\theta n}{3\pi^2}\sum_{y=1}^{u}c_y\phi_y r_y(n)} \\ &= m^2k^2 e^{-\frac{16\theta n}{3\pi}r_\odot(n)} \\ &= (n\log n)^4 e^{-4(\log n + \log\log n + \xi)} \\ &= e^{-4\xi}\end{aligned} \tag{18}$$

Then we have

$$\mathbb{P}_\tau(\widehat{\mathcal{H}^\tau}) \geq \gamma e^{-\xi} - e^{-4\xi} \tag{19}$$

Taking into account that $\xi$ is a function of $n$, the conclusion still holds.

Then, from Proposition 3, we know that $r_\odot \geq \frac{3\pi(\log n + \log\log n)}{4\theta n}$ is necessary to achieve the full view coverage of $\mathbb{M}$.

*3) Sufficient ESR for Full View Coverage of the Dense Grid:* First, we obtain the following proposition.

**Proposition 4.** *In CSN, if $n$ sensors are randomly and uniformly deployed in an unit square, and $r_\odot = cr_\odot(n)$ where $c > 1$, then*

$$\liminf_{n\to\infty}\mathbb{P}_\tau(\widehat{\mathcal{H}_\tau}) = 0 \tag{20}$$

*proof:* Let $\mathcal{F}_i$ denote the event that point $P_i$ in $\mathbb{M}$ is not covered, and $\mathcal{F}_{i,j}$ represents the event that orientation $O_j$ of $P_i$ is not $\theta$-viewed covered. Then, we obtain

$$\begin{aligned}\mathbb{P}(\widehat{\mathcal{H}}) &= \mathbb{P}(\bigcup_{i=1}^{m}\mathcal{F}_i) \leq \sum_{i=1}^{m}\mathbb{P}(\mathcal{F}_i) \leq \sum_{i=1}^{m}\sum_{j=1}^{k}\mathbb{P}(\mathcal{F}_{i,j}) \\ &= (n\log n)^2\prod_{y=1}^{u}(1 - \frac{4\theta\phi_y r_y(n)}{3\pi^2})^{c_y n} \\ &\sim (n\log n)^2 e^{-\frac{4\theta n}{3\pi}\theta r_\odot} \\ &= \frac{1}{(n\log n)^{2c^2-2}} \to 0\end{aligned} \tag{21}$$

for any $c > 1$.

Then the proof is completed, and from Proposition 3 and Definition 2 we know that $r_\odot \geq \frac{3\pi(\log n + \log\log n)}{4\theta n}$ is sufficient to achieve the full view coverage of $\mathbb{M}$.

*4) Critical ESR for Full View Coverage of the Operational Range:* Similar as the analysis in the i.i.d mobility model, we can each the following theorem.

**Theorem 3.** *Under the uniform deployment with 1-dimensional random walk mobility model, the critical ESR for mobile heterogenous CSNs to achieve asymptotic full view coverage is $R_\odot(n) = \frac{3\pi(\log n + \log\log n)}{4\theta n}$*

*D. Critical ESR Under Random Rotating Mobility Model*

Under the random rotating mobility model, we also study the sensing process and sensor's movement based on slots as the 1-dimensional random walk moble. We use $\mathcal{H}^\tau$ to denote the event that $\mathbb{M}$ is full view covered in time slot $\tau$, and $\mathbb{P}_\tau(\mathcal{H}^\tau)$ denotes the corresponding probability. Similarly, we define the critical ESR for 1-dimensional random walk model.

**Definition 2.** *For mobile heterogeneous CSN with random rotating mobility model, $r_\odot$ is the critical sensing radius for $\mathbb{M}$ if*

$$\lim_{n\to\infty} \mathbb{P}^\tau(\mathcal{H}^\tau) = 1,\ if\ r_\diamondsuit \geq cr_\diamondsuit(n)\ \ for\ any\ c > 1$$

$$\lim_{n\to\infty} \mathbb{P}^\tau(\mathcal{H}^\tau) < 1,\ if\ r_\diamondsuit \leq cr_\diamondsuit(n)\ \ for\ any\ 0 < c < 1$$

*1) Failure Probability of an Orientation in $\mathbb{K}$ :* Similarly, we use $\mathcal{F}_{i,j}$ to denote the event that orientation $O_j$ of point $P_i$ is not $\theta$-viewed covered, and use $\mathbb{P}(\mathcal{F}_{i,j})$ as the corresponding probability. We denote the probability of the orientation $O_j$ of the direction set $\mathbb{K}$ of point $P_i$ is $\theta$-viewed covered by sensor S in group $G_y$ by $\mathbb{P}_{i,j,S}$. Then we obtain

$$\begin{aligned}\mathbb{P}_{i,j,S} &= \mathbb{P}(S\ falls\ in\ T_j) \times \mathbb{P}(S\ has\ proper\ orientation) \\ &= \pi r_y(n)^2 \times \frac{2\theta}{2\pi} \times \mathbb{P}(S\ has\ proper\ orientation)\end{aligned} \tag{22}$$

Then we will first calculate $\mathbb{P}(S\ has\ proper\ orientation)$, which is shorted for $\mathbb{P}(S)$ in the following.

The sensor has proper orientation means the supposed viewed direction can be sensed by this sense, that means, this direction locates in the sensing area of the sensor. Suppose $\overrightarrow{PS}$ is the supposed the direction. We denote, initially, the angle between the sensor's bisector and $\overrightarrow{SP}$ is $\sigma$, which is a variable random uniformly distributed from 0 to $2\pi$ according to the deployment pattern, (in this case, we always calculate the angle the bisector should move anticlockwise to get to $\overrightarrow{SP}$), and we denote the angle the sensor move in a time slot as $\delta$, which is also a random variable distributed uniformly from 0 to $2\pi$.

To make sure the sensor has a proper orientation, clearly its critical condition is that they meet $\overrightarrow{SP}$ on its way during the time slot. Then, we can know that when it move anticlockwise, the critical condition is that $\sigma \leq \delta$, when it move clockwise, the critical condition is that $\sigma + \delta \geq 2\pi$

Apply Probability Theory, we can calculate that in two situations, $\mathbb{P}(S)$ are same, which is

$$\mathbb{P}(S) = \frac{\phi_y}{\pi} - \frac{\phi_y^2}{4\pi^2} \tag{23}$$

$$proof:\ \ See\ \ Appendix$$

Then we have

$$\begin{aligned}\mathbb{P}_{i,j,S} &= \pi r_y(n)^2 \times \frac{2\theta}{2\pi} \times \mathbb{P}(S) \\ &= \frac{\theta\phi_y r_y(n)^2}{\pi}(1 - \frac{\phi_y}{4\pi})\end{aligned} \tag{24}$$

Then, $\mathbb{P}(\mathcal{F}_{i,j})$ can be easily calculated.

*2) Necessary ESR for Full view coverage of the Dense Grid:* Here, we use $\widehat{\mathcal{H}^\tau}$ denote the event that the dense grid $\mathbb{M}$ is not fully full view covered in the time slot $\tau$. We have the following technical lemma.

**Lemma 5.** *If $r_\diamondsuit = \sqrt{\frac{\log n + \log(\log n) + \xi(n)}{n\theta(1-\frac{\phi_y}{4\pi})}}$, and $m(n) = n\log n$, $k(n) = n\log n$, for fixed $\gamma < 1$,*

$$mk\prod_{y=1}^{u}[1 - \frac{\theta\phi_y r_y(n)^2}{\pi}(1 - \frac{\phi_y}{4\pi})]^{c_y n} \geq \gamma e^{-\xi} \tag{25}$$

*holds for all sufficient large $n$.*

*proof:* Using the same approaching for Lemma 3.

Now, we present the following proposition regarding the necessary condition.

**Proposition 5.** *In the mobile heterogeneous CSN with 1-dimensional random walk mobility model, if $r_\diamondsuit = \sqrt{\frac{\log n + \log(\log n) + \xi(n)}{n\theta(1-\frac{\phi_y}{4\pi})}}$ and the density of the dense grid $M$ is $m = n\log n$, the density of the orientation set $K$ is $k = n\log n$, then*

$$\liminf_{n\to\infty} \mathbb{P}_\tau(\widehat{\mathcal{H}^\tau}) \geq e^{-\xi} - e^{-4\xi}$$

*where $\xi = \lim_{n\to\infty}\xi(n)$*

*proof:* The technique used here is similar to that used in the proof of Proposition 1, and we present the main steps here. We first study the case where $r_\star = \sqrt{\frac{\log n + \log(\log n) + \xi}{n\theta(1-\frac{\phi_y}{4\pi})}}$, for a fix $\xi$.

$$\begin{aligned}\mathbb{P}_\tau(\widehat{\mathcal{H}^\tau}) \geq& \sum_{P_i\in\mathbb{M}}\sum_{O_j\in\mathbb{K}} \mathbb{P}_\tau(\{O_j\ of\ P_i\ is\ uncovered\}) \\ &- \sum_{P_i\in\mathbb{M}}\sum_{\substack{O_j\neq O_t \\ O_j,O_t\in\mathbb{K}}} \mathbb{P}_\tau(\{O_j\ and\ O_t\ of\ P_i\ are\ uncovered)\end{aligned} \tag{26}$$

And then use similar approaches as before, we can bound the two terms on the right correspondingly. Then we have

$$\mathbb{P}_\tau(\widehat{\mathcal{H}^\tau}) \geq \gamma e^{-\xi} - e^{-4\xi} \tag{27}$$

Furthermore the result holds for when $\xi$ changes, thus we prove the necessary part.

*3) Sufficient ESR for Full view coverage of the Dense Grid::* Similarly, we obtain the following proposition.

**Proposition 6.** *In CSN, if $n$ sensors are randomly and uniformly deployed in an unit square, and $r_\diamondsuit = cr_\diamondsuit(n)$ where $c > 1$, then*

$$\liminf_{n\to\infty} \mathbb{P}_\tau(\widehat{\mathcal{H}_\tau}) = 0 \tag{28}$$

*proof:* Use the same technique as Proposition 2 and Proposition 4. Then we have $r_\diamondsuit \geq \sqrt{\frac{\log n + \log(\log n)}{n\theta(1-\frac{\phi_y}{4\pi})}}$ is sufficient to achieve the full view coverage of $\mathbb{M}$.

TABLE I
COMPARISON OF CRITICAL ESR FOR FULL VIEW COVERAGE AND FULL COVERAGE

| | $I.I.D\ Mobility\ Model$ | $1 - Dimensional\ Random\ Walk\ Model$ |
|---|---|---|
| $Full\ View\ Coverage$ | $\sqrt{\frac{\log n + \log(\log n)}{n\theta}}$ | $\frac{3\pi(\log n + \log\log n)}{4\theta n}$ |
| $Full\ Coverage$ | $\sqrt{\frac{\log n + \log(\log n)}{n}}$ | $\frac{3(\log n + \log\log n)}{4n}$ |

*4) Critical ESR for Full View Coverage of the Operational Range:* Similar as the analysis in the i.i.d mobility model, we can each the following theorem.

**Theorem 4.** *Under the uniform deployment with random rotating mobility model, the critical ESR for mobile heterogenous CSNs to achieve asymptotic full view coverage is* $R_{\diamondsuit}(n) = \sqrt{\frac{\log n + \log(\log n)}{n\theta(1-\frac{\phi_y}{4\pi})}}$

## IV. IMPACT OF MOBILITY AND HETEROGENOUS ON SENSING ENERGY CONSUMPTION

### A. Impact of Mobility

We consider the impact of mobility and sensors are considered to have critical ESR, that it they can be viewed as non-directional sensor, with radius $r_y = r_{\star}$, $r_y = r_{\odot}$,$r_y = r_{\diamondsuit}$, $(y = 1, 2, ..., u)$ under i.i.d., 1-dimensional random walk, and random rotating correspondingly. We use the area the sensor covers to represent the energy consumption of it.

We have the following results

(a) Under I.I.D. Mobility Model:

$$\overline{E}_{i.i.d} = \Theta(\frac{\log n + \log\log n}{n}) \tag{29}$$

(b) Under 1-Dimensional Random Walk Mobility Model:

$$\overline{E}_{r.w.} = \Theta((\frac{\log n + \log\log n}{n})^2) \tag{30}$$

(c) Under Random Rotating Mobility Model:

$$\overline{E}_{r.r.} = \Theta(\frac{\log n + \log\log n}{n}) \tag{31}$$

The i.i.d mobility model is actually quasi-static, as it make no change on the whole area the sensor covers. So the energy consumption $\overline{E}_{stat}$ equals to that in i.i.d mobility model. Therefore, we have

$$\overline{E}_{r.w.} = \Theta(\frac{\log n + \log\log n}{n}) \times \overline{E}_{stat}$$
$$\overline{E}_{r.r.} = \overline{E}_{stat}$$

Which indicates that 1-dimensional random walk mobility model can decrease the energy consumption in CSNs. And this improvement sacrifices the timeliness of the detection. As we divide the sensing process into time slots. The delay to achieve the full view coverage is upper bounded by $\Theta(1)$. This is a tradeoff between energy consumption and the delay.

However, for random rotating mobility, the energy consumption is the same as when sensors are stationary, yet, it still cause a delay upper bounded by $\Theta(1)$, due to the division of the time slots. In this case, it make no improvement for the network, no matter for energy consumption or timeliness. Thus, this is a bad movement pattern which should be avoided.

### B. Impact of Heterogeneity

The results of wang get the impact of heterogeneity on i.i.d., and 1-dimensional random walk mobility models, which can apply to our work well. And it shows that, heterogeneity won't make any difference for i.i.d and stationary CSNs, and will slightly increase the energy consumption for 1-dimensional random walk. And for random rotating mobility model, it is the same as stationary ones, thus heterogeneity has no impact on it, either.

### C. Comparison the Results With Those to Achieve Full Coverage

Here we compare the mainly results in our work with the ones in ??, which mainly focus on the asymptotic full coverage with non-direction sensors, and it shows in Chart 1.

From this table we can find that, under same mobility model, the critical ESR for full view coverage and full coverage are in the same order. In other words, we only need to multiply the critical ESR with some constant to improve the performance of the network to achieve the full view coverage in CSNs. This give us an important insight about the CSN design.

## V. CONCLUSION

In this paper, we have studied coverage in mobile and heterogeneous camera sensor networks. Specifically, we have investigated asymptotic coverage under uniform deployment model with i.i.d. and 1-dimensional random walk mobility model and with random rotating model, respectively. Mobility is found to decrease sensing energy consumption under the random walk and i.i.d, while increase the energy consumption under random rotating models. On the other hand, we demonstrate that heterogeneity increases energy consumption under 1- dimensional random walk mobility model but imposes no impact under the i.i.d. model. Still we need to avoid rotating. There are several directions for future work. First I would like to investigate the k-full view coverage in camera sensor network, then I want to consider the connectivity and coverage problem at the same time, and also includes the obstacle problem in this work.

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