Singleton Spectrum Mobility Games With Incomplete Information

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Abstract

In cognitive radio networks (CRNs), Secondary Users (SUs) can access Primary Users' (PUs') idle spectrums but the availability of spectrums is dynamic due to PUs' uncertain activities. In this paper, we investigate such spectrum mobility by proposing *Singleton Bayesian Spectrum Mobility Games* based on the *Singleton Congestion Games*, where each SU distributively re-selects one switch-to (and available) channel which can bring it the maximum SINR when the spectrum environment varies, accounting for other SUs' switching strategies at the same time. Unlike previous game-theoretic schemes for handling the spectrum mobility that assume SUs' complete knowledge of the CRN, we present our scheme in two information scenarios. We first demonstrate the proposed game in the complete-information scenario and prove the existence of pure Nash equilibriums. Then the game is extended to the incomplete-information scenario with the existence of Bayesian equilibriums. Besides, the other major contribution of this paper is that we provide a polynomial algorithm for finding the *social optimal equilibrium* which can optimize the (expected) overall performance of the entire CRN in terms of SUs' average SINR among all possible equilibriums. Numerical results show that the gap between the *social optimal equilibrium* and the centralized social optimal result (obtained by centralized algorithms) is very small (less than 2dB) even in the worst case.

I. Introduction

Cognitive Radio (CR) has been a promising paradigm for reliving the shortage of spectrum resources. Secondary Users (SUs) in the CR network are able to sense the states of channels possessed by Primary Users (PUs) and gain opportunities to access their spectrums when the channels are not occupied by PUs. However, when PUs appear on their licensed channels, SUs should cease their usage of PUs' spectrums or the interference caused by SUs should be less than a certain threshold [1]. As a result, the availability of the spectrums depends on PUs' activities, which causes the *spectrum mobility* for SUs. In order to handle the spectrum mobility, an SU should be able to switch channels quickly (channel switching in face of the spectrum mobility is also known as *spectrum handover* or *spectrum handoff*) in order to avoid significant interference from the licensed users and maintain consistency in the spectrum usage [2]. As a result, an efficient channel re-selection scheme is desperately needed in order to enable SUs to switch to the most suitable (and available) channel when the spectrum environment changes.

Centralized schemes [3], [4] for channel selection in face of spectrum mobility are one consideration for this problem since such schemes can be easily exploited to optimize the performance of the entire CR network, yielding the social optimal results. Unfortunately, centralized schemes are not suitable for CR networks in general due to its distributed nature [5]. By comparison, distributed schemes are quite flexible since they require no central control entities, and various market-driven models can be applied to the channel switching problem in a distributed manner such as auction-theoretic approaches [6], [7], pricing schemes [10], [11], contract-theoretic mechanisms [8], [9], etc. Among these possible models, game theory is especially suitable for handling the spectrum mobility since SUs need to distributively select their switch-to channels when spectrum availability varies, while accounting for the possible interference brought by the selection of other SUs at the same time, which is essentially a game among SUs in the CRN.

Spectrum mobility management was first proposed as the *spectrum mobility game* in [2], where *congestion games* [13] were exploited to establish the scheme. It enables SUs to distributively select their spectrum routing plans in PUs' channels in order to minimize the congestion they experience during the routing process, considering the tradeoff between the benefits and costs of the spectrum handover. However, **homogeneous** SUs and channels were assumed in the spectrum mobility games, which implies the complete-information scenario and doesn't fit the practical situations well. Besides, the proposed game only accounts for individual profits, neglecting the overall performance of the whole CRN. Those drawbacks are common in other literatures which apply game theory to handling the spectrum mobility [14], [15].

In order to address the above problems, we establish the channel switching problem as the *Singleton Bayesian Spectrum Mobility Game*. Our scheme has the following features.

• **Different Information Scenarios**: We first establish our model on the complete-information scenario with **heterogeneous** SUs and channels, and further extend our scheme the incomplete-information scenario using the Bayesian game, where each SU's information is private. Therefore, our scheme can better characterize the practical situations.

• **Social Optimal Equilibrium**: In this paper, we not only prove the existence of Nash (Bayesian) Equilibrium in both information scenarios but also provide a polynomial algorithm to calculate the social optimal Nash (Bayesian) equilibrium which can yield the optimal *social welfare* among all the possible equilibriums.

Our paper is organized as followings. We will present the physical model, and establish the Singleton Bayesian spectrum mobility game in section II. Singleton spectrum mobility games in the complete-information scenario will be first demonstrated in section III, and we further extend our model to the incomplete-information scenario in section IV. Finally, simulation results and the conclusion will be given in section V and VI.

II. System Model

A. Physical Model

We consider a cognitive network composed of M SUs (denoted by SU_k, $k \in \mathcal{M} = \{1, 2, \cdots, M\}$) and N heterogeneous and orthogonal channels with the same bandwidth that is possessed by PUs (denoted by C_i, $i \in \mathcal{N} = \{1, 2, \cdots, N\}$). Each SU has a pair of sender and receiver. When an SU exploits one (or singleton) channel (and only one at the same time in our model), it suffers the channel noise and the interference from other SUs in the same channel. For simplicity, only Additive White Gaussian Noise (AWGN) is assumed to exist in each channel, shown by σ_i^2. Besides, the interference caused by SU_k in the channels is denoted by I_k, which is private information. For the tractability of analysis, we assume that the signal *received* by each SU's receiver is of the same level P. Thus the SINR received by SU_k through channel C_i is shown by:

$$\eta_k|_{A(k)=i} = \frac{P}{\sigma_i^2 + \sum_{n \in \mathcal{M} \backslash \{k\}: A(n)=i} I_n}, \tag{1}$$

where $A(k) = i$ means that SU_k chooses channel C_i as its media and $\mathcal{M} \backslash \{k\}$ indicates the set $\mathcal{M}$ excluding the set $\{k\}$. Besides, we denote the sum of SUs' interference in C_i by W_i, and W_i^{-k} is W_i excluding SU_k's interference.

However, SUs can transmit on those channels only when they are not occupied by PUs, otherwise SUs must cease their usage of the current channels and handover to other idle ones, which causes the *spectrum mobility* for SUs. We assume that the handover cost and handover time are negligible in our model and there're no central control entities in the CR network. In our model, considering PUs' unpredictable activities in many cases and the dynamics of the number of SUs, we hold that those SUs need to sense the states of the channels (busy or idle) and determine the selection of these channels periodically and distributively in order to maximize their SINR as much as possible while accounting for the possible interference from SUs who make the same selection, which is based on the *Singleton Bayesian Spectrum Mobility Game* shown in the section II-C.

B. Congestion Games

Since our *Singleton Bayesian Spectrum Mobility Game* is inspired by the well-known *Congestion Games* (CG), we first introduce the CG briefly in this subsection. In the CG, some **homogeneous** and selfish players need to choose a subset of all common resources in order to attain a certain goal and minimize their own costs for exploiting the chosen facilities. The cost of each resource only depends on the number of players choosing the same resource with a non-decreasing cost function normally. If each player only chooses one resource in the CG, the game is also referred as the *Singleton Congestion Game* (SCG) [18]. Provided that each player exerts different congestion on the traffic or has different cost functions in the CG, the game becomes the *Weighted Congestion Game* (WCG) or the *Player-specific Congestion Game* (PCG) [20]. Previous literatures have shown that congestion games have some nice properties such as the existence of pure Nash Equilibrium (NE) (except for some WCGs and PCGs), etc. As a result, congestion games have been a widely-adopted tool in communications like selfish routing in the network [17], spectrum sharing [16] and so on.

C. Singleton Bayesian Spectrum Mobility Game Model

Our *Singleton Bayesian Spectrum Mobility Game* is denoted by a tuple $\mathcal{G} = \{M, N, \mathbf{T}, \mathbf{A}, \mathbf{p}, \eta\}$, which is explained as the following:

• $\mathbf{T}$ is the SUs' *Type Space* given by $\mathbf{T} = \mathbf{T_1} \times \mathbf{T_2} \times \cdots \times \mathbf{T_M}$ where $\mathbf{T_k}$ is the type space for SU_k. A *Type Profile* T is an element in $\mathbf{T}$, shown by $T = (t_1, t_2, \cdots, t_M)$ where t_k $(\forall k \in \mathcal{M})$ is type of SU_k. Besides, when SU_k is of type t_k, the interference caused by it is $I_k(t_k)$. In our incomplete-information model, each SU is only aware of its own type and the distribution of the types among all SUs without knowing exactly others' interference information.

• $\mathbf{A}$ is the *Strategy Space* of the game $\mathcal{G}$ given by $\mathbf{A} = \mathbf{A_1} \times \mathbf{A_2} \times \cdots \times \mathbf{A_M}$ where $\mathbf{A_k}$ is the strategy space for SU_k. Its element $A = (A_1, A_2, \cdots, A_M)$ is called a *Strategy Profile* of the game $\mathcal{G}$, and $A_k(t_k)$ $(\forall k \in \mathcal{M})$ denotes the strategy of SU_k of type t_k. Since each SU only needs to select one channel, the game $\mathcal{G}$ is singleton.

• $\mathbf{p}$ is the *Type Distribution* over type space $\mathbf{T}$, given by

$$\mathbf{p} = (p(t_1, t_2, \cdots, t_M))_{T \in \mathbf{T}}.$$

We assume that SUs' type distribution $\mathbf{p}$ is independent, i.e.,

$$p(\hat{t}_1, \hat{t}_2, \cdots, \hat{t}_M) = \prod_{k \in \mathcal{M}} p_k(\hat{t}_k),$$

where $p_k(\hat{t}_k)$ is probability that SU_k is of type $\hat{t}_k$, shown by:

$$p_k(\hat{t}_k) = \sum_{T \in \mathbf{T}: t_k = \hat{t}_k} p(t_1, t_2, \cdots, t_M).$$

Note that the summation notation in the above equation need to be modified into integral symbol when the type distribution is continuous.

• M, N and η coincides with the explanation in II-A. Note that the SINR η is actually the *payoff function* of the game $\mathcal{G}$.

In our *Singleton Bayesian Spectrum Mobility Game*, each SU distributively select one channel from the available (idle) channel set based on the prediction or *belief* of interference from other SUs in order to optimize their received SINR (payoff). With the increasing number of SUs who choose the same channel, the individual SINR (payoff) decreases. From the above description of the proposed game, we can easily observe the correspondence with SCG except that the cost minimization in SCG should be the payoff maximization in our game and that SUs' information is private in our game instead of the complete information in SCG. This correspondence offers us intuition that some properties in SCG could also apply in our proposed game, which is demonstrated in the following two sections.

III. Singleton Spectrum Mobility Games with Complete Information

Before we investigate the singleton spectrum mobility games with incomplete information, it's necessary and significant to study the complete-information scenario as the basis of the incomplete-information situation. In the complete-information scenario, each SU's interference level is common knowledge among all SUs and not related to its type (denoted by I_k $(\forall k \in \mathcal{M})$).

A. Existence of Nash Equilibriums

We first demonstrate the definition of the *Nash Equilibrium* in the singleton spectrum mobility games with complete information, and we only consider *Pure Nash Equilibriums* throughout this paper.

Definition 1 (Nash Equilibrium): A strategy profile $A^* = (A_1^*, A_2^*, \cdots, A_M^*)$ is a Nash Equilibrium if for any SU_k $(k = 1, 2, \cdots, M)$ and its any strategy $A_k \in \mathbf{A_k}$,

$$\eta(A_k^*; A_{-k}^*) \geq \eta(A_k; A_{-k}^*),$$

where A^*_{-k} is the strategy profile A^* except A^*_k.

Intuitively, a Nash Equilibrium is stable state which no SUs have the incentive to violate unilaterally. Based on the above definition and the property of SCG (presented in [17]) that a Nash Equilibrium exists in SCG, we can similarly conclude the following theorem.

Theorem 1: There's at least one Nash Equilibrium in the singleton spectrum mobility games with complete information.

Proof: We prove this theorem by offering an algorithm for finding the Nash Equilibrium and demonstrating its correctness, which is shown in *Algorithm 1*.

Algorithm 1 Find the Nash Equilibrium in the complete-information scenario

```
1: W_i = 0, θ_{k,i} = 0 ∀i ∈ 𝒩, k ∈ ℳ;
2: Sort I_1, I_2, ···, I_M in the descending order;
3: for k = 1 : M do
4:     l = arg max_{i∈𝒩} P/(σ_i^2 + W_i);
5:     θ_{k,l} = 1;
6:     W_l = W_l + I_k;
7: end for
8: END.
```

In *Algorithm 1*, $\theta_{k,i}$ is an indication of whether SU_k chooses C_i (1 for yes and 0 otherwise). Note that rearrangement of SUs' indices may be required to correspond with the order of the interference caused by them. We then prove that *Algorithm 1* can obtain a Nash Equilibrium in the complete-information scenario using the *Mathematical Induction*.

When $k = 1$, choosing channel $l = \arg\max_{i\in\mathcal{N}} \frac{P}{\sigma_i^2+W_i}$ is obvious the best strategy for SU_1, and there're no other SUs who have pre-occupied any channels when SU_1 chooses its switch-to channels. Therefore, the switching result obtained after $k = 1$ forms a Nash Equilibrium.

Suppose that the switching result obtained after $k = n - 1$ $(n \geq 2)$ is an NE, and we consider the situation when $k = n$. Apparently, choosing $l = \arg\max_{i\in\mathcal{N}} \frac{P}{\sigma_i^2+W_i}$ will be no doubt the best strategy for SU_n. It must be pointed that the selection of SU_n will only influence the SUs on the same channel (i.e., channel C_l), and SUs on other channels have no incentives to change their strategies since any changes of those SUs from the the NE obtained after $k = n-1$ to other channels will result in the loss of payoffs, which is based on the definition of NE and the fact that the congestion level in channel l is even higher than that after $k = n - 1$. Then we only need to prove that the SUs who have chosen channel l before SU_n's selection have no incentives to modify their switch-to channels, which is shown below.

From $I_1 \geq I_2 \geq I_3 \geq \cdots \geq I_M$, we can define:

$$I_{l_1} \geq I_{l_2} \geq \cdots \geq I_{l_{M^l}},$$

where M^l is the number of SUs who choose to switch to channel l after $k = n$, and I_{l_j} $(j = 1, 2, \cdots, M^l)$ is the interference brought by the j-th SU who selects channel l. Specially, $l_{M^l} = n$, and thus

$$I_n - I_{l_j} \leq 0$$

. Further from $l = \arg\max_{i \in \mathcal{N}} \frac{P}{\sigma_i^2 + W_i}$, we have:

$$\frac{P}{\sigma_i^2 + W_i} \leq \frac{P}{\sigma_l^2 + W_l} \leq \frac{P}{\sigma_l^2 + W_l + I_n - I_{l_j}} \ \forall i \neq l,$$

which means that channel l is still the best strategy for any SUs who have already chosen to switch to channel l even after $k = n$.

Based on the above discussions, we have proved that SUs in the CRN after $k = n$ have no incentives to change their strategies obtained from the algorithm, and thus the switching result obtained after $k = n$ is a NE. So far, *Theorem 1* has been proved through the *Mathematical Induction*. ■

B. Computation of the Social Optimal Nash Equilibrium

Theorem 1 implies that there might be more than one Nash Equilibriums in the singleton spectrum mobility games with complete information, and thus the comparison among these Equilibriums should be made based on a certain criterion, one of which is the *Social Welfare*.

Definition 2 (Social Welfare): The social welfare is the sum of all SUs' payoffs, i.e.,

$$SC = \sum_{k \in \mathcal{M}} \eta_k.$$

Social welfare in our model indicates the average SINR of all SUs in the CRN, which reflects the overall performance of the whole CRN. Based on the proof of *Theorem 1*, we propose the *Algorithm 2* using *Dynamic Programming* [21], [12] to compute the Nash Equilibrium which could maximize the social welfare in the complete-information scenario.

We divide the entire programming into $M \times N$ stages, and each stage is denoted by $stage_{k,i}$ $(\forall i \in \mathcal{N},\ k \in \mathcal{M})$, which means that the algorithm is determining whether SU_k should select C_i. Backward induction is applied in our algorithm and thus the sequence of programming would be

$$stage_{M,N} \rightarrow stage_{M,N-1}, \cdots, stage_{M-1,N}, \cdots, stage_{1,1}.$$

Note that after every N stages, an SU finishes its selection and we call such SUs the "programmed SUs". The *state variable* $s_{k,i}$ means the sum of interference from SUs that choose C_i **before** $stage_{k,i}$. Denote $f_{k,i}(s_{k,i})$ the optimal social welfare gained from $stage_{k,i}$ to $stage_{M,N}$, which is given by:

$$\begin{cases} f_{k,i}(s_{k,i}) = \max_{\theta_{k,i}}\{f_{k,i+1}(0) + v_{k,i}(\theta_{k,i}, s_{k,i})\}\ if\ i \neq N \\ f_{k,N}(s_{k,N}) = \max_{\theta_{k,i}}\{f_{k+1,1}(0) + v_{k,N}(\theta_{k,N}, s_{k,N})\} \end{cases} \tag{2}$$

where $v_{k,i}(\theta_{k,i}, s_{k,i})$ indicates the improvement in the social welfare at $stage_{k,i}$ if we choose $\theta_{k,i}$ as the strategy, which is given by (3).

$$v_{k,i}(\theta_{k,i}, s_{k,i}) = [\sum_{\substack{m=k \\ \widehat{\theta}_{m,i}^{(k,i)}(s_{k,i})=1}}^{M} \frac{P}{\sigma_i + s_{k,i} + I_k - I_m} - \sum_{\substack{m=k+1 \\ \widehat{\theta}_{m,i}^{(k,i)}(s_{k,i})=1}}^{M} \frac{P}{\sigma_i + s_{k,i} - I_m}]\theta_{k,i}. \tag{3}$$

In (3), $\widehat{\theta}_{m,i}^{(k,i)}(s_{k,i})$ is an element in the "temporarily optimal strategy profiles **after** $stage_{k,i}$ under state $s_{k,i}$" (denoted by $\widehat{\Theta}^{(k,i)}(s_{k,i})$), which temporarily records the best strategy when the state of $stage_{k,i}$ is $s_{i,k}$, excluding the stages **before** $stage_{k,i}$ and can be obtained using the *forward induction* in DP.

Besides, the state transfer equation is:

$$s_{k,i} = s_{k-1,i} + \theta_{k-1,i} I_{k-1}. \tag{4}$$

According to (2), (3) and (4), we design *Algorithm 2*.

Algorithm 2 Find the Social Optimal Nash Equilibrium in the complete-information scenario

1: **Initializing Stage**
2: Sort $I_1, I_2, \cdots, I_M$ in the ascending order;
3: $I_{MAX} = \sum_{k \in \mathcal{M}} I_k$; $\theta^*_{k,i} = 0 \ \forall k \in \mathcal{M}, i \in \mathcal{N}$;
4: **for** $s_{M+1,1} = 0 : \delta : I_{MAX}$ **do**
5: $\quad f_{M+1,1}(s_{M+1,1}) = 0;\ Path_{M+1,1}(s_{M+1,1}) = 0;$
6: **end for**
7: **Dynamic Programming Stage**
8: **for** each $k = M : 1,\ \ i = N : 1,\ s_{k,i} = 0 : \Delta : I_{MAX}$ **do**
9: $\quad$ **if** Constraint 1 and Constraint 2 are satisfied **then**
10: $\quad\quad$ Compute $f_{k,i}(s_{k,i})$ according to (2);
11: $\quad\quad$ Compute $Path_{k,i}(s_{k,i}) = \arg\max_{\theta_{k,i}} f_{k,i}(s_{k,i})$;
12: $\quad$ **else**
13: $\quad\quad$ $f_{k,i}(s_{k,i}) = f_{k,i+1}(0)$ (or $f_{k,N}(s_{k,N}) = f_{k+1,1}(0)$);
14: $\quad\quad$ $Path_{k,i}(s_{k,i}) = 0$;
15: $\quad$ **end if**
16: $\quad$ **if** $i = 1$ and $\sum_{n=1}^{N} \widehat{\theta}^{(k,i)}_{k,n}(s_{k,i}) = 0$ **then**
17: $\quad\quad$ $Path_{k,l}(s_{k,i}) = 1$,
$\quad\quad$ $(l = \arg\max_{j \in \mathcal{N}} \frac{P}{\sigma_j^2 + W_j^{(k,i)}(s_{k,i})})$;
18: $\quad\quad$ Update the value of $f_{k,i}(s_{k,i})$ accordingly;
19: $\quad$ **end if**
20: **end for**
21: **Output Stage**
22: **for** $i = 1 : N$ **do**
23: $\quad$ $s_i = 0$;
24: $\quad$ **for** $k = 1 : M$ **do**
25: $\quad\quad$ $\theta^*_{k,i} = Path_{k,i}(s_i);\ s_i = s_i + Path_{k,i}(s_i) I_k$;
26: $\quad$ **end for**
27: **end for**
28: END.

In *Algorithm 2*, Δ is the step length of the loop from 0 to I_{MAX}, $Path_{k,i}(s_{k,i})$ records the best strategy at $stage_{k,i}$ under $s_{k,i}$, and $W_j^{(k,i)}(s_{k,i}) = \sum_{m \in \mathcal{M}} \widehat{\theta}^{(k,i)}_{m,j}(s_{k,i}) I_m$ denoting the sum of interference from "programmed SUs" that choose C_j according to the temporarily optimal strategy profiles $\widehat{\mathbf{\Theta}}^{(k,i)}(s_{k,i})$. The two constraints in step 9 are used for ensuring the singleton property and that the solution is a Nash Equilibrium, which is shown below.

Constraint 1: An SU can only select one channel, (or the SU hasn't chosen any channels according the temporarily optimal strategy profiles), i.e.,

$$\sum_{n=i+1}^{N} \widehat{\theta}_{k,n}^{(k,i)}(s_{k,i}) = 0.$$

Constraint 2: At $stage_{k,i}$, the selection of C_i from all channels must bring the maximum payoff to SU_k, i.e.,

$$i = \arg\max_{j \in \mathcal{N}} \frac{P}{\sigma_j^2 + W_j^{(k,i)}(s_{k,i})}.$$

Besides, step 16-19 are used for guaranteeing that every SU can access one channel, which is also an important consideration for obtaining the Nash Equilibrium. The correctness of *Algorithm 2* is based on the following theorem.

Theorem 2: *Algorithm 2* can compute the social optimal Nash Equilibrium in the complete-information scenario.

Proof: The optimality is guaranteed by the optimal properties of DP algorithms, which we would not cover in this report. The conclusion that the result obtained by *Algorithm 2* is a Nash Equilibrium is based on *Theorem 1*, and we will show the correspondence of the dynamic programming procedures with *Algorithm 1*.

Observing the programming sequence of *Algorithm 2*, we can find that it firstly programmes SU_M with all channels (this covers N stages), then SU_{M-1} with all channels, $\cdots$, SU_1 with all channels. Note that SUs are sorted in a ascending order according to the interference they cause in the channels, so *Algorithm 2* actually programmes from the SU with the greatest interference to the SU with the least interference. which corresponds the sequence of *Algorithm 1*. Besides, in each stage of *Algorithm 2*, constraint 2 must be satisfied as a premise, i.e., at $stage_{k,i}$, the selection of C_i from all channels must bring the maximum payoff to SU_k, which corresponds with the greedy selecting strategies in *Algorithm 1* (note that *Algorithm 2* only considers this constraint as a premise but not the only selecting consideration as in *Algorithm 1* since DP also considers the social optimality). Besides, *Algorithm 2* also guarantees that every SU can (and can only) choose one channel (step 16-19). The above discussion has shown that the programming procedures in DP entirely cover the process in *Algorithm 1*, which ensures that the results obtained through *Algorithm 2* is a Nash Equilibrium together with the proof of *Theorem 1*. So far, *Theorem 2* has been proved. ■

Then we focus on the time complexity of *Algorithm 2*. The computational expense in one stage under a certain state is $O(MN)$ since we need to handle the two constraints, and the loop from 0 to I_{MAX} contains $O(M)$ computations. Hence the overall complexity of *Algorithm 2* is $O(M^3N^2)$. In addition, the space complexity is $O(M^2N)$.

IV. Singleton Bayesian Spectrum Mobility Game

Now we further consider the incomplete-information scenario where an SU isn't aware of others' interference information but know their type distribution. We first define the *Bayesian Nash Equilibrium* as:

Definition 3 (Bayesian Nash Equilibrium): A strategy profile $A^* = (A_1^*, A_2^*, \cdots, A_M^*)$ is a Bayesian Nash Equilibrium if for any SU_k ($k = 1, 2, \cdots, M$) and its any type $t \in \mathbf{T_k}$, $A_k^*(t)$ satisfies:

$$A_k^*(t) = \arg \max_{A_k \in \mathbf{A_k}} \mathbb{E}\{\eta(A_k; A_{-k}^*)|t_k = t\},$$

where $\mathbb{E}\{\cdot\}$ is the notation for the mathematical expectation.

In the incomplete-information scenario, the definition of social welfare is extended to *expected social welfare*, given by

$$\mathbb{E}\{SC\} = \sum_{k \in \mathcal{M}} \frac{P}{\sigma_{A_k}^2 + \mathbb{E}\{\sum_{n \in \mathcal{M} \setminus \{k\}: A_n = A_k} I_n\}}.$$

We then give a simple algorithm to compute the social optimal Bayesian Nash Equilibrium in order to maximize the expected social welfare. This algorithm is the derivation of the study of SCG [19]. Still, the summation notation in step 2 of *Algorithm 3* should be changed to integral symbol when the type distribution is continuous. The correctness of *Algorithm 3* can be proved in *Theorem 3*.

Algorithm 3 Find the Social Optimal Bayesian Nash Equilibrium in the incomplete-information scenario

1: **for** each $k = 1 : M$ **do**
2: Compute $\mathbb{E}\{I_k(t_k)\} = \sum_{T \in \mathbf{T}} p(t_1, t_2, \cdots, t_M) I_k(t_k)$;
3: **end for**
4: Compute the social optimal NE A^* using *Algorithm 2* by replacing I_k with $\mathbb{E}\{I_k(t_k)\}$;
5: Set A^* to be the social optimal Bayesian Nash Equilibrium in the incomplete-information scenario;
6: END.

Theorem 3: *Algorithm 3* can compute the social optimal Bayesian Nash Equilibrium for the singleton spectrum mobility games with incomplete information.

Proof: The optimality of *Algorithm 3* can be directly obtained from the the optimality of *Algorithm 2*, and we only prove that A^* derived in *Algorithm 3* is a Bayesian Nash Equilibrium for the proposed game.

The expected payoff for SU_k of type t is:

$$\mathbb{E}\{\eta_k^t(A, \mathbf{p})\} = \frac{P}{\sigma_{A_k(t)}^2 + \mathbb{E}\{W_{A_k(t)}^{-k}|t_k = t\}}.$$

We consider the contradiction and assume that A^* is not a Bayesian Nash Equilibrium for the proposed game. According to the definition of the Bayesian Nash Equilibrium, there exists an SU (e.g., SU_k) of type t who could improve its payoff when it chooses C_l ($l \neq A_k^*$), i.e.,

$$\eta_k^t(A', \mathbf{p}) > \eta_k^t(A^*, \mathbf{p}),$$

where A' is similar to A^* except that A_k^* is replaced by l. This means that

$$\frac{P}{\sigma_l^2 + \mathbb{E}\{W_l^{-k}|t_k = t\}} > \frac{P}{\sigma_{A_k^*}^2 + \mathbb{E}\{W_{A_k^*}^{-k}|t_k = t\}}. \tag{5}$$

Step 4 in *Algorithm 3* corresponds with a new complete-information game, where

$$\eta_k(A') = \frac{P}{\sigma_l^2 + \sum_{n \in \mathcal{M} \setminus \{k\}: A'_n = l} I_n}. \tag{6}$$

In the above equation,

$$\sum_{n\in\mathcal{M}\backslash\{k\}:A_n'=l} I_n = \sum_{n\in\mathcal{M}\backslash\{k\}:A_n'=l} \mathbb{E}\{I_n(t_n)\} = \sum_{n\in\mathcal{M}\backslash\{k\}:A_n'=l} \mathbb{E}\{I_n(t_n)|t_k=t\}.$$

This is due to the assumption that SUs' type distribution is independent of others', and we further derive

$$\begin{aligned}\sum_{n\in\mathcal{M}\backslash\{k\}:A_n'=l} I_n &= \sum_{n\in\mathcal{M}\backslash\{k\}:A_n'=l} \sum_{T\in\mathbf{T}:t_k=t} p(T|t_k=t) I_n(t_n)\\ &= \sum_{T\in\mathbf{T}:t_k=t} p(T|t_k=t) \sum_{n\in\mathcal{M}\backslash\{k\}:A_n'=l} I_n(t_n)\\ &= \mathbb{E}\{W_l^{-k}|t_k=t\}.\end{aligned}$$

Taking the above equation to (5) and observing (6), we derive:

$$\eta_k(A^{'}) > \eta_k(A^*),$$

which means that A^* is not a Nash Equilibrium for the corresponding complete-information game, contradicting to to *Theorem 2*. Hence *Theorem 3* has been proved. ■

V. Simulation Results

In our simulation, we set the received power $P = 10^{-3}$W. The AWGN variance σ_i^2 and the interference caused by each SU are uniformly distributed within $[10^{-8}, 10^{-7}]$W and $[0.5\times10^{-7}, 0.5\times10^{-6}]$W respectively. Each data point in the following simulations is the average of 100 experiments.

We first simulate *Algorithm 2* in the complete-information scenario. Figure 1 illustrates how the average SINR (which refelcts the social welfare as mentioned above) of all SUs in the CRN varies with M and N. It can be obviously observed that the average SINR decreases with M, and we can segment each line in Figure 1 into three parts according the decreasing rate. At the beginning, the N-to-M ratio (or resource-to-demand ratio) is relatively small and the average SINR drops slowly since the spectrum resources are sufficient and the participation of more SUs won't significantly degrade the network performance. Besides, the larger N is, the slower the line drops, which further confirms the above explanation. When M reaches a certain value and the N-to-M ratio further drops, the decreasing rate is accelerated, which means that the CRN with such N-to-M ratios is highly sensitive to the number of SUs and a new SU's participation will significantly degrade the average SINR due to the restricted resources. However, when M is extremely large and the N-to-M ratio is very low, the decreasing trend slows down and the sensitivity of the CRN is not very high, which indicates that the CRN reaches a "saturated state". It should be pointed out that although saturated state implies the CRN is able to accommodate more SUs without degrading the system performance much, the SINR is very low in such a state, which might not fulfil SUs' requirements of SINR.

In Figure 2, we compare the results obtained through *Algorithm 1* and *Algorithm 2*, and further compare the results of our decentralized scheme with the social optimal results which is obtained through **centralized algorithms** (we refer it as *Centralized Social Optimality*). The observation of Figure 2 reveals that our *Social Optimal Nash Equilibrium* yields less social welfare than centralized social optimality generally, which can be seen as the *Cost of Anarchy* [22] since the centralized scheme can neglect SUs' selfish nature. To better compare the gap between the social optimal equilibrium and the centralized social

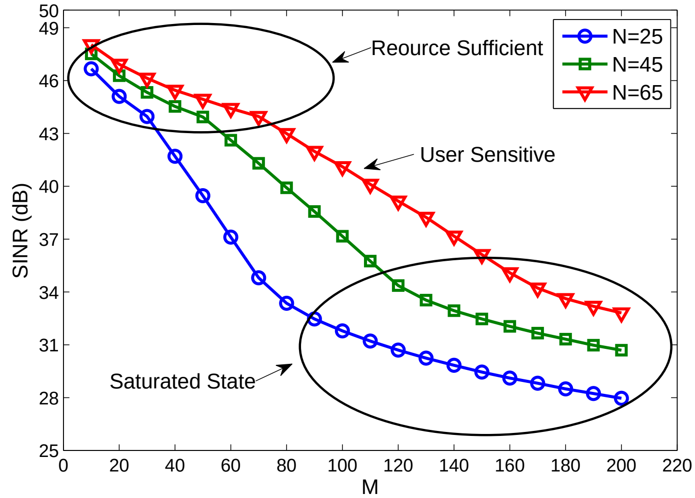

Fig. 1. Average SINR (which reflects social welfare) varying with M and N.

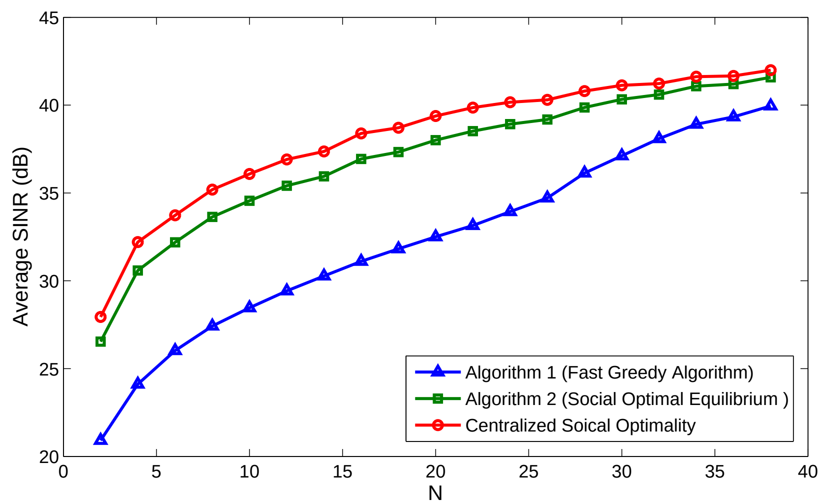

Fig. 2. Comparison of the average SINR between *Algorithm 1*, *Algorithm 2* and *Centralized Social Optimality* ($M = 40$).

optimality, we illustrate the gap in quantities in Figure 3. Obviously, the gap gradually shrinks with the increase of available channels ($N \geq 2$). When N is very large (i.e., the N-to-M ratio is very high), the gap almost disappears. Even in the worst case ($N = 2$), the gap is less than 2dB. Many other simulations with different M are also executed and the gap is still not large (less than 2dB) even if we set $M = 500$ (this corresponds extremely low N-to-M ratio, which is rare in the practical situations). As a result, our distributed scheme shows no obviously inferior performance than centralized scheme and the social optimal Nash equilibrium obtained through our scheme can approximate the centralized social optimal result well in the practical situations. Besides, note that the gap between *Algorithm 1* (the fast greedy algorithm for a equilibrium) and *Algorithm 2* (dynamic programming for the social optimal equilibrium) is also small when N is relatively large, which means that *Algorithm 1* might also work in some situations as a fast approximate algorithm.

Figure 4, we simulate *Algorithm 3* and compare the average SINR of SUs in the CRN in different

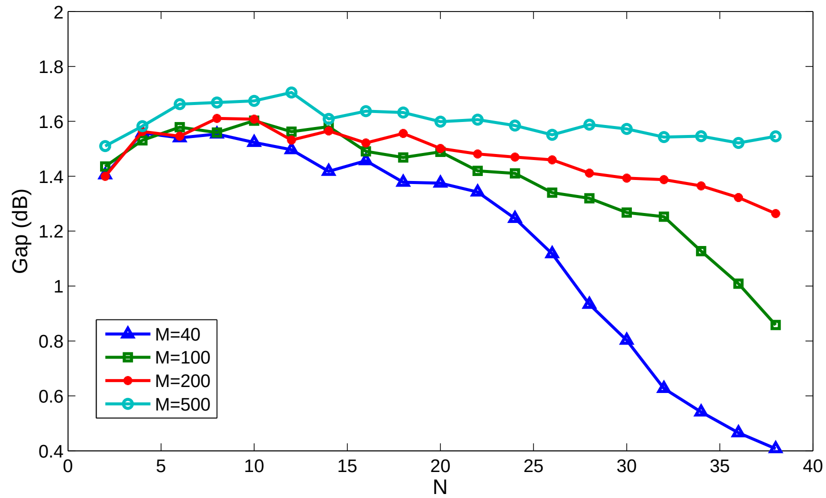

Fig. 3. The gap between the *Social Optimal Equilibrium* and the *Centralized Social Optimality*.

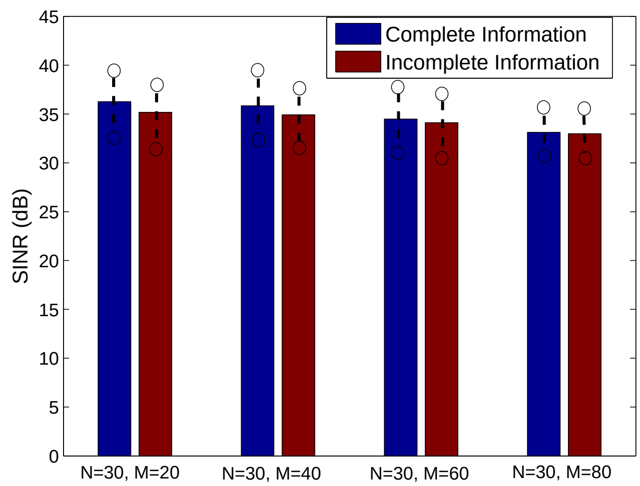

Fig. 4. Comparison of the average SINR between the complete and incomplete-information scenario.

information scenarios, and the dotted lines shows the range of the average SINR within 100 experiments. Apparently, the CRN can gain better social welfare in the complete-information scenario, which indicates the advantage of full information. However, when the number of SUs increases, the advantage is obscure since SUs' *real* type distribution is closer to the probability distribution.

VI. Conclusion

In this paper, we investigate the channel selection problem for heterogeneous SUs when the availability of PUs' channels varies and SUs' information is private. We formulate it as the *Singleton Bayesian Spectrum Mobility Game* where each SU distributively chooses one channel while accounting for others' selections. We first prove the existence of a pure Nash Equilibrium in the complete-information scenario and design an algorithm to derive the social optimal Nash Equilibrium in order to maximize the average SINR in the whole CRN. The extension to the incomplete-scenario is further given, and we also provide an algorithm for computing the social optimal Bayesian Nash Equilibrium. Simulation results show that the gap between the social optimal equilibrium obtained in our scheme and the social optimal results obtained by centralized algorithms is not large (less than 2dB) even in the worst case and that our scheme with incomplete information can obtain similar results as that in the complete-information scenario when the number of SUs is large enough.

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