Heterogeneous Multicast Networks with Wireless Helping Networks

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Abstract. Previously, it has been shown that wired infrastructures such as optical networks can improve the capacity of ad hoc wireless networks significantly. However, sometimes these wired infrastructures are too expensive or even infeasible. In this paper, we use wireless helping networks to enhance the throughput performance of ad hoc networks. We focus on heterogeneous multicast networks with wireless helping networks. The heterogeneity refers to the inhomogeneity of the distribution of the nodes. The helping networks are neither the sources nor the destinations of data flow. They only serve as relays of the data. The wireless helping networks can be regularly placed or randomly uniformly distributed or mobile. We derive achievable throughput for all these three cases. We also make a comparison between them and pure ad hoc networks without helping networks to see the contribution of wireless helping networks.

1 Introduction

There have been great interests in the scaling laws of wireless networks since the seminal work [1]. In that paper, Gupta and Kumar show that a throughput of $\Theta(1/n\sqrt{\log n})$ is achievable. In [2], Franceschetti et.al show us a throughput of $\Theta(1/\sqrt{n})$ is achievable via percolation theory.

Besides unicast, multicast is also considered in the literature[3]. Heterogeneity is also considered in wireless networks. For instance, [4] and [5] study the capacity of wireless networks with Inhomogeneous Poisson Process (IPP) distribution and give the upper bound and lower bound for the networks respectively. The impact of mobility on wireless networks is first discussed in [13]. In that paper, Grossglauser and Tse show that mobility can significantly increase the throughput capacity of wireless networks to $\Theta(1)$ at the cost of large delay. Garetto et.al [15][16] combine mobility and heterogeneity and derive the upper and lower bound for throughput capacity of mobile heterogeneous networks. The delay capacity tradeoff is considered in [18][17] and generally capacity can only increase at the cost of large delay. Li et.al [19] and Zhang et.al [20][21] study the impact of directional antennas on capacity and delay of wireless ad hoc networks.

In the above works, all nodes are assumed to share one common communication channel and one same wireless channel bandwidth. However, the corresponding capacity scaling for these kind of pure ad hoc wireless networks are pessimistic. The per node throughput usually goes to zero as the number of nodes goes to infinity. Although mobility can somehow increase the network capacity, it will also incur the large delay. Thus, a kind of *hybrid wireless networks* is proposed. The hybrid networks consist of both wireless ad hoc networks and wired base stations. In [11], Kozat and Tassiulas study the throughput capacity of hybrid wireless networks where both normal ad hoc nodes and access points are randomly distributed. In [9], Zemlianov and Veciana study the hybrid wireless networks with random ad hoc nodes and fixed base stations. In [8], Liu et.al show that under the k-nearest-cell strategy, if the number of base stations is $\Omega(\sqrt{n})$, then the network throughput will gain a significant enhance. In [12], Li et.al consider hybrid networks with so called L-maximum-hop strategy and show that [8] is actually a special case for that. Zhang et.al [10] consider hybrid wireless networks with directional antennas. Li et.al [7] consider the hybrid networks with asymmetric traffic patterns and network areas. Mao et.al [14] take multicast into consideration for hybrid networks.

However, this kind of hybrid wireless networks will need wired infrastructures which are very costly and even infeasible under some conditions. For example, in the battle field, constructing wired infrastructures previously is impossible. As a result, some scholars suggest to replace the wired infrastructures with wireless helping networks with large bandwidth (related with the number of nodes). For example, Li et.al [6] derive an achievable throughput of the asymmetric networks with wireless helping networks. This kind of wireless helping network is more realistic and less costly compared to wired base stations. The wireless helping network is neither sources of data nor destinations of data. They only serve as relay for transmissions. In addition, they are usually equipped with large bandwidth and thus more powerful than normal nodes. There is no doubt that the existence of these powerful wireless helping nodes will enhance the capacity performance of the original ad hoc network significantly.

So far, some works have been done about the multicast in hybrid networks with wired infrastructures or wired networks. But, to the best of our knowledge, few previous works have taken heterogeneous multicast with wireless helping networks into consideration. For wireless helping network, although its bandwidth will scale with the number of nodes, it is still finite. In contrast, the bandwidth of wired base stations is infinite. This is an essential difference since the transmission on the wireless helping networks will also take time. Besides, unlike [14], the multicast considered in this paper is heterogeneous i.e. the cluster clients (destinations) are more likely to be located nearby their corresponding cluster heads (sources).

In this paper, we will study the achievable throughput of heterogeneous multicast network with wireless helping network. The wireless helping network is assumed to have a large bandwidth (scales with n). The network area is a rectangle and thus asymmetric. We consider three cases: the wireless helping network is (i) regularly placed (ii) randomly distributed (iii) mobile. We derive achievable throughput for all the three cases respectively, which are the main results of this paper. We attempt to find out the extra throughput a heterogeneous multicast network can gain with the help of powerful wireless helping networks. We will also make a comparison between the three cases and pure ad hoc network without helping networks under a special case.

The rest of this paper is organized as follows. In Section 2, we describe the system model, including network topology and interference model. In Section 3 and Section 4, we describe the routing strategy and derive the corresponding achievable throughput for heterogeneous multicast network with regular wireless helping network and heterogeneous multicast network with random wireless helping network, respectively. In Section 5, we describe the transmission scheme and achievable throughput of heterogeneous multicast network with mobile wireless helping network. In Section 6 we show an achievable throughput for pure heterogeneous multicast networks without helping networks. In Section 7, we compare the results in previous sections and discuss (i)whether the wireless helping network will improve the throughput or not (ii)whether the mobility of helping network will improve the throughout or not . Finally, we conclude the paper in Section 8.

2 System Model

2.1 Network Model

We consider a heterogeneous multicast network to be the normal network. Specifically, there are $n^h(h > 0)$ cluster heads in the network. Those heads are served as sources of data flows. Each head has $n^c(c > 0)$ clients which are the destinations of the data from this head. The network area is a rectangle with width n^{a_1} and length n^{a_2}, where $a_2 \geq a_1 > 0$. In the following, we denote the direction of the short side of the rectangle as the direction of x axis and the direction of the long side of the rectangle as the direction of y axis. We denote the network as A. The n^h cluster sessions are independently distributed on the network area. Each head is uniformly distributed on the area. Once a head is distributed on the position ξ, all the clients of this head will be distributed independently around it with probability density fuction (PDF) $f(z, \xi)$. The function $f(z, \xi)$ induces heterogeneity into the network model and is achieved in the following way:

$$f(z,\xi) = \frac{s(|z-\xi|)}{\int_A s(|z-\xi|)dz}$$

where $s()$ is a positive, monotonically decreasing, continuous function defined on the interval $[0, \infty)$. We further assume $s()$ satisfy the property: $\lim_{\rho\to\infty} \rho^{2+\varepsilon} s(\rho) = c_1$,where c_1 and ε are two positive constants. Since we have:

$$\int_A s(|z-\xi|)dz \leq \int_{\mathbb{R}^2} s(|z|)dz = 2\pi \int_0^\infty \rho s(\rho) d\rho$$

and

$$\int_A s(|z-\xi|)dz \geq \int_0^1 \frac{1}{4} 2\pi\rho d\rho s(\rho) \geq \frac{\pi}{2} \int_0^1 \rho s(\rho) d\rho$$

we can say that $f(z, \xi)$ and $s(|z - \xi|)$ are of the same order i.e. there exists two positive constants $\underline{c}$ and $\overline{c}$ such that for all z and ξ in the network area, we have: $\underline{c}s(|z - \xi|) \leq f(z, \xi) \leq \overline{c}s(|z - \xi|)$.

In addition, there are $m = n^b(b > 0)$ helping nodes in the network area. These helping nodes can be either regularly placed on the network area or randomly distributed on the network area. If they are regularly placed, we call the corresponding network **heterogeneous multicast network with regular wireless helping network**, which will be discussed in Section 3. If the helping nodes are uniformly and independently distributed on the network area, we call the corresponding network **heterogeneous multicast network with random wireless helping network**, which will be discussed in Section 4. Since the distribution of random wireless helping network is uniform, we will see that the analysis is very similar to regular wireless helping network.

The wireless helping network can also be mobile. We call this model **heterogeneous multicast network with mobile wireless helping network**. This is specified as follows. Among the m helping nodes, each will have a home point. The m home points are regularly placed in the network area. Then each helping node will move within a circle centered at its home point independently. The radius of these circles are all n^r. In each circle, the probability density of the corresponding helping node is assumed to be uniform. The movement of each helping node is a stationary ergodic random process. This model will be discussed in Section 5.

2.2 Definition of Gaussian Channel Model[1]

The maximum transmission rate from transmitter X_i to its receiver X_j is given by:

$$R_{ij} = W \log(1 + SINR_{ij})$$

where W is the bandwidth of the channel and $SINR_{ij}$ is the signal to noise and interference ratio i.e. $SINR_{ij} = \frac{P_i/d_i^\gamma}{N+\sum_{k\neq i} P_k/d_k^\gamma}$. P_i is the transmission power of each transmitter. d_k is the distance from any simultaneously active transmitter to the receiver X_j. γ is the attenuation exponent. As usual, we assume $\gamma > 2$.

2.3 Definition of Protocol Model[1]

Nodes are assumed to employ a common transmission range, R. Then node i will transmit data successfully to node j iff:

(i) The distance between node i and j is no more than R, i.e., $d_{ij} \leq R$

(ii) For any other simultaneously active transmitters k, $d_{kj} \geq (1 + \triangle)R$

where $\triangle$ is a positive constant.

In the following sections, we will use Gaussian Channel Model to analyze static wireless helping networks in Section 3, Section 4 and Section 6. However, in Section 5, we will use Protocol Model to analyze the mobile helping network for simplicity. In fact, since Protocol Model has some kind of feasibility under Gaussian Channel Model, our analysis for mobile wireless helping networks can be easily extended to Gaussian Channel Model. Protocol Model is only for analysis convenience.

3 Achievable Throughput of Heterogeneous Multicast Network with Regular Wireless Helping Network

In this section, we study the achievable throughput of heterogeneous multicast network with regular wireless helping network. We assume the bandwidth of the normal network to be W_1, a positive constant. This bandwidth is used for either the communications between normal nodes or the communications between normal nodes and helping nodes. However, the bandwidth of the helping network is W_2, an variable related with n. This bandwidth is only used for communications between helping nodes. Then we transmit the data flow from the cluster heads to their corresponding clients with the help of helping nodes. Firstly, we transmit the data from the heads to a nearby help node. Secondly, we transmit the data from this help node to a help node near the destined clients. Thirdly, we transmit the data from that help node to the corresponding clients. The first step is called uplink while the third one is called downlink. We further divide the bandwidth of normal nodes into uplink bandwidth W_3 and downlink bandwidth W_4, where W_3 and W_4 are two positive constants i.e. $W_1 = W_3 + W_4$. This will ensure that there is no interference between uplink transmissions and downlink transmissions.

We first derive the achievable throughput for the network under the assumption $b > a_2 - a_1$ carefully. After this, we will show that another achievable throughput can also be gained via a similar way of analysis for the case $b \leq a_2 - a_1$.

We tessellate the network area into small squares (cells) with side length $l = \sqrt{\frac{n^{a_1+a_2}}{n^b}} = n^{\frac{a_1+a_2-b}{2}}$. The constraint $b + a_1 > a_2$ will guarantee the feasibility of this tessellation. Then we just put the n^b helping nodes at the center of each cell such that one cell has exactly one helping node.

By using a TDMA transmission scheme and an equal power for every normal nodes and an equal power for every helping nodes, every cell can achieve a $\Theta(1)$ transmission rate at uplink (first step) or downlink (third step) and a $\Theta(W_2)$ transmission rate at the second step. In Step I and III, we only allow transmission within the same cell. In Step II, we allow transmission between adjacent cells. This concluded as the following lemma. The proof can be found in [6].

Lemma 31 *Every cell can achieve a $\Theta(1)$ transmission rate at the first step and third step. Every cell can achieve a $\Theta(W_2)$ transmission rate at the second step.*

Now we are ready to derive the throughput of the step I (uplink), step II, and step III (downlink), respectively. The lowest one among these three will be the bottleneck of the overall multicast transmission and is therefore the throughput of the whole network.

Step I: From the heads (sources) to the helping network.

In Step I, for every head, it must be located in a cell. We transmit the data from this head to the helping node belonging to this cell.In order to derive an achievable throughput for Step I, we need an upper bound for the number of heads in each cell. This is stated in the following lemma.

Lemma 32 *For every cell, there are at most $\Theta(\max\{n^{h-b}, \log n\})$ heads inside it w.h.p.*

Proof. The proof is a standard application of Chernoff bound.

For any particular cell, denote X the number of heads inside it. Then for any positive sequence x_n, we have:

$$\mathbb{P}(X \geq x_n) \leq \frac{\mathbb{E}(e^X)}{e^{x_n}} \leq \frac{(1+(e-1)n^{-b})^{n^h}}{e^{x_n}} \leq \exp((e-1)n^{h-b} - x_n)$$

The last step uses the fact that $1+x \leq e^x$. Denote event E_1 as {There exists a cell such that the number of heads inside it is larger than x_n}. Then we have:

$$\mathbb{P}(E_1) \leq \frac{n^{a_1+a_2}}{l^2} \exp((e-1)n^{h-b} - x_n) = n^b \exp((e-1)n^{h-b} - x_n)$$

If $h > b$, then $x_n = \Theta(n^{h-b})$ will ensure that $\mathbb{P}(E_1) \to 0$. Else, $h \leq b$, then $x_n = \Theta(\log n)$ will also ensure that $\mathbb{P}(E_1) \to 0$. Thus we complete the proof.

Hence, we get an achievable throughput for Step I (uplink):

$$\lambda_1 = \Theta\left(\min\left\{n^{b-h}, \frac{1}{\log n}\right\}\right)$$

Step II: Helping network relay.

In this Step II, we transmit the data through helping network in the following way. This routing strategy is the same as [6]. Suppose there is a head client pair. The coordinate of the cell the head lies in is (x_1, y_1) and the coordinate of the cell the client lies in is (x_2, y_2). Every coordinate can also represent a helping node. Firstly, we transmit the data from (x_1, y_1) to (x_1, y_2) through the line parallel to the y axis. Then we transmit the data from (x_1, y_2) to (x_2, y_2) though the line parallel to the x axis. Each time, we only allow the transmission between two adjacent helping nodes. And by using multihop and TDMA, we can transmit the data successfully.

In order to get an achievable throughput for Step II, we need to get an upper bound on the number of data flows that go across each cell. This is done by the following lemma.

Lemma 33 *For every cell, the number of data flows that go across it is at most w.h.p.:*

$$\Theta(\min\{n^h, \max\{n^{c+h+\frac{a_1-a_2-b}{2}}, n^{h+\frac{a_2-a_1-b}{2}}, \log n\}\})$$

Proof. Consider a particular cell, denoted as B. Suppose the coordinate of the B is (x_1, y_1). Denote F the rectangle consists of all the cells with y coordinate y_1. We call this rectangle the **horizontal rectangle** of this cell. Denote G the rectangle consists of all the cells with x coordinate x_1. We call this rectangle the **vertical rectangle** of this cell. Denote X the number of cluster sessions which have at least one client in F. Denote Y the number of heads inside G. Then the number of data flows is no more than $X+Y$. Let E be a certain client of a certain head H.

Now we consider X.

Firstly, we consider the case that the distance between H and F is larger than l. Denote this case (event) as E_2. Denote E_3 the event {The distance between H and F is within the interval $[\rho, \rho + d\rho]$}, where $0 < \rho < \sqrt{2}n^{a_2}$. We have:

$$\mathbb{P}(E_3) \leq 2\frac{d\rho}{n^{a_2}}$$

Connect H and arbitrary point in the boundary of F to form a line. Denote θ the angle between this line and the y axis. Denote E_4 the event {client E is inside F}. Then we have:

$$\mathbb{P}(E_4|E_3) \leq 2\int_0^{\frac{\pi}{2}} l\frac{\rho d\theta}{\cos^2\theta}\bar{c}s\left(\frac{\rho}{\cos\theta}\right) = 2l\rho\bar{c}\int_0^{\frac{\pi}{2}} \frac{1}{\cos^2\theta}s\left(\frac{\rho}{\cos\theta}\right)d\theta$$

Since $\lim_{\rho\to\infty}\rho^{2+\varepsilon}s(\rho) = c_1$, there must exist a positive constant c_2, such that for all positive ρ: $\rho^{2+\varepsilon}s(\rho) \leq c_2$. Substitute this into the above equation we have:

$$\mathbb{P}(E_4|E_3) \leq \frac{2l\bar{c}c_2}{\rho^{1+\varepsilon}}\int_0^{\frac{\pi}{2}}(\cos\theta)^{\varepsilon}d\theta = c_{10}\frac{l}{\rho^{1+\varepsilon}}$$

where c_{10} is a positive constant. So:

$$\mathbb{P}(E_4, E_2) \leq \int_l^{\sqrt{2}n^{a_2}} c_{10}\frac{l}{\rho^{1+\varepsilon}}2\frac{d\rho}{n^{a_2}} = \frac{2c_{10}l}{n^{a_2}}\int_l^{\sqrt{2}n^{a_2}}\frac{d\rho}{\rho^{1+\varepsilon}}$$

If $a_1 + a_2 \geq b$, then

$$\mathbb{P}(E_4, E_2) \leq \frac{2c_{10}l}{n^{a_2}} \frac{1}{\varepsilon l^{\varepsilon}} = \frac{c_{11}l^{1-\varepsilon}}{n^{a_2}} \quad (1)$$

Else $a_1 + a_2 < b$, then

$$\mathbb{P}(E_4, E_2) \leq \frac{c_{12}l}{n^{a_2}}$$

c_{11} and c_{12} are two positive constants.

Secondly, we consider the case that the distance between H and F is no more than l. We denote this case as E_5. Then we have:

$$\mathbb{P}(E_5) \leq \frac{3l}{n^{a_2}}$$

By consider only the distance in x direction, we get the following inequality:

$$\mathbb{P}(E_4|E_5) \leq \int_0^{n^{a_1}} 2\overline{c}s(\rho)ld\rho = c_{13}l$$

where c_{13} is a positive constant. So:

$$\mathbb{P}(E_4, E_5) \leq \frac{3c_{13}l^2}{n^{a_2}} \quad (2)$$

If $a_1 + a_2 < b$, we will use (2). Else if $a_1 + a_2 \geq b$, then we just use $\mathbb{P}(E_4, E_5) \leq \mathbb{P}(E_5) \leq \frac{3l}{n^{a_2}}$.

Combine the above two formulas (1) and (2), if $a_1 + a_2 \geq b$, then:

$$\mathbb{P}(E_4) \leq c_{11}\frac{l^{1-\varepsilon}}{n^{a_2}} + \frac{3l}{n^{a_2}} \leq c_{14}\frac{l}{n^{a_2}}$$

where c_{14} is a positive constant.

Denote E_6 the event {There exists at least one client of head H inside F}. Denote $p' = \mathbb{P}(E_6)$. Then we have:

$$p' \leq c_{14}n^c\frac{l}{n^{a_2}} = c_{14}n^{c+\frac{a_1-b-a_2}{2}}$$

Then by Chernoff bound, for any positive sequence x_n, we have:

$$\mathbb{P}(X \geq x_n) \leq \exp((e-1)p'n^h - x_n) \leq \exp((e-1)c_{14}n^{c+h+\frac{a_1-b-a_2}{2}} - x_n)$$

Denote E_7 the event {There exists a cell, the number of cluster sessions which have at least one client inside the horizontal rectangle of this cell is no less than x_n}. Then:

$$\mathbb{P}(E_7) \leq \frac{n^{a_1+a_2}}{l^2}\exp((e-1)c_{14}n^{c+h+\frac{a_1-b-a_2}{2}} - x_n) = n^b\exp((e-1)c_{14}n^{c+h+\frac{a_1-b-a_2}{2}} - x_n)$$

When $c + h + \frac{a_1-b-a_2}{2} > 0$, we let $x_n = \Theta(n^{c+h+\frac{a_1-b-a_2}{2}})$. When $c + h + \frac{a_1-b-a_2}{2} \leq 0$, we let $x_n = \Theta(\log n)$. Thus we can ensure that $\mathbb{P}(E_7) \to 0$.

Now we consider Y. Since the n^h heads are uniformly and independently distributed on the network area, by using Chernoff bound, for any positive sequence y_n, we have:

$$\mathbb{P}(Y \geq y_n) \leq \exp((e-1)\frac{l}{n^{a_1}}n^h - y_n) \leq \exp((e-1)n^{h+\frac{a_2-a_1-b}{2}} - y_n)$$

Denote E_8 the event {There exists a cell, the number of heads inside the vertical rectangle of this cell is no less than y_n}. Then we have:

$$\mathbb{P}(E_8) \leq n^b\exp((e-1)n^{h+\frac{a_2-a_1-b}{2}} - y_n)$$

When $h + \frac{a_2-a_1-b}{2} > 0$, we let $y_n = \Theta(n^{h+\frac{a_2-a_1-b}{2}})$. When $h + \frac{a_2-a_1-b}{2} \leq 0$, we let $y_n = \Theta(\log n)$. This will ensure that $\mathbb{P}(E_8) \to 0$.

Denote E_9 the event {There exists a cell such that the number of data flows go across this cell is no less than $x_n + y_n$}. Then we have:

$$\mathbb{P}(E_9) \leq \mathbb{P}(E_8) + \mathbb{P}(E_7) \to 0$$

And $x_n + y_n = \Theta(\max\{n^{c+h+\frac{a_1-b-a_2}{2}}, n^{h+\frac{a_2-a_1-b}{2}}, \log n\})$.

Similarly, when $a_1 + a_2 < b$, we will get the same conclusion with $x_n + y_n = \Theta(\max\{n^{h+\frac{a_2-a_1-b}{2}}, n^{c+h+\frac{a_1-a_2-b}{2}}, \log n\})$

Besides, since the total amount of data flows is just n^h, it is obvious that the number of data flows go across any cell will be no more than n^h. Hence, we complete the proof.

Therefore, we've got an achievable throughput for Step II.

$$\lambda_2 = \Theta\left(W_2 \max\left\{n^{-h}, \min\left\{n^{-c-h+\frac{a_2+b-a_1}{2}}, n^{-h+\frac{a_1+b-a_2}{2}}, \frac{1}{\log n}\right\}\right\}\right)$$

Step III: From the helping network to clients (destinations).

In order to derive an achievable throughput of Step III, for every cell, we need an upper bound for the number of cluster sessions which have at least one client inside it. This is got by the following lemma.

Lemma 34 *For every cell, the number of cluster sessions which have at least one client inside it is no more than w.h.p.:*

$$\Theta\left(\min\{n^h, \max\{\log n, n^{c+h-b}\}\}\right)$$

Proof. Let's consider a particular cell B, a particular head H and a client of H named E. Firstly, we consider the case that the distance between H and B is larger than l. Denote this case as E_{10}. Denote E_{11} the event $\{$The distance between H and B is within the interval $[\rho, \rho + d\rho]\}$, where $l < \rho < \sqrt{2}n^{a_2}$. Then we have:

$$\mathbb{P}(E_{11}) \leq \frac{1}{n^{a_1+a_2}}(\pi(\rho + d\rho)^2 - \pi\rho^2 + 4l d\rho) = \frac{2\pi\rho + 4l}{n^{a_1+a_2}} d\rho$$

Denote E_{12} the event $\{E$ is inside $B\}$. Then we have: $\mathbb{P}(E_{12}|E_{11}) \leq \bar{c}s(\rho)l^2$

So:

$$\mathbb{P}(E_{12}, E_{10}) \leq \bar{c} \int_l^{\sqrt{2}n^{a_2}} s(\rho) l^2 \frac{2\pi\rho + 4l}{n^{a_1+a_2}} d\rho$$

Since $\rho^{2+\varepsilon}s(\rho) \leq c_2$ for any positive ρ, we have:

$$\mathbb{P}(E_{12}, E_{10}) \leq \frac{\bar{c}l^2}{n^{a_1+a_2}} \int_l^{\sqrt{2}n^{a_2}} \frac{c_2}{\rho^{2+\varepsilon}}(2\pi\rho + 4l)d\rho = \frac{\bar{c}l^2}{n^{a_1+a_2}} \left(2\pi c_2 \int_l^{\sqrt{2}n^{a_2}} \frac{d\rho}{\rho^{1+\varepsilon}} + 4lc_2 \int_l^{\sqrt{2}n^{a_2}} \frac{d\rho}{\rho^{2+\varepsilon}}\right)$$

When $a_1 + a_2 \geq b$, then $l \to \infty$ or l is a positive constant, so:

$$\mathbb{P}(E_{12}, E_{10}) \leq \frac{\bar{c}l^2}{n^{a_1+a_2}} \left(\frac{2\pi c_2}{\varepsilon l^\varepsilon} + \frac{4lc_2}{(1+\varepsilon)l^{1+\varepsilon}}\right) = c_3 \frac{l^{2-\varepsilon}}{n^{a_1+a_2}}$$

When $a_1 + a_2 < b$, then $l \to 0$, so:

$$\mathbb{P}(E_{12}, E_{10}) \leq c_3' \frac{l^2}{n^{a_1+a_2}}$$

Secondly, we consider the case that the distance between H and B is no more than l. Denote this case as E_{13}. Then we have:

$$\mathbb{P}(E_{13}) \leq \frac{1}{n^{a_1+a_2}}(5l^2 + \pi l^2) = \frac{(5+\pi)l^2}{n^{a_1+a_2}}$$

So:

$$\mathbb{P}(E_{12}|E_{13}) \leq \bar{c} \int_0^{\frac{\sqrt{2}}{2}l} 2\pi\rho s(\rho) d\rho$$

We first consider the case $a_1 + a_2 \geq b$. In this case, we have $\mathbb{P}(E_{12}|E_{13}) \leq 1$. So, $\mathbb{P}(E_{12}, E_{13}) \leq \frac{c_5 l^2}{n^{a_1+a_2}}$, where c_5 is a positive constant. Hence:

$$\mathbb{P}(E_{12}) \leq c_3 \frac{l^{2-\varepsilon}}{n^{a_1+a_2}} + c_5 \frac{l^2}{n^{a_1+a_2}} \leq c_6 \frac{l^2}{n^{a_1+a_2}}$$

where c_6 is a positive constant.

Denote E_{14} the event $\{$There exists a client of H inside $B\}$. Let $p = \mathbb{P}(E_{14}) \leq c_6 n^c \frac{l^2}{n^{a_1+a_2}} = \frac{c_6 l^2}{n^{a_1+a_2-c}}$.

Denote X the number of cluster sessions which have at least one client inside cell B. Then by Chernoff bound, for any positive sequence x_n, we have:

$$\mathbb{P}(X \geq x_n) \leq \frac{\mathbb{E}(e^X)}{e^{x_n}} = \exp((e-1)pn^h - x_n) \leq \exp((e-1)c_6 l^2 n^{c-a_1-a_2+h} - x_n)$$

Denote E_{15} the event $\{$There exists one cell such that the number of cluster sessions which have at least one client inside this cell is no less than $x_n\}$. Thus we have:

$$\mathbb{P}(E_{15}) \leq \frac{n^{a_1+a_2}}{l^2}\exp((e-1)c_6 l^2 n^{c+h-a_1-a_2} - x_n) = n^b \exp((e-1)c_6 n^{c+h-b} - x_n)$$

When $c+h \leq b$, we let $x_n = \Theta(\log n)$. When $c+h > b$, we let $x_n = \Theta(n^{c+h-b})$. Thus by letting $x_n = \Theta(\max\{\log n, n^{c+h-b}\})$, we will ensure that $\mathbb{P}(E_{15}) \to 0$.

Similarly, when $a_1 + a_2 < b$, we will get the same result.

Since the overall number of cluster sessions is just n^h, it is obvious that, for every cell, the number of cluster sessions which have at least one client inside it is no more than n^h. Hence, we finish the proof.

Therefore, an achievable throughput of Step III is :

$$\lambda_3 = \Theta\left(\max\left\{n^{-h}, \min\left\{\frac{1}{\log n}, n^{b-c-h}\right\}\right\}\right)$$

Now we've finished the discussion of the throughput of all the steps and we arrive at one main result:

Theorem 31 *When $b > a_2 - a_1$, an achievable throughput for heterogeneous multicast networks with regular wireless helping networks is:*

$$\lambda = \Theta(\min\{\lambda_1, \lambda_2, \lambda_3\})$$

where

$$\lambda_1 = \min\left\{n^{b-h}, \frac{1}{\log n}\right\}$$
$$\lambda_2 = W_2 \max\left\{n^{-h}, \min\left\{n^{-c-h+\frac{a_2+b-a_1}{2}}, n^{-h+\frac{a_1+b-a_2}{2}}, \frac{1}{\log n}\right\}\right\}$$
$$\lambda_3 = \max\left\{n^{-h}, \min\left\{\frac{1}{\log n}, n^{b-c-h}\right\}\right\}$$

Now we consider the case $b \leq a_2 - a_1$. The analysis is quite similar to the case $b > a_2 - a_1$.

When $b \leq a_2 - a_1$, similar to [6], we tessellate the long side n^{a_2} into n^b intervals of length n^{a_2-b}. In this way, we get n^b small rectangles (cells) of which the length is n^{a_2-b} and width is n^{a_1}. Then we locate the n^b help nodes on the center of the n^b cells and every cell will has exactly one help node inside it. After that, we use the same transmission scheme as for the case $b > a_2 - a_1$. The complete transmission also consists of three steps, uplink, help node relay and downlink.

Now we begin to analyze the achievable throughput. Firstly, for Step I, following the same procedure as Lemma 3.2, we can still get an achievable throughput of $\lambda_1 = \Theta(\min\{n^{b-h}, \frac{1}{\log n}\})$. For Step II, since all the cells have the same y coordinate now, we simply let $\lambda_2 = \Theta(Wn^{-h})$ as an achievable throughput. For Step III, by following the similar procedure as Lemma 3.4, we can still get an achievable throughput of $\lambda_3 = \Theta(\max\{n^{-h}, \min\{n^{b-c-h}, \frac{1}{\log n}\}\})$. Now we get an achievable throughput for the overall network.

Theorem 32 *When $b \leq a_2 - a_1$, an achievable throughput for heterogeneous multicast networks with regular wireless helping networks is:*

$$\lambda = \Theta(\min\{\lambda_1, \lambda_2, \lambda_3\})$$

where

$$\lambda_1 = \min\left\{n^{b-h}, \frac{1}{\log n}\right\}$$
$$\lambda_2 = W_2 n^{-h}$$
$$\lambda_3 = \max\left\{n^{-h}, \min\left\{\frac{1}{\log n}, n^{b-c-h}\right\}\right\}$$

4 Achievable Throughput of Heterogeneous Multicast Network with Random Wireless Helping Network

In this section, we study the achievable throughput of heterogeneous multicast network with random wireless helping network. We first consider the case $b > a_2 - a_1$. The analysis of the case $b \le a_2 - a_1$ is quite similar.

Similar to [6], we first tessellate the network area with small squares (cells) whose side length is $l' = \sqrt{\frac{n^{a_1+a_2}\log m}{m}}$. The constraint $b > a_2 - a_1$ will ensure the success of this tessellation and we will have the following lemma.

Lemma 41 *For every cell, there exists at least one helping node inside it w.h.p.*

The proof of this lemma is just a standard application of Chernoff bound and we omit it here.

The transmission scheme of heterogeneous multicast network with random wireless helping network is just the same as that of heterogeneous multicast network with regular wireless helping network. It also consists of three steps: uplink, helping network relay and downlink. As a result, the analysis is quite similar.

Step I: From the heads (sources) to the helping network.

For tessellation with side length l', we have the following lemma which is quite similar to Lemma 3.2.

Lemma 42 *For every cell, there is at most $\Theta(\max\{n^{h-b}\log n, \log n\})$ heads inside it w.h.p.*

As a result, an achievable throughput for Step I is:

$$\lambda'_1 = \Theta\left(\min\left\{\frac{n^{b-h}}{\log n}, \frac{1}{\log n}\right\}\right)$$

Step II: Helping network relay.

In Step II, we use the same transmission route as that in Section 3. Similar to Lemma 3.3, we have the following lemma.

Lemma 43 *For every cell, the number of data flows that go across it is at most:*

$$\Theta\left(\min\left\{n^h, \max\left\{n^{c+h+\frac{a_1-a_2-b}{2}}\sqrt{\log n}, n^{h+\frac{a_2-a_1-b}{2}}\sqrt{\log n}, \log n\right\}\right\}\right)$$

Therefore, an achievable throughput for Step II is:

$$\lambda'_2 = \Theta\left(W_2 \max\left\{n^{-h}, \min\left\{\frac{n^{-c-h+\frac{a_2+b-a_1}{2}}}{\sqrt{\log n}}, \frac{n^{-h+\frac{a_1+b-a_2}{2}}}{\sqrt{\log n}}, \frac{1}{\log n}\right\}\right\}\right)$$

Step III: From the helping network to clients (destinations)

Similar to Lemma 3.4, we have the following lemma:

Lemma 44 *For every cell, the number of cluster sessions which have at least one client inside it is no more than:*

$$\Theta(\min\left\{n^h, \max\left\{n^{c+h-b}\log n, \log n\right\}\right\})$$

Thus, an achievable throughput for Step III is:

$$\lambda'_3 = \Theta\left(\max\left\{n^{-h}, \min\left\{\frac{n^{b-c-h}}{\log n}, \frac{1}{\log n}\right\}\right\}\right)$$

So we come to the following result:

Theorem 41 *When $b > a_2 - a_1$, an achievable throughput for heterogeneous multicast networks with random wireless helping networks is:*

$$\lambda' = \Theta(\min\{(\lambda'_1, \lambda'_2, \lambda'_3)\})$$

where

$$\lambda'_1 = \min\left\{\frac{n^{b-h}}{\log n}, \frac{1}{\log n}\right\}$$

$$\lambda'_2 = W_2 \max\left\{n^{-h}, \min\left\{\frac{n^{-c-h+\frac{a_2+b-a_1}{2}}}{\sqrt{\log n}}, \frac{n^{-h+\frac{a_1+b-a_2}{2}}}{\sqrt{\log n}}, \frac{1}{\log n}\right\}\right\}$$

$$\lambda'_3 = \max\left\{n^{-h}, \min\left\{\frac{n^{b-c-h}}{\log n}, \frac{1}{\log n}\right\}\right\}$$

For the case $b \leq a_2 - a_1$, we can derive the following theorem in a similar way:

Theorem 42 *When $b > a_2 - a_1$, an achievable throughput for heterogeneous multicast networks with random wireless helping networks is:*

$$\lambda' = \Theta(\min\{\lambda_1', \lambda_2', \lambda_3'\})$$

where

$$\lambda_1' = \min\left\{\frac{n^{b-h}}{\log n}, \frac{1}{\log n}\right\}$$
$$\lambda_2' = W_2 n^{-h}$$
$$\lambda_3' = \max\left\{n^{-h}, \min\left\{\frac{n^{b-c-h}}{\log n}, \frac{1}{\log n}\right\}\right\}$$

5 Achievable Throughput of Heterogeneous Multicast Network with Mobile Wireless Helping Network

In this section, we study the case the wireless helping network is mobile as described in Section 2. For heterogeneous multicast network with this kind of mobile helping network, we show a feasible transmission scheme and derive its corresponding achievable throughput. Recall that the movement of each node is within a circle centered at its home point and the radius of this circle is n^r. In the following, we will only analyze the case $b > a_2 - a_1$. One can easily analyze the case $b \leq a_2 - a_1$ in a similar manner.

Since $b > a_2 - a_1$, similar to Section 3, we can tessellate the network area into small squares (cells) whose side length is $l = n^{\frac{a_1+a_2-b}{2}}$. Then we place the home points of the $m = n^b$ helping node at the center of these m small cells. In the following, we further assume that $\frac{a_1+a_2-b}{2} < r < a_1$. Otherwise, if $r > a_1$, then the move range is larger than the network area and this is not proper. If $r \leq \frac{a_1+a_2-b}{2}$, then the movement area of each helping node is just constrained within its corresponding small cell. Then there will be no significant difference with the regular wireless helping network which is discussed in Section 3.

Since $r < a_1$, we can further tessellate the network area into big squares (cells) whose side length is $l_1 = \frac{n^r}{10}$. Then each big cell will have $(\frac{l_1}{l})^2 = \Theta(n^{2r-a_1-a_2+b})$ small cells inside it.

The transmission scheme also consists of three steps: uplink, helping network relay and downlink. In the following, we will discuss them respectively. Note that for the mobile model in this section, we will analyze problems under the Protocol Model defined in Section 2. And we will choose different transmission ranges R for the three steps. We also assume the different steps use different bandwidth, so there will be no interference between different steps.

Step I: Uplink.

In order to choose a proper transmission range for uplink, we need to know the minimum distance between any two heads. This problem is solved by the following lemma.

Lemma 51 *For any positive sequence d_n such that $d_n = o(n^{\frac{a_1+a_2}{2}-h})$, the distance between any two heads will be larger than d_n, w.h.p.*

Proof. Denote E' the event {The distance between any two heads is larger than d_n}. Then we have:

$$\mathbb{P}(E') \geq \prod_{k=1}^{n^h-1}(1 - \frac{k\pi d_n^2}{n^{a_1+a_2}}) \geq 1 - \sum_{k=1}^{n^h-1} \frac{k\pi d_n^2}{n^{a_1+a_2}} = 1 - \frac{\pi d_n^2}{n^{a_1+a_2}}\frac{1}{2}(n^h-1)n^h \geq 1 - \frac{\pi}{2}\frac{d_n^2}{n^{a_1+a_2}}n^{2h} \to 1$$

With Lemma 5.1, we can get an achievable throughput for Step I. This is concluded in the following lemma.

Lemma 52 *An achievable throughput for Step I is: $\lambda_1^{(m)} = \Theta(1)$ when $b > 2h$; $\lambda_1^{(m)} = o(n^{b-2h})$ when $b \leq 2h$. Where $o(n^{b-2h})$ can be any small quantity compared to n^{b-2h}.*

Proof. First, let us consider the case $r < \frac{a_1+a_2}{2} - h$. If so, we choose the transmission range to be $R_T = \Theta(n^r)$. Let $d_n = \sqrt{R_T n^{\frac{a_1+a_2}{2}-h}}$. According to Lemma 5.1, the distance between any two heads is larger than d_n. Since $d_n = \omega(R_T)$, the transmissions from different heads will not interfere with each other. Consider any head H and the big cell it belongs to B. For any helping node G whose home point is inside B, we have: $\mathbb{P}(|H - G| \leq R_T) \geq c_{15}$, where c_{15} is a positive constant. Hence, every head will get a $\Theta(1)$ throughput.

Second, we consider the case $r \geq \frac{a_1+a_2}{2} - h$. If so, we choose the transmission range to be $R_T = o(n^{\frac{a_1+a_2}{2}-h})$. Let $d_n = \sqrt{R_T n^{\frac{a_1+a_2}{2}-h}}$. According to Lemma 5.1, similarly, we can ensure that there will be no interference between two simultaneous transmitting heads. We also use the notations in the first case. Then we have: $\mathbb{P}(|H-G| > R_T) \leq 1 - \frac{R_T^2}{4n^{2r}}$.

Since there are totally $n^{2r-a_1-a_2+b}$ helping nodes in the big cell B, denoting these helping nodes as $G_i (1 \leq i \leq n^{2r-a_1-a_2+b})$, then we have:

$$\mathbb{P}(|H-G_i| > R_T, \forall 1 \leq i \leq n^{2r-a_1-a_2+b}) \leq \left(1 - \frac{R_T^2}{4n^{2r}}\right)^{n^{2r-a_1-a_2+b}} \leq \exp\left(-\frac{R_T^2}{4n^{2r}} n^{2r-a_1-a_2+b}\right) = \exp\left(-\frac{1}{4} R_T^2 n^{-a_1-a_2+b}\right)$$

Therefore,

$$\mathbb{P}\left(\exists 1 \leq i \leq n^{2r-a_1-a_2+b}, |H-G_i| \leq R_T\right) \geq 1 - \exp\left(-\frac{1}{4} R_T^2 n^{-a_1-a_2+b}\right) \tag{3}$$

Then, if $b > 2h$, we have $R_T^2 n^{-a_1-a_2+b} = o(n^{b-2h})$. And we can always choose R_T sufficiently close to $n^{\frac{a_1+a_2}{2}-h}$, such that $R_T^2 n^{-a_1-a_2+b} \to +\infty$. Then the right hand side of (3) will tend to 1 as n goes to infinity. Thus, every head get an throughput of $\Theta(1)$. Note that the case $b > 2h$ will cover the first case $r < \frac{a_1+a_2}{2} - h$, so as long as $b > 2h$, throughput of $\Theta(1)$ is always achievable.

Next, if $b \leq 2h$, note the fact that $\lim_{x\to 0} \frac{e^x-1}{x} = 1$, the right hand side of (3) is asymptotically $o(n^{b-2h})$. Thus, for every head, a throughput of $o(n^{b-2h})$ is achievable, where $o(n^{b-2h})$ can be any small quantity compared to n^{b-2h}. Hence we complete the proof.

From now on, we choose the $o(n^{b-2h})$ in Lemma 5.2 to be $\frac{n^{b-2h}}{\log n}$, hence an achievable throughput for Step I is:

$$\lambda_1^{(m)} = \Theta\left(\min\left\{1, \frac{n^{b-2h}}{\log n}\right\}\right)$$

Step II: Helping network relay

Different from the static case, now the helping nodes are mobile. So we can allow transmission between two helping nodes only when they are close enough. This will enhance the performance of achievable throughput. Besides when r tends to be $\frac{a_1+a_2-b}{2}$, the movement of the helping nodes will be restricted within its corresponding small cell, thus the throughput will be the same as the static case. All of the above assertions will be shown in the following and we will see that throughput of Step II will get a significant enhance compared to the static case in Section 3 and 4.

In order to derive achievable throughput for Step II, we first need to know the transmission ability between two adjacent big cells. This is done by the following lemma.

Lemma 53 *Every two adjacent big cells can support $\mu = \Theta(W_2 n^{2r-a_1-a_2+b})$ communication rate, where W_2 is the data rate for successful transmission between two help nodes.*

Proof. For Step II, we choose the transmission range to be $R'_T = \Theta(n^{\frac{a_1+a_2-b}{2}}) = o(n^r)$. Consider two adjacent big cells, B_1 and B_2. G_i is any help node whose home point is within $B_i, i = 1, 2$. Similar to [16], we allow transmission between G_1 and G_2 only when (i) the distance between G_1 and G_2 is no more than R'_T i.e., $|G_1 - G_2| \leq R'_T$. (ii) for any other help node G, we have $|G - G_i| \geq (1+\triangle)R'_T, i = 1, 2$. Then we have,in the order sense: $\mathbb{P}(|G_1 - G_2| \leq R'_T) = \frac{R'_T}{n^{2r}}$. And regardless of the position of G_1 and G_2, we have:

$$\mathbb{P}(|G - G_i| \geq (1+\triangle)R'_T, i = 1, 2, \forall G \neq G_1, G_2) \geq \left(1 - \frac{2((1+\triangle)R'_T)^2}{n^{2r}}\right)^{1000n^{2r-a_1-a_2+b}} \geq 1 - 2000n^{b-a_1-a_2}(1+\triangle)^2 R'^2_T$$

So, denote E'' the event that G_1 and G_2 are permitted to communicate, combining the above inequalities, we have:

$$\mathbb{P}(E'') \geq \left(1 - 2000n^{b-a_1-a_2}(1+\triangle)^2 R'^2_T\right) \frac{R'^2_T}{n^{2r}} = \Theta\left(n^{a_1+a_2-b-2r}\right)$$

In addition, there are totally $n^{2r-a_1-a_2+b}$ helping nodes in each big cell. As a result, two adjacent big cells can support communication rate of $\Theta((n^{2r-a_1-a_2+b})^2 W_2 n^{a_1+a_2-b-2r}) = \Theta(W_2 n^{2r-a_1-a_2+b})$. Thus we finish the proof.

To derive the achievable throughput for Step II, we need to bound the number of data flows which go across every adjacent two big cells. Similar to Section 3, we have the following lemma.

Lemma 54 *(i)The number of data flows which go across two vertically adjacent big cells is at most w.h.p.*

$$\Theta\left(\max\{n^{h+r-a_1}, \log n\}\right)$$

(ii)The number of data flows which go across two horizontally adjacent big cells is at most w.h.p.

$$\Theta\left(\min\{n^h, \max\{n^{c+r+h-a_2}, \log n\}\}\right)$$

With the help of Lemma 5.3 and Lemma 5.4, we get an achievable throughput for Step II.

$$\begin{aligned}\lambda_2^{(m)} &= \Theta\left(\min\left\{\frac{W_2 n^{2r-a_1-a_2+b}}{\max\{n^{h+r-a_1}, \log n\}}, \frac{W_2 n^{2r-a_1-a_2+b}}{\min\left\{n^h, \max\left\{n^{c+r+h-a_2}, \log n\right\}\right\}}\right\}\right) \\ &= \Theta\left(W_2 n^{2r-a_1-a_2+b} \max\left\{n^{-h}, \min\left\{n^{-c-r-h+a_2}, n^{a_1-h-r}, \frac{1}{\log n}\right\}\right\}\right)\end{aligned}$$

Observe this result, we can see that $\lambda_2^{(m)}$ is a monotonically increasing function with respect to r, as long as the previous assumption $\frac{a_1+a_2-b}{2} < r < a_1$ holds. Besides, when r tends to $\frac{a_1+a_2-b}{2}$ i.e., the mobility of helping nodes tends to disappear, the throughput of Step II, $\lambda_2^{(m)}$, will tend to be:

$$\Theta\left(W_2 \max\left\{n^{-h}, \min\left\{n^{-c-h+\frac{a_2-a_1+b}{2}}, n^{\frac{a_1-a_2+b}{2}-h}, \frac{1}{\log n}\right\}\right\}\right)$$

which is the exact achievable throughput for Step II in static case which has been derived in Theorem 3.1.
Step III: Downlink

Now we begin to derive an achievable throughput for downlink. Here, we will not make use of the mobility characteristic of help nodes and mobility is even a disadvantage in our transmission scheme. However, as previously said, mobility will increase the throughput of Step II, so in practice, one should make a tradeoff according to specified parameters.

For the transmission scheme, we will simply exploit a round-robin TDMA scheme. That is, we divide all the big cells into a constant number of groups such that transmissions of different big cells of the same group will not interfere with each other. Hence, each big cell gets a constant transmission rate. Now, all we need to do is to bound the number of multicast sessions which have at least one client inside a big cell. Similar to Lemma 3.4 in Section 3, we have the following lemma.

Lemma 55 *For every big cell, the number of multicast sessions which have at least one client inside it is no more than w.h.p.:*

$$\Theta\left(\min\left\{n^h, \max\left\{n^{h+c-a_1-a_2+2r}, \log n\right\}\right\}\right)$$

.

With the help of Lemma 5.5, we can get an achievable throughput for Step III:

$$\lambda_3^{(m)} = \Theta\left(\max\left\{n^{-h}, \min\left\{n^{a_1+a_2-c-h-2r}, \frac{1}{\log n}\right\}\right\}\right)$$

Now we have finished the discussion of all the three steps, and we arrive a main result:

Theorem 51 *When $b > a_2 - a_1$, $\frac{a_1+a_2-b}{2} < r < a_1$, an achievable throughput for heterogeneous multicast network with mobile wireless helping network is:*

$$\lambda^{(m)} = \Theta(\min\{\lambda_1^{(m)}, \lambda_2^{(m)}, \lambda_3^{(m)}\})$$

where

$$\begin{aligned}\lambda_1^{(m)} &= \min\left\{1, \frac{n^{b-2h}}{\log n}\right\} \\ \lambda_2^{(m)} &= W_2 n^{2r-a_1-a_2+b} \max\left\{n^{-h}, \min\left\{n^{-c-r-h+a_2}, n^{a_1-h-r}, \frac{1}{\log n}\right\}\right\} \\ \lambda_3^{(m)} &= \max\left\{n^{-h}, \min\left\{n^{a_1+a_2-c-h-2r}, \frac{1}{\log n}\right\}\right\}\end{aligned}$$

6 Achievable Throughput of Pure Heterogeneous Multicast Network without Helping Network

In this section, we study the case that heterogeneous multicast network is not equipped with wireless helping network. The routing strategy is the same as the Step II in Section 3 and the corresponding analysis is similar.

If $h \leq a_2 - a_1$, then we simply let the achievable throughput be $\lambda_p = \Theta(n^{-h})$. This is obviously feasible by means of TDMA. Next, we focus on the case $h > a_2 - a_1$.

Firstly, as usual, we tessellate the network area with small squares (cells) whose side length is $l'' = \sqrt{\frac{n^{a_1+a_2}\log n^h}{n^h}}$. Then the following lemma holds:

Lemma 61 *For every cell, there is at least one head inside it w.h.p.*

Then we use the same routing strategy as Step II in Section 3. For each head-client pair, we first transmit the data flow vertically and then transmit it horizontally. By following the same procedure in Lemma 3.3, we can derive an achievable throughput and this is summarized in the following theorem:

Theorem 61 *An achievable throughput for heterogeneous multicast network without wireless helping network is:*

when $h > a_2 - a_1$:

$$\lambda_p = \Theta\left(\max\left\{n^{-h}, \min\left\{\frac{n^{-c+\frac{a_2-h-a_1}{2}}}{\sqrt{\log n}}, \frac{n^{\frac{a_1-h-a_2}{2}}}{\sqrt{\log n}}\right\}\right\}\right)$$

when $h \leq a_2 - a_1$:

$$\lambda_p = \Theta(n^{-h})$$

7 Discussion

In this section, we make a comparison between heterogeneous multicast network with wireless helping network and heterogeneous multicast network without wireless helping network. From Theorem 3.1, Theorem 3.2, Theorem 4.1, Theorem 4.2 and Theorem 5.1, we can see that with helping network, the achievable throughput is influenced by many parameters: the number of cluster heads, the number of clients for each session, the number of helping nodes, the length and width of the network area, the mobility radius for mobile helping network. But it is not influenced by the heterogeneity parameter ε. From Theorem 6.1, we can see that, without helping network, the achievable throughout is also influenced by a series of parameters. Now we make a comparison to see when the helping network will enhance the throughput performance significantly in the order sense and when will the mobility of helping network improve the throughput performance.

In subsection 7.1, we mainly discuss the case when the helping network is regular, but the result also holds when the helping network is random since $\log n$ factor will not influence the behavior of throughput heavily. In subsection 7.2, we use a special case to compare the performance between three kinds of networks: (i) heterogeneous multicast network without helping network, (ii) heterogeneous multicast network with regular helping network, (iii) heterogeneous multicast network with mobile helping network.

7.1 Multicast heterogeneous network with regular helping network and multicast heterogeneous network without helping network: a comparison

Case I: $b - c \leq 0$.

In this case, no matter the helping network is regular or random, $\lambda_3 = \Theta(n^{-h})$. Because of this bottleneck, no matter how large bandwidth the helping network has, the overall throughput can not be better than a standalone multicast network without helping network.

Case II: $b - c > 0$.

In this case, with helping network with sufficiently large W_2, λ_2 will not a bottleneck. In addition, $\lambda_3 = \min\{\frac{1}{\log n}, n^{b-c-h}\}$ and $\lambda_1 = \min\{n^{b-h}, \frac{1}{\log n}\}$. As a result, with sufficiently large W_2, the overall throughput is $\min\{\lambda_1, \lambda_3\} = \Theta(\min\{\frac{1}{\log n}, n^{b-c-h}\})$. According to Theorem 6.1, when $h \leq a_2 - a_1$, this is obviously better than the standalone throughput without helping networks, $\lambda_p = \Theta(n^{-h})$.

If $h > a_2 - a_1$, we can further the situation into two cases:

If $c \leq a_2 - a_1$, according to Theorem 6.1 we have $\lambda_p = \Theta(\frac{n^{a_1-h-a_2}}{\sqrt{\log n}})$. Then if $h \geq 2(b-c) + a_2 - a_1$, it is easy to see that $\min\{\lambda_1, \lambda_2\} \leq \lambda_p$. So there will be no enhancement with helping network. Else if $h < 2(b-c) + a_2 - a_1$, with sufficiently large W_2, the helping network will improve the throughput significantly.

If $c > a_2 - a_1$, according to Theorem 6.1 we have $\lambda_p = \Theta(\max\{n^{-h}, \frac{n^{-c+\frac{a_2-h-a_1}{2}}}{\sqrt{\log n}}\})$. It is easy to prove that when $b \leq \frac{a_2+b-a_1}{2}$, there will be no enhancement with helping network. However, when $b > \frac{a_2+b-a_1}{2}$, with sufficiently large W_2, there will be significant enhancement with helping network.

The above discussion can be summarized by the following corollary:

Corollary 71 *With sufficiently large W_2, helping network can enhance the throughput of heterogeneous multicast network significantly only when one of the following cases holds:*

- *(i)* $b > c, h \leq a_2 - a_1$
- *(ii)* $b > c, h > a_2 - a_1, c \leq a_2 - a_1, h < 2(b-c) + a_2 - a_1$
- *(iii)* $b > c, h > a_2 - a_1, c > a_2 - a_1, b > \frac{a_2+h-a_1}{2}$

Then one may ask how large W_2 can be considered as sufficiently large in Corollary 7.1? In fact, specifically, we can let $W_2 = n^x$, where x is any positive constant. Then $\lambda_2 = \Theta(n^{x-h})$ must be achievable for Step II. In order to avoid letting λ_2 as a bottleneck, we can choose x as follows:

When $c < b < c + h$, choose $x \geq b - c$, thus the achievable overall throughput is $\lambda = \Theta(n^{b-c-h})$.

When $b \geq c + h$, choose $x \geq h$, thus the achievable overall throughput is $\lambda = \Theta(\frac{1}{\log n})$.

By choosing W_2 like above, we can say W_2 is sufficiently large and Corollary 7.1 holds.

Thereby, to expect a significant enhance in the throughout performance with helping networks, one should ensure that at least one condition in Corollary 7.1 holds.

7.2 Comparison between three kinds of networks: a specified case

Now we consider three kinds of networks:

- (i) Heterogeneous multicast network without helping network
- (ii) Heterogeneous multicast network with regular helping network
- (iii) Heterogeneous multicast network with mobile helping network

We consider a particular case as an example. The parameters are given as follows:$a_1 = 1, a_2 = 3, b = 9, h = 7, c = 5$. The bandwidth of the powerful wireless helping network is assumed to be n i.e., $W_2 = n$.

Then according to Theorem 3.1, Theorem 5.1 and Theorem 6.1, we get an achievable throughput λ_p,λ_s,λ_m for networks (i)(ii)(iii) as follows, respectively:

$$\lambda_p = \Theta(n^{-7}), \lambda_s = \Theta(n^{\frac{-11}{2}}), \lambda_m = \Theta\left(\min\left\{\frac{n^{-5}}{\log n}, n^{2r+6}\max\left\{n^{-7}, n^{-r-9}\right\}, \max\left\{n^{-7}, n^{-8-2r}\right\}\right\}\right)$$

, where $-2.5 < r < 1$ is the mobility radius exponent of the mobile helping network.

From the result of the above computation, we can see that with the help of regular helping network, the normal network can perform better. Furthermore, for some particular mobility radius exponent r, the mobility of helping network can also enhance the performance compared to static helping network. This is shown in Figure 1. In Figure 1, for simplicity, we ignore the $\log n$ term in the expression of λ_m since this will not influence the overall throughput heavily.

From Figure 1, we can see that when $-2.5 < r < -1.25$, mobile helping network performs better than regular helping network. Particularly, when $-2.5 < r < -2$, the throughput will increase as the mobility radius exponent r increases. However, when $r > -1.25$, mobile helping network performs worse than regular helping network due to the bottleneck of downlink in mobile helping network.

8 Conclusion

In this paper, we study the throughput of heterogeneous multicast networks with helping networks. We discuss with quite general conditions. For instance, the network area can be a rectangle and not necessarily a square. The growing speed of every parameter is an arbitrary power of n. We discuss for three cases: helping network is regular, helping network is randomly distributed, helping network is mobile. We also discuss the case when the heterogeneous multicast network is standalone without helping network. We then make a comparison between different networks and see that under certain conditions, mobile helping network is better than static helping network, static helping network is better than having no helping network, in the sense of throughput. If the helping network is wired but not wireless, we can just let $W_2 \to \infty$ i.e. helping network relay is not a bottleneck, to get the result.

As you can see, our analysis and result provide fundamental insight to heterogeneous multicast networks with helping networks.

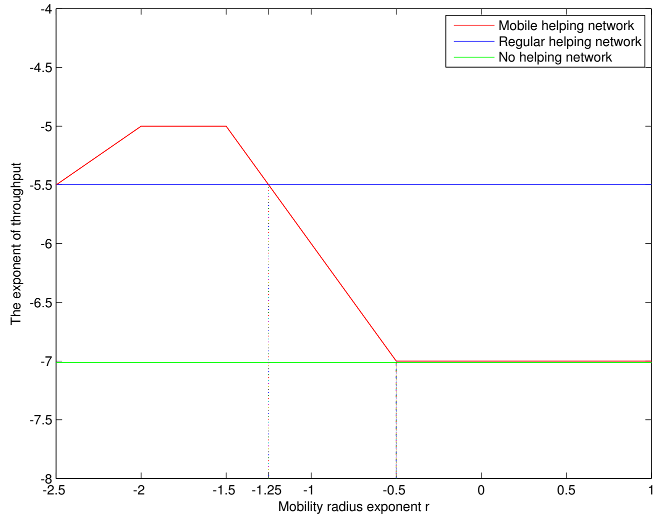

Fig. 1. Throughput Comparison between Different Helping Networks: A special case

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References

1. P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, pp. 388404, March 2000.
2. M. Franceschetti, O. Dousse, D. N. C. Tse, and P. Thiran, "Closing the Gap in the Capacity of Wireless Networks via Percolation Theory,"*IEEE Transactions on Information Theory,*vol. 53, no. 3, pp. 1009-1018, 2007.
3. X.-Y. Li, "Multicast capacity of wireless ad hoc networks,"*IEEE/ACM Trans. Netw.,*vol. 17, no. 3, pp. 950-961, Jun. 2009.
4. G. Alfano, M. Garetto, and E. Leonardi,"Capacity scaling of wireless networks with inhomogeneous node density: Upper bounds,"*IEEE J. Sel. Areas Commun.,*vol. 27, no. 7, pp. 1147-1157, Sep. 2009.
5. G. Alfano, M. Garetto, and E. Leonardi,"Capacity scaling of wireless networks with inhomogeneous node density: Lower bounds,"*Proc. IEEE INFOCOM,*2009, pp. 1890-1898.
6. Pan Li, Yuguang Fang,"On the Throughput Capacity of Hetergeneous Wireless Networks,"*IEEE Transactions on Mobile Computing,*2011.
7. P. Li and Y. Fang, "Impacts of topology and traffic pattern on capacity of hybrid wireless networks,"*IEEE Transactions on Mobile Computing,*8(12):1585-1595, December 2009.
8. B. Liu, Z. Liu, and D. Towsley, "On the capacity of hybrid wireless networks,"*Proceeding of the IEEE International Conference on Computer Communications (INFOCOM03),*an Francisco, California, USA, March 2003.
9. A. Zemlianov and G. Veciana, "Capacity of ad hoc wireless networks with infrastructure support,"*IEEE Journal on Selected Areas in Communications,*23(3), March 2005.
10. G. Zhang, Y. Xu, X. Wang, and M. Guizani, "Capacity of hybrid wireless networks with directional antenna and delay constraint,"*IEEE Transactions on Communications,*58(7):2097-2106, 2010.
11. U. Kozat and L. Tassiulas, "Throughput Capacity of Random Ad Hoc Networks with Infrastructure Support,"*Proc. ACM MobiCom,*June 2003.
12. P. Li, C. Zhang, and Y. Fang, "Capacity and Delay of Hybrid Wireless Broadband Access Networks,"*IEEE J. Selected Areas in Comm.,*special issue on broadband access networks, vol. 27, no. 2, pp. 117-125, Feb. 2009.
13. M. Grossglauser and D. Tse, "Mobility increases the capacity of ad hoc wireless networks,"*IEEE/ACM Transactions on Networking,*vol. 10, pp. 477-486, 2002.
14. XuFei Mao, Xiang-Yang Li, and Shao-Jie Tang, "Multicast Capacity for Hybrid Wireless Networks," *ACM MobiHoc,*2008.
15. Michele Garetto, Paolo Giaccone, Emilio Leonardi, "Capacity Scaling in Ad Hoc Networks with Heterogeneous Mobile Nodes: the Subcritical Regime,"*ACM/IEEE Transactions on Networking,*Vol.17, No.6, pp.1888-1901, ISSN: 1063-6692, December 2009
16. Michele Garetto, Paolo Giaccone, Emilio Leonardi, "Capacity Scaling in Ad Hoc Networks with Heterogeneous Mobile Nodes: the Supercritical Regime,"*IEEE/ACM Transactions on Networking,*Vol.17, No.5, pp.1522-1535, October 2009

17. Pan Li, Yuguang Fang, Jie Li, and Xiaoxia Huang, “Smooth Trade-offs Between Throughput and Delay in Mobile Ad Hoc Networks,” *IEEE Transactions on Mobile Computing,* vol. 11, no. 3, pp. 427-438, March 2012.
18. M. J. Neely and E. Modiano, “Capacity and Delay Tradeoffs for Ad-Hoc Mobile Networks,” *IEEE Transactions on Information Theory,* vol. 51, no. 6, pp. 1917-1937, June 2005.
19. Pan Li, Chi Zhang, and Yuguang Fang, “The Capacity of Wireless Ad Hoc Networks Using Directional Antennas,” *IEEE Transactions on Mobile Computing,* vol. 10, no. 10, pp. 1374-1387, October 2011.
20. G. Zhang, Y. Xu, Xinbing Wang, X. Tian, J. Liu, X. Gan, H. Yu, L. Qian, “Multicast Capacity for Hybrid VANETs with Directional Antenna and Delay Constraint,” *IEEE Journal on Selected Areas in Communications,* vol. 30, no. 4, pp. 818-833, April 2012.
21. G. Zhang, Y. Xu, Xinbing Wang, M. Guizani, “Capacity of Hybrid Wireless Networks with Directional Antenna and Delay Constraint,” *IEEE Transactions on Communications,* vol 58, no. 7, pp: 2097-2106, 2010.
22. X.Cao, J.Zhang, G.Zhang, L.Fu, X.Wang, “Heterogeneous Multicast Networks with Wireless Helping Networks” Techinical Report.