Project Report : Two-dimensional Contract Theory in Cognitive Radio Networks

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Abstract

While the spectrum resource of modern society is more and more insufficient, Cognitive Radio, which allows the Secondary Users (unlicensed users, SU) to access the licensed spectrum, is a promising solution to make the utilization of spectrum resource more efficient. Among many different paradigms of cognitive radio, market-driven spectrum trading has been proved to be an efficient way to deal with Cognitive Radio problems. In this paper, we consider the problem of spectrum trading with single primary user (PU) who has multiple spectra selling his idle spectra to multiple SUs in multiple types. Since there is only one PU, so it is a monopoly market, in which the PU sets the prices, powers and time for the spectrum he sells, just as a monopolist. SUs as customers choose the spectrum with exact price, power and time to buy. We model it as a two-dimensional power-time-price contract which is much different from the usual contract because the time could either be a strategy that an SU could decide to choose itself or a type which is not decided by SUs. We first discuss the situation in which the time is set as the strategy and we will prove that it can derive a feasible contract with some conditions. Then we will discuss the second situation in which the time is set as a type. In this situation, because the SU has two kinds of types, so it's difficult to make it become a feasible contract, however we will provide a solution to deal with this problem.

1 Introduction

Cognitive Radio (CR) has been viewed as a novel and promising approach for solving the scarcity in spectrum resource and inefficiency in spectrum usage. The key point about CR networks is the concept of dynamic spectrum sharing where the unlicensed cognitive radio users can opportunistically share the spectrum if the licensed users don't use it.

There are several comprehensive surveys on CR techniques [1], different spectrum sharing models [2], and challenges and issues in designing dynamic spectrum access networks [3]. And spectrum sharing whose aim is to satisfy the requirements of both primary and secondary users is an important part in CR networks. In [4], a game-theoretic adaptive channel allocation scheme was proposed for CR networks. In [5], a two tier dynamic spectrum allocation system was analyzed and in [6], a hybrid game approach which contains both cooperative behavior and competitive behavior was proposed. Further, Liu *et al.* proposed some special applications of spectrum sensing/accessing such as localization [7] [8] and monitoring [9].

However, all the above works are focusing on the technical aspect of spectrum sharing while we focus on the economic aspect in spectrum sharing which consider the incentive issue. This economic aspect which is also referred to as spectrum trading recently has been studied by many researchers. Such as in [10], Niyato *et al.* discussed the concept of spectrum trading in the context of different spectrum sharing models and outlined different forms of spectrum trading. In [11], a non-cooperative game based pricing scheme was proposed for uplink power control in CR networks. Among many papers about CR, the spectrum trading problem is often dealt with auction which is proved as a good way to solve the problem, such as in [12] [13] [14] [15]. But as an auction, the complexity is huge, the truthful issue always caused problems and it is not necessary to bid in every slot in a relatively static networks. Specifically, we consider the issue of spectrum trading between single PU and multiple SUs in a cognitive radio network, where we focus on the attribute of spectrum trading through the notions of *power* and *time*.

We model the trading process at a monopoly market, in which the PU acts as monopolist who sets the powers, time and prices for the spectrum he sells according to the first type *distance* of a SU in the first part. And in the second part, the PU sets the powers and prices according to the two kinds of types, *distance* and *time*, of a SU. For this purpose, we introduce the concept of contract in economics which has been used for solving CR spectrum trading problem in [19]. Contracts have been studied extensively in economics, (see [16] [17] [18]) while it was introduced into CR environment just in recent years. In [19], Lin Gao studied a monopolist-dominated quality-price contract, proposed the necessary and sufficient conditions for the contract and derived the optimal contract in this framework. In [20], a contract with the insurance is approached and they utilize insurance theory in spectrum trading in CR networks and model the market game as a four-stage Bayesian game. But all these papers only consider the situation that there is only one kind of type in the contract framework to identify the SUs and SUs only have one kind of strategy set. In this paper we propose a two dimensional contract which allows the SUs have two kinds of types or two different strategy sets. We will prove that this contract is feasible with some constraints in such a two dimensional framework and can derive the optimal contract as in [19].

The rest of this paper is organized as follows. In Section II we provide the system model and contract formulation. In Section III, we prove that a two dimensional contract with a power strategy set and a time strategy set is feasible. In Section IV, we study a two dimensional contract with two kinds of types. And finally, we conclude our work in Section V.

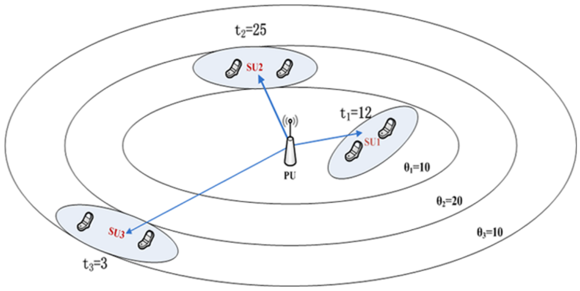

Figure 1: An example of CR network with single PU and multipe SUs.

2 System Model

2.1 PU Model

According to the system model in [19], we create a similar but different model, in which the cognitive radio network includes a primary network and a secondary network. There is only one primary user (PU) who has multiple spectra in the primary network. And we assume that the PU has several idle spectrum bands or channels at a particular time. The secondary network consists of a set of secondary transmitter and receiver pairs where each secondary transmitter does not always have or want to exchange packets with the receiver. We refer such a pair as secondary user (SU). And the PU is willing to sell his residual channels to SUs to gain more profit. An example of CR network with single PU and multiple SUs is shown in Figure 1 (the t_1, t_2 and t_3 represent the time and we will use δ to denote them in the rest of this paper).

First we use the model that the PU, as a monopolist, sets the the power, time and prices for SUs. We will discuss the situation in which time is a type which is not controlled by both PU and SUs in later. We obtain a set of all powers denoted by p $\in \Omega$, a set of all time denoted by $\delta \in \Delta$ and a set of all prices denoted by $\pi \in \Pi$. The SU decides whether to buy a channel and which power and time pair he is going to buy.

The power and time both have their lower-bounds and upper-bounds and can not be negative or infinite. We define them as $P_{min} \leq p \leq P_{max}$ and $\Delta_{min} \leq \delta \leq \Delta_{max}$. This definition is important because later we will see that only in this way this model could derive a feasible contract.

To get a feasible contract, we set a connection of δ and p to constrain them:

$$\delta \cdot p = k \tag{1}$$

where k is a constant. In reality, this equation means that the SU has limited energy and we will derive a feasible contract with this constraint. Further we will discuss the situation without this constraint in the second part of this paper in which the time is treated as a *type*, not *strategy*.

We define a cost function like in [19] as the expense of PU when SUs occupy and employ the channel. A little different, the function now consists of a fixed cost (i.e. the leasing fee of channel license) and a power and time specific cost (i.e. the interference caused by SUs' transmission). So we write it like this:

$$C(p,\delta) = C_0 + M(p,\delta) \tag{2}$$

where $C_0 > 0$ is the fixed cost and $M(p,\delta)$ is the power and time specific cost. Here we assume that $M_p(p,\delta) > 0$, $M_{pp}(p,\delta) \geq 0$, $M_\delta(p,\delta) > 0$, $M_{\delta\delta}(p,\delta) \geq 0$ and $M_{p\delta}(p,\delta) > 0$

We define the revenue of PU for selling one channel as $R(p,\delta)$ which is just the difference between the selling price and the cost:

$$R(p,\delta) = \pi(p,\delta) - C(p,\delta) \tag{3}$$

And also in the situation that a SU chooses to buy nothing, we denote $p = \delta = 0$ and the PU's revenue is also zero.

2.2 SU Model

Similar to that in [19], we set the first type θ according the Shannon-Hartley theorem:

$$\Phi(p) = W \log_2{(1 + p \cdot \frac{L_i}{I_i + J_i + \sigma^2})} \tag{4}$$

where W is the channel bandwidth, σ^2 is the noise variance, L_i is the path loss factor between the transmitter and receiver of SU i, I_i and J_i are respectively the interference come from the transmission of PU. Without loss of generality, we assume W = 1 and σ^2 is identical for all SUs.

We also use the expression $\frac{L_i}{I_i+J_i+\sigma^2}$ to denote the SUs' distance type, which is the first kind of type we will discuss. We refer an SU i as a type-θ SU if $\frac{L_i}{I_i+J_i+\sigma^2} = \theta$ and denote the set of all SUs' distance type as Θ, which can be either a discrete set or a continuous region.

And the SUs' type, both distance type and time type which we will see in the second part are all private information. But here to make the question simple, we assume the PU has some statical information about them.

Then we define a valuation function of a type-θ SU denoted by $V(\theta, p, \delta)$:

$$V(\theta, p, \delta) = w \log_2{(1 + p \cdot \theta)} \cdot \log_2{(1 + A \cdot \delta)} \tag{5}$$

where $w > 0$ is a predefined parameter to balance the unit and set the gain. A is also a parameter which has some constraints and we will see them later. p and δ are strategies that SUs can choose and represent power and time respectively. $p \in [P_{min}, P_{max}]$ and $\delta \in [\Delta_{min}, \Delta_{max}]$. The term $\log_2{(1 + A \cdot \delta)}$ means that if the SU chooses a relative long time to transmit, it will suffer the risk of conflict with PU and also it will suffer more interference, so the valuation's increasing will slow down. Obviously the valuation satisfy the condition that

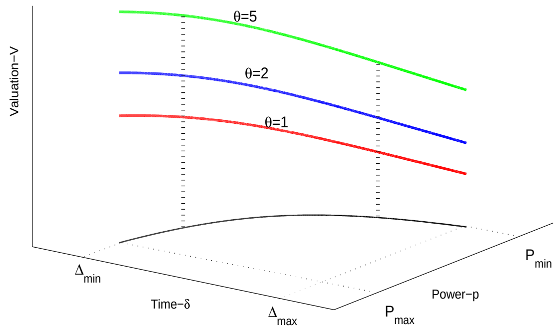

Figure 2: An illustration of valuation in 3 dimension.

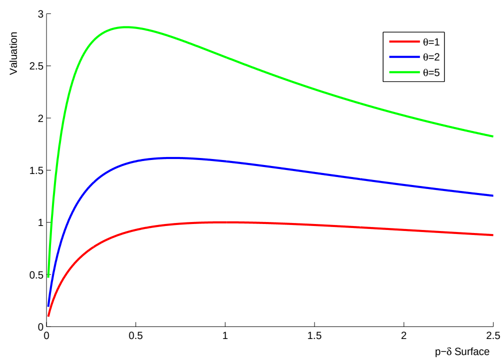

Figure 3: An illustration of valuation with $p \cdot \delta = k$.

$V(\theta, p, \delta) > V(\theta, 0, 0) = 0$. Figure 2 presents an illustration of the valuation function in 3 dimension.

We define the utility of a type-θ SU as:

$$U(\theta, p, \delta) = V(\theta, p, \delta) - \pi(p, \delta) \quad (6)$$

Here, we assume the SUs are all selfish and rational and maximizing his utility is the only task for each SU.

3 Feasible Contract

According to the contract theory, we define the *two-dimensional power-time-price contract* as:

$\Psi = \{(p(\theta), \delta(\theta), \pi(p(\theta), \delta(\theta)))|\forall\theta \in \Theta\}$, which can be simplified as $\Psi = \{(p(\theta), \delta(\theta), \pi(\theta))|\forall\theta \in \Theta\}$.

And a feasible contact is a set of power, time and price combinations. For every type θ, the SU will have an exact combination of power and time at an exact price to buy. And it is his best choice, which means he will not purchase anything else. According to the contract theory, it must be incentive compatible (IC), so we could write it like this:

$$V(\theta, p(\theta), \delta(\theta)) - \pi(\theta) \geq V(\theta, p(\theta'), \delta(\theta')) - \pi(\theta'), \forall\theta' \neq \theta \quad (7)$$

Because every SU is rational, so from Eq. (7) we could easily derive:

$$V(\theta, p(\theta), \delta(\theta)) - \pi(\theta) \geq V(\theta, 0, 0) - \pi(0) = 0 \quad (8)$$

So for a feasible contract the PU's utility function will be:

$$R = \sum_{\theta \in \Theta} N_\theta(\pi(\theta) - C(p(\theta), \delta(\theta))) \quad (9)$$

where N_θ is the number of type-θ SUs. Here we assume that PU has sufficient channels to lease to SUs.

To simplify the question, we assume there are finite types which are $\theta_1, \theta_2, \cdots \theta_T$, and further more we assume that $0 < \theta_1 < \theta_2 < \cdots < \theta_T$. We also rewrite N_{θ_t}, $p(\theta_t)$, $\delta(\theta_t)$ and $\pi(\theta_t)$ as N_t, p_t, δ_t, and π_t respectively for simplicity.

As discussed in [19], a feasible contract need an essential property for SUs' valuation: for a given δ increment , the valuation increment for a higher type SU is greater than that for a lower one. And the author refer to it as increasing preference property (IP) which formally can be written as the following proposition.

Proposition 1 - (IP property): For any type $\theta > \theta'$, $\delta > \delta'$ and $p < p'$, or $p' = \delta' = 0$ while $p \neq 0$ and $\delta \neq 0$ the following condition holds:

$$V(\theta, p, \delta) - V(\theta, p', \delta') > V(\theta', p, \delta) - V(\theta', p', \delta') \quad (10)$$

Proof. First we consider $p' = \delta' = 0$ and it is obviously right. Then we need to prove that $V_{\theta\delta}(\theta, p, \delta) > 0$ and just use the Spence-Mirrlees condition or single crossing property [21] . Because of the constraint that $p \cdot \delta = k$ we set before, the proof is like this:

$$V(\theta, p, \delta) = w \cdot \log_2 (1 + \frac{k}{\delta} \cdot \theta) \log_2 (1 + A \cdot \delta)$$

$$V_\theta(\theta, p, \delta) = \frac{w}{\ln 2} \cdot \frac{k}{\delta + k \cdot \theta} \log_2 (1 + A \cdot \delta)$$

$$V_{\theta\delta}(\theta, p, \delta) = \frac{w}{\ln 2} \cdot \frac{k}{(\delta + k \cdot \theta)^2} \left[\frac{A}{\ln 2} \cdot \frac{\delta + k\theta}{1 + A\delta} - \log_2 (1 + A \cdot \delta) \right]$$

Then we just need to make the terms in the bracket larger than zero. So we define $f(\theta, \delta) = \frac{A}{\ln 2} \cdot \frac{\delta + k\theta}{1 + A\delta} - \log_2 (1 + A \cdot \delta)$ and continue our proof:

$$f_\theta(\theta, \delta) = \frac{A}{\ln 2} \cdot \frac{k}{1 + A\delta} > 0$$

$$f_\delta(\theta, \delta) = \frac{A}{\ln 2} \cdot \frac{-(A\delta + Ak\delta)}{(1 + A\delta)^2} < 0$$

We find that $f(\theta, \delta)$ increases when θ increases and decreases when δ increases. In another word, the $f(\theta, \delta)$ is monotone increasing for θ and monotone decreasing for δ. So we find:

$$\lim_{\delta \to 0} f(\theta_1, \delta) = \frac{Ak\theta_1}{\ln 2} > 0$$

$$\lim_{\delta \to \infty} f(\theta_T, \delta) \to -\infty$$

So an interval $\delta \in [\Delta_{min}, \Delta_{max}]$ that satisfies $V_{\theta\delta}(\theta, p, \delta) > 0$ exists. That's why we set the $\delta \in [\Delta_{min}, \Delta_{max}]$ condition in the beginning of this paper. Here we derive $V_{\theta\delta}(\theta, p, \delta) > 0$, then using the fundamental theorem of calculus we have:

$$\begin{aligned} & V(\theta, p, \delta) - V(\theta, p', \delta') - V(\theta', p, \delta) + V(\theta', p', \delta') \\ &= \int_{\delta'}^{\delta} V_\delta(\theta, \frac{k}{y}, y) dy - \int_{\delta'}^{\delta} V_\delta(\theta', \frac{k}{y}, y) dy \\ &= \int_{\theta'}^{\theta} \int_{\delta'}^{\delta} V_{\theta\delta}(x, \frac{k}{y}, y) dy \cdot dx > 0 \end{aligned}$$

So for $\theta > \theta'$, $\delta > \delta'$ and $\delta \in [\Delta_{min}, \Delta_{max}]$, the equation above is always positive. Q.E.D.

Figure 3 in which the abscissa axis stands the $p \cdot \delta = k$, shows the illustration of the IP property in 2D. And the right of the axis stands for a large p and the internal we set above is on the left of the axis.

And there are still some lemmas necessary for a feasible contract, we will present them as follow.

Lemma 1: For any feasible contract $\Psi = \{(p_t, \delta_t, \pi_t)|\forall\theta \in \Theta\}$, $\delta_i \geq \delta_j$ if and only if $\pi_i \geq \pi_j$.

Proof. Using the IC constraint in Eq. (7), we can prove this lemma.
$\rightarrow$: First we prove that if $\delta_i \geq \delta_j$, then $\pi_i \geq \pi_j$. And before we use the IC constraint we need to prove that $V_\delta(\theta, p, \delta) > 0$:

$$\begin{aligned}
&V_\delta(\theta, p, \delta) = V_\delta(\theta, \frac{k}{\delta}, \delta) \\
&= \frac{w}{\ln 2} \cdot \frac{-\frac{k\theta}{\delta^2}}{1+\frac{k\theta}{\delta}} \log_2(1 + A\delta) + \frac{w}{\ln 2} \cdot \frac{A}{1+A\delta} \log_2(1 + \frac{k\theta}{\delta}) \\
&= \frac{w}{\ln 2}\left[\frac{A}{1+A\delta}\log_2(1+\frac{k\theta}{\delta}) - \frac{Ak\theta}{\delta+k\theta}\frac{\log_2(1+A\delta)}{A\delta}\right] \\
&> \frac{wA}{\ln 2}\left[\frac{1}{1+A\delta}\log_2(1+\frac{k\theta}{\delta}) - \frac{k\theta\ln 2}{\delta+k\theta}\right] \\
&= \frac{wA}{\ln 2}\frac{[\log_2(1+\frac{k\theta}{\delta}) - Ak\theta\ln 2] + k\theta[\log_2(1+\frac{k\theta}{\delta}) - \ln 2]}{(1+A\delta)(\delta+k\theta)}
\end{aligned}$$

We set two sufficient conditions to make sure that the $V_\delta(\theta, p, \delta) > 0$, the first is $k\theta_1 > (2^{\ln 2} - 1)\delta_{max}$ and the second is that $Ak\theta_T < 1$. And then we use the IC constraint, if $\delta_i \geq \delta_j$ then:

$$0 \geq V(\theta_j, p_j, \delta_j) - V(\theta_j, p_i, \delta_i) \geq \pi_j - \pi_i$$

So we could easily derive $\pi_i \geq \pi_j$.
$\leftarrow$: Then we prove if $\pi_i \geq \pi_j$, then $\delta_i > \delta_j$. Also we use the IC constraint and as a type-θ_i SU, we have:

$$V(\theta_i, p_i, \delta_i) - V(\theta_i, p_j, \delta_j) \geq \pi_i - \pi_j \geq 0$$

And as we proved above, the $V(\theta, p, \delta)$ is a strictly monotone increasing function on δ in our conditions. So we find that $\delta_i \geq \delta_j$. Q.E.D.

Lemma 2: For any feasible contract $\Psi = \{(p_t, \delta_t, \pi_t)|\forall\theta \in \Theta\}$, if $\theta_i > \theta_j$, then $\delta_i > \delta_j$.

Proof. Using the proof of contradiction, we assume that there exist $\theta_i > \theta_j$ and $\delta_i < \delta_j$. And using the IP property in Eq. (10), we have:

$$V(\theta_i, p_j, \delta_j) + V(\theta_j, p_i, \delta_i) > V(\theta_j, p_j, \delta_j) + V(\theta_i, p_i, \delta_i)$$

As discussed in [19], type θ_i and θ_j SU must also satisfy the IC constraints which will violate our assumption, so if $\theta_i > \theta_j$, then $\delta_i > \delta_j$. Q.E.D.

Lemma 3: For any feasible contract $\Psi = \{(p_t, \delta_t, \pi_t)|\forall\theta \in \Theta\}$, the following conditions hold:

- $\Delta_{min} \leq \delta_1 \leq \delta 2 \leq \cdots \leq \delta_T \leq \Delta_{max}$

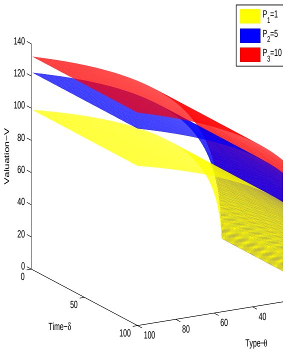

Figure 5: An illustration of valuation function with 2 kinds of types.

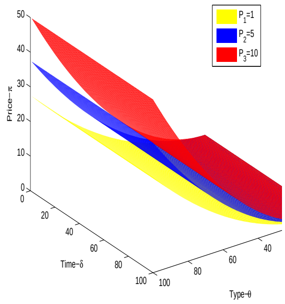

Figure 6: An illustration of $\pi(\delta, \theta, p)$ function with 2 kinds of types.

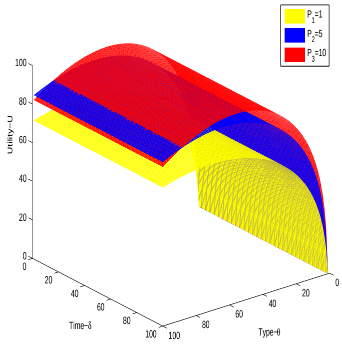

Figure 7: Convergence process to achieve the extremum and maximum point (DCC).

- $0 \le \pi_1 \le V(\theta_1, p_1, \delta_1)$, and
- For all $m = 2, 3, \cdots, T$,

$$\pi_{m-1} + B \le \pi_m \le \pi_{m-1} + C \tag{11}$$

where $B = V(\theta_{m-1}, p_m, \delta_m) - V(\theta_{m-1}, p_{m-1}, \delta_{m-1})$ and $C = V(\theta_m, p_m, \delta_m) - V(\theta_m, p_{m-1}, \delta_{m-1})$.

And the proof procedure is similar to that in [19], so we skip it.

Now we obtain a feasible two-dimensional power-time-price contract though it has some constraints and need some conditions. It can be proved that this feasible contract has all the property of a feasible contract and exists a optimal contract, but it is not the main point of this paper and has been done in the previous work.

4 Double Type Contract

As we discussed above, we treat the time as a strategy which an SU can control himself. Despite of the constraints needed to satisfy, in real world, there is another condition that the SU can not set his transmission time himself (i.e. the SUs are some relay points). So in this chapter, we reset the time as the type of an SU, not the strategy. So now the time is similar to θ we discussed in the first part and the PU will set the power and price according to the θ and δ. We use the same equation as the SUs' valuation function and because the θ and δ are two independent and separated variables, we could easily find $V_\theta(\theta, p, \delta) > 0, V_{\theta\theta}(\theta, p, \delta) < 0, V_\delta(\theta, p, \delta) > 0, V_{\delta\delta}(\theta, p, \delta) < 0$ and $V_{\delta\theta}(\theta, p, \delta) > 0$.

It is not reasonable to make connection between θ and δ like what we did in Eq (1) which makes it difficult to make this double type contract suit the general contract formation. Because we could not derive the IP constraint, we try to think and solve it in reverse. First we assume that the p is distributed value, which means we have a set of p like: $P_{min} \leq p_1 \leq p_2 \leq \cdots \leq p_T \leq P_{max}$.

Then we redefine this problem as type-p SUs choose θ and δ which means now we think the p as a parameter which is known. However, this problem can not be solved as the first part of this paper because there is no connection between θ and δ and we can not derive the IP property as in Eq. (10). So we need some tricks here. We assume that we have already known the price function and rewrite the π function as:

$$\pi(\theta, \delta, p) = \mu(\theta, \delta) \cdot (\alpha + \beta p^2) \tag{12}$$

where we assume that $\pi_\theta(\theta, \delta, p) > 0$, $\pi_{\theta\theta}(\theta, \delta, p) > 0$, $\pi_\delta(\theta, \delta, p) > 0$, $\pi_{\delta\delta}(\theta, \delta, p) > 0$ and $\pi_{\delta\theta}(\theta, \delta, p) > 0$. So the SUs' utility will be like this:

$$U(\theta, \delta, p) = V(\theta, \delta, p) - \pi(\delta, \theta, p); \tag{13}$$

Because the $\pi(\delta, \theta, p)$ is a concave function and the $V(\theta, \delta, p)$ is a convex function, so for each fixed p_i there must be at least one point making the $U(\theta, \delta, p)$ maximum. And because the derivative of the utility function is continuous, so there will be at least one point both extremum and maximum. We refer such a maximum to a *double-type contract combination* denoted by $DCC_i(p_i, \delta_i, \theta_i)$. And you will see this more clearly in the Fig. 4 , Fig. 5 and Fig. 6.

And here we also need some lemmas, we present them as follow:

Lemma 4: There will not be a DCC_i and a DCC_j holding the conditions that: $p_i \neq p_j, \delta_i = \delta_j$ and $\theta_i = \theta_j$.

Lemma 4 indicates that there won't a (θ_i, δ_i) pair that have more than one maximum for all value of p, which can be regard as a special version of IC constraint.

Proof. Because all DCC points are maximum points and the SUs' utility function's derivative is continuous, a DCC_i must satisfy the following conditions:

$$U_\theta(\theta_i, \delta_i, p_i) = 0, U_\delta(\theta_i, \delta_i, p_i) = 0$$

And we could calculate that:

$$U_\theta(\theta, \delta, p) = \frac{w}{\ln 2} \cdot \frac{p}{1 + p\theta} \log_2(1 + A\delta) - \mu_\theta(\theta, \delta)(\alpha + \beta p_i^2)$$

$$U_{\theta p}(\theta, \delta, p) = \frac{w}{(\ln 2)^2} \cdot \frac{1}{(1 + p\theta)^2} \log_2(1 + A\delta) - 2\mu_\theta(\theta, \delta)\beta p$$

$$U_{\theta pp}(\theta, \delta, p) < 0, U_{\theta p}(\theta, \delta, 0) = \frac{w\theta}{(\ln 2)^2} \log_2(1 + A\delta) > 0,$$

$$and \lim_{p \to \infty} U_{\theta p}(\theta, \delta, p) \to -\infty.$$

So there is existing a interval $[P_{min}, \infty)$ that make the $U_\theta(\theta, \delta, p)$ monotone decrease. And same to $U_\delta(\theta, \delta, p)$, that's why we set the P_{min}. Q.E.D.

Lemma 5: For a DCC_i, there will not be a $p_j \neq p_i$ that make the $U(\theta_i, \delta_i, p_j) > U(\theta_i, \delta_i, p_i)$.

Lemma 5 indicates that if it is a DCC_i, then the p_i will be the optimal strategy for $U(\theta_i, \delta_i, p)$ which also means the $U(\theta_i, \delta_i, p_i)$ is the maximum for $U(\theta_i, \delta_i, p)$. Of course this proof need some constraints and we will present them later.

Proof. We first calculate the derivative of p.

$$U_p(\theta_i, \delta_i, p) = \frac{w}{\ln 2} \cdot \frac{\theta_i}{1 + p\theta_i} \log_2(1 + A\delta_i) - 2\mu(\theta_i, \delta_i)\beta p$$

$$U_{pp}(\theta_i, \delta_i, p) = -\frac{w}{\ln 2} \cdot \frac{\theta_i^2}{(1 + p\theta_i)^2} \log_2(1 + A\delta_i) - 2\mu(\theta_i, \delta_i)\beta < 0$$

Because $U_p(\theta_i, \delta_i, 0) = \frac{w\theta_i}{\ln 2} \log_2(1 + A\delta_i) > 0$ and $\lim_{p\to\infty} U_p(\theta_i, \delta_i, p) \to -\infty$. We assert that the utility function will first increase and then decrease on p, so there must be an optimal point in our condition. And because a DCC_i is a maximum point, we have:

$$\begin{aligned} &U_\theta(\theta_i, \delta_i, p_i) \\ &= \frac{w}{\ln 2} \cdot \frac{p_i}{1 + p_i\theta_i} \log_2(1 + A\delta_i) - \mu_\theta(\theta_i, \delta_i)(\alpha + \beta p_i^2) = 0 \end{aligned}$$

We set $\mu(\theta, \delta) = \theta^2\gamma(\delta)$ and $\alpha = 0$. Then we will find at this point:

$$\begin{aligned} &U_p(\theta_i, \delta_i, p_i) \\ &= \frac{w}{\ln 2} \cdot \frac{\theta_i}{1 + p_i\theta_i} \log_2(1 + A\delta_i) - 2\mu(\theta_i, \delta_i)\beta p_i \\ &= \frac{\theta}{p_i} \cdot U_\theta(\theta_i, \delta_i, p_i) = 0 \end{aligned}$$

So we assert that there will not be a $p_j \neq p_i$ that make the $U(\theta_i, \delta_i, p_j) > U(\theta_i, \delta_i, p_i)$ Q.E.D.

Now, we calculate all the (θ_i, δ_i) pairs which are exactly corresponding to (p_i, π_i) and satisfy the basic property of contract theory. But what we must point out is that it's not a general contract because of the Lemma 5 is a sufficient condition not a necessary and sufficient condition. It can ensure that there will at least one double-type SU at $p_i \in [P_{min}, P_{max}]$, but can not ensure that all double-type SUs will have a feasible $p \in [P_{min}, P_{max}]$ to purchase.

And for such a situation, one solution is making the number of different p larger and try to cover more (θ, δ) pairs. And then we select the pairs which exist in our data-center (because we have a statistical information of SUs). If you calculate enough p, the contract will be very close to an absolute feasible contract. But theoretically it could not be a absolute feasible contract which covers all the (θ, δ) pairs like in the first part of this paper .

5 Conclusion

In this paper, we study the spectrum trading with single PU and multiple SUs and model this trading process as monopoly market, in which the PU acts as monopolist and the SUs act as consumers. We provide a two-dimensional power-time-price contract which is offered by the PU and consists of a set of power-time-price combinations each intended for a consumer type in the first part. We propose the necessary and sufficient conditions for the contract to be feasible. Further we consider this power-time-price contract with two kinds of types which means time act as a type not strategy of a SU. And finally we provide a feasible solution to this two-dimensional power-time-price contract and propose the necessary and sufficient conditions for it.

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