Interference's Impact on Critical Power

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1 instruction

Wireless communication systems consist of nodes which share a common wireless medium. Signals intended for a receiver can cause interference at other receiver nodes, resulting in reduced signal to noise ratio at the latter receivers. It is thus of interest to regulate the transmitter power around a level adequate for the transmitted signals to reach their intended receivers, while causing minimal interference for other receivers sharing the same channel. To achieve this objective, many iterative power control algorithms have been developed.

We examine the problem from the following perspective. Assume that nodes in the network cooperate in routing each others data packets, perhaps in a distributed fashion, as in mobile ad hoc networks, which are formed by a group of mobile nodes communicating with each other over a wireless channel without any centralized control. However, there is a strong interference source in the very area. A critical requirement is that each node in the network has a path to every other node in the network, i.e., the network is connected. We determine the critical power at which each node needs to transmit so as to guarantee asymptotic connectivity of the network, i.e., when the number of nodes is large.

More precisely, we consider the following problem: Let $\mathscr{D}$ be a disc in $\mathbb{R}^2$ normalized to have unit area. Let $\mathscr{G}(n, r(n))$ be the network formed when n nodes are placed randomly in $\mathscr{D}$ with a uniform probability distribution, and independently of each other. Meanwhile, a strong interference source of same channel locates in the very area. Suppose the power of interference is P_I and the nodes share the critical power P. Let the range of each nodes transmission be $r(n)$, i.e., if x_k is the location of node k, nodes i and j can communicate in one hop if $||x_i - x_j|| \le r(n)$(The norm used is the Euclidean norm). $r(n)$ can be determined by P considering certain signal-to-interference-noise ratio. Then the problem is to determine P guarantees that $\mathscr{G}(n, r(n))$ is asymptotically connected with probability one, i.e., the probability that $\mathscr{G}(n, r(n))$ is connected, denoted by $P_c(n, r(n))$, goes to one as $n \to \infty$, considering the influence from the strong interference source. We show that if $P = \frac{c \log n + \kappa(n)}{n(1-\varrho P_I)}$ then $P_c(n, r(n)) \to 1$ if and only if $\kappa(n) \to \infty$, and c,ϱ are positive constants. From the result, we can find that the power of nodes increases with stronger interference source. However, when the interference is strong enough, the change of the power of nodes may be less significant with same boost of interference.

2 Necessary Condition

Firstly, we try to determine the range of nodes and interference source according to their critical transmission power in different circumstances. In order to guarantee transmission, the $SINR$

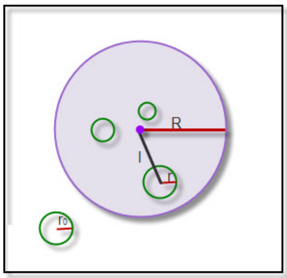

Figure 1: The network model

must satisfy the following equation:

$$SINR = \frac{Pr^{-2}}{N + I + P_I l^{-2}} \geq \alpha \tag{1}$$

where N is supposed to be Gaussian white noise, I is the influence from the neighborhood and $P_I l^{-2}$ comes from the strong interference source. Here, we assume that the path loss in the model is 2. Since the environment is relatively stable and the nodes are distributed uniformly, we can reasonably assume that N and I are constant and $N + I = C_1$.Therefore, we can find that the transmission ranges r of the nodes vary according to distance from the interference source l.

From the constraint above, we can directly obtain the transmission range of the interference source R and the node without such source r as

$$\begin{aligned} R &= \sqrt{\frac{P_I}{\alpha C_1}} \\ r_0 &= \sqrt{\frac{P}{\alpha C_1}} \end{aligned} \tag{2}$$

When the interference locates near the node, the transmission range is

$$r = \sqrt{\frac{P}{\alpha(P_I l^{-2} + C_1)}} \tag{3}$$

Theorem 1. If $P = \frac{c \log n + \kappa(n)}{n(1 - \varrho P_I)}$, then

$$\lim_{n \to \infty} \inf P_f(\mathscr{G}) \geq e^{-\kappa}(1 - e^{-\kappa}) \tag{4}$$

where $\kappa = \lim_{n \to \infty} \kappa(n)$.

In the certain model, the expected coverage area of all the nodes is

$$S = \int_0^R \pi r^2 (2\pi l) \mathrm{d}l + (1 - \pi R^2)(\pi r_0^2) \tag{5}$$

The first term of the left side can be simplified as

$$
\begin{aligned}
S_1 &= \int_0^R \pi r^2 (2\pi l)\mathrm{d}l \\
&= \int_0^R \frac{2\pi^2 P l \mathrm{d}l}{\alpha(P_I l^{-2} + C_1)} \\
&= \frac{2\pi^2 P}{\alpha} \int_0^R \frac{l^3 \mathrm{d}l}{C_1 l^2 + P_I} \\
&= \frac{\pi^2 P}{\alpha}(R^2 - \frac{P_I}{C} ln \frac{C_1 R^2 + P_I}{P_I})
\end{aligned}
\tag{6}
$$

Since $R = \sqrt{\frac{P_I}{\alpha C_1}}$

$$
S_1 = \frac{\pi^2 P_I P C_2}{\alpha} \tag{7}
$$

Similarly, the second term

$$
S_2 = (1 - \pi \frac{P_I}{\alpha C_1})\pi \frac{P}{\alpha C_1} \tag{8}
$$

Thus, we can obtain the expected coverage of each node.

$$
S = (1 - \varrho P_I) C_3 P \tag{9}
$$

where $\varrho = \frac{\pi}{C_1} \log \frac{\alpha+1}{\alpha} > 0$, $C_3 = \frac{\alpha C_1}{\pi} > 0$.

Let $P_f(\mathscr{G})$ denote the probability that $\mathscr{G}(n, r(n))$ is not fully disconnected. Then

$$
\begin{aligned}
P_f(\mathscr{G}) &\geq \sum_{i=1}^{n} P(\text{node } i \text{ is the only isolated nodes}) \\
&\geq \sum_{i=1}^{n} P(\text{node } i \text{ is isolated}) - \sum_{i=1}^{n} \sum_{j \neq i}^{n} P(\text{node } i \text{ and } j \text{ are isolated})
\end{aligned}
\tag{10}
$$

Respectively, we can evaluate the two terms on the right side. For the first term, we have

$$
P(\text{node } i \text{ is isolated}) = \prod_{j \neq i}^{n} P(\text{node } i \text{ is isolated from node } j) \quad = (1 - S)^{n-1} \tag{11}
$$

Then we can bound the first term that for $\theta < 1$,

$$
\sum_{i=1}^{n} P(\text{node } i \text{ is isolated}) \geq \theta e^{-\kappa} \tag{12}
$$

Thus, for two points P_i and P_j in $\mathscr{G}$, we obtain that

$$
P(\text{node } i \text{ and } j \text{ are isolated}) = n(n-1)[(1-4S)(1-2S)^{n-2} + (4S - S)(1 - \frac{5}{4})^{n-2} \leq (1+\epsilon)e^{-2\kappa} \tag{13}
$$

for all $n > N(\epsilon, \theta, \kappa)$. Therefore

$$
P_f(\mathscr{G}) \geq \theta e^{-\kappa} - (1+\epsilon)e^{-2\kappa} \tag{14}
$$

for all $n > N(\epsilon, \theta, \kappa)$. Now, consider the case where κ is a function $\kappa(n)$ with $\lim_{n\to\infty} \kappa(n) = \bar{\kappa}$. Then, for any $\epsilon > 0$, $\kappa(n) < \bar{\kappa} + \epsilon$ for all $n > N'(\epsilon)$. Also, the probability of disconnectedness is monotone decreasing in κ. Hence

$$P_f(\mathscr{G}) \geq \theta e^{-(\bar{\kappa}+\epsilon)} - (1+\epsilon)e^{-2(\bar{\kappa}+\epsilon)} \tag{15}$$

for $n \geq \max N(\epsilon, \theta, \bar{\kappa} + \epsilon), N'(\epsilon)$. Taking limits

$$\lim_{n\to\infty} \inf P_f(\mathscr{G}) \geq \theta e^{-(\bar{\kappa}+\epsilon)} - (1+\epsilon)e^{-2(\bar{\kappa}+\epsilon)} \tag{16}$$

Since this holds for all $\epsilon > 0$ and $\theta < 1$, the result follows.

3 Sufficient Condition

Let E_i denote the event where node i is disconnected, i=1,2,,n. Using the union bound, with$P = \frac{c \log n}{n(1-\varrho P_I)}$, we have

$$\begin{aligned} P(\bigcup_{i=1}^{n} E_i) &\leq \bigcup_{i=1}^{n} P(E_i) \\ &= n(1-S)^{n-1} \\ &\leq ne^{(n-1)[-(1-\varrho P_I)C_3 P]} \\ &\leq \frac{n}{n^c} \to 0(\text{when } n \to \infty) \end{aligned} \tag{17}$$

Therefore, $P = \frac{c \log n + \kappa(n)}{n(1-\varrho P_I)}$ is sufficient to guarantee connectivity of the whole area.

4 Another Possible Model

In the previous model, we assume that the power of interference and noise in the neighborhood can be represented by a constant. In fact, the interference from other nodes can be affected by the nodes' power, i.e. for any node j, the interference from other nodes can be presented as

$$I = \sum_{i \neq j} P d_i^{-2}$$

where d_i is the distance between the node i and node j. Since $\sum_{i\neq j} d_i^{-2}$ converges when n goes to infinity and the nodes are uniformly distributed, I has a linear relationship with P, that is, $I = kP$. But such a small parameter may cause great complexity later on. The calculation of expected area is almost the same, but we need to replace C_1 with $N + kP$. The power of nodes must satisfy the following equation:

$$\frac{C_4 P(kP + N) + C_5 P P_I}{(N + kP)^2} = \frac{\log n}{n} \tag{18}$$

and the result can be very complicated. However, we can still simplify the problem by a new threshold with a little allowance ϵ, so that

$$SINR = \frac{Pr^{-2}}{I + P_I l^{-2}} \geq \alpha + \epsilon = \alpha'$$

then the relationship between P and P_I can be interpreted as

$$P = \Theta\left(\frac{P_I}{\frac{n}{\log n} - C_6}\right) \tag{19}$$

where C_6 is a constant.