Project Report:Truthful Spectrum Auction for Secondary Networks

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1 ABSTRACT

Auctions represent a natural mechanism for allocating the spectrum, generating an economic incentive for the licensed user to relinquish channels. A severe limitation of existing spectrum auction designs lies in the oversimplifying assumption that every non-licensed user is a single-node or single-link secondary user. While such an assumption makes the auction design easier, it does not capture practical scenarios where users have multihop routing demands. For the first time in the literature, we propose to model non-licensed users as secondary networks (SNs), each of which comprises of a multihop network with end-to-end routing demands. We aim to design truthful auctions for allocating channels to SNs in a coordinated fashion.

2 INTRODUCTION

Recent years there was a substantial growth in wireless technology and applications, which rely crucially on the availability of bandwidth spectrum. Traditional spectrum allocation is prone to inefficient spectrum utilization in both temporal and spatial domains: large spectrum chunks remain idling while new users are unable to access them. Such an observation has prompted research interest in designing a secondary spectrum market, where new users can access a licensed channel when not in use by its owner, with appropriate remuneration transferred to the latter.

In a secondary spectrum market, a spectrum owner or primary user (PU) leases its idle spectrum chunks (channels) to secondary users (SUs) through auctions [1], [2]. SUs submit bids for channels, and pay the PU a price to access a channel if their bids are successful. A natural goal of spectrum auction design is truthfulness, under which an SUs best strategy is to bid its true valuation of a channel, with no incentive to lie. A truthful auction simplifies decision making at SUs, and lays a foundation for good decision making at the PU.

A unique feature of spectrum auction design is the need of appropriate consideration for wireless interference and spatial reuse of channels. A channel can be allocated to multiple SUs provided that they are far apart, with no mutual interference. Optimal channel assignment for social welfare maximization is equivalent to the graph coloring problem, and is NP-hard [3], even assuming truthful bids are given for free. Existing works on spectrum auctions mostly focus on resolving such a challenge (e.g., [4], [5]) while assuming the simplest model of a SU: a single node, or a single link, similar to a single hop transmission in cellular networks [2], [4], [5].

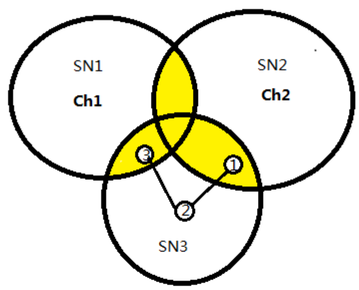


Figure 1: A secondary spectrum market with 3 SNs and 2 channels

After extensive research, auction design for single-hop users, each requesting a single channel, has been relatively well understood. However, a practical SU may very well comprise of multiple nodes forming a multihop network, which we refer to as a secondary network (SN). These include scenarios such as users with multihop access to base stations, or users with their own mobile ad hoc networks. SNs require coordinated end-to-end channel assignment, and in general benefit from multi-channel diversity along its path. The SN model subsumes the SU model as the simplest special case.

Fig. 1 depicts three co-located SNs, SN1, SN2 and SN3, which have interference with one another, because their network regions overlap. The primary network (PN) has two channels, Ch1 and Ch2, which have been allocated to SN1 and SN2, respectively. Now SN3 wishes to route along a two-hop path 1-2-3. Under existing single-channel auctions for SUs, SN3 cannot obtain a channel, because each channel interferes with either SN1 or SN2. However, a solution exists by relaxing the one channel per user assumption, and assigning Ch1 to link 1-2 and Ch2 to the link 2-3. In general, taking multichannel, multihop transmissions by SNs into consideration can apparently improve channel utilization and social welfare. Note here that the model in which an SN bids for multiple channels is inapplicable, because due to the unawareness of other SNs information, an SN cannot know the number of channels to bid for, to form a feasible path.

3 PRELIMINARIES

3.1 Truthful Auction Design

An auction allocates items or goods (channels in our case) to competitive agents with bids and private valuations. We adopt wi as nonnegative valuations of each agent i, which is often private information known only to the agent itself. Besides determining an allocation, an auction also computes payments/charges for winning bidders. We denote by p(i) and bi the payment and bid of agent i, respectively. Then the utility of i is a function of all the bids:

$$u_i(b_i, b_{-i}) = \omega_i - p(i) \tag{1}$$

where b_{-i} is the vector of all the bids except b_i. We first adopt some conventional assumptions in economics here. We assume that each agent i is selfish and rational. A selfish agent is one that acts strategically to maximize its utility. An agent is said to be rational in that it always prefers the outcome that brings itself a larger utility. Hence, an agent i may lie about its valuation, and bid $b_i \neq w_i$ if doing so yields a higher utility

Truthfulness is a desirable property of an auction, where reporting true valuation in the bid is optimal for each agent i, regardless of other agents bids. If agents have incentives to lie, other agents are forced to strategically respond to these lies, making the auction and its analysis complex. A key advantage of a truthful auction is that it simplifies agent strategies. Formally, an auction is truthful if for any agent i with any $b_i \neq w_i$, any b_{-i}, we have

$$u_i(\omega_i, b_{-i}) \geq u_i(b_i, b_{-i}) \tag{2}$$

Theorem 1. Let $P_i(b_i)$ be the probability of agent i with bid b_i winning an auction. An auction is truthful if and only if the followings hold for a fixed b_{-i} $P_i(b_i)$ is monotonically non-decreasing in b_i; Agent i bidding b_i is charged $b_i P_i(b_i) - \int P_i(b)db$ Given Theorem 1, there must be an crucial bid b_i^*, such that the agent i will win if he bids at least b_i^*.

3.2 System Model

We assume there is a set of SNs. Each SN has deployed a set of nodes in a geographical region, and has a demand for multihop transmission from a source to a destination. A PN has a set of channels, C, available for auctioning in the region. We refer to SNs as agents and the PN as the auctioneer. Each node within an SN is equipped with a radio that is capable of switching between different channels. SNs do not collaborate with each other, and nodes from different SNs are not required to forward traffic for each other.

We assume nodes from each SN i form a connected graph $G^i(E^i; V^i)$, which also contains node locations. We use node and link for the connectivity graphs and vertex and edge for the conflict graph introduced later. To better formulate the joint routing-channel assignment problem, we incorporate the concept of network flows. Let u^i be a node in SN i and s^i, d^i be the source and the destination in SN i. We use l^i_{uv} to denote the link from node u^i to node v^i belonging to SN i, and f^i_{uv} to denote the amount of flow on link l^i_{uv}. Later we connect d^i back to s^i with a virtual feedback link l^i_{ds}, for a compact formulation

of the joint optimization IP.
We define a conflict graph $H(\xi_H; \nu_H)$, whose vertices correspond to links from all the connectivity graphs. We use $(l^i_{uv}; l^i_{pq})$ to denote an edge in EH, indicating that link l^i_{uv} and link l^i_{pq} interfere if allocated a common channel. Before the auction starts, each SN i submits to the auctioneer a compound bid, defined as $B_i = (G_i(\xi_i; \nu_i); s_i; d_i; b_i)$. Then the conflict graph can be centrally obtained by the auctioneer. We denote by wi the private valuation of SN i for a feasible path between s and d, and p(i) its payment. bi, wi and p(i) all represent monetary amounts. Note that we assume agents only have incentives to lie about their valuations.
Hence, for the joint routing-channel assignment problem we have the Channel Interference Constraints:

$$x(c, l^i_{uv}) + x(c, l^j_{pq}) \leq 1 \tag{3}$$

We also need Flow Conservation Constraints:

$$\sum f^i_{uv} = \sum f^i_{vu} \tag{4}$$

Assuming each channel has the same unit capacity 1, we have the Capacity Constraints:

$$\sum f^i_{uv} \leq \sum x(c, l^i_{uv}) \leq 1 \tag{5}$$

which also ensures that a link can be assigned a single channel only.

4 TRUTHFUL AUCTION DESIGN

4.1 Channel Allocation

As discussed before, the key to designing a truthful auction is to have a non-decreasing allocation rule. Prices can then be calculated by the critical bids to make the auction truthful. A greedy allocation is adopted in Algorithm 1. Assume channels are indexed by 1; 2; ...C. For a simple heuristic auction, we first compute the shortest path for each agent as its end-to-end path. Let $I_s(i)$ be the set of SNs that interfere with i along the path. We define the virtual bid of SN i as

$$\phi(i) = \sum \frac{b_i}{m \cdot I_s(ij)} \tag{6}$$

The rationale behind scaling the bid by $|I_s(i)|$ is to take is interference with other agents into consideration, for heuristically maximizing social welfare. Then we greedily assign minimum indexed available channels along the paths to each link, according to a non-increasing order of virtual bids $\phi(i)$.
The steps are as follows:
(1)Compute the shortest path for each agent as its end-to-end path
(2)Calculate virtual bid of SN i$I_s(i)$ is the set of SNs that interfere with i along the path,j means the number j hop and total m):
(3)Assign minimum indexed available channels along the paths to each link

Fig. 2 shows an example to illustrate the channel assignment procedure. There are four SNs, a, b, c and d, where $\phi(a) > \phi(b) > \phi(c) > \phi(d)$. Two channels are available for allocation. In the figure, two intersecting links also interfere with each other. If two links from two different SNs intersect, they cannot be allocated with the same channel. The algorithm first assigns Channel 1 to SN a. As a result, it cannot assign Channel 1 to the first link of SN b, which receives Channel 2 instead, as shown in Fig. 2b, leaving SN c without a channel it is impossible to assign either channel to cs first link. However, SN d wins, and receives a channel assignment along its path without introducing interference to a or b.

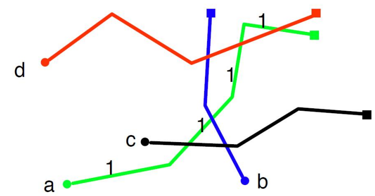
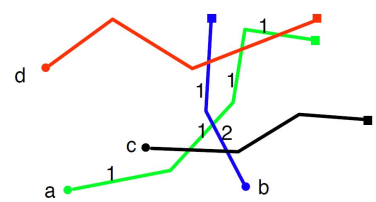


Figure 2:



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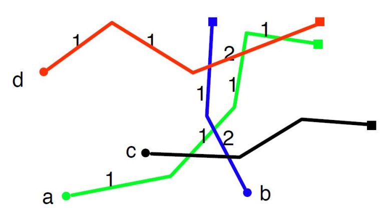

Figure 4:

4.2 Payment Calculation

The steps to calculate payment are as follows:
(1)In every hop, calculate the average bid $S(i) = \frac{1}{n} \cdot \sum \frac{b_j}{m}$
(2)Agent is payment can be computed as follows:

$$p(i) = \sum S(i) \tag{7}$$

Theorem 2. The auction in Algorithms 1 and 2 is truthful and individually rational. Define $r(b_i) = 0$ when agent i doesnt receive a channel and $r(b_i) = 1$ when agent i receives a channel When $\omega_i^* < \omega_i$,

- $r(\omega_i^*) = 0$,$r(\omega_i) = 0$ no incentive to lie
- $r(\omega_i^*) = 1$,$r(\omega_i) = 0$ impossible
- $r(\omega_i^*) = 0$,$r(\omega_i) = 1$ rational, no incentive to lie
- $r(\omega_i^*) = 1$,$r(\omega_i) = 1$ the critical bidder does not change

When $\omega_i^* > \omega_i$,

- $r(\omega_i^*) = 0$,$r(\omega_i) = 0$ no incentive to lie
- $r(\omega_i^*) = 1$,$r(\omega_i) = 0$ negative utility $p(i) > \omega(i)$
- $r(\omega_i^*) = 0$,$r(\omega_i) = 1$ impossible
- $r(\omega_i^*) = 1$,$r(\omega_i) = 1$ the critical bidder does not change

Thus, the auction we designed is truthful.

5 FUTURE WORK

Improve the performance guarantee of the randomized auction, by proving a tighter bound on social welfare approximation

6 REFERENCES

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