# Spectrum Trading in Cognitive Radio Networks and Partial Spectrum Trading between Primary Networks

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*Abstract*—**This is the Project Report for Wireless Network Communication under the guide of Pro. Xinbing Wang and Phd. Xiaohua Tian. Cognitive Radio is a promising technology to solve the inefficiency of the usage of spectrum resource. In my project, I study spectrum trading in Cognitive Radio Network (CRN). A large number of papers have been read and several advanced approaches are studied and related works are introduced. Moreover, an interesting problem about the partial spectrum sharing between the primary networks are raised and an tentative step to study this problem, which includes a Sealed First-price Auction and Supermodular Game, is given. Based on the results, I provide some discussions and some future works for the problem are proposed in the end.**

## I. Introduction

With the development of the wireless communication and the smart mobile personal devices, spectrum is becoming more and more precious. It seems that that people are facing the problem of spectrum shortage in a short time. However, from some works [1] [2] [3], several conclusions can be drawn: (1) some frequency bands in the spectrum are largely unoccupied most of the time; (2) some other frequency bands are only partially occupied; (3) the remaining frequency bands are heavily used. More recent measurement studies have shown that the licensed spectrum bands are severely underutilized at any given time and location [4] [5], mainly due to the traditional command-and-control type spectrum regulation that has prevailed for decades. According to Federal Communications, Commission (FCC) [6], temporal and geographical variations in the utilization of the assigned spectrum range from $15\%$ to $85\%$. Although the traditional fixed spectrum assignment policy generally served well in the past, the increasing demand of spectrum resource in the coming days will cause the problem of spectrum scarcity. Thus, we can see that the real problem is inefficiency of spectrum usage rather than spectrum scarcity in quantity. In other words, a more effective and flexible technology or scheme for spectrum usage should be figured out.

Cognitive radio, which is short for CR, has been proposed as a promising technology to sense and utilize the precious wireless spectrum resources. With software-defined radio (SDR) and smart antennas, CR is a highly context-aware intelligent radio, which is able to autonomously reconfigure its parameters in order to learn form and adapt to the communication environment. The flexible feature of CR makes it possible to utilize the spectrum hole and to improve the efficiency of spectrum resource significantly.

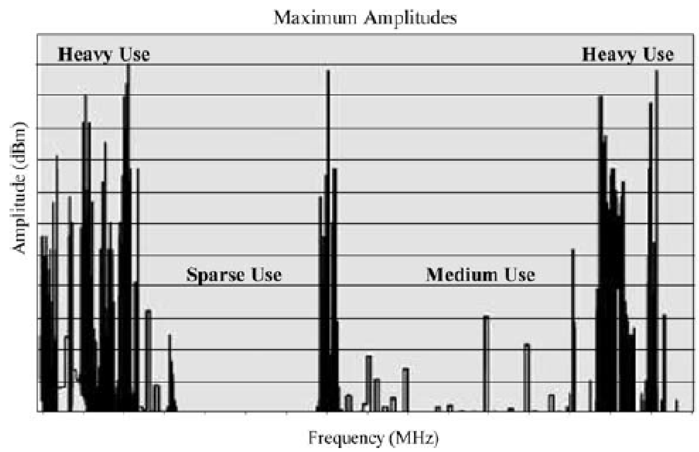


Fig. 1. The Statistic for Spectrum Utilization by FCC

Nowadays, CR is considered key to resolving the soon-to-occur spectrum scarcity problem. As it is mentioned above, under such a command-and-control spectrum policy, each spectrum band is assigned to a group of licensed users, which are given an exclusive spectrum usage right for a specific type of service and radio device. At a particular time and specific geographic location, the band may be not being utilized by those users and this kind of spectrum is called spectrum hole. CR can mitigate the spectrum scarcity problem by enabling dynamic spectrum access (DSA), which allows unlicensed users to find out the spectrum hole and utilize them opportunistically under the condition that they do not cause much interference to the licensed spectrum users' communications. In the context of DSA and the common hierarchical model, the licensed users are called primary users (PUs) and the CR users are called secondary users (SUs).

The realization of CRN is a challenging issue since it is related to business, regulatory policy and perhaps the most difficult part, the technologies for a strong wireless communication systems including effective hardware and software. Moreover, the problems about the transition from the current spectrum policy and technology to more advanced ones that CRN requires should be considered as well. Roughly, in terms of the technology part of CRN, there are four aspects of problems to be solved, namely spectrum sensing, spectrum management, spectrum mobility and spectrum sharing. In this project, I mainly focus on Spectrum Trading, which is related to spectrum management and spectrum sharing.

Spectrum Trading is the process of selling and purchasing the spectrum resource between the primary users and the secondary users. Because of the fluctuating nature of the available spectrum resource and the interference caused by the access of the secondary users, the primary users have to face the QoS degradation and the CRN may be denied by them. Spectrum Trading can solve this problem because in the scheme of trading, primary users can gain some profits after leasing the spectrum to the secondary users. This kind of incentives can provide the secondary users opportunity to access this underutilized spectrum. Although the economic models are highly strong tools to investigate spectrum trading, it is not just about economics because we have to consider many technological factors, such as the operational capability of the equipments in the network, limited overhead and capacity of the networks as well as locations in some multi-hop problems.

In terms of the architecture of cognitive radio networks, there are two kinds, namely centralized and distributed. In the centralized network, there exist base agents, operators or other centers to manage the behaviors of the entities in CRN or act as the mediator in the whole system. In distributed ones, no such centers or agents exist. In my project, I consider the former one. Centralized model can have significant advantages. First, with the help of infrastructure, SUs do not need to perform spectrum sensing or price negotiation with PUs. It does not require high intelligent equipments on PUs and SUs, and PUs' negotiation with the agent is less complex than that they directly participate in auctions or other trading schemes. Second, agents or operators with higher transmission power and larger spectrum sensing coverage can offer SUs with more spectrum trading opportunities. Third, current standards or infrastructures make agent's functions highly possible, like BS in 802.22 or AP in 802.11 network. In my project, when BS or AP act as agents, they are not considered as SUs in our model, since they do not purchase spectrum for themselves, although BS and AP are generally considered as SUs under some other circumstances. Thus, the centralized model with agents or other infrastructures is applicable. Forth, the agents can act as the manager of the licensed spectrum of primary networks and can make profits by leasing the spectrum to secondary users. In other words, there is incentive for agents to act as the retailer of spectrum in cognitive radio networks.

Partial Spectrum Sharing is one of the most interesting feature of cognitive radio. Most of the literature now focus on the partial spectrum sharing between the PU and SU. In [8], it proposed a kind of partial spectrum sharing between two licensed networks. However, it fails to discuss the situations about the sharing of the spectrum white space between the licensed networks. In [7], it does discuss the partial spectrum sharing between two wireless operator; however, it is not about cognitive networks. In my project, I have made an initiative step towards the partial spectrum sharing of spectrum white space between two primary networks.

The rest of the paper is organized as follows. In section II, we present the related works I have read in my project and simply discussed their contributions. Following that, in section III, I propose a problem about partial spectrum sharing between the primary networks. Afterwards, the system model for the problem is set up. In section IV, some mathematical results are given and the tentative step of studying the problem is proposed. In section IV-D, some future work for my project is given. In section V, I conclude this project.

## II. Related Work

### A. Cognitive Radio and Dynamic Spectrum Access

In [9] there is an introduction to cognitive radio techniques. [10] depicts different spectrum sharing models, and [11] points out the challenges and issues in designing dynamic spectrum access networks. In order to realize an effective spectrum management in CRN, many advanced models and approaches were proposed to study the spectrum sharing, spectrum access and pricing. In [12], in order to gives the highest utility to the cognitive radio users, an optimization problem was formulated for spectrum access to obtain the solution and it proposed a distributed algorithm which is based on a potential game formulation as well. In [13], Markov chain is used to analyze the quality-of-service (QoS) performance in a cognitive radio system with the hierarchical architecture of primary users and secondary users.

### B. Spectrum Trading in Cognitive Radio Networks

As it is mentioned above, spectrum trading is becoming more and more attractive in the field of cognitive networks and many advanced approaches which may be derived from the economic theories have been proposed. The literature [14] is a highly excellent survey about spectrum trading in cognitive radio networks. It points out the basic concept of spectrum trading, discussed several spectrum sharing models with one seller, multiple sellers or no permanent sellers, and discusses different forms of spectrum trading, the related research issues, and the variable solution approaches including Microeconomic Approach, Classical Optimization Approach as well as game theory and auction mechanism. In [15], multiple POs selling spectrum opportunities to multiple SUs in the spectrum trading and the price is the major issue determining the value or worth of the spectrum. Because of the evolution and the dynamic behavior of SUs, an evolutionary game is formulated , a noncooperative game is used to study the competition among the PUs. In [16], in order to improve the network's performance, a joint power/channel allocation scheme was proposed that used a distributed pricing strategy. In [17], a non-cooperative game based pricing scheme was proposed for uplink power control in cognitive radio networks.

What is more, market-driven auction is a promising approach to allocate spectrum resources and determine prices in spectrum trading. In [18], a bandwidth auction model was set up to study of dynamic spectrum sharing. In [19], an auction mechanism was applied to spectrum sharing among users who use spread spectrum signaling technology to access the spectrum.In [20], a truthful and computationally efficient spectrum auction mechanism, VERITAS is proposed. In [21], it showed that the primary users' revenue can be maximized by introducing a specially designed spectrum auction framework. In [22], a multi-auctioneer progressive auction framework was proposed to improve the efficiency of spectrum usage with multiple PUs and SUs.

In addition, contract-based dynamic spectrum sharing is also a hot topic in the study of spectrum trading in cognitive radio networks. In [23], the difference between the types of the spectrum is discussed and SUs are incentive to purchase PUs' spectra considering the quality-price designed for their own types. Moreover, in [24], under the CRNs modeled as a labor market, cooperative relay is achieved between the primary users and secondary users with incorporated contract theory.

## III. System Model

### A. Physical Model

We consider a cognitive radio network with N SUs (denoted as $SU_1, SU_2, \cdots, SU_k$, where $k$ is a positive constant integer) and two Primary Networks each of whom has a Primary Base Station (denoted as $PS_1, PS_2$, respectively). The base station manage the spectrum resource in each primary network and the spectrum of each primary network is denoted as $C_1$, $C_2$, respectively. The SUs are the unlicensed users and they can utilize the spectrum resource under the permission of $PS_1, PS_2$. What is more, there exists a Spectrum Policy Server (SPS) who acts as the mediator of the whole system and manages a part of spectrum white space denoted as $C$, which is not occupied by any of the users and not overlapped with $C_1$ and $C_2$.

In our model, the spectrum has been divided into channels (the amount of spectrum in one channel is 1 unit and spectrum of a primary network, $C_i, (i = 1, 2)$, equals to $C_i, (i = 1, 2)$ channels, respectively)and each channel can be accessed by only one users. PUs spectrums can be reused, and we focus on time reuse, which means that one PUs channel is allowed to be accessed by multiple SUs during one time slot. So we use TDMA in our model.

### B. Spectrum Trading

In order to assure the incentives of this cognitive network, the Primary Base Stations can charge some money from secondary users who purchases the access of spectrum. From [27], the price of the access of one channel in one time slot is denoted as $p_1, p_2$, respectively for the two primary networks and the cost (including the increasing interference and the fees for maintaining the equipments and so on) of each PS

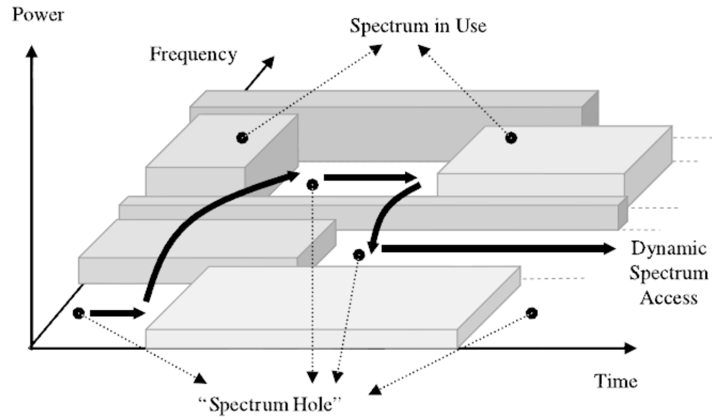


Fig. 2. Illustration for Spectrum Hole and Dynamic Spectrum Access

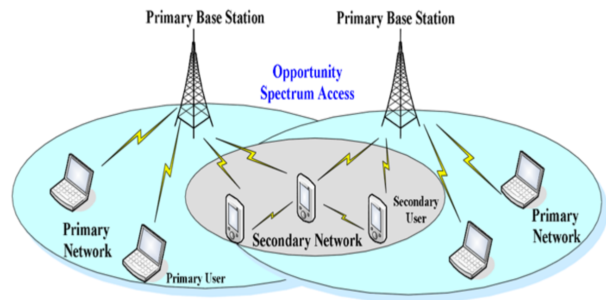


Fig. 3. Illustration for the physical model

is denoted as $m_1, m_2$. Moreover, the quality of the service of $PS_i$ is denoted as $W_i$.

Moreover, we have the following assumptions:

- The SUs in the systems are homogeneous.
- The SUs can only utilize the spectrum resource with the management of $PS$.
- The $PS$ can lease the spectrum $C$ from SPS and sell it to the SUs just as selling the original spectrum $C_1$ or $C_2$

From SUs point of view, the service of the agent is acceptable only if the price are reasonable and each SU determines to accept the service of PS with certain acceptance probability. So, we have to design an appropriate function of acceptance probability $A(p)$ reflecting SUs rational behavior. Easily, we can get the following nature of $A(p)$.

$$\frac{\partial A_i(p_i, W_i)}{\partial p_i} < 0$$

$$\frac{\partial A_i(p_i, W_i)}{\partial W_i} > 0$$

$$\lim_{p_i \to 0} A_i(p_i, W_i) = \alpha$$

$$\lim_{p_i \to \infty} A_i(p_i, W_i) = 0$$

Here $i = 1, 2$. $\alpha$ is a constant and $0 < \alpha < 1$. From [25] and [26], we use the choose the exponential expression for the

SU's acceptance probability for $PS_i$:

$$A_i(p_i) = \alpha e^{\frac{-p_i}{W_i}} \quad (1)$$

Moreover, after we take two prices $p_1, p_2$ into consideration, the function of SU's acceptance probability for $PS_i$ can be derived as :

$$A_i(\mathcal{P}) = \alpha e^{-\frac{p_i W_j}{p_j W_i}} \quad (2)$$

Here $\mathcal{P}$ means the price set containing $p_1$ and $p_2$. $i \in \{1, 2\}$, $j \in \{1, 2\}$ and $i \neq j$. In this work, we consider the services of the PSs are the same, which means that $W_i = W_j$.What is more, we consider that there are N SUs in this system and the demand (denoted as $D_i$) to $PS_i$ from the SUs is defined as the number of the SUs who determine to accept the service from $PS_i$. ($k$ denotes the index of a secondary user, $k = 1, 2, \cdots, N$)

$$D_i(\mathcal{P}) = \sum_{k=1}^{N} A_i(\mathcal{P}) = N \cdot A_i(\mathcal{P}) \quad (3)$$

obviously, if the demand of $PS_i$ is not larger than its spectrum $C_i$, then the profit of $PS_i$ is $R_i = D_i(\mathcal{P}) \cdot (p_i - m_i)$.

### C. Auction for the Spectrum White Space

As it is mentioned above, there exists one part of spectrum $C$ in this system which is not licensed to any of the users and can be leased to PBs. In this case, the PBs act as the retailers or operators and resell this spectrum to the SUs. The answer why the SUs is unable to access the spectrum $C$ without the management of PSs is that SUs have limited equipments and they can't sense the right spectrum white space independently. In my work, I consider a simple Sealed First-price Auction for SPS to allocate spectrum $C$.

- Step 1: In the beginning of each time slot, the two PSs bid with bidding prices for one unit spectrum, $r_1, r_2$, respectively.
- Step 2: SPS acts as auctioneer to choose the PS with higher bidding price to utilize part of or all of the spectrum while denying the other PS.

The advantages of the auction mechanism is simple and it can guarantee the fairness between different PSs.

## IV. Price War between Primary Base Stations

We consider a situation that there are too many SUs in the system and most of they is likely to buy the access of spectrum from PSs; as a result, the original part of spectrum occupied by a PS may be not enough to meet the whole demand. In short, the demand exceeds the supply. Then the PS may act as a retailer and lease the spectrum white space from SPS and resell it to the SUs if and only if the new profit exceeds the original one. Since the PS needs to pay for the leased spectrum from the whole spectrum white space $C$, it should set a appropriate biding price to win this auction and to maximize its profits. Obviously, there exist a trade-off in this problem. On one hand, if the PS set too low a bidding price in this auction, the probability to win the auction is small and the expect profit will decrease as well. On the other hand, if the PS set too high a bidding price, the profit will be small because PS has to pay the high bid to the SPS. Thus, it is necessary to study the bidding and pricing strategy of the PS.

### A. Incentive for Partial Spectrum Sharing

First, I prove that there is incentive for the PSs to participate in the spectrum auction.

*Proof:* In terms of the incentive of one PS to participate in the auction, we just need to consider the profit and pricing as well as bidding strategy of this PS. Thus, we can assume that the price of the other PS is constant. For simpleness without loss of generality, we ignore the other PS's price in the following calculation. We consider the situation of $PS_i, i \in \{1, 2\}$. In the following discussion, $R_i^o$ denotes the profit of $PS_i$ only using its occupied spectrum $C_i$ to trade while $R_i^r$ denotes the profit of $PS_i$ using its occupied spectrum $C_i$ and spectrum white space $C$ to trade.
(1) When $D_i < C_i$, $N \cdot A_i(\mathcal{P} < C_i$, then we get $p_i > W_i \ln \frac{\alpha W_i N}{C_1}$.
In this case, we have

$$R_i^o = D_i(p_i) \cdot (p_i - m_i) \quad (4)$$

Let $\frac{\partial R_i^o}{\partial p_i} = 0$, we can get the best pricing response in this case

$$p_i^* = W_i + m_i \quad (5)$$

Thus, $R_{imax}^o = N W_i^2 \alpha e^{-\frac{W_i + m_i}{W_i}}$

(2) When $D_i > C_i$, $N \cdot A_i(\mathcal{P} > C_i$, then we get $p_i < W_i \ln \frac{\alpha W_i N}{C_1}$.
In this case, we assume that $PS_i$ wins the auction and can lease the spectrum $C$ to resell it to SUs. Then we have

$$\begin{aligned} R_i^r &= C_i(p_i - m_i) + (D_i(p_i) - C_i) \cdot (p_i - m_i - r_i) \quad (6) \\ &= D_i(p_i)(p_i - m_i - r_i) + c_i r_i \quad (7) \end{aligned}$$

Let $\frac{\partial R_i^r}{\partial p_i} = 0$, we can get the best pricing response in this case

$$p_i^{r*} = W_i + m_i + r_i \quad (8)$$

Thus, $R_{imax}^r = N W_i^2 \alpha e^{-\frac{W_i + m_i + r_i}{W_i} + c_i r_i}$. (3)However, in the case of demand exceeds supply, we have to modify situation one. When $m_i + W_i < W_i \ln \frac{\alpha W_i N}{C_i}$, $D_max = C_i$, in this case, with $p_i = W_i \ln \frac{\alpha W_i N}{C_i}$, $R_{imax}^e = C_i(W_i \ln \frac{\alpha W_i N}{C_i} - m_i)$
Then if the profits with partial spectrum sharing is larger than the original ones, we have the following conditions:

$$\begin{cases} R_{imax}^e < R_{imax}^r \\ p_i^{r*} < W_i \ln \frac{\alpha W_i N}{C_1} \end{cases}$$

then $r_i < W_i \ln \frac{\alpha W_i N}{C_1} - W_i - m_i$. Thus, when $r_i \in (0, W_i \ln \frac{\alpha W_i N}{C_1} - W_i - m_i)$, the PS should choose the partial spectrum sharing scheme to pursue larger profit.

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From the proof, we also have the following results:

- When the market has only one PS, which means that there is only one bidder in the auction, the best response $r_i \rightarrow 0$, $R^r_{imax} \rightarrow R^o_{imax}$
- When the PS choose the maximal profitable bidding price, $r_i = W_i \ln \frac{\alpha W_i N}{C_1} - W_i - m_i$, $R^r_{imax}$ degenerates to $R^e_{imax}$ and this auction is meaningless to the $PS_i$

### B. Auction and Game

Since I have proofed that there is incentive for the PS to participate in the auction managed by the SPS. Because only the winner in the auction can lease the spectrum $C$ to resell to the SUs, which makes the profits of the PSs different from those without auction, it is necessary to study the payoff function of the under this auction. If we denote the payoff function of $PS_i$ as $U_i$, this problem can be expressed as finding:

$$(p_i^*, r_i^*) = \arg\max_{(p_i, r_i)} U_i(\mathcal{P}, \mathcal{R}) \tag{9}$$

$\mathcal{P}$, and $\mathcal{R}$ denote the price set and the bidding price set of the two PSs, respectively. Since this auction is a first-price sealed auction is an *Incomplete Information Static Game*, which means that a PS's bidding price is unknown to the other. In order to derive the payoff function in this game, in terms of $PS_i$, we assume that the other PS's bidding price $r_j$ is uniformly distributed on $(0, r_{max})$ and $r_{max} = W_j \ln \frac{\alpha W_j N}{C_j} - W_j - m_j$.

Moreover, the probability that $PS_i$ losses this game is denoted as $P_{loss}$ and $P_{loss} = \frac{r_j}{r_{max}}$. Then we have

$$\begin{aligned} U_i(\mathcal{P}, \mathcal{R}) =& P_{loss} R_i^o + (1 - P_{loss}) R_i^r \\ =& \frac{r_j}{r_{max}} C_i (p_i - m_i) + \\ & (1 - \frac{r_j}{r_{max}})[D_i(p_i - m_i - r_i) + C_i r_i] \\ =& C_i [\frac{r_j}{r_{max}}(p_i - m_i) + (1 - \frac{r_j}{r_{max}}) r_i] + \\ & \alpha e^{-\frac{p_i W_j}{p_j W_i}} \cdot N W_i (1 - \frac{r_j}{r_{max}})(p_i - m_i - r_i) \end{aligned} \tag{10}$$

Then we let $\frac{\partial U_i}{\partial p_i} = 0$. We can get the best response price :

$$p_i^* = \frac{C_i r_j}{r_{max}} + \alpha e^{-\frac{p_i W_j}{p_j W_i}} \cdot N W_i (1 - \frac{r_j}{r_{max}})[(-\frac{W_j}{p_j W_i})(p_i - m_i - r_i) + 1] \tag{11}$$

Therefore, if $PS_j$ has set up its parameters in this game and $r_i$ is given, the the best response price of $PS_i$ is fixed. So we can reduce this original game finding $(p_i^*, r_i^*)$ to a reduced game only finding $(r_i^*)$.

$$r_i^* = \arg\max_{r_i} U_i(\mathcal{P}, \mathcal{R}) \tag{12}$$

### C. Existence of the Nash Equilibrium

In order to find a Nash Equilibrium of this game, we first introduce the concept of *Supermodular Game* [27]. Supermodular Game is better known as games with strategic complementarities. First, the supermodular game needs no more assumptions in our model, which will render the system more applicable compared to the Bayesain non-cooperative game. Second, supermodular reduces the complexity of the mathematical work to find the NE convergence since we consider only the increasing or reducing of the variable (bidding prices of PSs).

- **Increasing difference**: Function $f(x, y) : X \times T \longrightarrow \mathbb{R}$ has an *increasing difference* if $(x, t)$, if $f(x', t') - f(x, t') \geq f(x', t) - f(x, t)$ or $\frac{\partial^2 f(x,t)}{\partial x \partial t} \geq 0$, for all $x' > x$ and $t' > t$
- **Supermodular Game**: Each player i and its competitorsstrategy set $p_i$ and $p_i \in \mathbb{R}$, and payoff $R_i(p_i, p_{-i})$ has an increasing difference in $(p_i, p_{-i})$, i.e., incremental payoff to choose a higher strategy is increasing with competitorsstrategies.

We can derive the *Hybrid Partial Derivative* of the payoff function.

$$\begin{aligned} \frac{\partial^2 U_i}{\partial r_i \partial r_j} =& \frac{\partial}{partial r_j}[(1 - \frac{r_j}{r_{max}})(C_i - \alpha e^{-\frac{p_i W_j}{p_j W_i}} N W_i)] \\ =& (-C_i / r_{max} + \alpha e^{-\frac{p_i W_j}{p_j W_i}} N W_i / r_{max}) \\ =& \frac{1}{r_{max}}(\alpha e^{-\frac{p_i W_j}{p_j W_i}} N W_i - C_i) \geq 0 \end{aligned} \tag{13}$$

Thus, this game is a *Supermodular Game*. It has been proved that a Supermodular Game must have a Nash Equilibrium. The nature of increasing difference in this model is intrinsical. Since the best response price $p_j^*$ increases as the $r_j$ increases, $PS_i$ is more likely to increase its own price to compete with its competitor and the increasing profit is more than that without the advance in price of its competitor.

### D. Future work

In my work, I set up the *hierarchical network model* and some mathematical models. In order to realize the partial spectrum sharing between the primary networks, I propose a simple first-price auction to allocate this common shared part of spectrum. This allocation is simple and fair. However, there are several future work for me to do in this summer.

- **Extend this problem to multiple PSs other than a scenario with only 2 PSs.** In this work, I just consider the situation between two PSs; thus, it is highly necessary to extend this work to a multiple PSs situations.
- **An efficient algorithm should be proposed.** In my work, I propose a first-price sealed auction for the allocation of the spectrum $C$ and the game model is set up. Afterwards, this game is a supermodular game and must have a Nash Equilibrium. Due to limited time. I fail to come up with an efficient algorithm for the PSs to decide the price and bidding price in this game.
- **Consider more effective auction mechanism.** A highly simple auction mechanism is used in my discussion. I believe that there exits more effective auction mechanism for the partial spectrum sharing between the primary networks.

## V. Conclusion

Spectrum Trading in Cognitive Radio Networks is the topic of my project. I have read a large number of papers and studied several advanced approaches to deal with spectrum trading in CRN, a topic related to economics, regulatory policy as well as technology.

I investigate a partial spectrum sharing between two primary networks. The shared part of spectrum in the system is managed by SPS which has the highest priority and two PSs can access the common shared part by a first-price sealed auction. Thus, a game is formulated in this progress. After analyzing the problem under the mathematical model, I proved that the incentives for the PSs to participate in this auction exit and that this game is a Supermodular Game. Thus, a Nash Equilibrium must exist. Several future work for this problem is proposed.

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