Coverage in Wireless Ad-Hoc Sensor Networks

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Abstract

Wireless Sensor Networks(WSNs) have recently emerged as a premier research topic. One of the fundamental issues that arises in sensor networks, in addition to location calculation, tracking, and deployment, is coverage and connectivity problems in 2D and 3D WSNs. That why our group want to do some study of coverage and connectivity in wireless sensor networks. In this paper, we propose an optimal deployment pattern to achieve both full coverage and k-connectivity, and prove its optimality for all values of r_c/r_s in 2D and 3D WSNs, where r_c is the communication radius, and r_s is the sensing radius. We study the knowledge along with the history of the development of this area, from two-dimensional space to three-dimensional space and so on.

Keyword

Full Coverage, k-connectivity,Delaunay triangulation,Voronoi polyhedra.

1 Introduction

It is well-known that placing disks in the triangular lattice pattern is optimal for coverage on a plane. However, as the wireless sensor networks developing, it is now no longer enough to consider coverage alone when deploying a wireless sensor networks; connectivity must also be considered at the same time. While moderate loss in coverage can be tolerated by applications of wireless sensor networks, loss in connectivity can be fatal. Moreover, since sensors are subject to unanticipated failures after deployment, it is not enough to have a wireless sensor network just connected. And difference applications require different degrees of connectivity, so it is of significance to have a complete set of optimal patterns to meet different applications. But in real-world applications, three dimension networks have more practical importance than that of two dimension networks. For example, WSNs deployed in 3D networks aerial space can be used in supporting intelligent computer vision systems, helping overcome human paropsia, constructing aerial defense systems, and building aerosphere pollution monitoring systems,etc.

2 The best Coverage Problem

2.1 Target

In the above section, we introduce that the coverage and connectivity are important problem in WSNs , So we'd like to introduce several deployment pattern to achieve both coverage and connectivity. But before that, we will consider the best coverage problem.

Given a wireless sensor network, the best coverage problem aims to find a path connecting point s and point t that maximizes the smallest observability of all points on the path. While in worst-case coverage,attempts are made to quantify the quality of service by finding areas of lower observability from sensor nodes and detecting breach regions.

To start this problem, we introduce Delaunay triangulation and Voronoi diagram in sensor network coverage.

2.2 Assumptions and Definitions

We begin with definitions of the **Voronoi diagram** and the **Delaunay triangulation**. We assume that all wireless nodes are given as a set S of n vertices in a 2D space. Each node has some computational power. We also assume that there are no four vertices of S that are cocircular. A triangulation of S is a Delaunay triangulation, denoted by DeleST, if the circumcircle of each of its triangles does not contain any other vertices of S in its interior. A triangle is called the Delaunay triangle if its circumcircle is empty of vertices of S. The Voronoi region, denoted by VorepT, of a vertex p in S is a collection of two-dimensional points such that every point is closer to p than to any other vertex of S. The Voronoi diagram for S is the union of all Voronoi regions VorepT, where p 2 S. The Delaunay triangulation DeleST is also the dual of the Voronoi diagram: Two vertices p and q are connected in DeleST if and only if VorepT and VoreqT share a common boundary. The shared boundary of two Voronoi regions VorepT and VoreqT is on the perpendicular bisector line of segment pq. The boundary segment of a Voronoi region is called the Voronoi edge. The intersection point of two Voronoi edge is called the Voronoi vertex. When there are no four points of S that are cocircular, then every Voronoi vertex has only exactly three Voronoi edges incident on it. The Voronoi vertex is the ircumcenter of some Delaunay triangle. Figure 1 gives an example of the Voronoi Diagram and the Delaunay triangulation of a set of two-dimensional points.

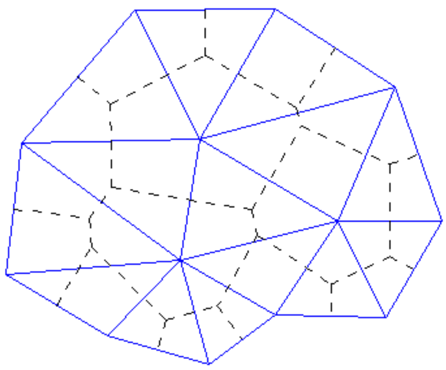


Figure 1: an example of a Voronoi diagram for a set of radomly placed sites

2.3 Implementation

In order to achieve deterministic coverage, a static network must be deployed according to a predefined shape. The predefined locations of the sensors can be uniform in different areas of the sensor field or can be weighted to compensate for the more critically monitored areas.

Algorithm Incremental Delaunay(V)
Input: set v of points in 2D domain
Output: Delaunay triangulation(DT)
1. add a appropriate triangle boudingbox to contain V (such as: we can use triangle abc, $a = (0, 3M)$,$b = (-3M, -3M)$,$c = (3M, 0)$, $M = Max(|x1|, |x2|, ...U|y1|, |y2|, ...))$
2.initialize DT(a,b,c) as triangle abc
3.for i$< -$ 1 to n
do (Insert(DT(a,b,c,v1,v2,...,vi-1), vi))
4.remove the boundingbox and relative triangle which cotains any vertex of triangle abc from $DT(a, b, c, v_1, v_2, ..., v_n)$ and return $DT(v_1, v_2, ..., v_n)$.

Algorithm Insert(DT(a,b,c,v1,v2,...,vi-1), vi)
1.find the triangle vavbvc which contains vi // FindTriangle()
2.if (vi located at the interior of vavbvc)
3. then add triangle vavbvi, vbvcvi and vcvavi into DT // UpdateDT()
FlipTest(DT, va, vb, vi)
FlipTest(DT, vb, vc, vi)
FlipTest(DT, vc, va, vi)
4.else if (vi located at one edge (E.g. edge vavb) of vavbvc)
5. then add triangle vavivc, vivbvc, vavdvi and vivdvb into DT (here, d is the third vertex of triangle which contains edge vavb) // UpdateDT()
FlipTest(DT, va, vd, vi)
FlipTest(DT, vc, va, vi)
FlipTest(DT, vd, vb, vi)
FlipTest(DT, vb, vc, vi)
6.return DT(a,b,c,v1,v2,...,vi)

Algorithm FlipTest(DT(a,b,c,v1,...,vi), va, vb, vi)
1.find the third vertex (vd) of triangle which contains edge vavb // FindThirdVertex()
2.if(vi is in circumcircle of abd) // InCircle()
3. then remove edge vavb, add new edge vivd into DT // UpdateDT()
FlipTest(DT, va, vd, vi)
FlipTest(DT, vd, vb, vi)

An example of a uniform deterministic coverage is the grid-based sensor deployment where nodes are located on the intersection points of a grid. In this case, the problem of coverage of the sensor field reduces to the problem of coverage of one cell and its neighborhood due to the symmetric and periodic deployment scheme. But in many situations, deterministic deployment is neither feasible nor practical. Another deployment option is to cover the sensor field with sensors randomly distributed in the environment.

Stochastic coverage is quite simple , that is insert the points into the set of points in some stochastic way. In the whole process it should maintain a Delaunay triangulation corresponding the current set of points.

Considering the situation that Point v_i is inserted, because $DT(v_1, v_2, ..., v_{i-1})$ which is made by all of the previous points is already a Delaunay triangulation , we should only consider the change brought by the Point v_i and adjust until$DT(v_1, v_2, ..., v_{i-1})Uv_i$ become a new Delaunay triangulation.

2.4 Complexity

The best known algorithms for the generation of the Voronoi diagram have $O(nlogn)$ complexities. We count the complexity according to the worst situation:

$$\begin{aligned} ComplexitiesT &= T(addboudingbox()) \\ &+ \sum T(insert(i), i = 1, .., n) \\ &+ T(removeboundingbox) \end{aligned}$$

addboudingbox() and removeboundingbox() don't change with t ,that means they are constant.So,

$T(addboudingbox()) = O(1)$,
$T(removeboundingbox()) = O(1)$.
$T = \sum(T(insert(i), i = 1, .., n)) + O(1) + O(1)$.

Then, consider the cost when point i is inserted

$$\begin{aligned} T(insert(i)) &= T(FindTriangle(i)) \\ &+ T(UpdateDT(i)) \\ &+ K * T(FlipTest(i)) \end{aligned}$$

$$\begin{aligned} T = \sum(T(FindTriangle(i)), i = 1, .., n) \\ + \sum(T(UpdateTD(i)), i = 1, .., n) \\ + K * \sum(T(FlipTest(i)), i = 1, .., n) \end{aligned}$$

Calculate the complexities one by one and finally in the worst situation is:

$$\begin{aligned} T &= O(n * n) + O(n) + K * (O(n * n) \\ &\quad + O(n) + O(n)) + O(1) + O(1) \\ &= O(n * n) \end{aligned}$$

This answer is different from the paper[1] we have studied. Later we know that others use different data structure to save the triangle diagram so that they reduce the complexities of FindTriangle() and FindThirdVertex(). That made the search of the third point can be done in $O(logn)$,not in $O(n)$. In this way, the whole cost is:

$$O(log1) + O(log2)+, ... + O(logn) = O(nlogn)$$

For example, DAG, Quad-edge can reach the complexity $O(nlogn)$. Although this algorithm was developed for a wireless ad-hoc sensor network, we have assumed a centralized control server, where nodes are connected using a gateway. Other control strategies such as distributed control systems are also feasible. It is possible to solve the problems presented in this paper in a decentralized approach. A natural course of study would be to compare the centralized coverage algorithm to distributed ones in terms of power consumption cost,a nd performance.

3 Full Coverage and k-connectivity in 2D space

3.1 Sensor Deploying

Now, we have discussed the best coverage problem, then we continue with the problem, taking connectivity into consideration. Here, in this section ,we will show how the deployment can help achieve both full coverage and connectivity in WSNs.

Deployment is a fundamental issue in Wireless Sensor Networks (WSNs) that affects many facets of network operation. According to the paper "Deploying Wireless Sensors to Achieve Both Coverage and Connectivity", we get the referenced conclusion that the asymptotic optimality of a deployment pattern(shown in Figure2) can achieve both coverage and 2-connectivity for all values of r_c/r_s.

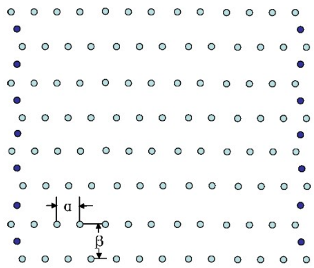


Figure 2: Strip-based deployment pattern to achieve coverage and 2-connectivity. The light-filled dots show the sensor locations that form the horizontal strip, while the dark-filled dots form the two vertical strips. Here, $\alpha = min\{r_c, \sqrt{3}r_s\}$ and $\beta = r_s + \sqrt{r_s^2 - \alpha^2/4}$ The vertical strip of sensors may be removed when $r_c/r_s \geq \sqrt{3}$.

We also extend the result of the paper "Low-coordination Topologies for Redundancy in Sensor Networks" and show that the strip-based deployment pattern (shown in Figure 3) is not only near-optimal but asymptotically optimal for achieving both full coverage and 1-connectivity. Moreover, its optimality holds not only for $r_c/r_s = 1$ but for all $r_c/r_s < \sqrt{3}$.

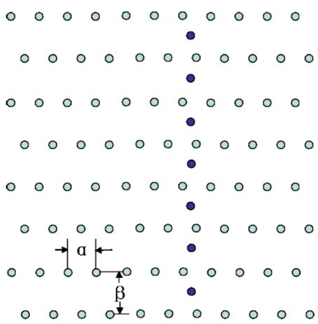
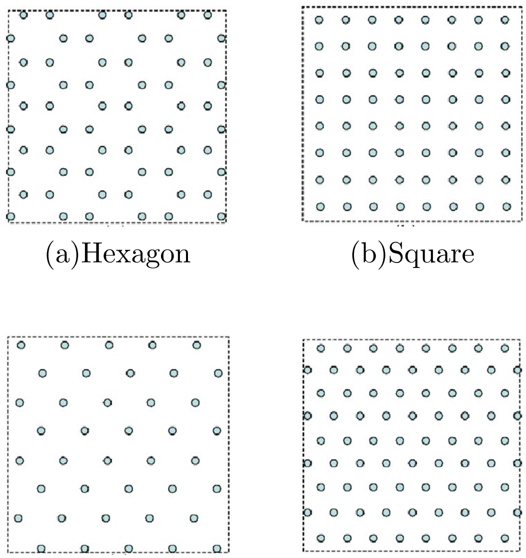


Figure 3: Strip-based deployment that is optimal for achieving coverage with 1-connectivity, when $r_c/r_s < \sqrt{3}$. The light-filled dots show the sensor locations that form the horizontal strip, while the dark-filled dots form the one vertical strip. Here, $\alpha = min\{r_c, \sqrt{3}r_s\}$ and $\beta = r_s + \sqrt{r_s^2 - \alpha^2/4}$.

However, in practice, wireless sensor networks are often desired to follow regular patterns due to at least two reasons (1)convenience of deployment and (2)to achieve a higher degree of connectivity. So, Four popular regular deployment patterns are hexagon, square grid, rhombus, and equilateral triangle, all of which are exhibited in Figure 4.



(a)Hexagon (b)Square

(c)Rhombus (d)Triangular Lattice

Figure 4: Four common regular patterns of deployment

Note that the triangular lattice pattern provides at least 6-connectivity, square grid provides at least 4-connectivity, rhombus provides at least 4 or 6 connectivity depending on its shape, and the hexagon provides at least 3-connectivity1. Connectivity aside, it would be interesting to know: (1) which of these four regular patterns is more efficient than the others (in terms of the number of sensors needed)? (2) what is the efficiency of these regular deployment patterns as compared to the optimal pattern? Toward these two questions, we establish the following:

- When $\sqrt{2} < r_c/r_s < \sqrt{3}$, the rhombus-based pattern is better than the other three. It requires upto 21% more sensors as compared to the optimal in this range of r_c/r_s.
- When $1.14 \leq r_c/r_s \leq \sqrt{2}$, the square pattern is better than the other three. It requires upto 60% more sensors than the optimal in this range of r_c/r_s.
- When $r_c/r_s \leq 1.14$, the hexagon pattern is better than the other three. It requires a constant number of sensors for $1 \leq r_c/r_s \leq 1.14$; it uses upto 44% more

sensors than the optimal in this range of r_c/r_s.

- When $r_c/r_s < 1$, the number of sensors needed by the hexagon pattern grows exponentially as compared with the optimal. Evidently, the number of sensors needed by the other three patterns are only worse when $r_c/r_s < 1$.

After that, we compare the number of nodes needed to provide both coverage and connectivity over a deployment region of size $1,000m \times 1,000m$ with $r_s = 30m$, and $24m \leq r_c \leq 75m$, when different patterns are used. To determine the number of nodes needed in each of the four regular patterns of deployment (hexagon, square, rhombus, and equilateral triangle), we divide the area of the deployment region by the maximum APN of the corresponding patterns. Figure 5 shows the results of our computation.

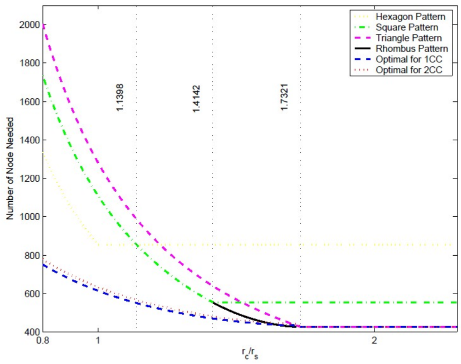


Figure 5: Number of nodes needed in the different patterns of deployment(hexagon, square, rhombus, triangle, and the optimal strip-based deployment patterns)to achieve both coverage and connectivity for various values of r_c/r_s, when sensors each with $r_s = 30m$ are deployed over a $1,000m \times 1,000m$ deployment region. The communication range r_c is varied from 24 m to 75 m

Eventually, we can find the strip-based deployment pattern is good to achieve both coverage and 2-connectivity, and proved its optimality. We can also find the optimality of a previously proposed strip-based deployment pattern to achieve coverage and 1-connectivity.

3.2 Disadvantage

It's easy to see that all the above we discuss about have several disadvantages. First of all, our algorithm only compare the distance of those kinds of model, moreover we only consider the situation of 2-dimensional space, but in real life it isn't practical. That's why we continue to study coverage of the 3-dimensional space.

4 Full Coverage and k-connectivity in 3D space

In this part, we study the problem that what is a optimal way to deploy sensor nodes in a three-dimension wireless sensor networks [1] that achieve full-coverage and different k-connectivity according to difference applications such that the number of nodes is minimized.

There are two sets of works related to this problem that we briefly discuss here, and further discussions will be followed.

One set of works is on sphere-covering problems in 3D Euclidean space in the area of discrete computational geometry. In 1887,Lord Kelvin conjectured that the deployment strategy generating Voronoi polyhedra that are 14-sided truncated octahedrons is the optimal strategy to the 3D covering problem. However, now the there has been proofed not on the optimality of this strategy(weaire-phelan structure is better). Note that Kelvins conjecture only considers coverage.

Another related work by S.Alam and Z.Haas is in the area of WSNs. This work considers both coverage and connectivity in 3D WSNs. It suggests a deployment pattern that creates the Voronoi tessellation of truncated octahedral cells in 3D space directly from Kelvins conjecture, but no proof is provided on the optimality. This deployment strategy can achieve 14-connectivity when the communication range is at least $\frac{4}{\sqrt{5}}$ times the sensing range.

4.1 Related work

There are two sets of work related to the connectivity and full-coverage deployment problem in WSNs.

- Sphere Covering and Packing in Discrete Computational Geometry

[1] However,this problem is hard and our knowledge of its answer is limited.So the research is not so deep, we mainly consider low-connectivity problems .

One closely related problem in discrete computational geometry is covering problem, especially sphere covering in 3D Euclidean space.

In 1887, Lord Kelvin provided a conjectured answer to the problem of What is the optimal way to fill a three dimensional space with cells of equal volume, so that the surface area (interface area) is minimum?.His answer sates that the Voronoi polyhedrons in the optimal covering strategy are 14-sided truncated octahedrons.

There are valuable efforts on the covering problem under certain conditions.One important condition is the spheres are placed following certain regularity. R.Bambah first proved that the least covering density of a 3D space by iden tical spheres is $\frac{5\sqrt{5}\pi}{24}$ (the definition of covering density will be given later).E.Barnesin and L.Fewin proved the same result in different ways.

Another closely related problem in discrete computational geometry is sphere packing in 3D Euclidean space. Sphere packing considers arrangements of non-overlapping identical spheres filling a space. There have been several works on the packing problem.One of the most famous results is known as Keplers conjecture. In 1611,Johannes Kepler conjectured the maximum possible density for sphere packing is $\frac{\pi}{\sqrt{18}}$ .There is no rigorous proof until 2005. T.C.Halesin accomplished the proof showing that no packing of identical spheres in 3D Euclidean space can have density greater than $\frac{\pi}{\sqrt{18}}$, which is the density of the face-centered cubic packing. None of the above efforts considers connectivity in 3D Euclidean space.

- Connectivity and Coverage in WSN Deployment

S.Alamand Z.Haasin suggested the sensor deployment pattern that creates the Voronoi tessellation of truncated octahedral cells in 3D space. The suggestion is directly from Kelvins conjecture. The numerical data in illustrates truncated octahedron tessellation is better than the tessellations of cube, hexagonal prism, and rhombic dodecahedron. However, the optimality proof for truncated octahedron tessellation is untouched. Besides the efforts focused on the optimal deployment strategy, there are some works in 3D sensor deployment addressing other issues related to coverage. A deployment algorithm is proposed torepaircoverage holes once they are discovered in a 3D volume.

4.2 Definitions and Notions

Similar to the context for optimal deployment pattern research in 2D, we consider that all sensors are of same type and have sphere-shaped communication domain with radius r_c and sensing field with radiusr_s .The deployment region is considered vast enough such that its boundary can be ignored. We discuss in Section V practical considerations which are beyond these mathematical abstractions. In the following, we introduce some important definitions.

DEFINITION 4.2.1 *Right Parallelepiped, Axle Set,F-diagonal,B-diagonal:*
A hexahedron is called aright parallelepiped if its bases are parallelograms aligned one directly above the other and has lateral faces that are rectangles. Any three edges of a parallelepiped are called an axle set if any two of them are not parallel. The diagonals of the parallelepiped faces are called F-diagonals. The body diagonal sofa parallelepiped are called B-diagonals.

DEFINITION 4.2.2 *Basic Lattice, Seed Parallelepiped:*
Given a right parallelepiped α,the set Λ is called a basic lattice generated by right parallelepiped α if Λ is composed of all the vertices generated by shifting α toitsthree-edges directions with shift distance being integer times the corresponding edge length. This right parallelepiped α is called the seed parallelepiped for Λ.

For example, if α is a unit cube with edge length equal to 1, we set its one vertex as the origin point and three lines passing through the axle set intersected at this vertex as the axes of a reference frame, then the basic lattice generated by α is the set of points with integer coordinates in this reference system. It is worth noting that one basic lattice may have different seed parallelepipeds, but it is determined as long as one of them is given.

DEFINITION 4.2.3 *Body-Centered Lattice:*
Given a basic lattice Λ generated by seed

parallelepiped α, point set Λ is called a body-centered lattice if it is composed of all points in Λ and all the center points of α in the process of generating Λ.

A body-centered lattice is called body-centered cubic lattice (bcc lattice in short) when its seed parallelepiped is a cube. Meanwhile a body-centered cuboid lattice will be generated if its seed parallelepiped is a cuboid. In this paper, we study regular lattices. A regular lattice Λ is either a basic lattice generated by its seed right parallelepiped α or a body-centered cuboid lattice generated by its seed cuboid α.

DEFINITION 4.2.4 *Coverage Lattice with Radius r:*
Given lattice Λ and spheres with radius r centering at each point in Λ, Λ is called a coverage lattice with radius r if every point in a 3D volume can be covered by at least one sphere.

DEFINITION 4.2.5 *Lattice Λ Pattern:*
Given sensors with sensing range r_s and a lattice , a sensor deployment scheme is called lattice pattern if sensors are deployed at each point in Λ andΛ is a coverage lattice with r_s. From Definition 4.2.5, when the term lattice pattern is used in this paper, full coverage is always implied.

DEFINITION 4.2.6 *Covering Density:*
If Λ is a coverage lattice with radius r and generated by seed parallelepiped α, then the ratio of the total volume of the spheres with radius r covering α to the volume of α is called covering density ofΛ with radius r, denoted by $\sigma(\Lambda, r)$.

Given a fixed r_s and two latticesΛ and Λ, if $\sigma(\Lambda, r_s) < \Lambda\ \sigma\ \Lambda,\ r_s$, then lattice$\Lambda$ pattern is better than lattice Λ pattern since less sensor nodes are needed in latticesΛ pattern to achieve full coverage.

DEFINITION 4.2.7 *Optimal Lattice Pattern:*
Given sensing range r_s, a latticeΛ pattern is called the optimal lattice pattern if $\sigma(\Lambda,\ r_s)$ is minimum among all regular lattice patterns. Sensor deployment patterns in 3D WSNs are numberless and can be complicated.

To find and prove the optimal deployment patterns to fully cover a 3D space among all possible patterns is very hard even when the connectivity is not considered. It has been noticed that many important natural constructs in 3D space show strong periodicity and homogeneity in their constructing components. One of the most universal and important structure with such properties is lattice. In this paper, we are to explore the optimal patterns among regular lattice patterns.

4.3 Lattice Pattern for 1- and 2-Connectivity

In this section, we first present optimal lattice patterns that achieve 1- and 2-connectivity, and then give a briefly proof.

Due to their symmetry, lattice patterns with exactly odd connectivity do not exist. We only need to consider those that achieve even connectivity. Naturally, the optimal lattice patterns that achieve 1-connectivity are optimal ones that achieve 2-connectivity.

4.3.1 Pattern Description

The proposed lattice patterns for 1- or 2-connectivity in 3D space are shown in Figure 6.

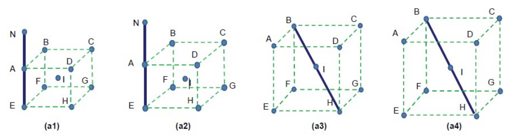


Figure 6: Lattice patterns that achieves 1- or 2-connectivity and full coverage.

- When $r_c/r_s < 4/3$, the pattern follows a body-centered lattice, denoted by $\lambda 2_1$, which is generated by a cuboid α with upper and bottom faces each with edge length $e_1 = \sqrt{(3r_{s2} - r_{c2} + r_s\sqrt{9r_{s2} - 2r_{c2}})/2}$; $e_2 = (3r_s + \sqrt{9r_{s2} - 2r_{c2}})/2$ and its center. The height of σ is r_c. This seed cuboid α and its center are illustrated by A, B, C, D, E, F, G, H, and I in Figure 6(a1). Any sensor is able to connect with its two neighbors along the direction of height, as illustrated by sensor A in Figure 6(a1).

- When $4/3 \leq r_c/r_s < 12/\sqrt{9+32\sqrt{3}}$, the pattern follows a body-centered lattice, denoted by $\Lambda 2_2$, which is generated by a cuboid α with upper and bottom faces each with edge length $e_3 = e_4 = \sqrt{4r_{s2} - r_{c2}/4}$ and its center. The height of α is r_c. This seed cuboid α and its center are illustrated by A, B, C, D, E, F, G, H and I in Figure 6(a2). Any sensor can connect with its two neighbors as illustrated by sensor A.

- When $12/\sqrt{9+32\sqrt{3}} \leq rc/rs < 2\sqrt{3}/\sqrt{5}$, the pattern follows a body-centered lattice, denoted by $\Lambda\ 2_3$, which is generated by a cube α with edge length $e_5 = 2r_c/\sqrt{3}$ and its center. This seed cube and its center is illustrated by A, B, C, D, E, F, G, H and I in Figure 6(a3). Any sensor is able to connect with its two neighbors along the direction of B-diagonal, as illustrated by sensor I in Figure 6(a3).

- When $2\sqrt{3}/\sqrt{5} \leq r_c/r_s$, the pattern also follows a body centered lattice, denoted byΛ_{2-4}, which is generated by a cube α with edge length $e_6 = 4r_s/\sqrt{5}$ and its center. This seed cube and its center is illustrated by A, B, C, D, E, F, G, H and I in Figure 6(a4). Any sensor is able to connect with its two neighbors along the direction of B-diagonal, as illustrated by sensor I in Figure 6(a4).

We note that some extra nodes are needed at the boundaries of 3D deployment volume for global connectivity when latticeΛ_{2-1} orΛ_{2-2} patterns are used. More discussions on this issue are provided blew ,and,before continued the discussing, we will first introduce a theorem.

THEOREM 4.3.1 To achieve 1- or 2-connectivity and full coverage in 3D space:

the body-centered latticeΛ_{2-1} pattern is an optimal regular lattice pattern when $r_c/r_s < 4/3$,

the body centered latticeΛ_{2-2} pattern is an optimal regular lattice pattern when $4/3 \leq r_c/r_s < 12/\sqrt{9+32\sqrt{3}}$,

the body-centered latticeΛ_{2-3} pattern is an optimal regular lattice pattern when $12/\sqrt{9}+32\sqrt{3} \leq r_c/r_s < 2\sqrt{3}/\sqrt{5}$,

and the body-centered latticeΛ_{2-4} pattern is an optimal regular lattice pattern when$2\sqrt{3}/\sqrt{5} \leq r_c/r_s$.

4.3.2 Proof

In this section, we will present the proof road map for the above theorem.

From Definition 4.2.7, to prove optimality is equivalent to find the lattice pattern with the least covering density $\sigma(,r)$, which is denoted by σ_{MIN}. To get σ_{MIN}, we need to consider all regular basic lattice patterns as well as regular body-centered lattice patterns, obtain $\sigma'_{MIN}\prime s$ for all cases, and then compare them.

Covering density is $4\pi r_s^3/(3V)$ for basic lattice patterns, and is $4\pi r_s^3/(3V)$ for body-centered lattice patterns, where V is the volume of the seed parallelepiped. To obtain σ'_{MIN} for each case, we are to obtain the maximum volume, which is denoted by V'_{MAX}, for each case, as is shown in equation (1), where x, y and z are the lengths of three non-parallel edges of the right parallelepiped and γ is the included angle of the bottom parallelogram.

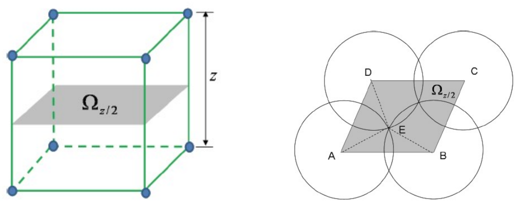
$$max f(x, y, z, \gamma) = xyz \sin\gamma \quad (1)$$

V'_{MAX} can be obtained by solving a nonlinear optimization problem (1) under constraints generated from full coverage and desirable connectivity.

We take the basic lattice situations as an example to proof theorem 4.3.1 according to the road map given above. In the proof, both coverage constraints and connectivity constraints are considered explicitly for (1).Coverage constraints are reflected by first properly choosing a certain face or a certain geometry point or both of the seed parallelepiped and then letting them be covered. This dimensionality reduction is important since it decides the number of constraints for the nonlinear optimization and thus decides the feasibility of solving it. Connectivity constraints are reflected by lengths of different set of edges, or F-diagonals, or B-diagonals of the seed parallelepiped

For Basic Lattice, We consider coverage constraints first, then connectivity constraints.

We denote the constraints for satisfying full coverage in this case by Cov-BL. As shown in Figure 7(a), we denote the parallelogram that is parallel to the bottom at $z/2$ in a seed parallelepiped by $\Omega_{z/2}$. Covering $\Omega_{z/2}$ is a necessary condition for full coverage. Now we



(a) (b)

Figure 7: (a)The parallelogram at the middle of height z is denoted by $\Omega_{z/2}$. (b) The most efficient way to cover the middle parallelogram at $z/2$.

show it is sufficient. Compared with other parallelograms parallel to $\Omega_{z/2}$, $\Omega_{z/2}$is the hardest to cover since the intersections of sensing spheres on this plane (intersections are disks) are smaller than those on other parallelograms. If the parallelogram $\Omega_{z/2}$ is covered, then any other parallelogram parallel to z/2 must be covered by larger intersection disks. Then we can transform the constraints to fully cover the seed parallelepiped to the constraints to fully cove $\Omega_{z/2}$.

The most efficient way to cover $\Omega_{z/2}$ is to let the overlapped area of any three disks (intersections of sensing spheres by the plane) be zero, as illustrated in Figure 7(b). Assume A is the origin point o at (0,0), B at (x,0) and D at $(y\cos\gamma, y\sin\gamma)$,then, we can get the condition Cov-BL as follow:

$$x^2 + y^2 - 2xy\cos\gamma \le (4r_s^2 - z^2)\sin^2\gamma \quad (2)$$

Now we consider the constraints for connectivity. The connection edge can be either one edge of the seed parallelepiped, or a F-diagonal, or a B-diagonal. Note that in basic lattice, if a F-diagonal or a B-diagonal is the connection edge, then at least two edges of the base are also be the connection edge. Hence, we only need to consider two cases here, namely,$x \le r_c$ or $z \le r_c$. Denote these two connectivity constraint by Con-BL-1 and Con-BL-2. The constraints of Cov-BL and Con-BL-1 then can be written as

$$\begin{cases} x^2 + y^2 - 2xy\cos\gamma \le (4r_s^2 - z^2)\sin^2\gamma \\ x \le r_c \end{cases} \quad (3)$$

Then we have solution as follows

- When $r_c/r_s \le \sqrt{2}, x = r_c, y = \sqrt{2}r_s, z = 2r_s\sqrt{\frac{4r_s^2-r_c^2}{8r_s^2-r_c^2}}$ and $\gamma = arccos(r_c/2\sqrt{2}r_s)$, $V'_{MAX-1} = r_c r_s\sqrt{4r_s^2 - r_c^2}$ and $\sigma'_{MIN-1} = 4\pi r_s^2/(3r_c^2\sqrt{4r_s^2 - r_c^2})$.
- When $\sqrt{2} \le r_c/r_s \le 4/\sqrt{5}$, $x = y = \sqrt{2}r_s$, $z = 2r_s/\sqrt{3}$ and $\gamma = \pi/3$, $V'_{MAX-1} = 2r_s^3$ and $\sigma'_{MIN-1} = 2\pi/3$.

The constraints of Cov-BL AND Con-BL-2 then can be written as

$$\begin{cases} x^2 + y^2 - 2xy\cos\gamma \le (4r_s^2 - z^2)\sin^2\gamma \\ z \le r_c \end{cases} \quad (4)$$

We have solution as follows.

- When $r_c/r_s \le 2/\sqrt{3}, x = y = \sqrt{3}\sqrt{4r_s^2 - r_c^2}, z = r_s$ and $\gamma = \pi/3$, $V'_{MAX-2} = 3r_c\sqrt{3}(4r_s^2 - r_c^2)/8$ and $\sigma'_{MIN-2} = 32\pi r_s^3/(9r_c\sqrt{3}(4r_s^2 - r_c^2))$.
- When $2/\sqrt{3} \le r_c/r_s \le 4/\sqrt{5}$, $x = y = \sqrt{2}r_s$, $z = 2r_s/\sqrt{3}$ and $\gamma = \pi/3$, $V'_{MAX-2} = 2r_s^3$ and $\sigma'_{MIN-2} = 2\pi/3$.

It is not necessary to consider the range $r_c/r_s \ge 4/\sqrt{5}$ Since the pattern proposed in [7] from Kevin's conjecture can achieve 14-connectivity in this range.

For other patterns, we can also make the same calculate to proof the theorem, The difficulty lies on the number of constraints and their complicated expressions. However, when general lattice patterns are considered, coverage constraints are difficult to get and they have more complicated expressions. To solve nonlinear optimization problems for such cases is hard. We conjecture that our proposed patterns here are also optimal among general lattice patterns. Its proof is our on-going work.

5 Conclusion

In this paper, we have designed a set of patterns for k-connectivity and full-coverage WSNs in both 2D and 3D space. In 2D space, We proposed a strip-based deployment pattern to achieve coverage and 2-connectivity, and briefly proved its optimality. We also proved the optimality of a previously proposed strip-based deployment pattern to achieve coverage and 1-connectivity. In 3D , we consider a proposed patterns which can save a significant number of sensor nodes. We are to explore the optimal deployment patterns to achieve full-coverage and multiple-connectivity in 2D and 3D space among more general regular patterns. Extending our research to more practical scenarios is also one of our future directions.

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