Report Three
Capacity, Coverage and Connectivity in Wireless Networks

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Contents

1 Introduction

In this final report, we provide a detailed exhibition of our work all through this semester.

2 Mobility Model

We summarize some commonly used mobility models to offer choices while evaluating the connectivity of different mobile networks. And they can be redefined and modified to evolve suitable models.

2.1 Random Walk Mobility Model

The Random Walk Mobility Model was developed to mimic the unpredictable moving pattern which is common in reality. In this mobility model, an mobile node(MN) moves from its current location to a new location by randomly choosing a direction and speed to travel. The new speed and direction are both chosen from pre-defined ranges, [*speedmin*, *speedmax*] and [0,2π] respectively. Each movement in the Random Walk Mobility Model occurs in either a constant time interval $\mathcal{T}$ or a constant distance traveled $\mathcal{D}$, at the end of which a new direction and speed are calculated.

Polya had proved that a random walk on a one or two-dimensional surface returns to the origin with probability one, which suggests that the random walk represents a mobility model that tests the movements of nodes around their starting points, without worry of the entities wandering away never to return. And this model is a memoryless pattern since MNs maintain no knowledge of their past locations and velocities.

2.2 I.I.D. Mobility Model

The process is divided into time-slots and at the very beginning of each time-slot each MN will be randomly and uniformly distributed and remains stationary during the rest of the time-slot.

This model is relatively idealistic and simple because it resembles the flat model in the sense that MNs are in fact static during a time-slot. There is actually no 'process' of mobility since the redistribution is completed instantaneously. And this model also maintains the memoryless property.

2.3 Random Way-point Mobility Model

The Random Way-point Mobility Model includes pause times between changes in direction and/or speed, which distinguishes it from the Random Walk model. An MN begins by staying in one location for a certain period of time (i.e., a pause time). Once this time expires, the MN chooses a random destination in the simulation area and a speed that is uniformly distributed between [*minspeed*, *maxspeed*]. The MN then travels toward the newly chosen destination at the selected speed. Upon arrival, the MN pauses for a specified time period before starting the process again.

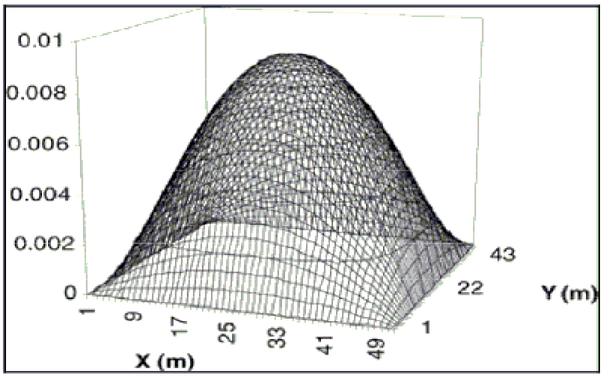

Fig 1: Possibility of Occurrence of Different Position

However, one problem with this model is that not all positions occur with the same probability as illustrated in **Fig** 1.

2.4 Gauss-Markov Mobility Model

The Gauss-Markov Mobility Model was designed to adapt to different levels of randomness via one tuning parameter. Initially each MN is assigned a current speed and direction. At fixed intervals of time, n, movement occurs by updating the speed and direction of each MN. Specifically, the value of speed and direction at the n^{th} instance is calculated based upon the value of speed and direction at the $(n-1)^{st}$ instance and a random variable using the following equations:

$$s_n = \alpha s_{n-1} + (1-\alpha)\bar{s} + \sqrt{(1-\alpha^2)}s_{x_{n-1}}$$
$$d_n = \alpha d_{n-1} + (1-\alpha)\bar{d} + \sqrt{(1-\alpha^2)}d_{x_{n-1}}$$

where s_n and d_n are the new speed and direction of the MN at time interval n; α, where $0 \leq \alpha \leq 1$, is the tuning parameter used to vary the randomness; $\bar{s}$ and $\bar{d}$ are constants representing the mean value of speed and direction as $n \to \infty$; and s_{n-1} and d_{n-1} are random variables from a Gaussian distribution. Totally random values (or Brownian motion) are obtained by setting $\alpha = 0$ and linear motion is obtained by setting $\alpha = 1$.

The Gauss-Markov Mobility Model can eliminate the sudden stops and sharp turns encountered in the Random Walk Model since it allows past experience to influence future movements, i.e. this pattern is with memory.

2.5 Hybrid Mobility Models

In [3], hybrid models are proposed such as hybrid random walk model, random direction model and discrete random direction model, all of which have one thing in common, they introduce a tuning parameter to vary the randomness, which enables some relatively general discussions.

As to these models, the hybrid random walk model and discrete random direction model are discrete since intermediate process in movements is ignored as the unit square is divided into cells. The random direction model has common ground with random walk model and their discrepancy is that one divides time into equivalent slots and the other divides the motion into multiple trips with equivalent distance.

3 Update of the Original Paper

We give a complete demonstration of the update of the paper—"Mobility Increases the Connectivity of Wireless Networks".

In the original paper, a tacit is employed to determine the area covered by a cluster member during a period. The angle between two segments of the track is regarded to be bound away from zero, otherwise the overlapped area can hardly go to $o(r(n))$.

In the updated paper, this problem is specifically targeted to yield more rigid results. It has been taken into account the probability of the event that the angle is larger than an infinitesimal. There are two main problems showing up in the development of the results which will be explained in details in the next section.

3.1 Main Difficulties

In this section, we will describe the two main difficulties.

3.1.1 Problem Concerning Random Variable n_o

The notation n_o denotes the number of certain cluster members which maintain a specific speed in a period. Recall its definition for accuracy:

$$i_o = \arg\min\{\frac{v^{(i)}}{\log_n np_i}, \forall i \in I\} \text{and} p_{i_o} = p$$

$$M_o = \{i | v_i = v^{(i_o)}, i = 1, 2, \ldots, n\}$$
$$n_o = |M_o|$$

Therefore, n_o is a random variable and its mathematical expectation is $\mathbb{E}(n_o) = np$.

In following proof, the formula below is employed

$$
\begin{aligned}
P^{\Lambda}_{f\_rw}(n,r(n)) &\geq \sum_{i=1}^{n} P^{\Lambda}(E_i') \geq \sum_{i\in M_o} P^{\Lambda}(E_i') \\
&\geq \sum_{i\in M_o}\left(P^{\Lambda}(E_i) - \sum_{j\neq i} P^{\Lambda}(E_{ij})\right) \\
&\geq \sum_{i\in M_o} P^{\Lambda}(E_i) - \sum_{i\in M_o}\sum_{j\neq i} P^{\Lambda}(E_{ij})
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
E_i' &= \{s_i \text{ is the only failed session in } \mathcal{G}_{rw}(n,r(n))\} \\
E_i &= \{s_i \text{ is a failed session in } \mathcal{G}_{rw}(n,r(n))\} \\
E_{ij} &= \{s_i \text{ and } s_j \text{ are failed sessions in } \mathcal{G}_{rw}(n,r(n))\}
\end{aligned}
$$

Being a random variable, n_o cannot be substituted into (1). The above formula needs modification.

3.1.2 Problem Concerning Random Variable n_m

This problem is similarly to the problem above and shows up in the proof of the sufficient part. But it is more complicated since it should take into account the correlation between various random variables. According to the definition:

$$
\begin{aligned}
\mathbb{V}_m &= \{i|v_i = v^{(m)}, i = 1,2,\ldots,n\} \\
\mathbb{U} &= \{m|\mathbb{V}_m \neq \emptyset, m \in \mathbb{M}\} \\
|\mathbb{V}_m| &= n_m
\end{aligned}
$$

In the following proof, we have

$$
\begin{aligned}
\sum_{i=1}^{n} P^{\Lambda}(E_i) &= \sum_{j\in\mathbb{U}}\sum_{i\in\mathbb{V}_m} P^{\Lambda}(E_i) \\
&\leq \sum_{j\in\mathbb{U}}\sum_{i\in\mathbb{V}_m} e^{-(1-\epsilon)2kv_i cr(n)n^d} \\
&= \sum_{j\in\mathbb{U}}\sum_{i\in\mathbb{V}_m} n^{-(1-\epsilon)cv_i/v_o} \\
&\leq \sum_{j\in\mathbb{U}}\sum_{i\in\mathbb{V}_m} n^{-(1-\epsilon)c\log_n(np_j)} \\
&\leq \sum_{j\in\mathbb{U}} n_j \cdot n^{-(1-\epsilon)c\log_n(np_j)} \\
&= \sum_{j\in\mathbb{U}} \frac{np_j + o(n)}{(np_j)^{(1-\epsilon)c}} \\
&= \sum_{j\in\mathbb{U}} \frac{1}{(np_j)^{(1-\epsilon)c-1}} + o(\frac{1}{n^{(1-\epsilon)c-1}})
\end{aligned} \tag{2}
$$

Actually, we suppose that $|\mathbb{V}_m| = n_m = np_m + o(n)$ and substitute it into the above formula. However, n_m itself is a random variable. Through, with high probability, n_m will be np_m, we may need more rigid approach.

3.1.3 Problem Concerning a Certain Probability

This problem shows up in the derivation of the critical transmission range for mobile k-hop clustered networks with random walk mobility.

In the lemma, we show

$$S_i^{\Lambda} \geq (1-\epsilon)2r(n)kv_i \quad \text{with probability} \quad P \geq 1 - \frac{2L^{\Lambda}}{\pi n^{d_o/2}}$$

where S_i^{Λ} is the total area covered by node i during its movement in the period Λ. And what follows is that

$$S_{i+j}^{\Lambda} \geq (1-\epsilon')4r(n)kv_i \quad \text{with probability} \quad P' = P \cdot P$$

where S_{i+j}^{Λ} denotes the total area covered by either node i or node j during their movement in period Λ.

In the derivation of $r(n)$, we are supposed to calculate the probability

$$\begin{aligned}
&P^{\Lambda}(\{s_i \text{ and } s_j \text{ are failed sessions in } \mathcal{G}_{rw}\}) \\
&\quad = P(\{s_i \text{ and } s_j \text{ are failed sessions in } \mathcal{G}_{rw}\} | \{S_{i+j}^{\Lambda} \geq (1-\epsilon')4r(n)kv_i\}) \\
&\qquad\quad \cdot P(\{S_{i+j}^{\Lambda} \geq (1-\epsilon')4r(n)kv_i\}) \\
&\qquad + P(\{s_i \text{ and } s_j \text{ are failed sessions in } \mathcal{G}_{rw}\} | \{S_{i+j}^{\Lambda} \leq (1-\epsilon')4r(n)kv_i\}) \\
&\qquad\quad \cdot P(\{S_{i+j}^{\Lambda} \leq (1-\epsilon')4r(n)kv_i\}) \\
&\quad \leq P(\{s_i \text{ and } s_j \text{ are failed sessions in } \mathcal{G}_{rw}\} | \{S_{i+j}^{\Lambda} \geq (1-\epsilon')4r(n)kv_i\}) \\
&\qquad + P(\{S_{i+j}^{\Lambda} \leq (1-\epsilon')4r(n)kv_i\}) \\
&\quad \leq \left(1 - (1-\epsilon_2)4r(n)kv^{(i_o)}\right)^{n^d} + \frac{4L^{\Lambda}}{\pi n^{d_o/2}} - \left(\frac{2L^{\Lambda}}{\pi n^{d_o/2}}\right)^2 \\
&\quad \leq e^{-4n^d(1-\epsilon_2)r(n)kv^{(i_o)}} + \frac{4L^{\Lambda}}{\pi n^{d_o/2}} - \left(\frac{2L^{\Lambda}}{\pi n^{d_o/2}}\right)^2 \\
&\quad = e^{-2(1-\epsilon_2)[\log(np)+(1+\log_n p)\kappa]} + \frac{4L^{\Lambda}}{\pi n^{d_o/2}} - \left(\frac{2L^{\Lambda}}{\pi n^{d_o/2}}\right)^2
\end{aligned} \tag{3}$$

And the probability that $\mathcal{G}_{rw}(n, r(n))$ has some node that is not connected in the period Λ is obtained by (1).

Using (2) in (1), we obtain

$$P_{f\_rw}(n,r(n)) \geq np\Big(1-\big(1+\epsilon_1\big)2kv^{(i_o)}r(n)\Big)^{n^d} - (np)^2 e^{-2(1-\epsilon_2)[\log(np)+(1+\log_n p)\kappa]} \\ -(np)^2\frac{4L^{\Lambda}}{\pi n^{d_o/2}} + (np)^2\left(\frac{2L^{\Lambda}}{\pi n^{d_o/2}}\right)^2 \tag{4}$$

According to the proof of the lemma, $d_o < d \leq 1$, then the last two terms of (3) will go to infinity as $n \to \infty$.

3.2 Approach to the Problems

In this section, I propose a method to solve the problem concerning n_o and demonstrate the attempts targeting the second problem.

3.2.1 Way to the Random Variable n_o Problem

The distribution of n_o can be determined. It follows the *binomial distribution* and its mathematical expectation is np.

The distribution of n_o can be exploited in the deduction of $P^{\Lambda}_{f\_rw}(n,r(n))$, which renders the following result

$$\begin{aligned} P^{\Lambda}_{f\_rw}(n,r(n)) &\geq \sum_{i=1}^{n} P^{\Lambda}(E'_i) \\ &\geq \sum_{i\in M_o} P^{\Lambda}(E'_i) \\ &\geq \sum_{i\in M_o}\Big(P^{\Lambda}(E_i) - \sum_{j\neq i} P^{\Lambda}(E_{ij})\Big) \\ &\geq \sum_{i\in M_o} P^{\Lambda}(E_i) - \sum_{i\in M_o}\sum_{j\neq i} P^{\Lambda}(E_{ij}) \\ &= \sum_{m=0}^{n}\Big(\Big(\sum_{i=0}^{m} P^{\Lambda}(E_i) - \sum_{i=0}^{m}\sum_{j=0}^{m} P^{\Lambda}(E_{ij})\Big)P(n_o=m)\Big) \\ &= \sum_{i=0}^{np} P^{\Lambda}(E_i) - \sum_{i=0}^{np}\sum_{j=0}^{np} P^{\Lambda}(E_{ij}) \end{aligned} \tag{5}$$

$$E'_i = \{s_i \text{ is the only failed session in } \mathcal{G}_{rw}(n,r(n))\}$$
$$E_i = \{s_i \text{ is a failed session in } \mathcal{G}_{rw}(n,r(n))\}$$
$$E_{ij} = \{s_i \text{ and } s_j \text{ are failed sessions in } \mathcal{G}_{rw}(n,r(n))\}$$

The stochastic distribution of n_o is incorporated into this formula, which naturally leads to np in the inequality and solves the problem.

3.2.2 Way to the Random Variable n_m Problem

To cope with this problem, we employ similar approach in the proceeding part.

Let $\mathbb{V}_m = \{i|v_i = v^{(m)}, i = 1,2,\ldots,n\}$, where $m \in \mathbb{M}$ and v_i is the velocity of the cluster-member node i. Hence, let $|\mathbb{V}_m| = n_m$, we have $\mathbb{E}(n_m) = np_m$. Define $\mathbb{U} = \{m|\mathbb{V}_m \neq \emptyset, m \in \mathbb{M}\}$ and $|\mathbb{U}| = u = \Theta(1)$. Using established results, with probability one we have that

$$\begin{aligned}
\sum_{i=1}^{n} P^{\Lambda}(E_i) &= \sum_{m\in\mathbb{U}} \sum_{i\in\mathbb{V}_m} P^{\Lambda}(E_i|i \in \mathbb{V}_m) \\
&= \sum_{m\in\mathbb{U}} \sum_{l=0}^{n} n_m P^{\Lambda}(E_i|i \in \mathbb{V}_m) P(n_m = l) \\
&= \sum_{m\in\mathbb{U}} \mathbb{E}(n_m) P^{\Lambda}(E_i|i \in \mathbb{V}_m) \\
&\leq \sum_{m\in\mathbb{U}} \mathbb{E}(n_m) e^{-(1-\epsilon)2kv_i cr(n)n^d} \\
&= \sum_{m\in\mathbb{U}} (np_m) n^{-cv_i/v_o} \\
&= \sum_{m\in\mathbb{U}} (np_m) n^{-c\log_n(np_m)} \\
&= \sum_{m\in\mathbb{U}} \frac{np_m}{(np_m)^c}
\end{aligned}$$

We incorporate into the expression the distribution of the random variable n_m and solve this problem.

3.2.3 Initial Attempts to Solve the Third Problem

Efforts were made to adjust several steps in the course of deduction.

(A) First, I tried to modify the probability of event that $S_i^{\Lambda} \geq (1-\epsilon)2r(n)kv_i$ occurred. In the proof, there stands

$$S_o^2 \leq \sum_{k=1}^{L^{\Lambda}} \frac{2r(n)}{|\sin\varphi_k|} \cdot 2r(n) \leq 4L^{\Lambda} r^2(n) \cdot \frac{1}{\sin\varepsilon}.$$

And we expect $r(n)/\sin\varepsilon$ to approach 0 as $n \to \infty$. In the original proof, $\varepsilon(n)$ is explicitly chosen to be $1/n^{d_0/2}$, which leads to $1/\sin\varepsilon < 1/\varepsilon^2 = n^{d_0}$. However, the probability $P(A) > 1 - \frac{C}{n^{d_0/2}}(d_0 < d)$ where the order of n is relatively small.

Thus we can choose $\varepsilon(n) = \frac{1}{n^{d_0}}$. Note that $\frac{1}{\sin\varepsilon} < \frac{1}{\varepsilon^t}$ holds for $\forall t > 1$. Then $r(n) \cdot \frac{1}{\sin\varepsilon} < \frac{\log n}{n^d} \cdot n^{td_0}$ will goes to 0 if $td_0 < d$. After the modification the probability is $P(A) > 1 - \frac{C}{n^{d_0}}(td_0 < d, t > 1)$

(B) I consider the second step in inequality (2). In inequality (2), two cases are taken into consideration that $S_{i+j}^{\Lambda} \geq (1-\epsilon')4r(n)kv_i$ and $S_{i+j}^{\Lambda} < (1-\epsilon')4r(n)kv_i$. The term that $\mathrm{P}(\{s_i$ and s_j are failed sessions in $\mathcal{G}_{rw}\}|\{S_{i+j}^{\Lambda} \leq (1-\epsilon')4r(n)kv_i\})$ is substituted with 1.

Maybe we need a more accurate bound on $\mathrm{P}(\cdot)$ above. However, it will introduce new probability while taking into account the event that S_{i+j}^{Λ} is larger than a certain value on the premise that event $S_{i+j}^{\Lambda} < (1-\epsilon')4r(n)kv_i$ occurs.

(C) In the proof of *Lemma 4.1*, L^{Λ} denotes the total number of intersections of track segments and $0 \leq L^{\Lambda} \leq C_k^2$. Actually, L^{Λ} is itself a random variable. Hence, it might worth efforts to look deeper into the movement of a cluster member during a period. The reveal of the stochastic property of this process would help to modify the probability.

There are some challenges to assess this stochastic process. It is easy to bound the probability of overlap between two track segments if they are separated by only one segment. As the number of segments between them increases, the task will get more demanding. So some technics are necessary to cope with this difficulty.

(D) I have try several other approaches which, however, proved to be vain to the extent that they could not solve the problem, either.

3.3 Simplify Mobility Model to Solve the Third Problem

The mobility model is simplified to cope with this problem. We assume that the cluster member will stick to its direction throughout the period. The modified definition of random walk mobility model is as follows:
Random Walk Mobility Model with Non-Trivial Velocities: Define a discrete random variable V the velocity of a node with the probability mass function $P\big(V = v^{(m)}\big) = p_m$ for all $m \in \mathbb{M}$, where the index set $\mathbb{M}$ is finite and invariant of n. We assume that $v^{(m)} = \omega\big(r(n)\big) = \Theta\Big(\sqrt{\frac{\log n}{n^{d'}}}\Big)$, for all $m \in \mathbb{M}$, where $d' < d$. This assumption, combined with the k-time-slots deadline that we will introduce next, implies that we are interested in the case when the velocity is fast enough so that nodes can move multiple transmission ranges before the deadline, for which we expect the scaling laws to differ substantially with that in stationary networks. In addition, we assume that p_m for all $m \in \mathbb{M}$ does not change with n, and further, we assume that there exists an index $m_\star$ such that for all sufficiently large n, $\frac{v^{(m)}}{\log_n np_m} \geq \frac{v^{(m_\star)}}{\log_n np_{m_\star}}$ for all m. We will see later that the probability of full connectivity will depend heavily on the dynamics of the nodes belonging to class $m_\star$. We then partition the data transmission process into time-slots with unit length. At the beginning of each period (i.e., every k slots; see *Transmission scheme* for the definition of a *period*) each member node randomly and indepen-

dently select a velocity $V = v$ according to the distribution of V, and uniformly and independently choose a random direction $\theta \in [0, 2\pi)$. The node then moves along this direction θ with the constant velocity v for the entire period.

Since the node will not alter its moving direction, there will be no overlaps between the area covered by the node in different time slots, which can substantially simplifies the analysis of this problem. And the proof just follows the one that deal with the i.d.d. case.

4 Connectivity Under Mobility-Restricted Model

We have explored the connectivity under mobility-restricted model. As for mobility-restricted model, that means the motion of nodes is confined within a certain area instead of the whole unit square. Such mobility models were studied in current literature. Hence, we would like to study the impact of this kind of mobility pattern on the connectivity of the network.

Unfortunately, our results show that the constraints on the scale of the area that a node can move make no difference to the connectivity of the networks. However, this result is more or less intuitive since in our proof of asymptotic connectivity, the most important issue is the area that a node can cover at an instant or over a time interval which dominates the course of proof. From this point of view, we know that the restraints on the scale of area a node can travel do not impact the area that a node cover. Especially, these models are actually based on i.d.d. mobility model which has a property that the probability that a node meet with another node has no relation with the scale of area this node can travel. Therefore, connectivity under mobility-restricted model remains the same.

5 Coverage In Mobile Wireless Sensor Networks

Coverage is also a fundamental concern in wireless sensor networks. We touched this topic at the late period of this project. Several results have been achieved and the work is still ongoing. The preliminary scratch has come into being. See "coverage_skeleton".

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Appendix

Solid and aggressive. Show desire to pursue new knowledge in this project.

Active in the course and try best to assistant with the work.

Take his responsibility in this group and high involvement into the project.