Buffer Size in Large-scale Waiting-allowed Wireless Networks with External Constraints

Project Report, Group 12
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***Abstract*—We consider a wireless network where waiting is allowed, caused by some external constraints. These constraints can be secondary users waiting for the availability of the communication channel in cognitive radio networks, or nodes turning off in a wireless sensor network for power saving. Waiting in these cases is much different from queuing delay, because it cannot be eliminated via improving the *original capacity* of the network.**

We assume data transmission in the network is in a multihop fashion, thus data buffer of adequate size is needed for each node in the network. We show that, in a supercritical case, the required buffer size is determined by the probability of waiting and message generation rate of each node in the network but has no relation to the size of the network, while in a subcritical case, the required buffer size increases as size of the network grows.

I. Introduction

Waiting is common in wireless networks where practical *constraints* exist. There can be various constraints in different network backgrounds. In [1], the constraint is energy saving. Nodes in a wireless sensor network switch between an active (on) and a sleeping (off) mode to save energy. During sleeping mode, a node cannot receive or transmit information, thus waiting is required in this case. In [2], the constraint for the secondary users in a cognitive radio network is the existence of primary users. When neighboring primary users are communicating, secondary users must remain silent, and waiting is needed until the primary users finish communication. In [3], the constraint is the mobility of each node, and waiting occurs when transmitter and the receiver is not within the transmission range. In these cases, where waiting exists, buffer for each node is required to

In a wireless network without these constraints, buffer is also required for queuing, i.e., to temporarily store the packets to transmit, and queuing delay is the waiting time between the point of entry of a packet in the transmit queue to the actual point of transmission. We use the term *original capacity* to denote the network capacity without external constraints, which can be increased by improving processing speed of each node or physical conditions of wireless channels. In this case, if the required workload of the network is constant, queuing delay and required buffer size decreases as the original capcity increases. Further, if the original capacity of the network tends to infinity, the required buffer size and queuing delay tends to zero.

The constraints mentioned above are external in some sense. The term "*external*" implies that waiting caused by these constraints cannot be eliminated via improving processing speed of each node or physical conditions of wireless channels. Correspondingly, the required buffer size does not tend to zero even if the original capacity tends to infinity. In this paper, we analyze the buffer size requirement for waiting-allowed wireless networks with external constraints.

We consider the network where each node has the same probability to wait, p, as a result of external constraints. We show that the buffer size requirement has a close relation to p. By utilizing the *percolation theory*, we can find a critical value for this p, $p_c(\lambda)$. If $p < p_c(\lambda)$, the network is in a *supercritical case* where there exists an unique infinite connected cluster at any time a.s. when the network size tends to infinity. In contrast, if $p > p_c(\lambda)$, the network is in a *subcritical case* where no infinite connected cluster exists a.s. when the network size tends to infinity. The buffer size requirements are quite different in the two cases. In the supercritical case, it is determined by the probability of waiting and message generation rate of each node in the network but has no relation to the size of the network, while in a subcritical case, the required buffer size increases as size of the network grows.

II. Network Model

In this section, we first define the model of the original network without external constraints. Then, we specify the impact on the original network by the external constraints.

A. Original Network

We consider a Poisson point process on $\mathbb{R}^2$ with constant point density λ. Locations of nodes in the original network are the points within the square region $\mathcal{B} = \left[-\frac{L}{2}, \frac{L}{2}\right]^2$. Let n denote the number of nodes in the network. According to the property of Poisson point process, $\frac{n}{\lambda L^2} \to 1$ as $L \to \infty$.

Each node covers a disk shaped area with radius r. To simplify the analysis, r is treated as a constant for all nodes. Let $X_i (1 \le i \le n)$ denote the random position of node v_i. Two nodes v_i and v_j can communicate with each other if and only if $||X_{v_i} - X_{v_j}|| \le 2r$, where $||X_{v_i} - X_{v_j}||$ is the Euclidean distance between v_i and v_j. Without loss of generality, we assume $2r = 1$.

The model of the original network is denoted by $\mathcal{ON}(\lambda, L)$. When $L \to \infty$, $\mathcal{B} \to \mathbb{R}^2$, the corresponding network is defined as $\mathcal{ON}_\infty(\lambda) = \lim_{L\to\infty} \mathcal{ON}(\lambda, L)$.

According to continuum percolation theory, there is a critical value for λ, λ_c, and there exists a unique infinite connected cluster in $\mathcal{ON}_\infty(\lambda)$(*original giant cluster*, denoted by $C(\mathcal{ON}_\infty(\lambda))$) if and only if $\lambda > \lambda_c$. To assure the majority of the network is connected, we make the following assumption.

Assumption 1 (On Node Density): In the original network, $\lambda > \lambda_c$.

In this paper, we mainly analyze communications of nodes within the original giant cluster. We denote the nodes belonging to the original giant cluster by the term *connected nodes*.

For a finite network, we define the original giant cluster as the largest connected cluster in the original network. The number of connected nodes n_c tends to a constant proportion to n, i.e., $\frac{n_c}{n} \to c_\lambda$ as $n \to \infty$, where c_λ is determined by λ.

B. External Constraints and Waiting

External constraints on the original network make each nodes switching between two states: *active state* and *waiting state*. During active state, a node can transmit or receive messages, while during waiting state it can neither transmit nor receive messages. The transmission between two nodes is possible only if both the transmitter and the receiver are active. For example, in a cognitive radio network, a secondary user is active if the neighboring primary users stop transmission and thus the channel is available; otherwise, the secondary user should keep waiting until the occurrence of available channel.

We assume the external constraints in the network are in a synchronized time-slotted manner with a slot length T_{EC}, which implies that the state of each node changes only at the beginning of a time slot. Further, the external constraints satisfy the following assumptions:

1) The realizations of active nodes vary from slot to slot, and are i.i.d. across slots.
2) The probability to be waiting is a constant p for all nodes in the network.
3) States of different nodes are i.i.d.

Without loss of generality, let $T_{EC} = 1$ in this paper. The network with external constraints is denoted by $\mathcal{CN}(\lambda, L, p)$, and define $\mathcal{CN}_\infty(\lambda, p) = \lim_{L\to\infty} \mathcal{CN}(\lambda, L, p)$. The respective sets of time active nodes during time slot t is denoted by $A(\mathcal{CN}(\lambda, L, p), t)$ and $A(\mathcal{CN}_\infty(\lambda, p), t)$.

If there is a path consisted of active nodes in time slot t between node u and v, we say u and v are *instantaneously connected*.

C. Traffic Pattern and Buffering

We only consider the traffic of nodes within $C(\mathcal{ON}(\lambda, L))$, i.e., the connected nodes.

Traffic Pattern of Connected Nodes: For each connected node in the network, as a source, it randomly chooses a permanent destination among other connected nodes, and this source-destination relationship does not change over time. Each node generates messages to its corresponding destination node in a multihop fashion at a constant rate of r_g, which does not vary among different nodes.

Buffering: In each hop, if the transmitter or the receiver is waiting, the message should be kept in the buffer of the transmitter until both nodes are active. As we define before, if a node(as a source) or its first intermediate node toward destination is waiting, it cannot send any message actually. Yet we can still assume the waiting source node "sends" messages at rate r_g but temporarily stored in the buffer of itself.

We present the definition of *original capacity*, and a basic assumption on the original capacity in this paper.

Definition 1 (Original Capacity): The original capacity is the maximum bits per second each connected node can send to its chosen destination node.

Assumption 2 (On the Original Capacity): The original capacity is large enough to be considered as infinity, compared to the actual transmission rate of each node.

As Assumption 2 states, the original capacity of the network is infinity, which implies that once a node and its next hop turn active, they can transmit and receive message without delay[1]. If all nodes in one path are active, the message can be transmitted from one end to the other without delay. This helps us focus on the effect of external constraints of the network.

Maximum Buffer Size in Each Time Slot: Since the original capacity is infinity, buffered message in each node is transimitted only at the beginnig of each time slot in a very small time interval. On the other hand, the message generation rate r_g is finite and constant, and thus smooth message buffering could happen during each time interval. Therefore, in each time slot, the size of occupied buffer in each node is maximum at the end of the time slot. For a connected node v, we use $S_v^{(L)}(t)$ to denote the occupied buffer size of v at the end of time slot t in $\mathcal{CN}(\lambda, L, p)$, and $S_v^{(\infty)}(t)$ denote the occupied buffer size of v at the end of time slot t in the occupied buffer size of v at the end of time slot t in $\mathcal{CN}_\infty(\lambda, p)$.

Message Slot: We assume the transmission path of each message does not change if the states of all the nodes in the network do not change. Therefore, it is easy to see that the messages generated by one node during one time slot must exist at the same node at the end of a time slot. We call the messages generated by u during time slot t whose destination is v a *message slot*, denoted by $m_{u\to v}^{(t)}$. If only the source or destination is specified, the notation is simplified as $m_{u\to}^{(t)}$ or $m_{\to v}^{(t)}$. If the generating time slot is not specified, the notations can be simplified as $m_{u\to v}$, $m_{u\to}$ and $m_{\to v}$.

III. Percolation in the Network with External Constraints

According to Assumption 1, the original network is in a supercritical case, where a giant component exists a.s. as the

[1]The propagation delay is omitted in the network. Since the original capacity is infinity, the queuing delay in each node is zero.

size of the network tends to infinity. With external constraints, at each time slot, we consider the connectivity of active nodes, or the *instantaneous connectivity*.

Since the the states of nodes are i.i.d., the distribution of active nodes in $\mathcal{CN}_\infty(\lambda, p)$ is according to a Poisson Point Process with constant point density $(1-p)\lambda$. Therefore, there exists a critical value for $p_c(\lambda) = 1 - \frac{\lambda_c}{\lambda}$ such that:

If $p < p_c(\lambda)$, $\mathcal{CN}_\infty(\lambda, p)$ is in a supercritical case, where there exists a unique infinite connected cluster of active nodes a.s. during each time slot. Let $C(\mathcal{CN}_\infty(\lambda, p), t)$ denote the infinite connected cluster of active nodes(active giant cluster) during time slot t;

If $p > p_c(\lambda)$, $\mathcal{CN}_\infty(\lambda, p)$ is in a subcritical case, where there does not exist a unique connected cluster of active nodes a.s. during each time slot.

Let $\theta(\lambda, p|active)$ denote the probability that an arbitrary active connected node belongs to active giant cluster in an arbitrary time slot, then we have

$$\theta(\lambda, p|active) \begin{cases} > 0, & p < p_c(\lambda) \\ = 0, & p > p_c(\lambda) \end{cases} .$$

Let $\theta(\lambda, p)$ denote the probability that an arbitrary connected node(without knowing its state) belongs to active giant cluster in an arbitrary time slot, then

$$\theta(\lambda, p) = (1-p)\theta(\lambda, p|active) \begin{cases} > 0, & p < p_c(\lambda) \\ = 0, & p > p_c(\lambda) \end{cases} .$$

Lemma 1 describes the relationship between $C(\mathcal{CN}_\infty(\lambda, p), t)$ and $C(\mathcal{ON}_\infty(\lambda))$.

Lemma 1: If $p < p_c(\lambda)$, $C(\mathcal{CN}_\infty(\lambda, p), t) \subseteq C(\mathcal{ON}_\infty(\lambda))$.

Proof: Since $\lambda > \lambda_c$, the original network $\mathcal{ON}_\infty(\lambda)$ is supercritical and $C(\mathcal{ON}_\infty(\lambda))$ is unique. Suppose there is a node $v \in C(\mathcal{CN}_\infty(\lambda, p), t)$ but $v \notin C(\mathcal{ON}_\infty(\lambda))$. Then $C(\mathcal{ON}_\infty(\lambda)) \cap C(\mathcal{CN}_\infty(\lambda, p), t) = \emptyset$, because otherwise v can connect to $C(\mathcal{ON}_\infty(\lambda))$ through the nodes belonging to both sets. Therefore, there exist two disjoint infinite connected clusters in $\mathcal{ON}_\infty(\lambda)$, which contradicts the fact that $C(\mathcal{ON}_\infty(\lambda))$ is unique. ∎

IV. Buffer Size Requirements in Supercritical Case

In this section, we study the buffer size requirements of active connected nodes in $\mathcal{CN}(\lambda, L, p)$. The main result is that the buffer size requirements of active connected nodes do not increase as the size of network grows, as stated in Theorem 1.

Theorem 1: For an arbitrary connected node w in $\mathcal{CN}_\infty(\lambda, p)$ with $p < p_c(\lambda)$, at the end of an arbitrary time slot t, it is achievable with some routing strategy that

$$0 < b_0 < \mathbb{E}(S_w^{(\infty)}(t)) < c_0 r_g < \infty, \quad (1)$$

where b_0, c_0 are constants irrelevant to the choosing of w and t, and r_g is the message generation rate of each connected node.

We first present a routing strategy, and then prove that under this strategy, buffer size requirement specified in Theorem 1 is achieved.

A. Ideal Routing Strategy

In Ideal Routing Strategy(IRS), we assume that each connected nodes knows the locations and current states of all the other connected nodes in the network.

For each connected node u, as a source whose destination is v, in time slot t, if $u \notin C(\mathcal{CN}(\lambda, L, p), t)$, then u stores the message being generated by itself in its buffer; if $u \in C(\mathcal{CN}(\lambda, L, p), t)$, then u sends the messages generated and being generated by itself to w via the intermediate nodes in $C(\mathcal{CN}(\lambda, L, p), t)$, where

$$w = \underset{w_i \in C(\mathcal{CN}(\lambda, L, p), t)}{\arg\min} ||X_w - X_v||,$$

i.e., the nearest node in $C(\mathcal{CN}(\lambda, L, p), t)$ to v. [2]

For messages in the buffer of u which are not generated by u, u selects the next hop x, which is the next hop of u in the path from u to the destination of the messages with the smallest number of hops in the original network $\mathcal{ON}(\lambda, L)$. [3] In this case, u is called a *buffering intermediate node*.

For each message slot m, we name the path consisting of its buffering intermediate node the *buffering path of* m, denoted by PB_m. The number of buffering intermediate node is denoted by NB_m. Then we give the definition of *buffering radius*.

Definition 2 (Buffering Radius): The buffering radius of message slot $m_{\to v}$, denoted by RB_m is

$$RB_m = \max_{w_i \in PB_m} ||X_{w_i} - X_v||.$$

Let $L \to \infty$, we get the IRS in $\mathcal{CN}_\infty(\lambda, p)$.

B. Finite Buffering Hops and Radius

Buffering can only happen in two cases: if currently the source $u \notin C(\mathcal{CN}_\infty(\lambda, p), t)$, then u should buffer the messages being generated by itself; if $u \in C(\mathcal{CN}_\infty(\lambda, p), t)$, the buffering intermediate nodes should buffer the messages generated by u.

Lemma 2: For a message slot $m_{\to v}$, the number of buffering intermediate nodes $NB_{m_{\to v}}$ and the buffering radius $RB_{m_{\to v}}$ satisfy

$$\mathbb{P}(RB_{m_{\to v}} \geq R) < \beta_1(R+1)e^{-\alpha_1 R},$$

where α_1 and β_1 are constant with $\alpha_1 > 0$ and $\beta_1 < \infty$, and

$$\mathbb{P}(NB_{m_{\to v}} \geq N) < \beta_2(\sqrt{N}+1)e^{-\alpha_2\sqrt{N}}$$

where α_2 and β_2 are constants with $\alpha_2 > 0$ and $\beta_2 < \infty$. The values of $\alpha_1, \alpha_2, \beta_1, \beta_2$ are irrelevant to the choosing of $m_{\to v}$.

Proof: Consider the distance between v and the first node in $RB_{m_{\to v}}$, w. According to IRS, the time when w receives the message, there is no node belonging to $C(\mathcal{CN}_\infty(\lambda, p), t)$ within the circle centered at v with radius $R = ||X_w - X_v|| -$

[2]If $v \in C(\mathcal{CN}(\lambda, L, p), t)$, then $w = v$.

[3]The path with the smallest number of hops from u to destination in $\mathcal{ON}(\lambda, L)$ is equivalent to the path with the smallest number of hops from u to destination in $C(\mathcal{ON}(\lambda, L))$. This is because u is not connected to any node outside $C(\mathcal{ON}(\lambda, L))$.

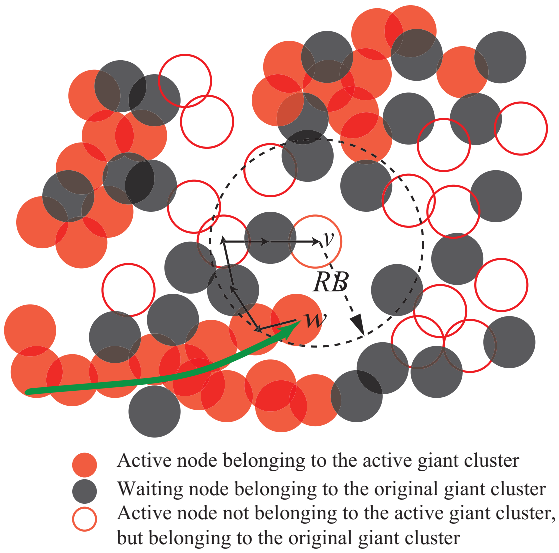


Fig. 1. Finite hops of Buffering

$\frac{1}{2}$[4], denoted by C_R. It implies that a vacant component[5] with radius at least R surrounds C_R. We assume v is at the origin, then the vacant component should cross the x axis outside C_R.

As shown in Figure 2, we draw a string of unit squares, beginning at $(R, 0)$ along x axis, denoted by $\{Sq_i\}$. Let N_i denote the number of vacant component intersecting Sq_i, and $\{V_{i,j}\}$ denote the jth vacant component intersecting Sq_i. By Lemma 4.5 in [5], the expected number of vacant component intersecting each square is a constant, denoted by $N_{vacant} = \mathbb{E}(N_i)$. Since it is supercritical case, in continuum percolation

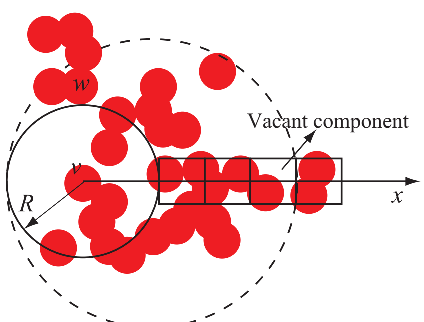


Fig. 2. Vacant component surrounding the circular region.

theory(see [5] Chapter 4), the diameter of an arbitrary vacant component $d(V)$ satisfies

$$\mathbb{P}(d(V)) \geq a) \leq c_{v1} e^{-c_{v2} a}$$

[4] $\frac{1}{2}$ is the radius of the coverage area of a node.

[5] A vacant component means a continuous area where no *active* node exists, see [5].

where c_{v1} and c_{v2} are constants with $c_{v1} < \infty$ and $c_{v2} > 0$. Then we have

$$\begin{aligned}
& \mathbb{P}(Sq_i \,\text{contains a vacant component surrouding}\, C_R) \\
\leq\ & \sum_{k=0}^{\infty} \mathbb{P}(N_i = k) \sum_{j=1}^{k} \mathbb{P}(V_{i,j} \,\text{surrounds}\, C_R) \\
\leq\ & \sum_{k=0}^{\infty} \mathbb{P}(N_i = k) \sum_{j=1}^{k} \mathbb{P}(d(V_{i,j}) \geq 2R + i - 1) \\
\leq\ & \sum_{k=0}^{\infty} \mathbb{P}(N_i = k) \sum_{j=1}^{k} c_{v1} e^{-c_{v2}(2R+i-1)} \\
\leq\ & c_{v1} e^{-c_{v2}(2R+i-1)} \sum_{k=0}^{\infty} \mathbb{P}(N_i = k) k \\
=\ & c_{v1} e^{-c_{v2}(2R+i-1)} \mathbb{E}(N_i) \\
=\ & N_{vacant} c_{v1} e^{-c_{v2}(2R+i-1)}
\end{aligned}$$

Therefore

$$\begin{aligned}
& \mathbb{P}(||X_w - X_v|| \geq R) \\
\leq\ & \sum_{k=1}^{\infty} \mathbb{P}(Sq_i \,\text{contains a vacant component surrouding}\, C_{R-\frac{1}{2}}) \\
\leq\ & \sum_{k=1}^{\infty} N_{vacant} c_{v1} e^{-c_{v2}(2R-1+i-1)} \\
\leq\ & \beta' e^{-\alpha' R}
\end{aligned}$$

where α' and β' are constants with $\alpha' > 0$ and $\beta' < \infty$.

In Figure 3, the shaded region contains a circuit of connected nodes(denoted by event E_d, where d is the side length of the square) with probability $\mathbb{P}(E_d) \geq 1 - \beta_1' d e^{-\alpha_1' d}$ where $\alpha_1' > 0$ and $\beta_1' < \infty$(see [1], Lemma 2). In this case, $PB_{m \to v}$ is within the square, and thus $RB_{m \to v} \leq \sqrt{2} d$.

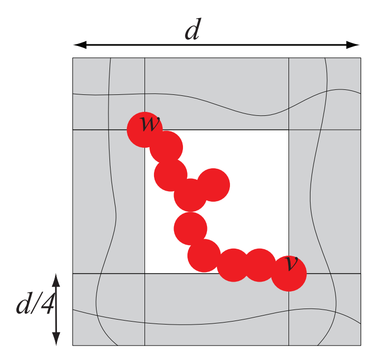


Fig. 3. w and v are enclosed in a circuit of connected nodes.

$$\begin{aligned}
& \mathbb{P}(RB_{m \to v} \geq R) \\
\leq\ & \mathbb{P}(\{||X_v - X_w|| \geq \frac{R}{2}\} \cup \overline{E_R}) \\
\leq\ & \mathbb{P}(||X_v - X_w|| \geq \frac{R}{2}) + 1 - \mathbb{P}(E_R) \\
\leq\ & \beta' e^{-\alpha' R/2} + \beta_1' R e^{-\alpha_1' R} \\
\leq\ & \beta_1 (R+1) e^{-\alpha_1 R},
\end{aligned}$$

where α_1 and β_1 are constant with $\alpha_1 > 0$ and $\beta_1 < \infty$.

$NB_{m\to v}$ is the number of hops of connected nodes in the shortest path between w and v. In the proof of Proposition 4 in [1], the authors show that there exist constants γ, $\beta_2' < \infty$ and $\alpha_2' > 0$ such that for $N > \gamma||X_w - X_v||$, $\mathbb{P}(NB_{m\to v} \geq N) \leq \beta_2'\sqrt{N}e^{-\alpha_2'\sqrt{N}}$. We have

$$\begin{aligned} & \mathbb{P}(NB_{m\to v} \geq N) \\ \leq \;& \mathbb{P}(||X_w - X_v|| \geq \frac{N}{\gamma}) \\ & +\mathbb{P}(NB_{m\to v} \geq N | N > \gamma ||X_w - X_v||) \\ \leq \;& \beta' e^{-\alpha' \frac{N}{\gamma}} + \beta_2'\sqrt{N}e^{-\alpha_2'\sqrt{N}} \\ < \;& \beta_2(\sqrt{N}+1)e^{-\alpha_2\sqrt{N}} \end{aligned}$$

■

Based on Lemma 2, Corollary 1 can be proved.

Corollary 1: For a message slot $m_{\to v}$ and a connected node w, there exist constants $\alpha_1 > 0$ and $\beta_1 < \infty$ such that

$$\mathbb{P}(w \in PB_{m\to v}) < \beta_1(||X_w - X_v|| + 1)e^{-\alpha_1||X_w - X_v||}.$$

Proof: From the definition of buffering radius $RB_{m\to v}$, $w \in PB_{m\to v}$ implies that $||X_w - X_v|| \leq RB_{m\to v}$. Applying Lemma 2, we have

$$\begin{aligned} \mathbb{P}(w \in PB_{m\to v}) &\leq \mathbb{P}(||X_w - X_v|| \leq RB_{m\to v}) \\ &< \beta_1(||X_w - X_v|| + 1)e^{-\alpha_1||X_w - X_v||}. \end{aligned}$$

■

C. Finite Message Existing Time

It can be proved that with IRS, the existing time of each message in the network is finite a.s., regardless of the size of the network. We present the following lemma with respect to the network with $L \to \infty$.

Lemma 3: Let T_m denote the existing time an arbitrary message slot m. Then

$$\mathbb{E}(T_m) < c_1 < \infty,$$

where c_1 is irrelevant to the choosing of m.

Proof: $T_m = T_{m,s} + T_{m,b}$, where $T_{m,s}$ is the time(or number of time slots) during which m is buffered in its source, and $T_{m,b}$ is the time m is buffered in PB_m. Since $p < p_c(\lambda)$, we have $\theta(\lambda, p) > 0$ and

$$\mathbb{E}(T_{m,s}) = \frac{1}{\theta(\lambda, p)}. \quad (2)$$

Now consider the single hop waiting-time in PB_m, denoted by T_{b1}. At the beginning of each hop, the transmitter is active, because it has just received m from its previous hop. The possibility to transmit at the first time slot(i.e. without waiting) is $1-p$; if m is buffered in the current hop for some time slots, then the possibility to transmit in the next time slot is $(1-p)^2$. We have

$$\begin{aligned} \mathbb{E}(T_{b1}) &= \sum_{k=1}^{\infty} p(1-p)^2(1-(1-p)^2)^{k-1}k \\ &= \frac{p}{(1-p)^2} \end{aligned}$$

From Lemma 2, $\mathbb{P}(NB_m \geq k) < \beta_2(\sqrt{k}+1)e^{-\alpha_2\sqrt{k}}$, then

$$\begin{aligned} \mathbb{E}(T_{m,b}) &= \sum_{k=0}^{\infty} \mathbb{P}(NB_m = k)k\mathbb{E}(T_{b1}) \\ &\leq \sum_{k=0}^{\infty} \mathbb{P}(NB_m \geq k)k\frac{p}{(1-p)^2} \\ &< \sum_{k=0}^{\infty} \beta_2(\sqrt{k}+1)e^{-\alpha_2\sqrt{k}}k\frac{p}{(1-p)^2} = c_1' < \infty. \end{aligned}$$

Therefore,

$$\mathbb{E}(T_m) = \mathbb{E}(T_{m,s}) + \mathbb{E}(T_{m,b}) < p + c_1' = c_1 < \infty.$$

■

Corollary 2: Let $M_{u\to}(t)$ denote the number of message slots generated by u existing in the entire network at the end of time slot t, then

$$\mathbb{E}(M_{u\to}(t)) < c_1 < \infty,$$

where c_1 is irrelevant to the choosing of u and t.

Proof: From Lemma 3, c_1 is an upper bound of the expected existing time for all message slots. Therefore, the average existing time $\overline{T}$ for message slots is less than c_1. Since the u generates one message slot during each time slot, by *Little's Law*,

$$\mathbb{E}(M_{u\to}(t)) = 1 \times \overline{T} < c_1.$$

■

Corollary 3: Let $M_{\to v}(t)$ denote the number of message slots whose destination is v existing in the entire network at the end of time slot t, then

$$\mathbb{E}(M_{\to v}(t)) =< c_1 < \infty,$$

where c_1 is irrelevant to the choosing of v and t.

Proof: Suppose the number of connected nodes is n. For an arbitrary connected node v as a destination, let NS_v denote the number of its corresponding source, then $\mathbb{E}(NS_v) = n \times \frac{1}{n} = 1$. Let $n \to \infty$, we also have $\mathbb{E}(NS_v) = 1$ in the infinite network. Then

$$\begin{aligned} \mathbb{E}(M_{\to v}(t)) &= \sum_{k=0}^{\infty} \mathbb{P}(NS_v = k) \sum_{i=1}^{k} \mathbb{E}(M_{u_i\to}(t)) \\ &< \sum_{k=0}^{\infty} \mathbb{P}(NS_v = k)kc_1 \\ &= c_1\mathbb{E}(NS_v) = c_1 < \infty. \end{aligned}$$

■

D. Finite Buffer Occupation

In this section we finally present the proof of Theorem 1.

Proof of Theorem 1: First consider the upper bound in Inequality 1. Let $M_{\to v}(w, t)$ denote the number of message slots buffered in w with destination v, and m_i denote one of the

$M_{\to v}(w,t)$ message slots. Applying Corollary 1 and Corollary 3,

$$
\begin{aligned}
& \mathbb{E}(M_{\to v}(w,t)) \\
\leq\; & \sum_{k=1}^{\infty}\mathbb{P}(M_{\to v}(t)=k)\sum_{i=1}^{k}\mathbb{P}(w\in PB_{m_i}) \\
<\; & \sum_{k=1}^{\infty}\mathbb{P}(M_{\to v}(t)=k)k\beta_1(||X_w-X_v||+1)e^{-\alpha_1||X_w-X_v||} \\
=\; & \beta_1(||X_w-X_v||+1)e^{-\alpha_1||X_w-X_v||}\mathbb{E}(M_{\to v}(t)) \\
<\; & c_1\beta_1(||X_w-X_v||+1)e^{-\alpha_1||X_w-X_v||}
\end{aligned}
$$

We divide the network into a collection of ring areas centered at w, each with a width of d, denoted by $\{R_i\}$ where R_i is the ith ring from the w. Let N_{R_i} denote the number of connected nodes in R_i, then $\mathbb{E}(N_{R_i}) < 2\lambda\pi id$.

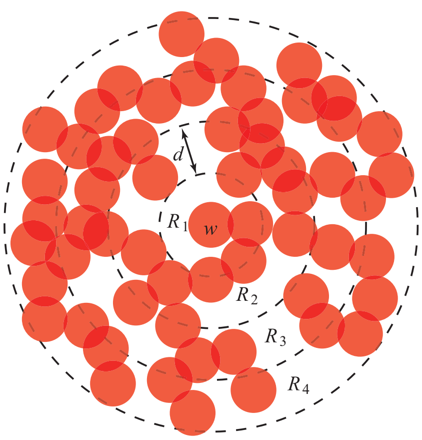


Fig. 4. Division of the network space into rings centered at w

Let M_{w,t,R_i} denote the number of message slots buffered in w at the end of time slot t with destinations within R_i, and v_j denote one connected node in R_i Then

$$
\begin{aligned}
& \mathbb{E}(M_{w,t,R_i}) \\
=\; & \sum_{k=1}^{\infty}\mathbb{P}(N_{R_i}=k)\sum_{j=1}^{k}\mathbb{E}(M_{\to v_j}(w,t)) \\
\leq\; & \sum_{k=1}^{\infty}\mathbb{P}(N_{R_i}=k)kc_1\beta_1(||X_w-X_{v_j}||+1)e^{-\alpha_1||X_w-X_{v_j}||} \\
\leq\; & \sum_{k=1}^{\infty}\mathbb{P}(N_{R_i}=k)kc_1\beta_1(id+1)e^{-\alpha_1(i-1)d} \\
=\; & c_1\beta_1(id+1)e^{-\alpha_1(i-1)d}\mathbb{E}(N_{R_i}) \\
<\; & 2\lambda\pi id\beta_1(id+1)e^{-\alpha_1(i-1)d}=\beta_4(id+1)ide^{-\alpha_1(i-1)d}
\end{aligned}
$$

Sum up the message slots to all the rings, and multiply the sum by the message generation rate r_g we finally have

$$
\begin{aligned}
\mathbb{E}(S_w^{(\infty)}(t)) &= r_g\sum_{i=1}^{\infty}\mathbb{E}(M_{w,t,R_i}) \\
&< r_g\sum_{i=1}^{\infty}\beta_4(id+1)ide^{-\alpha_1(i-1)d} \\
&= c_0 r_g < \infty
\end{aligned}
$$

For the the lower bound in Inequality 1, let $M_w(w,t)$ denote the number of message slots generated by and buffered in w at the end of time slot t and m is an arbitrary one of such message slots. Applying Equation 2 and $Little'sLaw$,

$$
\begin{aligned}
\mathbb{E}(S_w^{(\infty)}(t)) &> c_g\mathbb{E}(M_w(w,t)) \\
&= c_g\times 1\times\mathbb{E}(T_{m,s})) \\
&= \frac{c_g}{\theta(\lambda,p)}=b_0
\end{aligned}
$$

■

b_0, c_0, are determined by the p and λ, which does not depend on the size of the network. Theorem 1 indicates that the occupied buffer size in each connected node does not grow to infinity as the size of the network goes to infinity.

V. Unfinished Work:Buffer Size Requirements in Subcritical Case

If $(1-p)\lambda < \lambda_c$, the network is subcritical. In this case, the active giant does not exist during each time slot a.s. Correspondingly, in the optimal case, the length of buffering path should be asymptotically linear to the distance of the source and the destination. Therefore, when the size of the network goes to infinity, message existing time and the length of buffering path should also approaches to infinity.

In the optimal case, the required buffer size of each node should at least satisfy the following inequality.

$$
0 < b_0 < \lim_{L\to infty}\mathbb{E}(\frac{S_w^{(L)}(t)}{L}) < c_0 r_g < \infty,
$$

References

[1] O. Dousse, P. Mannersalo and P. Thiran “Latency of wireless sensor networks with uncoordinated power saing mechanisms,” in *Proc. ACM MobiHoc'04*, May. 2004.

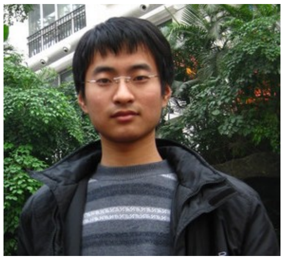
[2] W. Ren and Q. Zhao, “On the multihop delay in ad hoc cognitive radio networks,” *Technical Report (TR-09-2)*. http://www.ece.ucdavis.edu/ qzhao/TR-09-02.pdf., September 2009.

[3] Z. Kong and E. M. Yeh, “On the Latency for Information Dissemination in Mobile Wireless Networks,” in *Proc. ACM MobiHoc'08*, Hong Kong SAR, China, May. 2008.

[4] G. R. Grimmett, “Percolaiton,” Springer, 1999.

[5] R. Meester and R. Roy, “Continuum Percolation,” NewYork: Cambridge University Press, 1996.

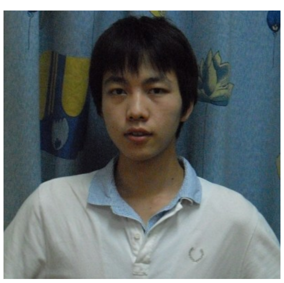
[6] P. Gupta and P. R. Kumar, “The capacity of wireless networks,” *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 388-404, Mar. 2000.



Yuanzhong Xu finds the topic of buffer size and points out what the results should be. He uses techniques in percolation theory to prove the results in the supercritical case.

Chenhui Zhai reads some papers on the issue of connectivity of wireless networks, especially those utilizing percolation theory. He also takes part in the proof of the result.

Yuxiang Cui studies some papers such as [1], and helps to construct the proof methods for the unfinished subcritical part.



Jing Huang reads some papers on connectivity, as well as capacity of wireless networks. He provides some valuable advice on the network model.