A Unifying Perspective on Multicast Capacity in Wireless Network with Spatial Inhomogeneity

***Abstract*—In this paper, we investigate the asymptotic multicast capacity of static wireless network with inhomogeneous node distributions per cluster and each cluster kernel desires to send identical packet to nodes of its cluster. In particular, we introduce a novel indicator *spatial variance* $\delta_{\mathcal{O}}$ to describe inhomogeneities over the deployment region. In addition, we construct an optimal network layout to achieve the maximum capacity of a cluster when $\delta_{\mathcal{O}}$ is upper bounded by some value. Finally, we provide an algorithm of the achievable capacity of such network configuration utilizing percolation theory.**

To the best of our knowledge, there is few work concerning spatial inhomogeneous of a cluster and non-uniform cluster size. Our analysis can generalize various previous results like [2]–[4], [12] obtained under homogeneous node distributions and traffic. By analyzing the constraints imposed by the wireless network, we provide a close form of the capacity upper bound of each cluster. Then we study the maximized achievable capacity given $\delta_{\mathcal{O}}$ is restricted. And at last, we propose a routing scheme for a special case. In addition, we find that non-uniform traffic can improve the achievable capacity in our network, which, in some case, can achieve the maximized capacity even as MANET $\Theta(W)$.

***Index Terms*—Static Wireless Ad-hoc Network, Capacity, Inhomogeneity.**

I. Introduction

Wireless networks are modeled as a set of nodes that send and receive messages over a wireless channel. Since the seminal work done by P. Gupta, P. R. Kumar [1], there is significant interest toward the asymptotic capacity of such networks when the number of nodes n grows. In [1], they prove that the achievable unicast capacity is $\Theta(W/\sqrt{n \log n})$ of static ad hoc network. Later M. Franceschetti, etc. [4] show a better result of $\Theta(W/\sqrt{n})$ utilizing percolation theory.

While the above studies are all based on unicast traffic, there are blast applications on some more generalized traffic patterns. In [16], S. Toumpis present capacity results for asymmetric, cluster, and hybrid wireless ad hoc networks, which first theoretically investigate non-uniform traffic. In [2], Li, etc. study cluster network, also known as multicast network. They provide a better achievable data rate $\Theta(W/\sqrt{kn \log n})$ when a single source sends identical packets to k randomly selected destinations, which generalize both unicast and broadcast [14] capacity. Other works falling into this class can be seen in [3], [8]. In [17], [18], they analyze the capacity of hybrid wireless network with n randomly distributed normal nodes and m regularly placed base stations connected via an optical network. On the other hand, some researchers also investigate the achievable capacity of wireless network with non-uniform traffic, i.e [19], [20].

Another line of research deals with inhomogeneous node distributions. In [11], the proposed approach can be utilized to analyze capacity for non-i.i.d. node distributions. However, their resulting capacity is similar to that derived in [1]. In [5], G.Alfano, etc. study the upper bound on the achievable capacity of networks comprising significant inhomogeneities in the node spatial distribution. They show that the network capacity is related with the minimal node intensity. And in [6], they propose novel scheduling and routing schemes which approach previously computed upper bounds.

To the best of our knowledge, the SD[1] pairs assumed in all the previous works are across the deployed region, which means the distance between source and destination scales linearly with the side length of the network area. However, SD pairs are likely to exist between nodes of shorter distance in many applications, e.g, in military battlefield, different commanders from different places must send their orders over a common wireless channel to their respect soldiers around them. In sensor network, a local scheduler also need to send packets to sensors around it. They exhibit both clustered characteristic and spatial inhomogeneities for that members belonged to the same cluster are are not uniformly distributed in the network area. Therefore, an estimation on the achievable capacity of these wireless networks is essential. Now the questions are as follows:

- How to model the non-uniform distributed SD pairs to approximate the above natural scenario?
- Is there an indicator that can describe the degrees of the inhomogeneity?
- What is the maximized data rate of such network and how to schedule the traffic to make it achievable.

To answer the above questions, we generate non-uniform distributed SD pairs with an IPP[2]. Specifically, we assume there are n_s clusters over the network deployed region $\mathcal{O}$. In each cluster, a header (also known as kernel) disseminates its packets to the other members generated by an IPP according to a dispersion density function $\phi(\cdot)$.

The main insight provided in this paper is that the extent of inhomogeneity can be described by a variable *spatial variance* $\delta_{\mathcal{O}}$ over the deployed region $\mathcal{O}$. And it is intuitive that a larger $\delta_{\mathcal{O}}$ usually indicates a higher degree of inhomogeneity.

Based on the proposed model, we first analyze various constraints imposed by the wireless network. These are some fundamental limits which can not be violated by arbitrary routing and scheduling policies. Then we unify these limitations and present an upper bound for the achievable capacity per

[1] source and destination

[2] inhomogeneous poisson process

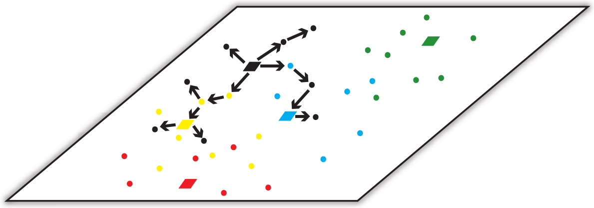

Fig. 1: Demonstration of Network topology. Members of the same cluster are labeled with the same cluster. The square nodes and circular nodes are kernels and members respectively.

cluster. Counter-intuitively, we find that the achievable data rate per cluster does not relate with its cluster size if the size of each cluster are identical. After that, we consider what is the maximized achievable capacity given that $\delta_{\mathcal{O}}$ is constrained. For studying the optimal case, we propose a *uniform cluster random model* and propose routing schemes to approach the previous upper bound. Therefore, we find the mathematical upper bounds can be achievable to some extent.

Main Contributions:

- We propose a novel cluster network model for investigating the non-uniform distributed SD pairs, which utilizes dispersion density function $\phi(\cdot)$ for describing inhomogeneity.
- We provide a fundamental restrictions on the achievable capacity $\lambda_i = O(1)$ for each cluster $\mathcal{C}_i$ $(1 \leq i \leq n_s)$ as follows:
$$\sum_{i=1}^{n_s} \lambda_i \sqrt{|\mathcal{C}_i|} \leq \frac{8\sqrt{2\pi \sum_{i=1}^{n_s} |\mathcal{C}_i|} WL}{\Delta c_0 c_2 c_4 c_5 \int_{\mathcal{O}} \sqrt{\phi(\xi)} d\xi}$$
where c_i is some constants and $|\mathcal{C}_i|$ is the size of cluster $\mathcal{C}_i$.
- When $\delta_{\mathcal{O}}$ is utilized to characterize inhomogeneity, we derive the maximized capacity in terms of it. We find that the achievable capacity per cluster does not vary with the cluster size if all the clusters are equal sized.
- Under the cluster random model, we propose a routing scheme for approaching the upper bound assisted by percolation theory.

The rest of the paper is organized as follows. In section II, we outline the network topology, transmission model, traffic model and some mathematical notations. In section III, we analyze various restrictions of the proposed network model. In section IV, we give a close form of the capacity upper bound per cluster. In section V, we provide a routing scheme for the achievable capacity for *unform random cluster model*. A discussion of the results is in section VI. Finally, we conclude this paper in section VII.

II. Network Model

A. Network Topology

We consider a large scale wireless ad hoc networks composed of n_s clusters distributed within a 2-dimensional torus $\mathcal{O}$ of side length L. For each cluster $C_i (1 \leq i \leq n_s)$, We first specify a *HPP*[3] to generate cluster kernel v_i, whose position is denoted as k_i. Then, each kernel v_i generates its cluster members according to an IPP whose intensity at ξ is given by $|\mathcal{C}_i|\phi_i(k_i, \xi)$, where $|\mathcal{C}_i|$ is the expected size(cardinality) of the cluster. We further assume that $|\mathcal{C}_i| \leq p$ and there exists a constant c_0 such that the number of cluster whose size is $\Theta(p)$ is larger than $c_0 n_s$.

And $N = \sum_{i=1}^{n_s} |\mathcal{C}_i|$ denotes the expected number of overall wireless nodes. The dispersion density function $\phi(k_i, \xi)$ satisfy the following properties.

(i) $\phi(k_i, \xi)$ is invariant under both translation and rotation with respect to v_i, which means $\phi(k_i, \xi)$ can be rewritten as $\phi(|k_i - \xi|)$.

(ii) $\phi(|k_i - \xi|)$ is a non-increasing function with respect to the Euclidean distance $|k_i - \xi|$.

(iii) Integrate $\phi(k_i, \xi)$ of ξ over the whole torus $\mathcal{O}$ equals 1, which means
$$\int_{\mathcal{O}} \phi(k_i, \xi) d\xi = 1.$$

Then we know the size of a cluster conforms to a poisson distribution with rate $|\mathcal{C}_i|$. Virtually, we can prove that the size of cluster $\mathcal{C}_i$ is between $|\mathcal{C}_i|/2$ and $2|\mathcal{C}_i|$ w.h.p. when $|\mathcal{C}_i|$ scales with N[4]. For what we concern is the scaling laws, $|\mathcal{C}_i|$ can be utilized to denote the number of nodes in cluster $\mathcal{C}_i$, which will not influence our results. In addition, we restrict $L = o(\sqrt{n_s})$, which means the distance between two adjacent kernels tends to zero when n_s goes to infinity. In our literature, we describe inhomogeneity of node distributions with *spatial variance* $\delta_{\mathcal{O}}$, defined as

$$\delta_{\mathcal{O}} = \int_{\mathcal{O}} \left(\phi(|\xi|) - \frac{\int_{\mathcal{O}} \phi(|\xi'|) d\xi'}{L^2} \right)^2 d\xi = \int_{\mathcal{O}} \phi^2(|\xi|) d\xi - \frac{1}{L^2}$$

We omit the term k_i for simplification of notations. It will not affect our results and liberate us from analyzing the tedious but useless border effect. Therefore we will always make such an assumption when discussing the properties of dispersion density function in the rest of the paper. According to the definition, we know in case of uniform distribution functions over the region $\mathcal{O}$, $\delta_{\mathcal{O}} = 0$ and larger inhomogeneity results in larger $\delta_{\mathcal{O}}$.

Finally, we specify a special point process for each cluster as *Uniform Cluster Random Model*. In this case, the dispersion density function is as follows:

$$\phi(|k_i - \xi|) = \begin{cases} \frac{1}{\pi R^2} & |\xi - k_i| \leq R \\ 0 & \text{otherwise} \end{cases}$$

where $R = \frac{L}{\sqrt{\pi(1+(L\delta_{\mathcal{O}}^{max})^2)}}$ and will be utilized frequently in the following parts. It is a special case of the point generating process and we can prove that it will lead to a maximized throughput for each clusters when $\delta_{\mathcal{O}}$ is upper bounded by $\delta_{\mathcal{O}}^{max}$. Fig.1 demonstrates such model with $n_s = 10$ clusters.

[3]homogeneous poison process

[4]Throughout this paper, $N = \sum_{i=1}^{n_s} |\mathcal{C}_i|$ is the total number of nodes in the deployed region and x scales with N means x can approach infinity when N goes to infinity

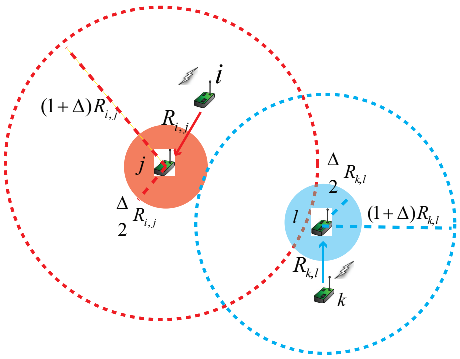

Fig. 2: Demonstration of two successful transmission satisfying the protocol model.

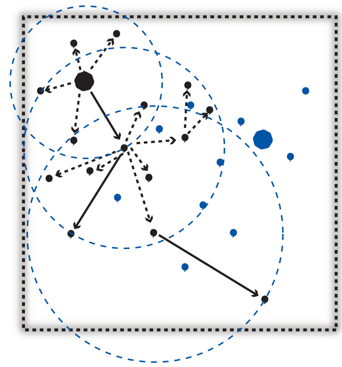

Fig. 3: Case I A relative larger uniform transmission range.

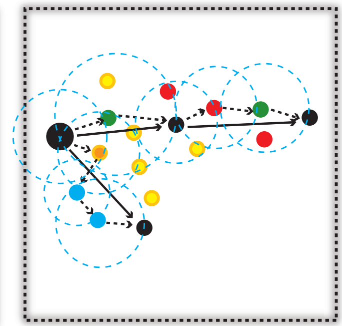

Fig. 4: Case II A relative shorter transmission range.

B. Transmission Model

All the wireless transceivers can communicate over a common channel of limited bandwidth W. We adopt the same transmission protocol model for interference proposed in [1], which derives from a well-known physical interference model:

$$\text{Capacity} = W \log(1 + \text{SINR}),$$

where SINR represents the signal to interference and noise ratio. The proposed model is identical to this model when analyzing the scaling laws, which is verified in [1]. A sender i can successfully transmit at W bit/second to a destination j when the Euclidean distance between any other concurrent transmitters and j is larger than $(1 + \Delta)R_{i,j}$, where $R_{i,j}$ is the Euclidean distance between i and j, $\Delta > 0$ defines a guard zone for a successful transmission, which is a constant independent of the position of i, j, k. We further assume that all the nodes within the transmission range can overhear the information. Fig.2 illustrates two concurrent transmissions.

C. Traffic Model

A multicast scenario is assumed where each cluster kernel generates data flows to their respect members i.e. in Fig.1. Therefore there are n_s *one to many* data flows existing in the wireless network. A multicast tree $\mathcal{T}_i$ spanning $|\mathcal{C}_i|$ nodes of cluster $\mathcal{C}_i$ as in Fig.1 can be constructed to indicate the data flows. In [2], they prove that the *Euclidean minimal spanning tree* $EMS\mathcal{T}_i$ will result in the optimal network performance. However, whether it can also lead to the optimal result under inhomogeneous node distributions are unknown, which will be discussed in the following sections. Note that the the communication between any SD pairs can also go through multiple members of other cluster as relays. Now we provide the definition of capacity.

Definition of Capacity: Let $\lambda_i(1 \leq i \leq n_s)$ denote the sustainable rate of data flow for cluster $\mathcal{C}_i$. A rate vector $\Lambda_{n_s} = (\lambda_1, \lambda_2, \ldots, \lambda_{n_s-1}, \lambda_{n_s})$ for all n_s clusters can be constructed. Assume that $\lambda = \min\{\lambda_1, \lambda_2, \ldots, \lambda_{n_s-1}, \lambda_{n_s}\}$. Then $\lambda = \Theta(f(n))$ is achievable if and if only if there exist deterministic constants $c > c' > 0$ such that

$$\lim_{n_s p \to \infty} \mathbf{Pr}(\lambda \geq cf(n)) < 1 \qquad \lim_{n_s p \to \infty} \mathbf{Pr}(\lambda \geq c' f(n)) = 1.$$

Then capacity λ is defined as the achievable rate for all clusters. Therefore, it will not cause any network backlog, which means $\lim_{t\to\infty} \sup_{1 \leq i \leq n_s} B_i(t) \leq O(1)$, $B_i(t)$ is the number of data units already generated in $\mathcal{C}_i$ which has not yet been delivered to all of its members at time t.

D. Mathematical Notations

Here we list some mathematical notations used frequently in our analysis.

- $h^b_{\mathcal{C}_i}$: Number of hops required of bit b sent to all members in cluster $\mathcal{C}_i$.
- ℓ^h_b: Length of transmission of bit b in its $h_{th}(1 \leq h \leq h^b_{\mathcal{C}_i})$ hop. We assume a uniform transmission range $r = O(L)$, such that $\ell^h_b = \Theta(r)$.
- $\delta_{h,b}$: Number of nodes that can overhear a packet during a transmission of bit b in its h_{th} hop.
- $D(\xi, R)$: Circular region centered at ξ with radius R.
- d_c: Critical distance between cluster centers and defined as $d_c = \Theta(\frac{L}{\sqrt{n_s}})$.
- $\mathcal{R}$: The influencing range of every kernel, which means that nodes outside such area have zero probability to help relay packets belonged to that kernel.

III. Some Restrictions Imposed by the Wireless Network

In this section, we will discuss several restrictions inherent in the proposed network model. Regardless of the routing policy, there are some tradeoffs that must be paid among number of hops, transmission range, limited radio resources and so force. Therefore, a through comprehension of the implicit relationships among them is constructive for deriving the information-theoretic upper bound of the achievable capacity. First, we will investigate some restrictions on network from a global aspect, that is, we do not consider the inner properties of a cluster.

A. Restrictions From Global Aspects

Since it consumes radio resources to forward a bit b to relays or destinations. The following lemma captures the tradeoffs among number of hops, transmission range, limited radio resources.

Lemma 1: **Constraint of Transmission Protocol model** Under the transmission protocol model, the following inequity must be hold for any routing scheme when the simulation time T is sufficient large.

$$\sum_{i=1}^{n_s}\sum_{b=1}^{\lambda_i T}\sum_{h=1}^{h^b_{\mathcal{C}_i}}\frac{\pi}{16}\Delta^2(\ell_b^h)^2 \leq WTL^2.$$

Proof: When T is sufficient large, the total number of bits communicated from kernel to its members in cluster $\mathcal{C}_i$ is $\lambda_i T$. According to our transmission protocol model, the Euclidean distance between any two concurrent receivers from different clusters must be greater than some value. Assume two current SD pairs $X_i \rightarrow X_k$ and $X_j \rightarrow X_l$ are active in the given time slot. Then according to [1],

$$|X_k - X_l| \geq \frac{\Delta}{2}\left(|X_i - X_k| + |X_j - X_l|\right).$$

Thus disks of radius $\frac{\Delta}{2}$ times the transmission range centered at the receiver can be viewed as a "guard region" that receivers of different data flows can not reside in. Such property also holds for broadcast that the transmission range is defined as the furthest node that can receive the packets. Let S_b^h be the overlapped area between the consumed area of bit b's h_{th} hop and the deployed region $\mathcal{O}$, then

$$S_b^h \geq \frac{\pi}{4}(\frac{\Delta\ell_b^h}{2})^2 = \frac{\pi\Delta^2(\ell_b^h)^2}{16}.$$

In our literature, radio resources can be regarded as the limited bandwidth W times the simulation time T and times the area of deployed region L^2 such that the following inequity must satisfy:

$$\sum_{i=1}^{n_s}\sum_{b=1}^{\lambda_i T}\sum_{h=1}^{h^b_{\mathcal{C}_i}}\frac{\pi}{16}\Delta^2(\ell_b^h)^2 \leq \sum_{i=1}^{n_s}\sum_{b=1}^{\lambda_i T}\sum_{h=1}^{h^b_{\mathcal{C}_i}} S_b^h \leq WTL^2.$$

■

We do not consider cooperative MIMO scheme as in [21], each wireless transceiver is only equipped with a single antenna, it can not transmit and receive signal at the same time. Therefore the number of wireless transceivers can also be regarded as wireless resources. The following lemma illustrates this property, which extends the result in [1] to broadcast scenario.

Lemma 2: **Constraint of Half Duplex** The following trivial inequity must be satisfied for sufficient large simulation time T.

$$\sum_{i=1}^{n_s}\sum_{b=1}^{\lambda_i T}\sum_{h=1}^{h^b_{\mathcal{C}_i}}\delta_{h,b} \leq NTW$$

Let $\overline{\phi}$ and $\underline{\phi}$ denote the maximized and minimum point intensity over $\mathcal{O}$, then in such a cluster dense regime $\overline{\phi}/\underline{\phi} \leq 4$ according to [5]. Given the order of transmission range $\Theta(r)$, how many nodes $\delta_{h,b}$ can overhear the packets is taken into consideration next.

Lemma 3: Given the transmission range r, the number of nodes that are forced to overhear the information in bit b's h_{th} hop is tight bounded by

$$\delta_{h,b} = \Theta(\max\{1, \frac{n_s p r^2}{L^2}\}).$$

Proof: When $r = \omega(\frac{1}{\sqrt{n_s p}/L})$, $\delta_{h,b}$ is comprised between $\frac{c_0\pi n_s p r^2}{2L^2}$ and $\frac{2\pi n_s p r^2}{L^2}$, which is a standard application of Chernoff bound and property of Riemann sum (one can refer to Theorem 1 in [5]).

Now we come to $r = \Theta(\frac{1}{\sqrt{n_s p}/L})$ case. Note that the node density is upper bound by a HPP with rate $\mu = \frac{\pi r^2 n_s p}{L^2}$. Therefore the number of nodes inside the circle exceeding n_0 is

$$\begin{aligned}\mathbf{Pr}(\delta_{h,b} < n_0) &\geq e^{-\mu}\sum_{i=0}^{n_0}\frac{\mu^i}{i!}\\ &\geq e^{-\mu}\left(e^{\mu} - \frac{e^{\theta\mu}\mu^{n_0+1}}{(n_0+1)!}\right) \quad 0<\theta<1\\ &\geq 1 - \frac{e^{(\theta-1)\mu}\mu^{n_0+1}}{(n_0+1)!}\end{aligned}$$

During the above derivation, we use Lagrange form of the remainder term. Therefore we know when $n_0 = \omega(\frac{n_s p r^2}{L^2})$, $\mathbf{Pr}(\delta_{h,b} < n_0) = 1$ w.h.p. For each transmission must cover at least one node, therefore $\delta_{h,b} \geq 1$ is a prerequisite. ■

We have completed our analysis of the limited resources. However, some local perspective inside a cluster is also deterministic of information-theoretic capacity upper bound, which we will investigate in the following part.

B. Restrictions from Local Aspect

The above two lemmas only depends on the limited resources provided. They do not consider the constraints of network topology and traffic patterns. In the following part, we will investigate the characteristics of traffic patterns defined in our model.

In [2], [3], EMST is widely investigated in multicast traffic and such a multicast spanning tree can lead to the best network performance in uniformly, non-clustered node distributions. However, whether EMST is still powerful in inhomogeneous, clustered regime are what we concern. To obtain the results, we will first introduce a theorem in [10].

> If f is the density of the probability function for picking points, then for large n and $d \neq 1$, the size of the EMST is approximately $c(d)n^{\frac{d-1}{d}}\int_{\mathbb{R}^d} f(x)^{\frac{d-1}{d}}dx$, where where c(d) is a constant depending only on the dimension d.

In our case, $d = 2$, thus there exists a constant c_0, such that the length of *Euclidean minimum spanning tree* (EMST) $|\mathcal{T}_i|$ for each cluster $\mathcal{C}_i$ is

$$|\mathcal{T}_i| = c_0\sqrt{p}\int_{\mathcal{O}}\sqrt{k(\xi)}d\xi.$$

Such a theorem is very useful in analysis of the length of EMST under arbitrary node distributions. Before that, we provide an algorithm for generating *ρ-simplified EMST* $\rho - EMST(\mathcal{T}_i)$ from arbitrary EMST($\mathcal{T}_i$) .

Algorithm 1 Generation of ρ-simplified EMST from an arbitrary EMST.
Input:$EMST(\mathcal{T}_i)$ Output:$EMST(\rho - \mathcal{T}_i)$

1: Label each $p+1$ nodes in tree $\mathcal{T}_i$ with number $1, 2, \ldots, p, p+1$.
2: Choose nodes with the smallest labeled number existed in $\mathcal{T}_i$.
3: Add the chosen nodes to $\rho - \mathcal{T}_i$. and discard all the nodes with distance smaller than ρ from the chosen node above, including itself.
4: Back to process (2) until no node is left in process 2.
5: Construct EMST with the remained nodes in $\rho - \mathcal{T}_i$.

Thus we know $\rho - EMST(\mathcal{T}_i)$ is a thinned version of $EMST(\mathcal{T}_i)$ and all the discarding nodes are within a distance of ρ from the remained nodes in $\rho - \mathcal{T}_i$. The following lemma will give a lower bound on the length of such simplified tree.

Lemma 4: Assume there are p nodes in a cluster following the dispersion intensity function $\phi(\cdot)$. Denote the overall length of the *ρ-simplified EMST* as $EMST(\rho - \mathcal{T}_i)$, then we can prove that

$$|\rho - \mathcal{T}_i| \geq c_0\sqrt{p}\left(\frac{\rho_{1,i}^2}{\rho} + c_5\int_{\mathcal{O}/S_{1,i}}\sqrt{\phi(|\xi - k_i|)}d\xi\right)$$

where $\rho_{1,i}$ is a sufficient solution for $\phi(\rho_{1,i}) = \omega(1/\pi\rho^2)$ and c_5 is a constant.

Proof: The point intensity after the thinning process determines the length of such simplified tree. First we want to specify three regions. For that the dispersion density function is invariant under rotations, the specified regions are all circular. Let $X(\xi, k_i, \rho)$ denote the number of members in circle of radius ρ centered at ξ of cluster $\mathcal{C}_i$ and $p\phi'(|\xi - k_i|)$ point intensity in $\rho - \mathcal{T}_i$.

- **Dense Region** ($S_{1,i} = O_1$) Nodes in this region are populous and we specify a radius ρ_1 for this circular region defined as $\phi(\rho_1) = \omega(1/\pi\rho^2)$. After the thinning process, the distribution of the remained nodes can be viewed as a HPP with point intensity $p\phi'(|\xi - k_i|) \geq \frac{p}{\pi\rho^2}$.
- **Sparse Region** ($S_{3,i} = \mathcal{O}/O_2$) Nodes in this region are so sparse that each circle of radius ρ can cover at most 1 node. We specify a radius $\rho_{2,i}$ for O_2 satisfying $\phi(\rho_{2,i}) = o(1/\pi\rho^2)$. The proposed algorithm can not thin nodes in this region. Therefore, $p\phi'(|\xi - k_i|) = \phi(|\xi - k_i|)$.
- **Partial Dense Region** ($S_{3,i} = O_2/O_1$) Nodes in this region are neither populous nor sparse. Actually, the node intensity distribution is $\Theta(1/\pi\rho^2)$. Therefore utilizing a similar method as in Lemma 3, we can prove that there exist a constant $c_4 > 0$ so that the point intensity $p\phi'(|\xi - k_i|) \geq c_4\frac{p}{\pi\rho^2}$.

Recall the overall length of EMST introduced above, the lower bound of $EMST(\rho - \mathcal{T}_i)$ is

$$EMST(\rho - \mathcal{T}_i) = c_0\int_{\mathcal{O}}\sqrt{p\phi'(|\xi - k_i|)}d\xi$$
$$\geq c_0\sqrt{p}\left(\frac{\rho_{1,i}^2}{\rho} + \int_{S_{2,i}} c_4\sqrt{\phi(\rho_{2,i})}d\xi + \int_{S_{3,i}}\sqrt{\phi(|\xi - k_i|)}d\xi\right)$$
$$\geq c_0\sqrt{p}\left(\frac{\rho_{1,i}^2}{\rho} + \min\{1, c_4\}\int_{\mathcal{O}/S_{1,i}}\sqrt{\phi(|\xi - k_i|)}d\xi\right)$$

Let $c_5 = \min\{1, c_4\}$, we complete our proof. Note that our result holds even when there is no $\rho_{1,i}$ or $\rho_{2,i}$ suitable for the equation, S_1 or S_2 or S_3 perhaps is empty set. ■

The above lemmas theoretically analyze the constraint imposed by wireless communications i.e. radio resources, traffic patterns, sensor distributions. In next section, we will utilize the above lemmas to derive the upper bound of capacity in such wireless network.

IV. Upper Bound of Multicast Capacity under Inhomogeneous Node Distributions

In this section, we will investigate the theoretical upper bound of multicast capacity under our model. Note that our results are unrelated with the scheduling/routing schemes. Therefore, it is instructive when we want to know the maximized capacity when we distribute wireless nodes in some regions.

Lemma 5: If the transmission range of each hop is on the order of $\Theta(\rho)$ in cluster $\mathcal{C}_i$, then the overall length of transmission of a bit b in this cluster is constrained as follows

$$\sum_{h=1}^{h_p^b} \ell_b^h \geq \Theta(EMST(\rho - \mathcal{T}_i)).$$

Proof: According to our definition, $EMST(\rho - \mathcal{T}_i)$ can achieve minimal total length of transmission to cover all the $p+1$ nodes in cluster $\mathcal{C}_i$ when the transmission range is bounded by $\Theta(\rho)$. For every routing scheme with transmission range $\Theta(\rho)$, there must exist a constant $c_1 > 0$, such that $\ell_b^h > c_1\rho$. Therefore,

$$\sum_{h=1}^{h_p^b} \ell_b^h \geq EMST((c_1\rho) - \mathcal{T}_i) = \Theta(EMST(\rho - \mathcal{T}_i)).$$

■

Summarize all the conditions listed above, we can provide an upper bound of capacity λ_i in each cluster $\mathcal{C}_i$.

Theorem 1: Under the assumptions of the proposed wireless network, the following tradeoffs must be satisfied by all scheduling policy.

$$\sum_{i=1}^{n_s} \lambda_i\sqrt{|\mathcal{C}_i|} \leq \frac{8\sqrt{2\pi\sum_{i=1}^{n_s}|\mathcal{C}_i|}WL}{\Delta c_0 c_2 c_4 c_5\int_{\mathcal{O}}\sqrt{\phi(\xi)}d\xi}$$

where λ_i represents the data rate achieved by cluster $\mathcal{C}_i$ and all the c_i are constants.

Proof: First, we must investigate the point intensity generated by the process. In our cluster dense regime, it is proved in [5] that the ratio of maximized node density $\overline{\phi}$ and minimal node density $\underline{\phi}$ is smaller than 4. Therefore, given the total number of nodes $\mathcal{N} = \sum_{i=1}^{n_s} |\mathcal{C}_i|$ generated by all cluster kernels, we can obtain $\frac{\mathcal{N}}{4L^2} \leq \underline{\phi} \leq \overline{\phi} \leq \frac{4\mathcal{N}}{L^2}$. And the transmission range r must guarantee there is at least 1 node receiving the packet. Then according to Lemma 3,

$$0 < \mathbf{Pr}(\delta_{h,b} > 0) \leq \mathbf{Pr}(\phi_{max} r^2 \geq \Theta(1)).$$

Substituting $\overline{\phi} \leq \frac{4\mathcal{N}}{L^2}$ into it, we can obtain that there exists a constant $c_2 > 0$ such that $r \geq c_2 L/\sqrt{\mathcal{N}}$ is a necessary condition. Now given the transmission range r, the number of nodes one hop covers $\delta_{h,b}$ can be lower bounded according to Lemma 3.when $r = \omega(L/\sqrt{\mathcal{N}})$, $\delta_{h,b} \geq 0.5\pi r^2 \phi_{min} = \frac{\pi \mathcal{N} r^2}{8L^2}$; when $r = \Theta(L/\sqrt{\mathcal{N}})$, $\delta_{h,b} \geq 1$. Therefore, there exists a constant $c_3 > 0$, such that $\delta_{h,b} \geq \frac{c_3 \mathcal{N} r^2}{L^2}$.

Then according to Lemma 2, we know

$$\begin{aligned}\sum_{i=1}^{n_s}\sum_{b=1}^{\lambda_i T}\sum_{h=1}^{h_p^b} 1 &\leq \frac{\sum_{i=1}^{n_s}\sum_{b=1}^{\lambda_i T}\sum_{h=1}^{h_p^b}\delta_{h,b}}{\frac{c_3 r^2(\sum_{i=1}^{n_s}|\mathcal{C}_i|)}{8L^2}} \\ &\leq \frac{8L^2\sum_{i=1}^{n_s}|\mathcal{C}_i|TW}{c_3 r^2\sum_{i=1}^{n_s}|\mathcal{C}_i|} \leq \frac{8L^2TW}{c_3 r^2}\end{aligned}$$

Then utilizing Lemma 1 and Cauchy inequality, we have

$$\begin{aligned}\sum_{i=1}^{n_s}\sum_{b=1}^{\lambda_i T}\sum_{h=1}^{h_p^b}\ell_b^h &\leq \sqrt{\sum_{i=1}^{n_s}\sum_{b=1}^{\lambda_i T}\sum_{h=1}^{h_p^b}(\ell_b^h)^2\sum_{i=1}^{n_s}\sum_{b=1}^{\lambda_i T}\sum_{h=1}^{h_p^b}1} \\ &\leq \sqrt{\frac{16WT\pi L^2}{\pi\Delta^2}\frac{8L^2TW}{c_3 r^2}} \\ &\leq \sqrt{\frac{128\pi W^2T^2L^4}{c_3\Delta^2 r^2}}. \qquad (1)\end{aligned}$$

Now we divide n_s clusters into two sets $\mathbb{S}_1$, $\mathbb{S}_2$ depending on the characteristic of their $r - EMST$.

$$\begin{aligned}&\mathbb{S}_1 = \{\mathcal{C}_i|\text{Partial Thinned Region } S_{2,i} \text{ exists}\} \\ &\mathbb{S}_2 = \{\mathcal{C}_i|\text{Partial Thinned Region } S_{2,i} \text{ does not exist}\}.\end{aligned}$$

Applying Lemma 4,5, we can obtain if $\mathbb{S}_1 \neq \emptyset$, there exists a constant c_4, such that

$$\begin{aligned}\sum_{i=1}^{n_s}\sum_{b=1}^{\lambda_i T}\sum_{h=1}^{h_p^b}\ell_b^h &\geq c_4\sum_{i=1}^{n_s}\sum_{b=1}^{\lambda_i T} EMST(r-\mathcal{T}_i) \\ &\geq c_0c_4\left(\sum_{\mathcal{C}_i\in\mathbb{S}_1}\sum_{b=1}^{\lambda_i T}\sqrt{\mathcal{C}_i}\left(EMST(r-\mathcal{T}_i)\right)+\right. \\ &\qquad \left.\sum_{\mathcal{C}_i\in\mathbb{S}_2}\sum_{b=1}^{\lambda_i T}\sqrt{\mathcal{C}_i}\int_{\mathcal{O}}\sqrt{\phi(|k_i-\xi|)}d\xi\right) \\ &\geq c_0c_4T\left(\sum_{i=1}^{n_s}\lambda_i\sqrt{\mathcal{C}_i}\right)\psi\left(\phi(\cdot),\mathbb{S}_1\right) \qquad (2)\end{aligned}$$

Where

$$\psi\left(\phi(\cdot),\mathbb{S}_1\right) = \min_{\mathcal{C}_i\in\mathbb{S}_1}\left\{\frac{EMST(r-\mathcal{T}_i)}{|\mathcal{C}_i|}\right\}.$$

Then substitute (1) into (2), we can obtain

$$\begin{aligned}\sum_{i=1}^{n_s}\lambda_i\sqrt{|\mathcal{C}_i|} &\leq \frac{1}{c_0c_4T\psi\left(\phi(\cdot),\mathbb{S}_1\right)}\sum_{i=1}^{n_s}\sum_{b=1}^{\lambda_i T}\sum_{h=1}^{h_p^b}\ell_b^h \\ &\leq \frac{8\sqrt{2\pi}WL^2}{\Delta c_0\sqrt{c_3}c_4 r\psi\left(\phi(\cdot),\mathbb{S}_1\right)} \qquad (3)\end{aligned}$$

Else if $\mathbb{S}_1 = \emptyset$:

$$\begin{aligned}\sum_{i=1}^{n_s}\sum_{b=1}^{\lambda_i T}\sum_{h=1}^{h_p^b}\ell_b^h &\geq c_4\sum_{i=1}^{n_s}\sum_{b=1}^{\lambda_i T} EMST(r-\mathcal{T}_i) \\ &\geq c_0c_4\sum_{i=1}^{n_s}\sum_{b=1}^{\lambda_i T}\sqrt{|\mathcal{C}_i|}\int_{\mathcal{O}}\sqrt{\phi(|\xi-k_i|)}d\xi \\ &= c_0c_4\int_{\mathcal{O}}\sqrt{\phi(\xi)}d\xi\left(\sum_{i=1}^{n_s}\lambda_i\sqrt{|\mathcal{C}_i|}\right) \qquad (4)\end{aligned}$$

Substitute (4) into (1), we obtain

$$\begin{aligned}\sum_{i=1}^{n_s}\lambda_i\sqrt{|\mathcal{C}_i|} &\leq \frac{1}{c_0c_4T\int_{\mathcal{O}}\sqrt{\phi(\xi)}d\xi}\sum_{i=1}^{n_s}\sum_{b=1}^{\lambda_i T}\sum_{h=1}^{h_p^b}\ell_b^h \\ &\leq \frac{8\sqrt{2\pi}WL^2}{\Delta c_0c_4 r\int_{\mathcal{O}}\sqrt{\phi(\xi)}d\xi} \\ &\leq \frac{8\sqrt{2\pi\sum_{i=1}^{n_s}|\mathcal{C}_i|}WL}{\Delta c_0c_2c_4\int_{\mathcal{O}}\sqrt{\phi(\xi)}d\xi} \qquad (5)\end{aligned}$$

During the above derivation, we utilize that $r \geq c_2\frac{L}{\sqrt{\mathcal{N}}}$ and in this case $\mathcal{N} = \sum_{i=1}^{n_s}|\mathcal{C}_i|$. Compare the results under the two cases, the only thing required to do is to prove that there exists a constant c_5 such that

$$\frac{L\sqrt{\sum_{i=1}^{n_s}|\mathcal{C}_i|}}{c_5\int_{\mathcal{O}}\sqrt{\phi(\xi)}d\xi} \geq \frac{L^2}{\sqrt{c_3}r\psi\left(\phi(\cdot),\mathbb{S}_1\right)}$$

Recall the results of Lemma 4, it is equivalent to prove

$$c_5\int_{\mathcal{O}}\sqrt{\frac{\phi(\xi)}{\sum_{i=1}^{n_s}|\mathcal{C}_i|/L^2}}d\xi \leq c_0\sqrt{c_3}\rho_{1,i}^2 + r\int_{\mathcal{O}/S_{1,i}}\sqrt{\phi(\xi)}d\xi$$

Note that $r \geq c_2\frac{L}{\sqrt{\sum_{i=1}^{n_s}|\mathcal{C}_i|}}$ and $\phi(\xi) \leq \overline{\phi} \leq \frac{4\sum_{i=1}^{n_s}|\mathcal{C}_i|}{L^2}$ and let $c_5 = \max\{c_2^2, 2c_0\sqrt{c_3}\}$ we complete our proof. ■

Theorem 2: If there are p members in each cluster $\mathcal{C}_i$, then the upper bound of achievable capacity λ is given as

$$\lambda \leq \min\left\{1, \frac{8\sqrt{2\pi}}{\Delta c_0c_4c_5}\frac{L}{\sqrt{n_s}\int_{\mathcal{O}}\sqrt{\phi(\xi)}d\xi}\right\}W \qquad (6)$$

Proof: Utilizing theorem 1 and the definition of capacity per cluster, we can directly obtain this result. Note that it means that

$$\lambda \leq O\left(\min\left\{1, \frac{L}{\sqrt{n_s}\int_{\mathcal{O}}\sqrt{\phi(\xi)}d\xi}\right\}W\right)$$

Lemma 6: **Constraint of our Traffic Model**
For any multicast spanning trees (*EMST*) with p nodes uniformly distributed within a circle of radius $\mathcal{L}$, let $||T||$ denote the total Euclidean length of such *EMST*, then we can prove that there exist a constant c_1 such that

$$c_1\sqrt{p}\mathcal{L} \leq ||T|| \leq 5\sqrt{2p}\mathcal{L},$$

which means $||T|| = \Theta(\sqrt{p}\mathcal{L})$.

Proof: First we derive the lower bound of $||T||$. In [9], they prove that $||T||$ is at least $\frac{\sqrt{3}}{2}$ times the *EMST* spanning of nodes. And in [10], we can know the *EMST* spanning of nodes is asymptotic to $\Theta(p^{1/2}\mathcal{L})$. Therefore we can obtain that there exist a constant c_1, such that

$$||T|| \geq c_1\sqrt{p}\mathcal{L}$$

Then we derive the upper bound of $||T||$. Now we use Prim's algorithm to construct *EMST* similar to *Lemma 10* in [2]: To begin with, each node is a separate part, we iteratively find a shortest edge to compose a lager part until one part is left. We utilize a square with side length $2\mathcal{L}$ to cover the whole circle. At each i_th ($1 \leq i \leq p$) step, there are $p+1-i$ parts remained. We partition the the square into $\lfloor\sqrt{p+1-i}\rfloor^2$ equal size subsquare with side length $\frac{2\mathcal{L}}{\lfloor\sqrt{p+1-i}\rfloor}$, therefore there exists at least one cell which contains more than 2 parts, which means the shortest edge connecting two parts in i_th step is at most $\frac{2\sqrt{2}\mathcal{L}}{\lfloor\sqrt{p+1-i}\rfloor}$. Therefore, the upper bound of $||T||$ is:

$$\begin{aligned} ||T|| &\leq \sum_{i=1}^{p} \frac{2\sqrt{2}\mathcal{L}}{\lfloor\sqrt{p+1-i}\rfloor} \\ &\leq 2\sqrt{2}\mathcal{L}(1+\int_1^{p-1}\frac{1}{\sqrt{x}}dx) \\ &= 2\sqrt{2}\mathcal{L}(1+2\sqrt{p-1}) \\ &\leq 5\sqrt{2p}\mathcal{L}. \end{aligned}$$

Finally, we know that $||T|| = \Theta(\sqrt{p}\mathcal{L})$. ■

V. How to distribute nodes in a cluster within the network

In this section, we study what distribution can achieve the maximized capacity per cluster when the variance of node distribution $\delta_{\mathcal{O}}$ is constrained. From previous section, we obtain that the length of EMST $|\mathcal{T}_i|$ determines the upper bound of achievable capacity per cluster. It is actually that a smaller value of $|\mathcal{T}_i|$ leads to a larger upper bound of capacity. Therefore it is intuitive to study what distributions can achieve the minimized value of $|\mathcal{T}_i|$ of cluster $\mathcal{C}_i$.

Theorem 3: Assume that the variance of node distribution $\delta_{\mathcal{O}} \leq \delta_{\mathcal{O}}^{max}$ for a cluster. Let $R = \frac{L}{\sqrt{\pi(\delta_{\mathcal{O}}^{max})^2L^2+1)}}$, then we can prove that the following dispersion density function can achieve the maximized upper bound of capacity.

$$\phi(|\xi - k_i|) = \begin{cases} \frac{1}{\pi R^2} & |\xi - k_i| \leq R \\ 0 & \text{otherwise} \end{cases}$$

■

Before proving this theorem, we need to prove another theorem.

Theorem 4: Assume that the dispersion density function $k(\xi, k_i)$ satisfies the following conditions:

1) $\phi(\xi, k_i)$ is invariant under both translation and rotation with respect to k_i, which indicates $\phi(\xi, k_i)$ can be rewritten as $\phi(|\xi - k_i|)$.
2) $\phi(|\xi - k_i|)$ is a non-increasing with respect to $|\xi - k_i|$.
3) The integration of $\phi(\xi, k_i)$ over the entire deployed region $\mathcal{O}$ equals 1.

$$\int_{\mathcal{O}} \phi(\xi, k_i)d\xi = \int_{\mathcal{O}} \phi(|\xi - \xi_k|)d\xi = 1$$

4) The integration of $\phi^2(\xi, k_i)$ over the deployed region $\mathcal{O}$ is upper bounded as follows:

$$\int_{\mathcal{O}} \phi^2(\xi, k_i)d\xi = \int_{\mathcal{O}} \phi^2(|\xi - \xi_k|)d\xi \leq \frac{1}{L^2} + (\delta_{\mathcal{O}}^{max})^2$$

Then we can prove that the following $\phi'(\xi, k_i)$ can minimize $\Upsilon(\phi(\cdot))$.

$$\phi'(|\xi - k_i|) = \begin{cases} \frac{1}{\pi R^2} & |\xi - k_i| \leq R \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

Where $\Upsilon(\phi(\cdot))$ is a real variable function with $\phi(\cdot)$ as a variable and $\Upsilon(\phi(\cdot)) = \int_{\mathcal{O}} \sqrt{\phi(\xi, k_i)}d\xi$.

Proof: To prove that $\phi'(\cdot)$ can achieve the minimal value of $\Upsilon(\phi(\cdot))$, we first transform the conditions and objective functions into Riemann sum. Here we do not explicitly explain how to conduct the transformation and only list the results. As to the thorough proof, one can refer to [13].

$$\sum_{i=1}^{N} \phi_i^n = \frac{1}{\Delta S} \quad \sum_{i=1}^{N} (\phi_i^n)^2 \leq \frac{1}{\pi R^2 \Delta S}$$

$$\Upsilon(\phi(\cdot)) = \Delta S \left(\sum_{i=1}^{N} \sqrt{\phi_i^n}\right)$$

During the above derivation, we assume that we equally divide the square $\mathcal{O}$ into n cells with area $\Delta S = \frac{L^2}{n}$ each. ϕ_i^n is the chosen point in cell i to approximate the value of $\phi(\xi, k_i)$ in cell i. It can be proved that when $N \to \infty$, the Riemann sums equal the integrations. We further assume that $\phi_i^n \leq \phi_j^n$ iff $i \geq j$.

In the following part, we utilize $\Phi^n = (\phi_1^n, \phi_2^n, \ldots \phi_{n-1}^n, \phi_{n-1}^n)$ during the proof and the optimal $(\Phi^n)' = ((\phi_1^n)', (\phi_2^n)', \ldots (\phi_{n-1}^n)', (\phi_{n-1}^n)')$ that can minimize $\Upsilon(\phi(\cdot))$ is

$$(\phi_i^n)' = \begin{cases} \frac{1}{\pi R^2} & 1 \leq i \leq n_c \\ \frac{1}{\Delta S} - \frac{n_c}{\pi R^2} & i = n_c + 1 \\ 0 & \text{otherwise} \end{cases}$$

where $n_c = \lfloor\frac{\pi R^2}{\Delta S}\rfloor$. To obtain this result, the proposed Algorithm 2 can convert arbitrary Φ^n to the $(\Phi^n)'$ and in every step of the transformation,it decreases the value of $\Upsilon(\phi(\cdot))$.

Algorithm 2 Conversion from Arbitrary Φ^n to Optimal $(\Phi^n)'$
Input:Φ^n Output:$\widetilde{\Phi^n} = (\Phi^n)'$

Require: Φ^n satisfies all the conditions listed above
Ensure: $\widetilde{\Phi^n}$ can be generated

1: $\widetilde{\Phi^n} \leftarrow \Phi^n$

2: Find three points $\widetilde{\phi_i^n}$, $\widetilde{\phi_j^n}$, $\widetilde{\phi_l^n}$ in $\widetilde{\Phi^n}$ satisfying the following conditions:

- For $k \leq i$, $\phi_k^n > \frac{1}{\pi R^2}$ and for $k > i$, $\phi_k^n \leq \frac{1}{\pi R^2}$
- For $k < j$, $\phi_k^n \geq \frac{1}{\pi R^2}$ and for $k \geq j$, $\phi_k^n < \frac{1}{\pi R^2}$
- For $k \leq l$, $\phi_k^n > 0$ and for $i > l$, $\phi_k^n = 0$

If ϕ_i^n can not be found, finish the program.

3: Find $\max\{\Delta_i\}$ and $\max\{\Delta_l\}$ satisfying the follow conditions:

$$\phi_i^n \geq \frac{1}{\pi R^2} + \Delta_i \quad \phi_j^n \leq \frac{1}{\pi R^2} - \Delta_i - \Delta_l \quad \phi_l^n \geq \Delta_l$$
$$(\phi_i^n - \Delta_i)^2 + (\phi_j^n + \Delta_i + \Delta_l)^2 + (\phi_l^n - \Delta_l)^2 \leq (\phi_i^n)^2 + (\phi_j^n)^2 + (\phi_l^n)^2$$
$$\Delta_i \leq \frac{\phi_j^n(\sqrt{\phi_i^n} - \sqrt{\phi_j^n})(\sqrt{\phi_i^n} - \sqrt{\phi_k^n})}{\sqrt{\phi_i^n}(1 + (\frac{\phi_i^n - \phi_j^n}{\phi_j^n - \phi_l^n})^2)(\sqrt{\phi_j^n} + \sqrt{\phi_l^n})}$$

4: $\widetilde{\phi_i^n} = \widetilde{\phi_i^n} - \Delta_i$, $\widetilde{\phi_j^n} = \widetilde{\phi_j^n} + \Delta_i + \Delta_l$, $\widetilde{\phi_l^n} = \widetilde{\phi_l^n} - \Delta_l$. Go back to step 2.

Step (4) in Algorithm 2 can decrease the value of $\Upsilon(\phi(\cdot))$, which is equivalent to prove

$$\sqrt{\phi_i^n - \Delta_i} + \sqrt{\phi_j^n + \Delta_i + \Delta_l} + \sqrt{\phi_l^n - \Delta_l} \leq \sqrt{\phi_i^n} + \sqrt{\phi_j^n} + \sqrt{\phi_l^n} \tag{8}$$

According to step (3), we can obtain

$$\Delta_l = \frac{\phi_i^n - \phi_j^n}{\phi_j^n - \phi_l^n}\Delta_i - \frac{\Delta_i^2 + \Delta_l^2 + \Delta_i\Delta_l}{\phi_j^n - \phi_l^n} \tag{9}$$

Rewrite (8) and Substitute (9) into it, we can obtain

$$\begin{aligned}
&\sqrt{\phi_i^n - \Delta_i} + \sqrt{\phi_j^n + \Delta_i + \Delta_l} + \sqrt{\phi_l^n - \Delta_l} \\
&-(\sqrt{\phi_i^n} + \sqrt{\phi_j^n} + \sqrt{\phi_l^n}) \\
\leq& -\frac{\Delta_i}{2\sqrt{\phi_i^n}} + \frac{\Delta_i + \Delta_l}{2\sqrt{\phi_j^n}} - \frac{\Delta_l}{2\sqrt{\phi_l^n}} + \delta(\Delta_i, \Delta_l) \\
\leq& \frac{\Delta_i}{2\sqrt{\phi_j^n}}\left(\frac{\sqrt{\phi_i^n} - \sqrt{\phi_j^n}}{\sqrt{\phi_i^n}} - \frac{\sqrt{\phi_j^n} - \sqrt{\phi_l^n}}{\sqrt{\phi_l^n}}\frac{\Delta_l}{\Delta_i}\right) + \delta(\Delta_i, \Delta_l) \\
\leq& \frac{\Delta_i}{2\sqrt{\phi_j^n}}\left(\frac{\sqrt{\phi_i^n} - \sqrt{\phi_j^n}}{\sqrt{\phi_i^n}} - \frac{\sqrt{\phi_j^n} - \sqrt{\phi_l^n}}{\sqrt{\phi_l^n}}(\rho + \frac{1 + \rho + \rho^2}{\phi_j^n - \phi_l^n}\Delta_i)\right) \\
&+\delta(\Delta_i, \Delta_l) \\
\leq& -\frac{\Delta_i(\sqrt{\phi_i^n} - \sqrt{\phi_j^n})(\sqrt{\phi_i^n} - \sqrt{\phi_l^n})}{2\sqrt{\phi_i^n\phi_j^n\phi_l^n}} + \\
&\left(\frac{1 + \rho + \rho^2}{2\sqrt{\phi_j^n\phi_l^n}(\sqrt{\phi_j^n} + \sqrt{\phi_l^n})} + \frac{1 + \rho^2}{2(\phi_j^n)^{1.5}}\right)\Delta_i^2 \\
\leq& -\frac{\Delta_i(\sqrt{\phi_i^n} - \sqrt{\phi_j^n})(\sqrt{\phi_i^n} - \sqrt{\phi_l^n})}{2\sqrt{\phi_i^n\phi_j^n\phi_l^n}} + \frac{\Delta_i^2(1 + \rho^2)(\phi_j^n + \phi_l^n)}{2\sqrt{\phi_j^n}\phi_l^n\phi_j^n} \\
\leq& 0
\end{aligned}$$

During the above derivation, $\delta(\Delta_i, \Delta_l)$ is the reminder term of Taylor Series, and we can prove that $\delta(\Delta_i, \Delta_l) \leq \frac{(\Delta_i + \Delta_l)^2}{2(\phi_j^n)^{1.5}}$. We also simplify our analysis using $\rho = \frac{\phi_i^n - \phi_j^n}{\phi_j^n - \phi_l^n}$.

Therefore, Algorithm 2 can reduce $\Upsilon(\phi(\cdot))$ until $\widetilde{\Phi^n} = (\Phi^n)'$. When $n \to \infty$, the Riemann sum becomes integration

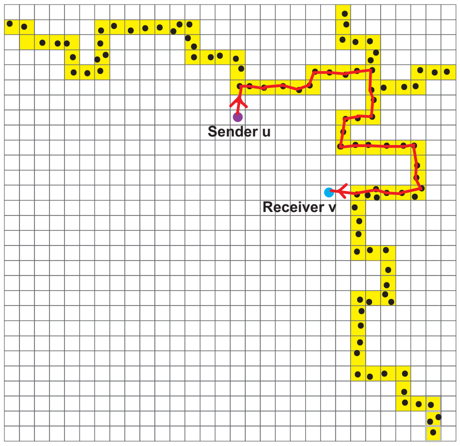

Fig. 5: Demonstration of Transmission Utilizing Information Highway. Only Used Path and Nodes displayed.

and due to the non-increasing characteristic of Φ^n, the respect $\phi(\cdot)$ is the same as (7). Therefore we complete our proof. ∎

When $\delta_{\mathcal{O}} \le \delta_{\mathcal{O}}^{max}$, the optimal node distribution $\phi'(\cdot)$ can achieve the maximized capacity per cluster. Recall theorem 2, the upper bound of achievable capacity is

$$\lambda \le \min\left\{\frac{8\sqrt{2}}{\Delta c_0 c_4 c_5}\frac{LW}{\sqrt{n_s}R}, 1\right\} W \quad R = \frac{L}{\sqrt{\pi(1+L^2(\delta_{\mathcal{O}}^{max})^2)}}$$

VI. How to schedule transmission in MAC layer for the achievable capacity under uniform cluster random model

In this section, we will use some results of percolation theory to construct the network to lower bounded the capacity of *Uniform Cluster Grid/Random Model*.

Theorem 5: In *Cluster Grid/Random Model*, When $R = \Omega(\frac{L(\log(n_s p))^2}{\sqrt{n_s}})$, we can wisely schedule the network, therefore the capacity of every cluster (multicast session) is lower bounded by $\Theta\left(\frac{L}{R\sqrt{n_s}}\right)$, thus we know $\Theta\left(\frac{L}{R\sqrt{n_s}}\right)$ is a tight bound of such wireless network.

In order to prove the theorem, we world introduce some lemmas. First, We give a definition of *information highway* in a wireless network system.

Definition 1: **Information Highway**
Assume there are n nodes distributed in a square of length L. We equally divide the square into $m * m$ cells. A cell is open if it contains at least one node. Link two adjacent cells with a line if both of them are open. A horizontal (vertical) path is a set of lines that cross the square from left to right (top to bottom). The set of disjoint horizontal paths and disjoint vertical paths is defined as *Information Highway*[5], which is also called backbones of wireless network. The yellow girds in Fig.5 shows a vertical and horizontal path.

As to our hypothesis, cluster members are distributed according to HPP within a circle of radius R. The next Lemma tells us that all of the nodes within the network can also be regarded as distributed according to HPP.

[5] Vertical path and Horizontal path are not required to be disjoint.

Lemma 7: Under both two models, let $r = \Theta(L/\sqrt{n_s p})$ and $\mathcal{N}(r)$ denote number of nodes within a circle of radius r over $\mathcal{O}$, then if $R = \Omega(d_c)$,

$$\mathbf{Pr}(\mathcal{N}(r) = k) \le 2e^{-\frac{\pi r^2 n_s p}{L^2}} \frac{(\frac{\pi r^2 n_s p}{L^2})^k}{k!}.$$

Proof: Let $\mathcal{A}(d, r_1, r_2)$ denote the overlapping area of two circles of radius r_1, r_2 with centers of distance d away, and $\mathcal{C}(r)$ denotes number of centers within radius r. Note that the distribution of cluster members is HPP within a circle of radius R, thus we could obtain

$$\begin{aligned}
&\mathbf{Pr}(\mathcal{N}(r) = k) \\
=& \sum_{m=0}^{n_s} \mathbf{Pr}(\mathcal{N}(r) = k | \mathcal{C}(r+R) = m)\mathbf{Pr}(\mathcal{C}(r+R) = m) \\
=& \sum_{m=0}^{n_s} (\int_0^{R+r} \sum_{\mathcal{S}} (\prod_{i=1}^{m} (e^{-\mu_1}\frac{\mu_1^{\upsilon_i}}{\upsilon_i!}))\frac{2x}{(R+r)^2}dx) e^{-\mu_2}\frac{\mu_2^m}{m!} \\
=& \int_0^{R+r} (\sum_{m=0}^{n_s} e^{-m\mu_1}\frac{(m\mu_1)^k}{k!} e^{-\mu_2}\frac{(\mu_2)^m}{m!})\frac{2x}{(R+r)^2}dx \\
\le& \int_0^{R+r} (e^{-\mu_1\mu_2}\frac{(\mu_1\mu_2)^k}{k!})\frac{2x}{(R+r)^2}dx \\
=& \int_0^{R+r} e^{-\frac{\mathcal{A}(x,R,r)n_s p}{L^2}} \frac{(\frac{\mathcal{A}(x,R,r)n_s p}{L^2})^k}{k!} \frac{2x}{(R+r)^2}dx
\end{aligned}$$

During the above derivation, we utilize the following notations to simplify our calculations:

$$\mu_1 = \frac{\mathcal{A}(x,R,r)p}{\pi(R+r)^2}, \mu_2 = \frac{\pi(R+r)^2 n_s}{L^2} \text{ and } \sum_{i=1}^{m}\upsilon_i = k$$

$$\mathcal{S} = \{\upsilon_0, \upsilon_1 \dots \upsilon_m | \sum_{i=1}^{m}\upsilon_i = k\}$$

Note that $R/r = \Omega(\sqrt{p})$, then we can obtain that

$$\mathcal{A}(x,R,r) = \begin{cases} \pi r^2 & \text{if } x < R-r \\ \frac{R^2(\theta_1 - \sin\theta_1) + r^2(\theta_2 - \sin\theta_2)}{2} & \text{if } R-r < x < R+r \\ 0 & \text{if } x > R+r \end{cases}$$

$\cos\theta_1 = \frac{R^2+x^2-r^2}{2xR}$ and $\cos\theta_2 = \frac{r^2+x^2-R^2}{2xr}$, then we substitute it into $\mathbf{Pr}(\mathcal{N}(r) = k)$ to obtain:

$$\mathbf{Pr}(\mathcal{N}(r) = k) \le 2e^{-\frac{\pi r^2 n_s p}{L^2}} \frac{(\frac{\pi r^2 n_s p}{L^2})^k}{k!}$$

∎

Lemma 8: We partition the square $\mathcal{O}$ into equal sub-square with side length $\tau L/\sqrt{n_s p}$ where τ is some constant, then we can make sure that the probability that at least one node inside a sub-square is larger than $1 - 2e^{\frac{\pi\tau^2}{4}}$ w.h.p.

Proof: First, assume r=$\frac{\tau L}{2\sqrt{n_s p}}$ then in every sub-square we can construct such a circle. According to Lemma7, the

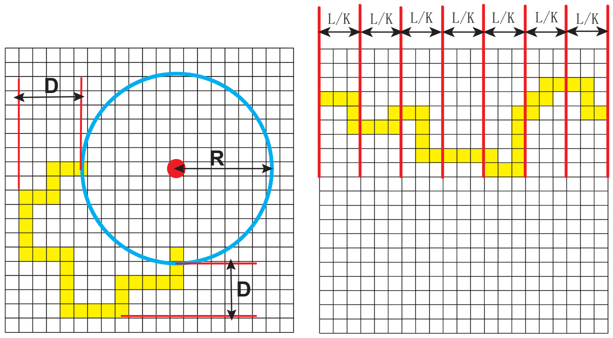

Fig. 6: Demonstration of influential range and a division of a percolation path

probability that at least one node inside the circle is greater than

$$\mathbf{Pr}(\mathcal{N}(r) \geq 1) \geq 1 - 2e^{-\frac{\pi r^2 n_s p}{L^2}} = 1 - 2e^{\frac{\pi \tau^2}{4}}$$

■

Therefore, according to Theorem 5 in [4], we can construct an *information highway* if the constant τ is large enough. Now we list some properties of such *information highway* originated from [4].

- there are $\Theta(\sqrt{n_s p})$ crossing paths from left to right and top to bottom, respectively.
- the length of each crossing path is tightly bounded by $\Theta(L)$.
- In each horizontal(vertical) rectangular of size[6] $L * (\kappa \log(n_s p) \frac{L}{\sqrt{n_s p}} - \epsilon_L)$, there are at least $\delta \log(n_s p)$ horizontal(vertical) highway paths w.h.p.
- the distance between two adjacent horizontal(vertical) path is at most $O(L \log(n_s p)/\sqrt{n_s p})$.
- the distance between two adjacent nodes on the same path is $O(L\sqrt{1/n_s p})$.
- there exists a spatial and temporal scheme that can achieve O(1) throughput on the highway. That is to say one node can relay O(1) bits to its adjacent nodes in a single time slot.

Based on the above characteristic of *information highway*, we provide a scheme to achieve a per cluster throughput of $\Theta(\frac{L}{\sqrt{n_s p}})$.

Now we will prove that such a scheme can achieve the lower bound of capacity in Theorem7. First we would upper bounded the number of clusters a cell need to serve transmission for.

Lemma 9: As for every cell, it can not serve for transmission of clusters with kernels $\sqrt{2}(R + \frac{(1+\kappa \log(n_s p))\tau L}{\sqrt{n_s p}})$ away from the cell, which also means $\mathcal{R} \leq \sqrt{2}(R + \frac{(1+\kappa \log(n_s p))\tau L}{\sqrt{n_s p}})$

Proof: We will refer to Fig.5 to prove this lemma. According to the properties of *information highway*, every path is constrained within a strip of width $(\kappa \log(n_s p) \frac{L}{\sqrt{n_s p}} - \epsilon_L)$. Thus

[6] κ and δ is some constant, $\epsilon_L = o(\log(n_s p) \frac{L}{\sqrt{n_s p}})$ and is to make $\kappa \log(n_s p) \frac{L}{\sqrt{n_s p}} - \epsilon_L$ an integer.

Algorithm 3 Multicast capacity of Uniform Cluster Random Model

1: Choose τ large enough so that $1 - 2e^{\frac{\pi \tau^2}{4}} > 5/6$. then we equally partition $\mathcal{O}$ into cells with side length $\tau L/\sqrt{n_s p}$. Thus there are $\lfloor \frac{\sqrt{n_s p}}{\tau} \rfloor * \lfloor \frac{\sqrt{n_s p}}{\tau} \rfloor$ cells and each cell in i_{th} row and j_{th} column is denoted by $s_{i,j}$.
2: Construct an information highway in the same way as in [4], which holds all of the properties listed above.
3: Construct an Euclidean spanning tree(EMST) for each cluster described in Lemma 6 to prove that the length of EMST is upper bounded by $5\sqrt{2p}R$.
4: As to every edge on EMST linking nodes u and v in cell s_{x_1,y_1}, s_{x_2,y_2}, respectively, find a nearest horizontal path in the highway system from s_{x_1,y_1}, node on the path with the same ordinate is considered access point of u. The same method for v to find its access point on vertical path.
5: We equally divide every single time slot into 3 phases as in Fig.5:
  - **Uplink**: Nodes drain its information to the access point.
  - **Highway link**: Relaying information to the respect access point of its destination.
  - **Downlink**: Delivering information from access point to nodes.

D in the figure is upper bounded by $(\kappa \log(n_s p) \frac{L}{\sqrt{n_s p}} - \epsilon_L)$. And the radius of such circle is R. From the figure we know the farthest cell that could be used is the black cell, a distance $\mathcal{R} \leq \sqrt{2}(R + \frac{(1+\kappa \log(n_s p))\tau L}{\sqrt{n_s p}})$ from the kernel. Therefore we prove the lemma. ■

Corollary 1: The number of cells that a cell serves can not exceed $4\pi(R/L + \frac{(1+\kappa \log(n_s p))\tau}{\sqrt{n_s p}})^2 n_s$

Proof: The proof is a directly use of Lemma 9 and Chernoff bound. ■

Lemma 10: Assuming that the distance between a sender and receiver is ℓ, then the number of hops for transmission is smaller than $O(\frac{c_4\sqrt{2n_s p}\ell}{L} + 6\kappa \log(n_s p))$ when utilizing Scheme I (3).

Proof: Assume s_{u_x,u_y} and s_{v_x,v_y} are two points on a horizontal path of *information highway*. Let $h(s_{u_x,u_y}, s_{v_x,v_y})$ denote the number of hops required for transmission from s_{u_x,u_y} to s_{v_x,v_y}. We can prove that there exists a constant c_4, such that

$$\mathbf{Pr}(h(s_{i,j}, s_{k,l}) \leq c_4|u_x - v_x|) \geq 1 - \delta(n_s p) \quad \lim_{n_s p \to \infty} \delta(n_s p) = 0 \tag{10}$$

Now we divide an aribitrary path into $K = \frac{L}{u_x - u_y}$ sub-paths as in the second figure in Fig.6 and use $\ell_i = \frac{L\wp(n_s p, i)}{K}(1 \leq i \leq K)$ to denote the length for transmission across the i_{th} sub-path. Then the proof of Eqn.10 is identical to prove the following inequity.

$$\lim_{n_s p \to \infty} \mathbf{Pr}(\ell_i = \Omega(\frac{L}{K})) = 0,$$

which is also identical to prove

$$\lim_{n_s p\to\infty} \mathbf{Pr}(\wp(n_s p, i) = \infty) = 0.$$

For each path on the highway system, the length of it is on the order of $\Theta(L)$ according to its properties. Thus there exist a constant c_3, such that the length of every path is upper bounded by $c_3 L$, which means $\sum_{i=1}^{K} \ell_i \leq c_3 L$. Taking the expectation on both sides, we can obtain

$$\sum_{i=1}^{K} \inf_{\wp(n_s p,i)=\infty} \{\wp(n_s p, i)\} \mathbf{Pr}(\ell = \Omega(L/K)) \frac{L}{K} \leq \sum_{i=1}^{K} \mathbf{E}[\ell_i] = \mathbf{E}[\sum_{i=1}^{K} \ell_i] \leq c_3 L.$$

Then we can obtain

$$\mathbf{Pr}(\ell = \Omega(L/K)) \leq \frac{c_3}{\inf_{\wp(n_s p,i)=\infty} \{\wp(n_s p, i)\}}$$

Taking $\delta(n_s p) = \frac{c_3}{\inf_{\wp(n_s p,i)=\infty}\{\wp(n_s p,i)\}}$, we complete the proof of Eqn.10.

Now consider two nodes of distance ℓ away with $s_{i,j}$ and $s_{k,l}$ serving as their access points, respectively. And $s_{m,n}$ as the intersection point for transmission. According to the properties of percolation path, every highway path is constrained within a strip of width $\kappa \log(n_s p) \frac{L}{\sqrt{n_s p}}$ and Eqn.10, such that

$$\begin{cases} \max\{|m - i|\} \leq c_4 |i - k| + \kappa \log(n_s p) \\ \max\{|n - j|\} \leq c_4 |j - l| + \kappa \log(n_s p) \\ \max\{\sqrt{(i-k)^2 + (j-l)^2}\} \leq \frac{\ell\sqrt{n_s p}}{L} + 2\kappa \log(n_s p) \quad \text{w.h.p} \end{cases}$$

Then the number of hops required for transmission with nodes ℓ away is upper bounded by

$$\begin{aligned} & \max\{|m - i| + |n - j|\} \\ \leq \; & c_4(|i - k| + |j - l|) + 2\kappa \log(n_s p) \\ \leq \; & c_4 \sqrt{2((i-k)^2 + (j-l)^2)} + 2\kappa \log(n_s p) \\ \leq \; & \frac{c_4 \sqrt{2 n_s p}\,\ell}{L} + 6\kappa \log(n_s p) \end{aligned} \quad (11)$$

■

Lemma 11: Given a cluster(multicast session), assume that cell s is within the radius R of the kernel. Then the probability that s is utilized for transmission for that kernel is less than $\frac{16\sqrt{2}c_0\tau^2 L}{\sqrt{n_s}R}$.

Proof: There are at least $\lfloor \frac{\sqrt{2n_s p}R}{\tau L} \rfloor^2 \geq (\frac{\sqrt{2n_s p}R}{\tau L} - 1)^2$ cells within the circle of radius R. Imitating the steps of constructing EMST in Lemma 6. Let $\mathcal{I}(s,i)$ be the indicator whether cell s is used in the i_{th} step and $\mathcal{P}$ be the probability that s is used in the whole process. Then the probability $\mathbf{Pr}(\mathcal{I}(s) = 1)$ is

$$\begin{aligned} \mathcal{P} \;\leq\; & \sum_{i=1}^{p} \mathbf{Pr}(\mathcal{I}(s,i) = 1) \\ \leq\; & \sum_{i=1}^{p} \frac{c_0}{(\frac{\sqrt{2n_s p}R}{\tau L} - 1)^2} (\frac{4c_4\sqrt{n_s p}\mathcal{R}}{\lfloor\sqrt{p+1-i}\rfloor L} + 6\kappa \log(n_s p)) \\ \leq\; & \sum_{i=1}^{p} \frac{2c_0\tau^2 L^2}{n_s p R^2} (\frac{4c_4\sqrt{2n_s p}}{\lfloor\sqrt{p+1-i}\rfloor L}(R + \frac{(1+\kappa\log(n_s p)\tau L)}{\sqrt{n_s p}}) \\ & + 6\kappa\log(n_s p)) \\ \leq\; & \frac{8\sqrt{2}c_0 c_4 \tau^2 L}{\sqrt{n_s}R^2}(R + \frac{2\kappa\log(n_s p)\tau L}{\sqrt{n_s p}}) + \frac{12 c_0\tau^2\kappa L^2 \log(n_s p)}{n_s R^2} \\ \leq\; & \frac{16\sqrt{2}c_0 c_4 \tau^2 L}{\sqrt{n_s}R} \end{aligned}$$

During the above derivation, we use $\mathcal{R} \leq \sqrt{2}(R + \frac{(1+\kappa\log(n_s p))\tau L}{\sqrt{n_s p}})$ from Lemma 9 and $R = \Omega(\frac{\log(n_s p)L}{\sqrt{n_s p}})$ according to our prerequisite. ■

Now we have upper bounded the probability that a cell serving for a cluster within its influential region $\mathcal{R}$. Then we will utilize Vapnik-Chervonenkis theory to prove our results as in [1]. Before that, we must know the VC-dimension of any multicast trees. From [2], we know the VC-dimension of EMST with p nodes is $\Theta(\log p)$.

Theorem 6: Given cluster radius R in a wireless network, we can prove that for arbitrary cell s,

$$\mathbf{Pr}(\#\text{of flows using s} \leq \frac{64\sqrt{2n_s}c_0 c_4 \pi\tau^2 R}{L}) \geq 1 - \delta(n_s p),$$

which means, every cell on the highway can serve for at most $\frac{64\sqrt{2n_s}c_0c_4\pi\tau^2 R}{L}$ clusters w.h.p.

Proof: First we will introduce Vapnik-Chervonenkis theory as follows:

For each sub-square s and the whole set of all the sub-square $\mathcal{O}$:

$$\mathbf{Pr}\left(\sup_{s\in\mathcal{O}} |\frac{\#\text{ of flows utilizing s}}{N} - \mathcal{P}| \leq \epsilon(n_s p)\right) > 1 - \delta(n_s p)$$

$$\text{when } N \geq \max\left\{\frac{8d}{\epsilon(n_s p)}\log\frac{13}{\epsilon(n_s p)}, \frac{4}{\epsilon(n_s p)}\log\frac{2}{\delta(n_s p)}\right\}$$

d is the VC-dimension of $\mathcal{O}$ and $d = \Theta(\log(n_s p))$, $\mathcal{P}$ is the probability that s is utilized to serve for the cluster when it is within the influential region of that kernel.

Then we substitute $\mathcal{P} \leq \frac{16\sqrt{2}c_0c_4\tau^2 L}{\sqrt{n_s}R}$ according to Lemma 11 into it to obtain

$$\mathbf{Pr}\left(\sup_{s\in\mathcal{O}} \frac{\#\text{ of flows utilizing s}}{N} \leq \frac{16\sqrt{2}c_0 c_4\tau^2 L}{\sqrt{n_s}R} + \epsilon(n_s p)\right) > 1 - \delta(n_s p)$$

Now let $\epsilon(n_s p) = \frac{16\sqrt{2}c_0c_4\tau^2 L}{\sqrt{n_s}R}$ and $\delta(n_s p) = \frac{2}{n_s p}$ and when

$$N \geq \max\left\{\frac{8d}{\epsilon(n_s p)}\log\frac{13}{\epsilon(n_s p)}, \frac{4}{\epsilon(n_s p)}\log\frac{2}{\delta(n_s p)}\right\},$$

we can satisfy the inequity. Now according to Lemma **??**, $N \geq \frac{\pi R^2}{2d_c^2} = \frac{\pi R^2 n_s}{2L^2}$, then we can know if

$$\frac{\pi R^2 n_s}{2L^2} \geq \frac{(\log(n_s p))^2 \sqrt{n_s} R}{2\sqrt{2} c_0 c_4 \tau^2 L}$$
$$R \geq \frac{(\log(n_s p))^2 L}{\sqrt{2n_s} \pi c_0 c_4 \tau^2},$$

the condition of the inequity can be satisfied. Then utilizing $N \leq \frac{2\pi R^2}{d_c^2} = \frac{2\pi R^2 n_s}{L^2}$ according to Lemma **??**, we can obtain the result. ■

Note that the constraint of R above means that only when R is sufficiently large, the upper bound of number of flows over a certain sub-square s could be determined by Vapnik-Chervonenkis theory. However, it does not mean that the upper bound of number of flows can not be determined for these cases, which is beyond the scope of this paper. Therefore we can obtain the constraint of *highwaylink* on capacity is

$$\lambda_{highwaylink} \geq \frac{WL}{64\sqrt{2n_s} c_0 c_4 \pi \tau^2 R} = \Theta(\frac{LW}{\sqrt{n_s} R}) \text{ when } R = \Omega(\frac{L(\log(n_s p))^2}{\sqrt{n_s}})$$

The above parts all deal with transmissions on information highway, nevertheless, we must analyze the constraint of transmission between nodes and its access point to complete the proof of our Scheme. First we introduce Lemma 3 in [4].

Lemma 12: Every node inside can achieve w.h.p. a rate to some node on the highway system of

$$\lambda_{uplink} = \lambda_{downlink} = \Omega((\log(n_s p))^{-3})$$

However, when $R = \Omega(\frac{L(\log(n_s p))^2}{\sqrt{n_s}})$, the bottleneck is caused by the highwaylink, when indicates:

$$\lambda = \min\{\lambda_{uplink}, \lambda_{downlink}, \lambda_{highwaylink}\} \geq \Theta(\frac{LW}{\sqrt{n_s} R}), \quad (12)$$

Therefore complete our proof.

VII. Upper Bound for SNCP Cluster Random/Grid Model

Theorem 7: Under

VIII. Conclusion

The conclusion goes here.

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