Report2 for Project on CS

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Abstract—-**This report is a summary of the work our group did in CS during last several weeks. We just pick up some important pieces of our work.We introduce some basic problem such as the relationship among m,n,k the noise and error, and some applications of CS.** *Index Terms*—-**Sparse representation, random measurement, ℓ_0 norm, ℓ_p norm, signal recovery, noise image-processing .**

I Introduction

A. Core of CS–the concept

Suppose a real-value, finite-length,one-dimensional, discrete-time signal x, which can be considered as an vector in $\mathrm{R^N}$. Then we can represent x by

$$x = \sum_{i=1}^{N} s_i \psi_i \qquad or \qquad x = \Psi s,$$

where $\{\psi_i\}_{i=1}^{N}$ is a basis of $N \times 1$ vectors in $\mathrm{R^N}$ and $\{s_i\}_{i=1}^{N}$ is the $N \times 1$ column vector. For example, we can represent x from the time domain into the frequency domain by the Fourier transform. Now, if the signal x is a linear combination of only K basis vectors then we say that x is K-sparse. That is, all of the s_i coefficients but K of them are zero. When there are just a few large coefficients and many small coefficients, we say x is compressible. In these algorithms, the K largest coefficients are located and the $(N - K)$ smallest ones are discarded[3]. We can have M measurements, where $N < M \ll N$[1][2] to process it. Now suppose that we have M vectors denoting $\{\phi_j\}_{j=1}^{M}$, where ϕ_j is one row vector in the $M \times N$ matrix Φ (= $\{\phi_j\}_{j=1}^{M}$). Arrange an $M \times 1$ vector y, make

$$y = \Phi x = \Phi \Psi s = \Theta s,$$

where Θ is an $M \times N$ matrix. This is the core of compressive sensing,which generates many questions for us to focus on and try to solve.

B. Questions

1.What's the relationship between m,n,k?
2.What happens if there is noise in the measurements?
3.How to solve the problem arose by the noise?
4.Different Ensembles of CS Matrices.
5.What's the application of CS?

II Relationship between m,n,k

Soppose we have an signal x with a very high degree of transform sparsity–only k non-zeros out of n coefficients, how big dose m have to be for the CS to work well? In many papers,they suggest the result: if an signal has a representation using only k non-zeros at randomly-chosen sites, something like $n \approx (3 \sim 4)k$ measurements would typically be needed. This is a result

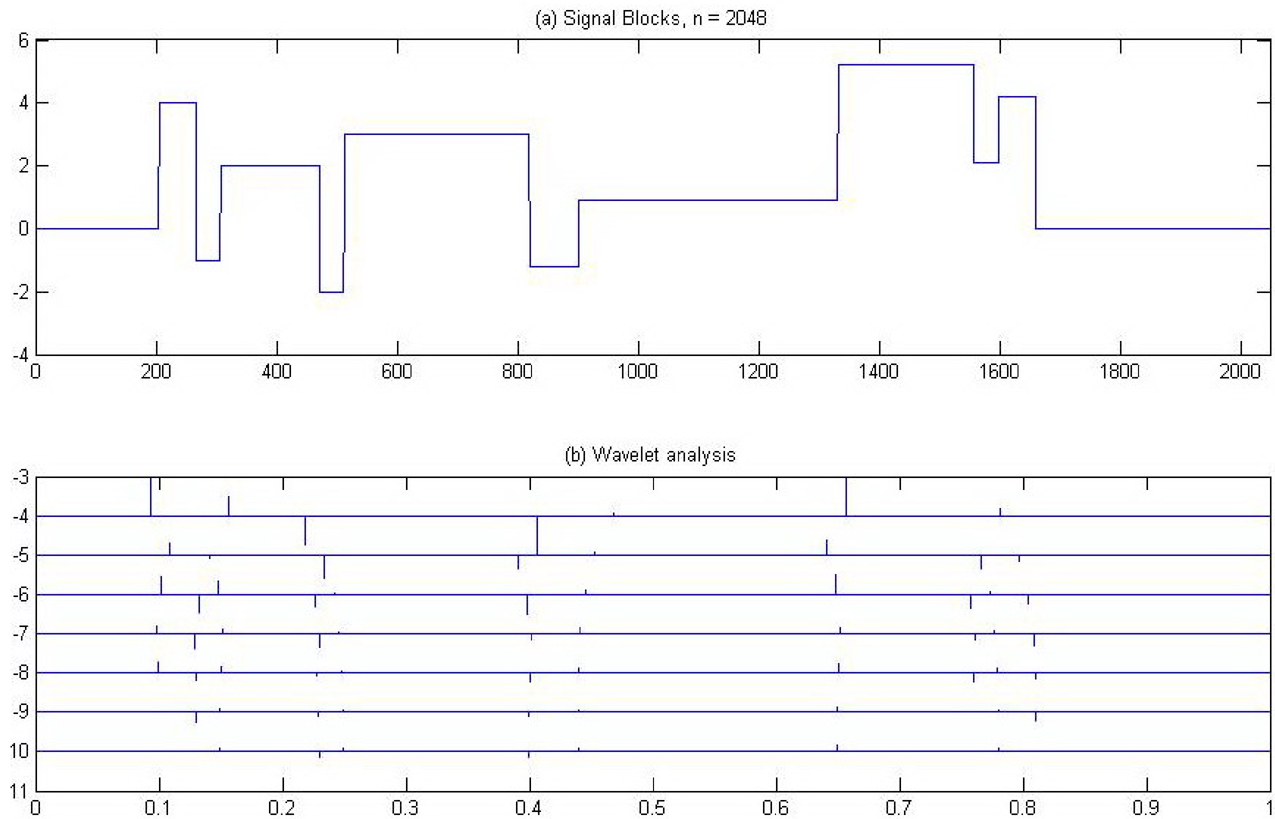

Figure 1: (a)Signal Blocks;(b)its expansion in a Haar wavelet basis.

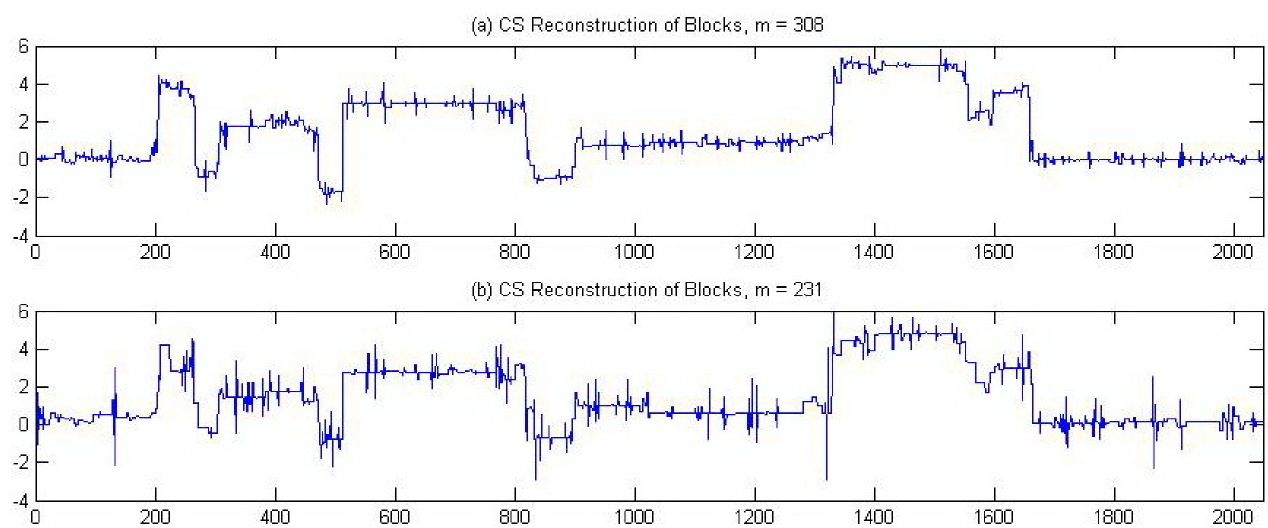

Figure 2: CS reconstructions of Blocks from (a) $m = 308$and (b)$m = 231$.

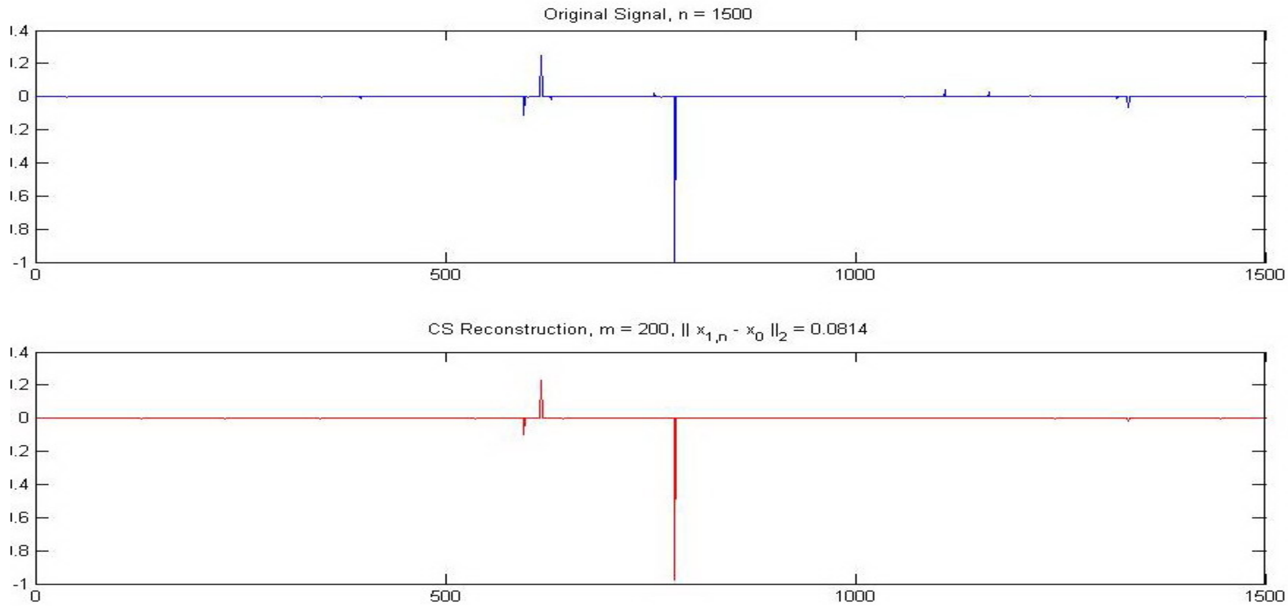

Figure 3: CS reconstruction of a signal with controlled l^p norm.

from the rule of thumb, that is to say, it's based on experiments.we also did some simulative experiments to validate it. We considered the object Blocks from the Wavelab package[1] as Figure 1 shows . The object is piecewise constant, and its Haar wavelet transform has relatively few nonzero coefficients. In fact, Blocks has k = 77 nonzero coefficients in a signal length m = 2048. We select $m = 3k = 231$ and $m = 4k = 308$.The reconstruction results shows in Figure 2. Clearly, their results are not bad and $m = 4k$works much better than $m = 3k$.We can say $m = 4k$work well if we compare both the construction result and the cost with the traditional way.

One of the necessary conditions for CS is that signal x must be K-sparse. But in practice, real signals will not typically have exact zeros anywhere in the transform, that is, those $(N - K)$ coefficients we discard are not zero,so what should we do to alleviate or even eliminate the effect? We usually judge the sparsity of a signal with its l^0 norm.It's powerful but not is also limited at the same time.So we consider its l^p norm.Since a signal x can have all entries nonzero,but still have small l^p $0 < p \leq 1$ norm.Besides, signals sparse in the l^0 case are also sparse in the l^0 sense.In Figure3 ,the original signal is not seriously l^0 sparse, but it's l^p and p here is $1/2$.We process it with CS and the reconstruction result show that most of the big-value coefficients or we can call them "important" coefficients are recovered well, but some of the very small coefficients are failed to recover.Clearly they are discarded by CS scheme.Then we measure the reconstruction error $\|\hat{x_{1,n}} - x\|_2 = 0.0814$,which can be tolerated.When n is fixed, the error decreases when we increase the value of m.

Error bound is another issue related to the m,n and p.It takes the form

$$\|\hat{x_{1,n}} - x\|_2 \leq C_p \cdot R \cdot (\frac{m}{\log(n)})^{1/2-1/p}$$

So we may want to know how large is the constant C_p.This question is theoretical in nature but almost impossible to solve by now, however, experiments with typical signals can be informative.This is a very creative way to see if the bound is tolerated or not.

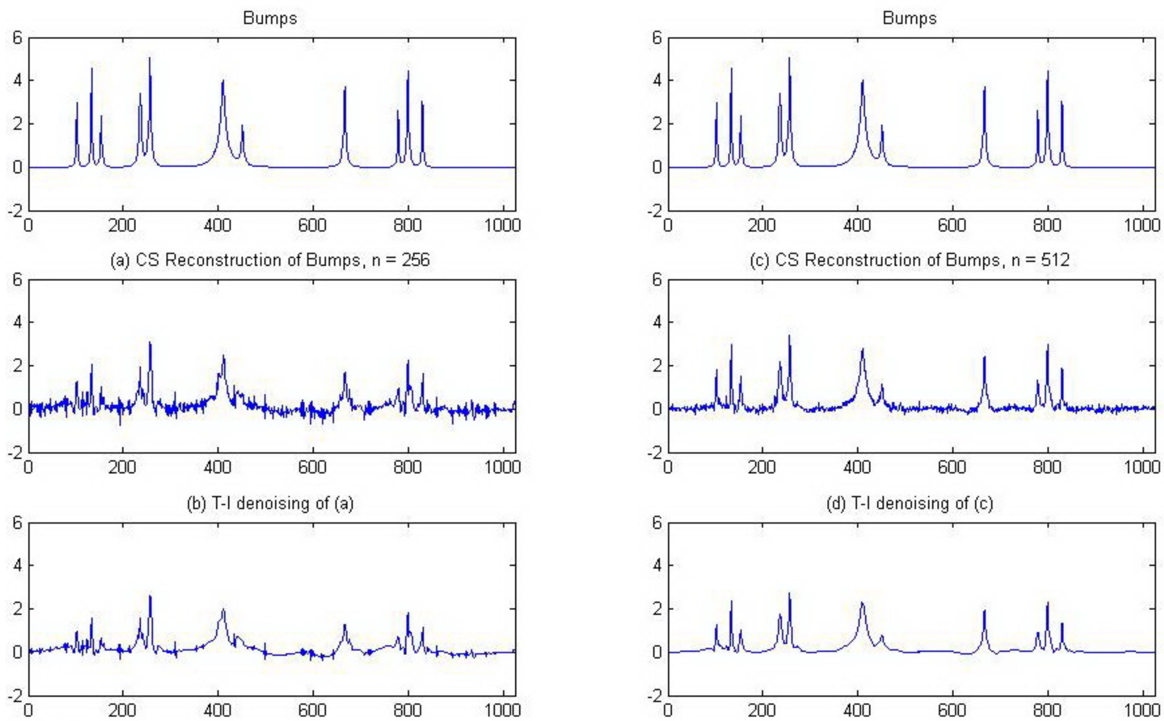

Figure 4: CS reconstruction of Bumps.

III NOISE

We can see there exist noise in the reconstruct signals even through there is no noise in the original signal or in the system.So when the object is undersampled, CS reconstructions are typically noisy.We considered the object Bumps from the Wavelab package. Figure 4 shows the result.Panel (a) and (b) is the original signal.We plot it for us to compare easily. Panel (c) shows the results of the reconstruction with $m = 256$ measurements. Panel (d) shows the result with $m = 512$. Clearly both results are 'noisy', with, understandably, the 'noisier' one coming at the lower sampling rate.

IV NOISE

Another situation is that the data actually are noisy.

$$y = \Phi\Psi^T x + z$$

where z is an arbitrary disturbance.We add zero-mean white gaussian noise to the Blocks.Figure 5 shows the result of CS under this scheme.

V APPLICATIONS

As mentioned at the report 1, CS has various applications on different realms,from imaging to communications, from network to integrated circuits. As some of us have strong interest in the digital image processing, so we pay more attention to the application of CS in the reconstruction of image. We'd like to introduce a very good paper "Compressive Imaging of Color Images". In this paper, the author proposes two novel concepts.One is a development of existing singel-pixel CS camera which is achieved by employing the Bayer color filter. Another is a novel CS reconstruction algorithm that employs joint sparsity models in simultaneously recovering R,G, B

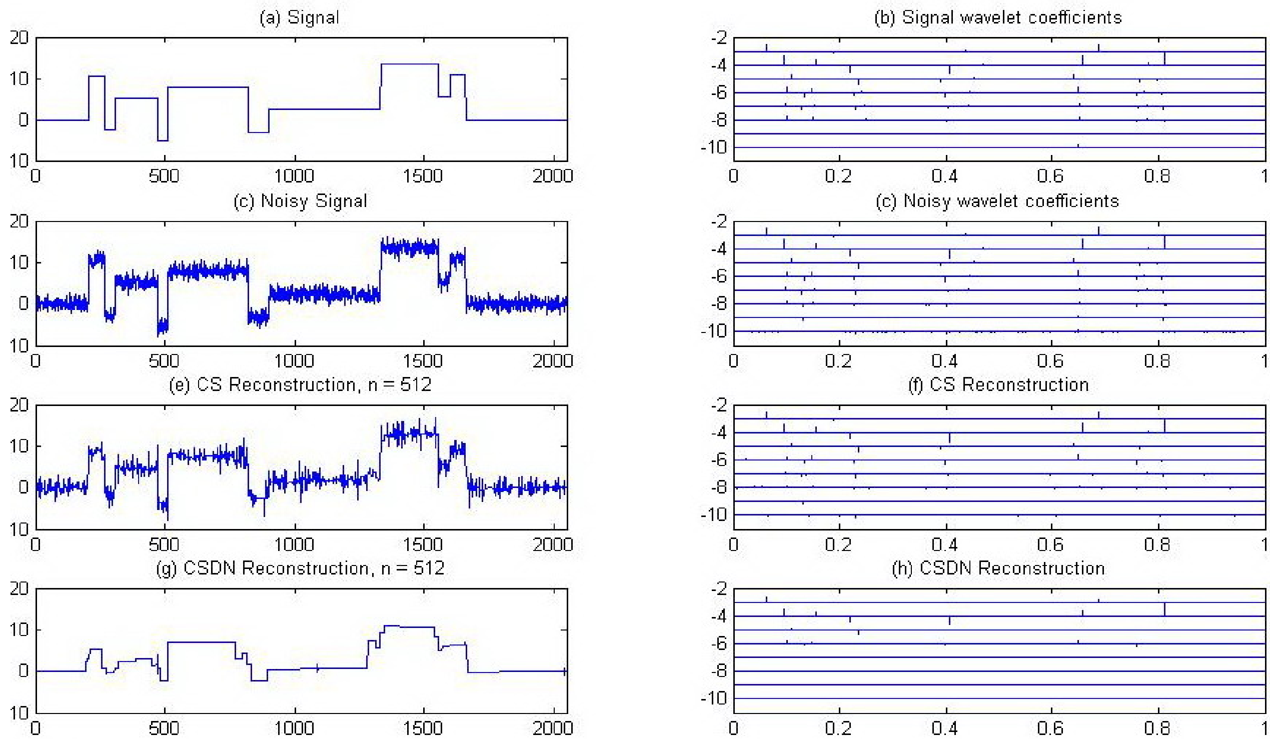

Figure 5: CS reconstruction of Bumps.

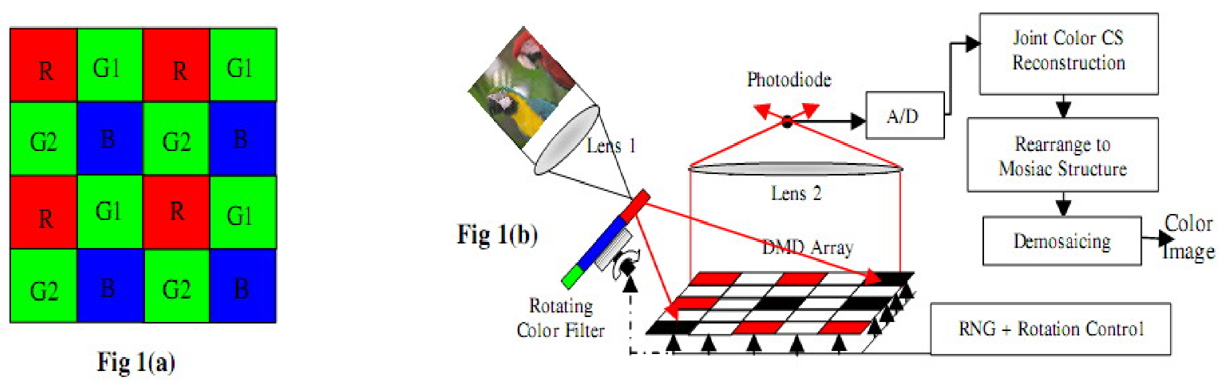

Figure 6:

channels from the compressive measurements. Now we will explain more details about the two new ideas in the paper. First, the application of Bayer color filter The whole process can be represented with a picture,as shown in Figure 6.

In the figure1(a),The mosaic structure is a "vitual Bayer filter" structure on the DMD(digital micro mirror) array.In the figure(b),from the figure we recognize that by placing an actual Bayer filter like that in figure1(a) between mirrors and rotating color filter(RCF),we get the R,G,B pixels.The pseudo-random number generator(RNG) make the acquistion of R,G,B planes seperately.Take the acquistion of R plane as an example Controling the RCF makes only red mirrors operated.Then RNG randomly choose some R mirrors to get the vector Φ_R .The we get

$$y_R = \Phi_R x_R \quad y_R \in R^{M_R \times 1}$$

Where,$\Phi_R = [\Phi_1 \Phi_2 \Phi_3 ... \Phi_M]^T$ is the random measurement matrix and $x_R$is sub-sampled images x,which get from mosaic cell "R" of the Bayer filter.By the similar methods,we can get the representation of G1,G2 and B.With the help of the Bayer filter compressive measurement,we reduce the data-rate and acquistion time per color images.The actual effective of the Bayer filter should be considered the actual Bayer filter and practical feasiblility. We can choose appropriate Bayer filter based on the characteristics of photos.

Second CS reconstruction algorithm
The novel reconstruction has a better qulity for the constructed images,comparing to the individual reconstructing R,G,B channels.
Joint R-G-B Reconstruction: A Baseline Algorithm
The reconstruction which based on the aligned pixels is one kind of common reconstructions.It can be interpreted by the content next. Let r,g,b be the rew R,G,B images,and $\theta_r, \theta_g, \theta_b$ be transform coefficients(in a basisΨ),then we can get

$$\theta_r = \theta^c + \theta_r^i = \Psi^T(r^c + r^i) = \Psi^T r$$

$$\theta_g = \theta^c + \theta_g^i = \Psi^T(g^c + g^i) = \Psi^T g$$

$$\theta_b = \theta^c + \theta_b^i = \Psi^T(b^c + b^i) = \Psi^T b$$

$\theta^c, \theta_r, \theta_g, \theta_b$derived from a common support Ω,which is non-zero coefficients,with cardinality $K^c, \theta_r^i, \theta_g^i, \theta_b^i$ are the sparse innovation components that are unique to each image. Then the author denotes $S = [\theta^c \;\; \theta_r^i \;\; \theta_g^i \;\; \theta_b^i]$has sparsity $K = K^c + K_r^i + K_g^i + K_b^i$,while the independent representation has a total sparsity of $\tilde{K} = K_r^i + K_g^i + K_b^i$. So the S ,namely the joint representation, can be recovered back by JSM(Joint Sparity Model) reconstruction method.

$$\hat{S} = \arg\min \|S\|_1 \quad s.t. y = \tilde{\Phi}\tilde{\Psi}S$$

Where $y = [y_r \; y_g \; y_b]^T \in R^{(M_r + M_g + M_b}$ and$\tilde{\Phi} = diag([\Phi_r \;\; \Phi_g \;\; \Phi_b]^T)$ and Obviously the R-G-B recovery is better than the naive approach.It minimum number of measurements required for faithful reconstruction is reduced.At the same time,for a fixed measurement,the fidelity of reconstructed image would be superior.But the method only can be applied to aligned pixels. for the random pixels positions,the E-JSM Extended Joint R-G-B Reconstruction) will have its own advantages. 1.2.2 Extended Joint R-G-B Reconstruction(E-JSM) The Bayer images can be represented as $S = [\theta^c \;\; \theta_R^i \;\; \theta_{G_1}^i \;\; \theta_{G_2}^i \;\; \theta_B^i]^T \in R^{5N/4}$,where θ^c is the common component extracted from a common support Ω_B of cardinality K_B^c and θ^is are innovation components.Then we can define a new E-JSM additive model considering a global common and all six pair-wise common components as follows: :

$$\theta_R = \theta^c + \theta_{RG_1}^c + \theta_{RG_2}^i + \theta_{RB}^c + \theta_R^{'i}$$

$$\theta_{G_1} = \theta^c + \theta_{RG_1}^c + \theta_{G_1G_2}^i + \theta_{G_1B}^c + \theta_{G_1}^{'i}$$

$$\theta_{G_2} = \theta^c + \theta_{RG_2}^c + \theta_{G_1G_2}^i + \theta_{G_2B}^c + \theta_{G_2}^{'i}$$

$$\theta_B = \theta^c + \theta_{RB}^c + \theta_{RG_1}^i + \theta_{G_2B}^c + \theta_B^{'i}$$

Under this mode we have a "sparser" joint representation as $S_E = [\theta^c S_E^c S_E^i]^T \in R^{11N/4}$,where, $S_E^c = [\theta^c_{RG_1}\theta^c_{RG_2}\theta^c_{RB}\theta^c_{G_1G_2}\theta^c_{G_1B}\theta^c_{G_2B}] \in R^{6N/4}$ is the vector of pair-wise common components and $S_E^i = [\theta'^i_R \ \ \theta'^i_{G_1} \ \ \theta'^i_{G_2} \ \ \theta'^i_B$ is a vector of new innovation component.The reconvery is similar to that discussed above.The full r,g,b images can then be obtained using any existing demosaicing techniques.

VI Further Work

We bring 5 questions in the beginning, they are very basic and fundamental.We talked about some of them in this report and maybe we won't study all of them.Recently we did very little job in the reconstruction algorithm.This is one of the things we will do in following days.

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