Project Report2
Capacity and Delay Tradeoffs in MANETs with Unicast and Multicast

Group 2
Xiaoyu Chu, Jia Guo, Zhang Li, Chaomin Ye

I. Introduction

Based on our former studies on capacity and delay in MANETs with Unicast, we step further to the field of multicast. In this report, a correcion for 2-hop relay capacity bound is given first, followed by some calculation and deduction to obtain the capacity-delay tradeoff for multicast, which is shown in the third section. More analysis on our results are provided in Section Four, where fantastic results are shown there. This report ends with our conclusion and our future work.

II. One Correction For 2-Hop Relay Capacity Bound

First of all, we have a correction for the 2-hop relay with redundancy capacity bound. In [2], Chenhui deduced the total network delay as $\mathbb{E}\{W\} = O(\sqrt{N \log K})$, with which we agree . However, he obtained the achievable per-node capacity under this algorithm as $\Omega(1/K\sqrt{N \log K})$ by treating this relay model as a continuous time M/M/1 queue with input rate $K\lambda_r$ and service rate μ_r, which we doubt here. This sub-queue view is shown in Fig. 1.

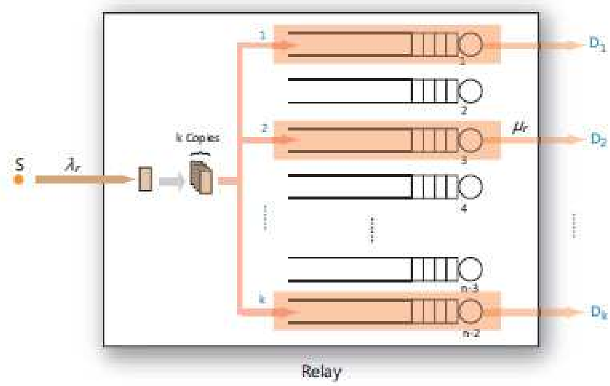

Fig. 1. The 'Virtual Duplication' Relay Model

As we can see from Fig. 1, Chenhui extended the inner side of a relay node, and allocate $n-2$ sub-queues in it to buffer packets intended for different destinations. When a packet arrives, it is duplicated to those K sub-queues corresponding to those K destinations. For this step, the incoming rate is said to be $K\lambda_r$. Then, when this relay meets a desired destination, assuming it is the K'th destination, it will delete this duplication from the sub-queue which is designed for the K'th destination. For this step, the outflow rate is said to be μ_r in [2]. As a result, the waiting time expectation is $E\{W_{rd}^i\} = 1/(\mu_r - K\lambda_r)$, which seems reasonable. However, the author fails to take the other relays' sub-queues for K'th destination into consideration. When the above relay meets the K'th destination, it transmits this packet, and delete it from the corresponding sub-queue of itself. Meanwhile, the duplications of this packet in the other relays' sub-queues become useless, since the K'th destination won't need to receive this packet any more, so the other relays who have this duplication will also delete it from their sub-queues. Therefore, the outflow rate of packet is not simply μ_r any more. Instead, it will be related with the number of K. Here, the unicast model becomes a special case, since the incoming packet won't be duplicated in the relay, so the 'virtual incoming rate' is never changed. However, the multi-cast model is different, in that we can't simply multiply K to the incoming rate and make no amendments to the outflow rate.

To illustrate this relay model more clearly, we won't use this 'virtual-duplication' any more, but just treat a relay node as a simple node, and its mission is saving a packet intended for its destinations. By the time all K destinations have received this packet from no matter which relay node, this relay will drop this packet from its buffer. The other relay nodes who have this packet also do the same thing at this moment. As a result, the incoming rate is still λ_r, and the outflow rate remains μ_r.

Until now, with the total network delay $E\{W\} = O(\sqrt{N \log K})$, we correct the per-node capacity to be $\Omega(1/\sqrt{N \log K})$. The corrected capacity and delay tradeoff between the 2-hop relay algorithm without and with redundancy is show below.

TABLE I
Capacity And Delay Tradeoff of 2-Hop Relay Algorithm

scheme	capacity	delay
2-hop relay w.o. redundancy	$O(\frac{1}{K})$	$O(N \log K)$
2-hop relay w. redundancy	$O(\frac{1}{\sqrt{N \log K}})$	$O(\sqrt{N \log K})$
multi-hop relay w. redundancy	$O(\frac{1}{N \log N})$	$O(\log N)$

From Table 1, we can deduce that the 2-hop relay algorithm satisfies $delay/rate \geq KN\log K$. In addition, since this ratio for 2-hop relay without redundancy is $KN\log K$, and for 2-hop relay with redundancy is $N\log K$, there is an improvement of the redundancy algorithm. But in the unicast model, there is no improvement from the no redundancy algorithm to the redundancy algorithm. A simple explanation for this question is that, in this multicast algorithm, since the redundancy lowers the delay of networks, the time packets staying in the buffers of the relays becomes shorter, which allows a faster incoming rate of packets to the relays. As a result, the delay-capacity ratio is improved comparing to that without redundancy.

III. Delay/Rate Tradeoffs in multicast model

Observing Table 1 can we see that the ratio under these three schemes are $NK\log K$, $N\log K$, $N(\log N)^2$ respectively, which led us to suppose the general relationship between delay and capacity is that their ratio is large than $N\log K$.

Consider a network with N users, and suppose all users receive packets at the same rate λ. A control protocol which makes decisions about scheduling, routing, and packet retransmissions is used to stabilize the network and deliver all packets to their respective K destinations while maintaining an average end-to-K delay[1] less than some threshold $\overline{W}$

Theorem 1: A necessary condition for any conceivable routing and scheduling protocol with K[2] destinations for transmitting that stabilizes the network with input rates λ while maintaining bounded average end-to-K delay $\overline{W}$ is given by:

$$\overline{W} \geq \Theta(N\log K)\frac{\lambda}{1-K\lambda} \tag{1}$$

Which equals to this following expression,

$$\begin{cases} \lambda = O(1/K), & \overline{W}/\lambda \geq \Theta(N\log K) \\ \lambda = \omega(1/K), & \overline{W} \geq \Theta(N\log K/K) \end{cases} \tag{2}$$

Proof: Suppose the input rate of each of the N sessions is λ, and there exists some stabilizing scheduling strategy which ensures an end-to-K delay of $\overline{W}$. In general, the end-to-K delay of packets from individual sessions could be different, and we define W_i as the resulting average delay of packets from session i. We thus have:

$$\overline{W} = \frac{1}{N}\sum_i \overline{W_i} \tag{3}$$

Now we count the number of transmission times for session i. Every time-slot, if this packet or its copies been transmitted to M different non-destination receivers, the count will be added by M. We define $\overline{R_i}$ as the *non-destination redundancy* which represents the final number of counting when the packet finally reaches the K^{th} destination and end its task, averaged over all packets from session i. That is, $\overline{R_i}$ is average number of non-destination transmissions for a packet from session i. Note that all packets are eventually received by the K destinations, so that $\overline{R_i}+K$ is the actual number of transmissions for packets from session i, and then the average number of successful packet receptions per time-slot is thus given by the quantity $\lambda\sum_{i=1}^{N}(\overline{R_i}+K)$. Since each of the N users can receive at most 1 packet per time-slot, we have:

$$\lambda\sum_{i=1}^{N}(\overline{R_i}+K) \leq N \tag{4}$$

Now consider a single packet p which enters the network from session i. This packet has an average delay of $\overline{W_i}$ and an average non-destination redundancy of $\overline{R_i}$. Let random variables W_i and R_i represent the actual delay and non-destination redundancy for this packet. We have:

$$\begin{aligned} \overline{W_i} &= \mathbb{E}\{W_i|R_i \leq 2\overline{R_i}\}Pr[R_i \leq 2\overline{R_i}] + \\ &\quad \mathbb{E}\{W_i|R_i \geq 2\overline{R_i}\}Pr[R_i \geq 2\overline{R_i}] \\ &\geq \mathbb{E}\{W_i|R_i \leq 2\overline{R_i}\}Pr[R_i \leq 2\overline{R_i}] \\ &\geq \mathbb{E}\{W_i|R_i \leq 2\overline{R_i}\}\frac{1}{2} \end{aligned} \tag{5}$$

Where the last inequality follows because $Pr[R_i \leq 2\overline{R_i}] \geq \frac{1}{2}$ for any nonnegative random variable R_i.

Consider now a virtual system in which there are $2\overline{R_i}$ users initially holding packet p, and let Z_m represent the time required for one of these users to enter the same cell as the m^{th} destination. Then let Z represent the time required for these users to enter all the K destinations, so we have $Z = \max\{Z_1, Z_2, \ldots, Z_K\}$. Note that the distribution of each Z_m is the same as $Pr[Z_m > w] = (1-\phi)^{[w]}$, in which $\phi = 1-(1-\frac{1}{C})^{2\overline{R_i}}$. And thus $\mathbb{E}\{Z_i\} = \frac{1}{\phi}$.

In order to connect this variable Z to our interest W_i, we develop another parameter W_i^{rest}, which represents the corresponding delay under the *restricted scheduling policy* that schedules packets as before until either the packet is successfully delivered to all K destinations, or the redundancy increases to $2\overline{R_i}$(where no more redundant transmissions are allowed). Since this modified policy restricts redundancy to at most $2\overline{R_i}$, the delay W_i^{rest} is stochastically greater than the variable Z, representing the delay in a virtual system with only one packet that is initially held by $2\overline{R_i}$ users. In addition, as the restricted policy is identical to the original policy whenever $R_i \geq 2\overline{R_i}$, hence $\mathbb{E}\{W_i|R_i \leq 2\overline{R_i}\} = \mathbb{E}\{W_i^{rest}|R_i \leq 2\overline{R_i}\}$.

Finally, we introduce the last much easier calculated continuous variable $\widetilde{Z}$, which is also the maximum of several ones - $\widetilde{Z} = \max\{\widetilde{Z}_1, \widetilde{Z}_2, \ldots, \widetilde{Z}_K\}$. For each of them has the same distribution as $Pr[\widetilde{Z}_m > w] = e^{-\gamma w} = (1-\phi)^w \leq (1-\phi)^{[w]} = Pr[Z_m > w]$, where $\gamma = \log\frac{1}{1-\phi}$.

So now we put the relationship among these three variables clearly as follows:

$$W_i^{rest} \succeq Z \succeq \widetilde{Z}^3 \tag{6}$$

[1]Here the delay represents the average time for one packet transmitting from the specific source until it reaches all the K destinations arranged to.

[2]K can be any value smaller than N, including constant order which covers the unicast condition.

[3]Because $Pr[Z > w] = 1 - Pr[Z \leq w] = 1 - \prod_{m=1}^{K} Pr[Z_m \leq w] \geq 1 - \prod_{m=1}^{K} Pr[\widetilde{Z}_m \leq w] = Pr[\widetilde{Z}]$, and according to the definition in [3], we have that Z is stochastically greater than $\widetilde{Z}$.

Further, from Appendix A, we have this useful inequality:

$$\mathbb{E}\{W_i|R_i \le 2\overline{R}_i\} \ge \inf_{\Theta} \mathbb{E}\{Z|\Theta\} \ge \inf_{\widetilde{\Theta}} \mathbb{E}\{\widetilde{Z}|\widetilde{\Theta}\} \quad (7)$$

where the conditional expectation is minimized over all conceivable events Θ*(for Z, while* $\widetilde{\Theta}$*for* $\widetilde{Z}$*) which occur with probability greater than or equal to 1/2.*

Until now we have to calculate the last value $\inf_{\widetilde{\Theta}} \mathbb{E}\{\widetilde{Z}|\widetilde{\Theta}\}$. From Lemma 8 in [1], whose result been put as follows:

For any nonnegative random variable X, we have:

$$\inf_{\{\Theta|Pr[\Theta]\ge\frac{1}{2}\}} \mathbb{E}\{X|\Theta\} = \mathbb{E}\{X|X<w\}2Pr[X<w] + w(1-2Pr[X<w]) \quad (8)$$

Where w is the unique real number such that $Pr[X < w] \le \frac{1}{2}$ *and* $Pr[X \le w] \ge \frac{1}{2}$.

Note that in the special case when $P(x)$ is continuous at $x = w$, then $Pr[X < w] = Pr[X \le w] = \frac{1}{2}$ and hence we get the simpler expression:

$$\inf_{\widetilde{\Theta}} \mathbb{E}\{\widetilde{Z}|\widetilde{\Theta}\} = \mathbb{E}\{\widetilde{Z}|\widetilde{Z} \le w\} \quad (9)$$

Now recall the distribution expression of $\widetilde{Z}$:

$$Pr[\widetilde{Z} \le z] = \prod_{m=1}^{K} Pr[\widetilde{Z}_m \le z] = (1-e^{-z\gamma})^K. \quad (10)$$

Then we get the value of w: $w = -\frac{1}{\gamma}\ln[1-(\frac{1}{2})^{\frac{1}{K}}]$.

Return to our question, we do the calculation as follows:

$$\begin{aligned}
\mathbb{E}\{\widetilde{Z}|\widetilde{Z} \le w\} &= \frac{\int_0^w x[(1-e^{-\gamma x})^K]'_x dx}{Pr[\widetilde{Z} \le w]} \\
&= 2x(1-e^{-x\gamma})^K|_0^w - 2\int_0^w (1-e^{-x\gamma})^K dx \\
&= w - 2\int_0^w \sum_{i=0}^{K} C_K^i(-1)^i e^{-ix\gamma} dx \\
&= w + 2\sum_{i=0}^{K} C_K^i(-1)^{i+1} \int_0^w e^{-ix\gamma} dx \\
&= -w + 2\sum_{i=1}^{K} \frac{1}{i\gamma} C_K^i (-1)^i [e^{-iw\gamma} - 1] \\
&\triangleq -w + 2X(K) \quad (11)
\end{aligned}$$

Note that $C_K^i = C_{K-1}^{i-1} + C_{K-1}^i$, we can calculate $X(K)$ by gradually reducing the variable K, as $X(1)$ can be easily obtained as $X(1) = -\frac{1}{\gamma}[e^{-w\gamma} - 1]$.

$$\begin{aligned}
X(K) - X(K-1) &= \sum_{i=1}^{K} \frac{1}{i\gamma} C_{K-1}^{i-1} (-1)^i [e^{-iw\gamma} - 1] \\
(\because \frac{1}{i} C_{K-1}^{i-1} = \frac{1}{K} C_K^i) &= \frac{1}{K\gamma} \sum_{i=0}^{K} C_K^i (-1)^i [e^{-iw\gamma} - 1] \\
&= \frac{1}{K\gamma}[(1-e^{-w\gamma})^K - (1-1)^K] \\
&= \frac{1}{2K\gamma} \quad (12)
\end{aligned}$$

Continue this recursion, we have that:

$$\begin{aligned}
X(K) &= X(1) + \frac{1}{2\gamma}(\frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{K}) \\
&= -\frac{1}{\gamma}[e^{-w\gamma} - 1] + \frac{1}{2\gamma}(\frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{K}) \\
&\triangleq -\frac{1}{\gamma}[e^{-w\gamma} - 1] + \frac{1}{2\gamma} f(K) \quad (13)
\end{aligned}$$

Wherein $f(K) = \Theta(\log K)$, see Appendix B. Connecting (9),(11) and (13) can we have that:

$$\begin{aligned}
\inf_{\widetilde{\Theta}} \mathbb{E}\{\widetilde{Z}|\widetilde{\Theta}\} &= \mathbb{E}\{\widetilde{Z}|\widetilde{Z} \le w\} \\
&= -w + 2X(K) \\
&= -\frac{1}{\gamma}\ln[1-(\frac{1}{2})^{\frac{1}{K}}] - \frac{2}{\gamma}(\frac{1}{2})^{\frac{1}{K}} + \frac{1}{\gamma} f(K) \\
&\triangleq \frac{1}{\gamma} g(K) \quad (14)
\end{aligned}$$

Wherein $g(K) = \Theta(\log K)$, see Appendix C. From the definitions of γ and ϕ, we have $\gamma = \log(1/(1-\frac{1}{C})^{2\overline{R}_i}) = 2\overline{R}_i \log(1+\frac{1}{C-1})$. Since $\log(1+x) \le x$ for any x, we have $\gamma \le 2\overline{R}_i/(C-1)$. Then using (5), (7) and (14) in (3) yields:

$$\begin{aligned}
\overline{W} = \frac{1}{N}\sum_{i=1}^{N} \overline{W}_i &\ge \frac{1}{2N}\sum_{i=1}^{N} \frac{1}{\gamma} g(K) \\
&\ge \frac{g(K)}{2N}\sum_{i=1}^{N} \frac{C-1}{2\overline{R}_i} \\
&\ge g(K)(C-1)\frac{1}{4N}\sum_{i=1}^{N} \frac{1}{\overline{R}_i} \\
&\ge g(K)\frac{C-1}{4}\frac{1}{\frac{1}{N}\sum_{i=1}^{N}\overline{R}_i} \quad (15)
\end{aligned}$$

Where (15) follows from Jensen's inequality, noting that the function $f(R) = \frac{1}{R}$ is convex, and hence $\frac{1}{N}\sum_{i=1}^{N} f(\overline{R}_i) \ge f(\frac{1}{N}\sum_{i=1}^{N}\overline{R}_i)$. Combining (15) and (4), we have:

$$\overline{W} \ge g(K)\frac{C-1}{4}\frac{\lambda}{1-K\lambda} = \Theta(N\log K)\frac{\lambda}{1-K\lambda} \quad (16)$$

wherein C has the same order as N. Proving the theorem. ■

IV. More analysis on the Theorem

In the last section, we get a complicated formula though tough deduction. We will analyze this formula and disinter it significance.

A. What is $\overline{W}/\lambda$

First, we notice that because $\overline{R_i} >= 0$ in formula (4), $\lambda < \frac{1}{K}$. It means we should never worry about that $1 - K\lambda$ could be a negative one.

Divide λ on both sides of formula (1), we get $\frac{\overline{W}}{\lambda} \geq \Theta(N \log K)\frac{1}{1-K\lambda}$. The left side is the tradeoff that we want to calculate. What troubles us is that in the right side, there is another λ. If $1 - K\lambda$ remain a constant as N and K growing into infinity, i.e. $\lambda = o(1/K)$, we will get a fantastic result as

$$\frac{\overline{W}}{\lambda} \geq \Theta(N \log K) \tag{17}$$

From this formula, we get the lower bound of $\overline{W}/\lambda$. It is the consideration of this lower bound that leads us to focus on Chen Hui's conclusion on 2-hop with redundancy model, which has a larger $\overline{W}/\lambda$ than $\Theta(N \log K)$. And we finally find the mistake in Chen Hui's logic and prove that $\overline{W}/\lambda$ could be smaller under 2-hop with redundancy model.

However, if $\frac{1}{1-K\lambda} = \omega(C)$, which means it would be infinitely large as K or N growing into infinity, we can never simply cancel $\frac{1}{1-K\lambda}$ in the right side of the formula. In this condition, $\overline{W}/\lambda = \omega((N \log K))$. Thus, if we want to attain the lower bound: $\Theta(N \log K)$, λ should be smaller than $\frac{1}{K} + o(\frac{1}{K})$. It means although we always expect a large λ, in fact, a larger λ may do harm on the tradeoff.

B. K has same order with constant

If $K = O(C)$, from formula (4), we get

$$\overline{W} \geq \Theta(N\frac{\lambda}{1-K\lambda}) \tag{18}$$

Obviously, λ cannot have same order as constant, instead, it must be lower than it. Thus, $1-K\lambda$ can be deleted. And Then we get

$$\frac{\overline{W}}{\lambda} \geq \Theta(N) \tag{19}$$

This result is same as Neely's in his paper on uni-cast transmission. Therefore, situation of a constant order destination almost has no difference comparing with uni-cast transmission. And this will validate our result from another angle.

C. The lower bound of $\overline{W}$

From formula (4), we can not only calculate the lower bound of $\overline{W}/\lambda$, but can also get something about $\overline{W}$. From the process of proof, we can see that

$$\frac{\lambda}{1-K\lambda} = \frac{1}{\frac{1}{N}\sum_{i=1}^{N}\overline{R}_i} \tag{20}$$

Obviously,$\overline{R}_i < N$. Thus,

$$\frac{\lambda}{1-K\lambda} > \frac{1}{N} \tag{21}$$

Put this result into our theorem, we get

$$\overline{W} \geq \Theta(\log N) \tag{22}$$

In this formula, we replace K into N because in most situation, K has the same order with N. So $\log N$ is the lower bound of $\overline{W}$. This means if we take some way to make the delay attain $\Theta(\log N)$, to decrease capacity won't benefit the tradeoff any more.

V. Conclusion and Future Work

In this paper, we first point out some tiny mistakes in Chen Hui's paper and correct it to achieve a better result. We also compare three basic models on uni-cast and multi-cast and find out some relationship. Then we broaden Neely's result of tradeoff on uni-cast model to multi-cast model. The process of proof is the highlight of this paper. Neely use a abstract conception on probability to make the calculation of tradeoff achievable. We borrow his train of thought and continue it with much more complicated math calculation skills. And we finally get a relationship of delay and capacity under multi-cast models. Some analysis has been done to show the significance of this formula. We find that the lower bound of $\overline{W}/\lambda$ is $\Theta(N \log K)$ and the lower bound of $\overline{W}$ is $\Theta(\log N)$.

After we build the relationship of delay and capacity, almost no space is left for us to promote our research in independent and identically distributed (i.i.d.) model. More over, i.i.d. model is the easiest one among all models on math calculation. Therefore, our future work may change direction to some other models, such as Markovian random walks model and so forth, which seem more complex and less mature.

Appendix A – Stochastic Processes

Lemma 1: Given that two nonnegative distributions G and F satisfied that G is stochastically less than F, i.e. for any variables X and Y having distributions G and F respectively, $P_r[X > w] \leq P_r[Y > w], \forall w \geq 0$.Then there must exist two variables X and Y having G and F distributions respectively, and $P_r[X \leq Y] = 1$

Proof: For a certain variable Y having distribution F, create a variable X satisfied $X = G^{-1}[F(Y)]$. Then

$$\begin{aligned} P_r[X \leq x] &= P_r\{G^{-1}[F(Y)] \leq x\} \\ &= P_r\{F(Y) \leq G(x)\} \\ &= P_r\{Y \leq F^{-1}[G(x)]\} \\ &= F\{F^{-1}[G(x)]\} \\ &= G(x) \end{aligned}$$

Thus X has the distribution of G. Furthermore, from the definition, $1 - G(x) \leq 1 - F(x)$, we have $G^{-1}(x) \leq F^{-1}(x)$ for any nonnegative value x.Then we have

$$X = G^{-1}[F(Y)] \leq F^{-1}[F(Y)] = Y$$

■

So as $W_i^{rest} \succeq Z$, there must exist a variable Z', which has the same distribution as Z but satisfies $W_i^{rest} \geq Z'$ all the time. Thus:

$$\begin{aligned} \inf_{\{\Psi|Pr[\Psi]\geq\frac{1}{2}\}} \mathbb{E}\{W_i^{rest}|\Psi\} &\geq \inf_{\{\Psi|Pr[\Psi]\geq\frac{1}{2}\}} \mathbb{E}\{Z'|\Psi\} \\ &= \inf_{\{\Theta|Pr[\Theta]\geq\frac{1}{2}\}} \mathbb{E}\{Z|\Theta\} \end{aligned}$$

Similar to Z and $\widetilde{Z}$, we have:

$$\inf_{\{\Theta|Pr[\Theta]\geq\frac{1}{2}\}} \mathbb{E}\{Z|\Theta\} \geq \inf_{\{\widetilde{\Theta}|Pr[\widetilde{\Theta}]\geq\frac{1}{2}\}} \mathbb{E}\{\widetilde{Z}|\widetilde{\Theta}\}$$

So we get the final wanted result:

$$\begin{aligned}\mathbb{E}\{W_i|R_i \leq 2\overline{R}_i\} &= \mathbb{E}\{W_i^{rest}|R_i \leq 2\overline{R}_i\} \geq \inf_{\Psi} \mathbb{E}\{W_i^{rest}|\Psi\} \\ &\geq \inf_{\Theta} \mathbb{E}\{Z|\Theta\} \geq \inf_{\widetilde{\Theta}} \mathbb{E}\{\widetilde{Z}|\widetilde{\Theta}\}\end{aligned}$$

APPENDIX B – ORDER OF $\sum_{k=1}^{n} \frac{1}{i}$

Solving this problem, we have to prove the following bound

Lemma 2:

$$\ln(n+1) < \sum_{k=1}^{n} \frac{1}{k} < \ln n + 1$$

Proof: Here we use the integral way as follows.

$$\begin{aligned}\frac{1}{k+1} &< \int_{k}^{k+1} \frac{1}{x} dx < \frac{1}{k} \\ \Rightarrow \sum_{k=1}^{n} \frac{1}{k+1} &< \sum_{k=1}^{n} \int_{k}^{k+1} \frac{1}{x} dx < \sum_{k=1}^{n} \frac{1}{k} \\ \Rightarrow \sum_{k=1}^{n} \frac{1}{k+1} &< \int_{1}^{n+1} \frac{1}{x} dx < \sum_{k=1}^{n} \frac{1}{k} \\ \Rightarrow \sum_{k=1}^{n} \frac{1}{k+1} &< \ln(n+1) < \sum_{k=1}^{n} \frac{1}{k}\end{aligned}$$

■

Hence we get the both bounds of $\sum_{k=1}^{n} \frac{1}{k}$ from above, and can easily obtain the order.

$$\sum_{k=1}^{n} \frac{1}{k} = O(\ln n) = O(\log n)$$

APPENDIX C – ORDER OF $g(K)$

First mention that $g(K) = f(K) - \ln[1-(\frac{1}{2})^{\frac{1}{K}}] - (\frac{1}{2})^{\frac{1}{K}-1}$, where $f(K) = \Theta(\log K)$.

When K has a constant order,i.e.,K does not increase to infinity when N does, then the result stands. Else, we see the order of $g(K)$ as $K \to \infty$. As $(\frac{1}{2})^{\frac{1}{K}-1} \to 2$, we only need to compare the orders of $f(K)$ (i.e. $\log k$) and $\ln[1-(\frac{1}{2})^{\frac{1}{K}}]$, which can be simplified to K and $\frac{1}{1-(\frac{1}{2})^{\frac{1}{K}}}$. As

$$\begin{aligned}\lim_{K\to\infty} \frac{K}{\frac{1}{1-(\frac{1}{2})^{\frac{1}{K}}}} &= \lim_{K\to\infty} \frac{1-(\frac{1}{2})^{\frac{1}{K}}}{\frac{1}{K}} \\ &= \lim_{K\to\infty} \frac{(\frac{1}{2})^{\frac{1}{K}} \ln 2(-\frac{1}{K^2})}{-\frac{1}{K^2}} \qquad (23) \\ &= \ln 2 \lim_{K\to\infty} (\frac{1}{2})^{\frac{1}{K}} = \ln 2\end{aligned}$$

Where (23) follows from the theorem in which $\lim_{n\to\infty} \frac{p(n)}{q(n)} = \lim_{n\to\infty} \frac{p'(n)}{q'(n)}$ holds when $p(n)$ and $q(n)$ satisfy that as n goes to infinity, they both tend to be zero.

Since the scaling ratio of K and $\frac{1}{1-(\frac{1}{2})^{\frac{1}{K}}}$ is a constant value, it implies that these two have the same order, so do $f(K)$ and $\ln[1-(\frac{1}{2})^{\frac{1}{K}}]$. Hence, $g(K) = \Theta(\log K)$.

REFERENCES

[1] M. Neely and E. Modiano, "Capacity and delay tradeoffs for ad-hoc mobile networks," *IEEE Transactions on Information Theory*, vol. 51, no. 6, pp. 1917C1937, June 2005.

[2] Hui Chen, "MotionCast: On the Capacity and Delay Tradeoffs."

[3] S. Ross. *Stochastic Process*. John Wiley & Sons, Inc., New York, 1996