

A Survey on Capacity of Wireless Networks

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Abstract—Studies of throughput capacity of wireless ad hoc networks attracts many attention since the year 2000. In these works, the fundamental capacity limits of wireless networks is studied. These results are of great importance because they are not only critical for theoretical analysis, but also provide guidelines in designing wireless ad hoc networks. Recently, the researches on multicast is proposed, which study the condition when a number of nodes in the networks are interested in identical information. Other related issues like mobility, hybrid networks and energy consumption is also studied in the multicast networks. Although these researches seems independent with each other, there are some interconnections between them and enable us to make a comprehensive survey. In this work, we present the recent works on multicast and the techniques on proving the results. In addition, we specifically provide the throughput-delay tradeoff in detail.

Index Terms—Capacity, Throughput, Multicast, Broadcast, Unicast, Scaling laws, Throughput-delay tradeoff, Wireless ad hoc networks

I. INTRODUCTION

Wireless ad hoc networks are useful when there is a lack of infrastructure for communication. Such a situation may arise in a variety of civilian and military contexts like sensor network applications and communicate in harsh environments. Since the seminal work by Gupta and Kumar [1], the study of wireless ad hoc networks has focused on understanding its fundamental capacity limits. The throughput capacity per source-destination (S-D) pair of random ad hoc network developed by Gupta and Kumar is $\Theta(1/\sqrt{n \log n})$, which is pessimistic because the capacity goes to 0 as the number of nodes in a fixed area $n \rightarrow \infty$. Since then there are three kinds of work that focus on the study of capacity.

One of the topics is to extend the original work done by Gupta and Kumar. This kind of work include completing the proofs on unicast [30], extending the number of receivers to the case of multicast [23] [24] and broadcast [35]. This kind of work are the extension of the unicast and the generalization of different kinds of transmission possibilities in the real networks. However, the results are also pessimistic because the per-node capacity tends to 0 as the number of nodes $n \rightarrow \infty$.

Another topic is on the trade-off between capacity and other network variables like delay and power consumption. In 2002 Grossglauser and Tse found that mobility can increase the throughput capacity [2]. The per node throughput can be bounded by a constant according to the 2-hop scheme

proposed in [2]. However, the end-to-end delay is very large when mobility is introduced. Followed by this work, there are a group of people work on the capacity-delay trade-off [41] [31]. There is also another issue that needs consideration in the networks: energy consumption. There are also many works on the capacity of energy-constrained networks [18]. Since we are quite interested in the throughput-delay tradeoff, we present this issue in section II. Due to limited time, we will not consider other types of tradeoff in our report.

Thirdly, other works are related with changing the ad hoc network model. The classical model is so called random homogeneous ad hoc networks. To have a change on this, some people studied arbitrary networks [4], some people studied inhomogeneous networks (clusters) [42] [10] [3], some people combined the cellular network and ad hoc network and worked on hybrid networks [16] [47] [46], some people let nodes to cooperate and built a hierarchical MIMO network [29], still others used network coding [38] [28] [33] [22] [14] [6] and MPR [32] [21] to improve the network capacity.

The paper is organized as follows. The next section we introduce the throughput-delay tradeoff in detail. We present the tradeoff in several models including both static and mobile ones. Furthermore, we also describe the tradeoff network models allowing hierarchical cooperation network and models integrating wireless and optical network. Section III is dedicated to describe the different system model and related definitions in our report. Three widely used interference models are introduced and some related terms are defined. In Section IV we offer the main works and results on capacity of static random ad hoc networks. Several different kinds of schemes that benefit the capacity is introduced. In Section V we turn our attention to arbitrary networks. Finally, Section VI concludes the paper and present our future work plan.

II. THROUGHPUT-DELAY TRADEOFF

Besides throughput, delay is actually another fundamental issue for measuring the network performance. From an application point of view, in several cases, a network with satisfactory throughput that can be obtained at the cost of increase in delay may not be useful. Plenty of papers focus on the throughput-delay tradeoff. In this section, we present several research result on this issue.

A. End-to-end Delay

The end-to-end delay is composed of four kinds as below.

(1) Processing delay - time for processing the packet by nodes including checking bit errors, determining next destination and so on. In some cases it can be ignored while in several systems it can be quite large due to reasons like complex encryption algorithms.

(2) Queuing delay - time when the packet waits for transmission in a queue (also called the buffer), depending on buffer size, traffic pattern and node density. During network congestion, if the packet is dropped queuing delay can be considered infinite.

(3) Transmission delay - time for putting bits into the channel. It is a function of the packet's size and has nothing to do with the distance between the source and the destination. It can be calculated according to the following formula.

$$D_t = N/R \quad (1)$$

where D_t is the transmission delay, N is the packet's length in bits and R is the transmission rate (say in bits per second(bits/s)).

(4) Propagation delay - time for propagation in the channel. It is given by the expression below.

$$D_{pr} = d/s \quad (2)$$

where d is the S-D distance and s is the propagation speed.

B. Definitions of Related Terms

Definition 1 (Dense network). *If the area of the network is fixed as the number of users increases, the network is called a dense network.*

Definition 2 (Extended network). *If the density of nodes in the network is fixed as the number of nodes increases, the network is called an extended network.*

Definition 3 (Delay of a packet). *The delay of a packet is the time it takes for the packet to reach its destination after it leaves the source.*

Definition 4 (Delay of a Scheme). *Let $D_{\Pi_n}^i(j)$ denote the delay of packet j of S-D pair i under policy Π_n , then the sample mean of delay (over packets that reach their destinations) for S-D pair i is*

$$\bar{D}_{\Pi_n}^i = \limsup_{k \rightarrow \infty} \frac{1}{k} \sum_{i=1}^n D_{\Pi_n}^i(j) \quad (3)$$

C. Throughput-delay Tradeoff in Static Wireless Networks

To our best knowledge, Bansal and Liu [48] conducted the first study on delay-capacity relationship in wireless networks. They constructed a model consisting of stationary S-D pairs and mobile relays. They considered a routing algorithm that could achieve throughput that is only a poly-logarithmic factor off from the optimal and studies its performance in terms of delay.

In this subsection, we considered static wireless networks. The paper [44] by Gamal et al. provided the throughput-delay

tradeoff both in static and mobile networks with the packet size scale as the transmission rate. According to the equation $D_t = N/R$, we can know that the transmission delay D_t is constant. They showed that $D(n) = \Theta(nT(n))$, for $T(n) = O(1/\sqrt{n \log n})$ which actually was obtained by Gupta-Kumar utilizing a random wireless network model with n static nodes in [1], where $D(n)$ and $T(n)$ are the average packet delay and throughput per node in a network of n nodes, respectively (also applicable in other equations if no extra notice). Moreover, in [37] Gamal et al. provided the optimal throughput-delay tradeoff in the static random network as $D(n) = \Theta(nT(n))$. This result was obtained by using a fluid model, in which the packets are allowed to be arbitrarily small. In [36] they gave that the result is still $D(n) = \Theta(nT(n))$ in the case of constant-size packets, where now $D(n)$ is the average delay per bit since constant-size packets necessitate the use of buffers in the network. Queuing delay at the source node was not considered in [36] [37] [44].

The paper [9] studies throughput and delay performance in random access MAC multi-hop wireless ad hoc networks with stationary nodes that is modeled as open G/G/1 queuing networks. G/G/1 queue is a single-server queue with arbitrary arrival process and arbitrary service time distribution, with infinite buffer. In the analysis, the queuing delays at source and intermediate nodes are included.

D. Throughput-delay Tradeoff in Mobile Wireless Networks

In this part, we discussed mobile wireless networks. By applying the random walk mobility model in [44], Gamal et al. proved that $D(n) = O(\sqrt{n}/v(n))$ (excluding the queuing delay), for $T(n) = \Theta(1)$ which is achieved in Grossglauser-Tse mobile network model [2], where $v(n)$ is the velocity of the mobile nodes. This delay result increases with n since in [2] the chance of the relay being closest neighbor with the destination is $1/n$. The scheme in [44] that achieve the optimal throughput-delay tradeoff varies due to the degree of node mobility, the number of hops and the transmission range. The paper [37] demonstrated $D(n) = \Theta(nT(n))$ for $T(n)$ up to $O(1/\sqrt{n \log n})$. Additionally, in [30], Franceschetti et al. used percolation theory to prove that the classical multi-hop strategy can achieve this optimal throughput-delay scaling when $T(n) = \Theta(1/\sqrt{n})$. Thus, in the interference-limited wireless networks model, $D(n) = \Theta(nT(n))$ exists for $T(n)$ up to $\Theta(1/\sqrt{n})$ which indicates that in this case the linear scaling of optimal packet delay respect to the aggregate throughput $nT(n)$. Neely and Modiano [41] indicated the tradeoff for independent and identically distributed (i.i.d.) mobility model in which each node is equally likely to be in any part of the network at each time slot, independent of the past. In other words, the network is totally reshuffled from one time slot to another. The paper [41] utilized a cell partitioned region and showed that $D(n)/T(n) \geq \Theta(n)$. They develop schemes that can achieve $\Theta(1)$, $\Theta(1/\sqrt{n})$, and $\Theta(1/n \log n)$ when the delay constraint is on the order of $\Theta(n)$, $\Theta(\sqrt{n})$, and $\Theta(\log n)$, respectively. In [43], by considering multi-user reception and taking the effect of fading

into account, Toupmpis and Goldsmith offered a scheme that a mobile relay will always use single-hop transmission to forward the messages directly to the destination to achieve $T(n) = \Theta(n^{(d-1)/2}/(\log n)^{5/2})$ when the delay is bounded by $O(n^d)$, where $0 < d < 1$, showing that an improved tradeoff $D(n)/(T(n))^2 = O(n(\log n)^5)$ without proof of optimality. If fading isn't incorporated, the throughput per node could be $T(n) = \Theta(n^{(d-1)/2}/(\log n)^{3/2})$. Another paper [45] that considered the i.i.d. mobility model with less restrictive network presented $D(n)/(T(n))^3 \geq \Theta(n/(\log n)^3)$. In sum, if ignoring logarithmic terms for roughly comparison and assuming that $D(n) = \Theta(n(T(n))^\alpha)$, we can see $\alpha = 1, 2, 3$ in [41], [43] and [45], respectively. We can also conclude that both the mobility model and the network setting exert influences on the throughput-delay tradeoff. In [45], X. Lin and N. Shroff also provided the limiting factor in the scheme of [41] and [43]. They proposed that in [41] the optimal choice for the distance from the last mobile relay to the destination l_b should be $\Theta(n^{-(1+2d)/6}(\log n)^{-1/2})$ when the mean delay is bounded by $\Theta(n^d)$, where $0 < d < 1$. Nonetheless, l_b is on the order of $O(1/\sqrt{n})$ which limits the result in [41]. They also indicated that, for the scheme in [43], the optimal choice for the number of hops from the last mobile relay to the destination node h_b should be $\Theta(n^{(1-d)/3}/\log n)$ when the mean delay is bounded by $\Theta(n^d)$, where $0 < d < 1$. However, h_b is always 1 which acts as limiting factor in the scheme of [43]. In [17], the author investigate the delay throughput tradeoffs in mobile ad hoc networks given a delay constraint D in all following models. They proved that the tradeoff are $T(n) = O(\sqrt{D/n})$ and $T(n) = O(\sqrt[3]{D/n})$ for the two-dimensional i.i.d. mobility model with fast mobility and slow mobility, respectively. Besides, the results are $T(n) = O(\sqrt[3]{D^2/n})$ and $T(n) = O(\sqrt[4]{D^2/n})$ for the one-dimensional i.i.d. mobility with fast mobility and slow mobility, respectively. In addition, other mobility models, such as random way-point mobility, hybrid random walk and Brownian mobility models, have also been studied by the research community.

E. Other results

Several papers researched wireless networks when hierarchical cooperation schemes is allowed. For static model, Ozgur et al. introduced a hierarchical cooperation scheme with distributed MIMO communication to achieve throughput per node $\Theta(n^{-\frac{1}{h+1}})$ for the dense network in [29], where h is the number of hierarchical levels. Therefore, $D(n)$ can be close to $\Theta(1)$ by increasing h . Nevertheless, in this scheme the end-to-end delay is much larger compared with those in multihop ones. In [19], a modified scheme was presented for the performance that $D(n) = \Theta(n(\log n)^2 T(n))$ where $T(n)$ (per node) lies between $\Theta(1/n)$ and $\Theta(1/\sqrt{n})$. Comparing static models using hierarchical cooperation schemes and mobility schemes, we can conclude that the two can achieve throughput of same order. Nevertheless, the delay scaling performance of the latter one may vary significantly due to different models.

Additionally, the paper [7] studies the throughput-delay

tradeoff in a network model that integrate wireless and optical network. The network consists of several randomly distributed normal nodes and some regularly placed base stations connected via an optical network. Two transmission modes, i.e., the infrastructure mode and the ad hoc mode share the same bandwidth of W bits/sec. They find that the average packet delay can be maintained as low as $\Theta(1)$ even when the per-node throughput is $\Theta(W)$.

III. DIFFERENT SYSTEM MODELS

In this section we will introduce different models for wireless ad hoc networks. The ad hoc networks consists of n nodes lying in a certain area. Capacity bounds in wireless networks are primarily concerned with the scaling laws of how fast information can be transported over distance with respect to the number of communicating nodes. Typically, there are two ways of letting the number of nodes n tend to infinity. One can either keep the area on which the network is deployed constant, and make the node density tend to infinity (termed *dense network*); or one can keep the node density constant, and increase the area to infinity (termed *extended networks*). For both of these settings, information theoretic bounds are obtained by allowing arbitrary (physical layer) cooperative relay strategies; while network theoretic bounds are usually derived through constructive methods by assuming certain relationship between link capacity and interference models, and applying graph theoretical results on the resulting weighted graphs. The shape of the area can be either a circle or a square. Usually we ignore the edge effect by assuming the square is located on the surface of a tours. Some works also consider the situation when the node are located in a d dimension area.

A. Wireless Channel Models

Here we present all the models found in the literature on wireless network capacity, namely the following three groups of models. First, the *protocol model* is the simplest of all and earned the earliest attention. It allows a successful transmission only when the receiving node is not in the interfering area of other transmissions. Second, the *physical model* sets a threshold on the *signal to interference plus noise ratio* (SINR) of the receiving signal. The transmission is successful only when the SINR is above the threshold, and fails otherwise. Third, the *generalized physical model* (or the *Gaussian channel model*) decides the transmission rate in terms of the SINR by using Shannon's capacity formula for a wireless channel with additive Gaussian white noise.

For both protocol and physical model the transmission rate from node X_i to X_j is

$$W_i = \begin{cases} W & \text{if successful} \\ 0 & \text{if unsuccessful.} \end{cases} \quad (4)$$

where W is the channel capacity. The difference between two models is the conditions for a successful transmission. For protocol model, there is a interference parameter $\Delta > 0$ that

is related to the successful transmission. The transmission from node X_i to node X_j is successful when:

- $|X_i - X_j| \leq r$
- For every other node X_k simultaneously transmitting over the same channel: $|X_k - X_j| \geq r$

where r is the transmission range of each node.

Under the physical model, the transmission is successful if

$$\text{SINR} = \frac{P_i |X_i - X_j|^{-\alpha}}{N + \sum_{k \neq i, k \in S} P_k |P_k - P_j|^{-\alpha}} \geq \beta \quad (5)$$

where β is the threshold of SINR, N is the power of noise and $\alpha > 0$ is the signal loss exponent.

In generalized physical model, the direct transmission rate W_i between any two node X_i and X_j depends on SINR as

$$W_i = B \log_2 \left(1 + \frac{P_i |X_i - X_j|^{-\alpha}}{BN_0 + \sum_{k \neq i, k \in S} P_k |P_k - P_j|^{-\alpha}} \right) \quad (6)$$

where B is the bandwidth of the wireless communication channel and $N_0/2$ is the noise spectral density. The generalized physical model is more realistic than the other two models. However, it is also problematic because the transmission rate goes to infinite when two nodes are placed very close to each other. This can be solved by upper bounding the received power at each node.

B. Definitions of Related Terms

In this paper we use Knuth's notation: Given two functions $f(n) \geq 0$ and $g(n) \geq 0$, $f(n) = O(g(n))$ means $\limsup_{n \rightarrow \infty} f(n)/g(n) = c < \infty$; $f(n) = \Omega(g(n))$ is equivalent to $g(n) = O(f(n))$; $f(n) = \Theta(g(n))$ means $f(n) = O(g(n))$ and $g(n) = O(f(n))$. Also, we use the abbreviation *w.h.p.* to stand for *with high probability*, which indicate the probability tends to 1 as the number of nodes $n \rightarrow \infty$.

Definition 5 (Transport Capacity of Arbitrary Networks). *Given any set of successful transmissions taking place over time and space, let us say that network transports one bit-meter when one bit has been transported a distance of one meter toward its destination. The sum of products of bits and distances is the indicator of a network's transport capacity.*

Definition 6 (Feasible Throughput). *A throughput of $\lambda(n)$ bits per second for each node is feasible if there is a spatial and temporal scheme for scheduling transmissions, such that by operating the network in a multihop fashion and buffering at intermediate nodes when awaiting transmission, every node can send $\lambda(n)$ bits per second on average to its chosen destination node. That is, there is a $T < \infty$ such that in every time slot $[(i-1)T, iT]$ every node can send $T\lambda(n)$ bits to its corresponding destination node.*

Definition 7 (The Throughput Capacity of Random Wireless Networks). *The throughput capacity of the class of random networks is of order $\Theta(f(n))$ bits per second if there are*

deterministic constants $c > 0$ and $c' < +\infty$ such that

$$\begin{aligned} \lim_{n \rightarrow \infty} \Pr(\lambda(n) = cf(n) \text{ is feasible}) &= 1 \\ \liminf_{n \rightarrow \infty} \Pr(\lambda(n) = c'f(n) \text{ is feasible}) &< 1. \end{aligned} \quad (7)$$

Here the probability is computed using all possible connected random networks formed by n nodes distributed in the area.

IV. RANDOM NETWORKS

In this section we discuss works dealing with random networks. In random networks, the nodes are either independently and uniformly distributed (i.i.d.) or placed according to some probability distribution (usually in forms of clusters). They are called *homogeneous* and *inhomogeneous networks* respectively. We mainly consider the static networks in this section, while other issues like mobility and delay will be discussed independently.

A. Random Homogeneous Networks: Unicast, Multicast and Broadcast

The ground breaking work [1] done by Gupta and Kumar studied the arbitrary networks and show that transported capacity is $\Theta(W\sqrt{An})$ bit-meters per second under both protocol model and physical model, with a fixed area size A . More importantly, [1] studies the capacity of random networks, which shows under protocol model, the upper bound $O(\frac{W}{\sqrt{n \log n}})$ can be achieved in a circle of unit area. However, under physical model, the same lower bound $\Omega(\frac{W}{\sqrt{n \log n}})$ can be achieved, while the proved upper bound is $O(\frac{W}{\sqrt{n}})$. A gap of order $1/\sqrt{\log n}$ exists under the physical model.

In [30], the authors achieved the same upper bound in [1] under physical model and proves the throughput capacity $\Theta(\frac{1}{\sqrt{n}})$. In this work, the authors proposed a scheme called "highway system", in which nodes that can carry information across the network at constant rate using short hops. The rest of the nodes access the highway system using single hops of longer length.

All the papers above deal with the situation of *unicast*, when the source node randomly choose one destination node to transmit data. Still there are other cases when some information need to be *broadcast* so that every node in the network would like to know. In [35], the authors for the first time considered this situation. Using protocol model, the aggregated capacity bound of $\Theta(W/\max(1, \Delta^d))$ (where W is the channel capacity, Δ is the interference parameter and d is the number of dimensions) in random homogeneous dense network can be achieved. The technique to prove the upper bound is using *minimum connected dominate set*. This technique is widely used in proving upper bound in multicast capacity. The result in [35] shows us that the aggregated capacity of broadcasting is at most a constant, with the per-node throughput $\Theta(W/n)$.

While unicast and broadcast are two extremes of the transmission in ad hoc networks, another more general issue

of transmission is *multicast*. Multicast means the number of destination is between the two extremes: 1 node or $n - 1$ nodes. That is, the number of destination nodes $1 \leq k \leq n - 1$. In [23], the authors for the first time discovered a threshold $k = \frac{a^2}{r^2}$ (note $\frac{a^2}{r^2} = \frac{n}{\log n}$ is a special case to guarantee the connectivity w.h.p.) on the number of destination nodes for every source node, where a is the side length of 2-dimensional area and r is the transmission range. When $k = O(\frac{a^2}{r^2})$, the multicast capacity is similar to unicast; when $k = \Omega(\frac{a^2}{r^2})$, the multicast capacity is similar to broadcast. The results is as follows:

$$\Lambda_k(n) = \begin{cases} \Theta(\frac{a}{r} \cdot \frac{W}{\sqrt{k}}) & \text{when } k = O(\frac{a^2}{r^2}) \\ \Theta(W) & \text{when } k = \Omega(\frac{a^2}{r^2}). \end{cases} \quad (8)$$

The same work also shows us the capacity result of multicast in d -dimensional space.

Another paper deals with multicast networks is [24]. Traditionally we considered all n nodes in the network have some packets to transfer to a certain number of destination nodes. In this paper, the authors considered another scenario when there are only $n_s = n^\epsilon$ source nodes, each sending data to $n_d n^{1-\epsilon}$ destination nodes. In this scenario, it is also very convenient to reduce the multicast case to broadcast (with $n_s = 1$ source node and $n_d = n - 1$ destination nodes). However, it is somewhat confusing to consider the unicast case. The explanation given in [24] is when $n_s = n - 1$ and $n_d = 1$, the model is the same as a unicast with $\Theta(n)$ source nodes. But in this case, the model is more reasonable to be considered as a “*converge cast*” because all $n - 1$ source nodes choose the same destination node. Still, the proved network capacity is $\Theta(\frac{\sqrt{n^\epsilon}}{\sqrt{\log n}})$ w.h.p. with a per flow throughput capacity of $\Theta(\frac{1}{\sqrt{n^\epsilon \log n}})$ w.h.p. This result is a simple extension of arguments in [1].

Also in [24] the authors used a unique method to achieve the upper bound. The routing architecture is called the *multicast comb*, which is constructed independent of the senders and receiver locations. Both senders and receivers complete the multicast tree by attaching themselves to the comb using shortest path routing. In this case, I think the multicast comb is very similar with the technique highway system used in [30]. They are all restrict the main data stream in a *backbone* network and all other nodes not belong to the backbone transmit/recieve data to/from it. It is a simple but useful technique that can be used in constructively prove the capacity upper bound.

Recently Gaussian channel model has gained more and more attention because it is more closed to reality. Li et al. have studied multicast case under Gaussian channel model [12]. The authors showed that when $k \leq \theta_1 \frac{n}{(\log n)^{2\alpha+6}}$ and $n_s \geq \theta_2 n^{1/2+\beta}$, the capacity of each multicast session can achieve is at least $c_8 \frac{\sqrt{n}}{n_s \sqrt{k}}$ w.h.p, where $\alpha > 2$ is the attenuation exponent, θ_1, θ_2 and c_8 are some constant factors and $\beta > 0$ is a positive real number. Because of the complexity

of the Gaussian channel model, there are still many aspect of multicast that need further study.

In order to generalized the three different interference models, i.e. protocol model, physical model and generalized physical model, [27] introduced a new concept called *transmission arena*. By introducing this concept, the authors found a new way to prove the upper bound of the throughput capacity, regardless of the interference model that is used. Although the arena makes the proofs of the upper bound become simple and clear, it is not capable to constructively prove the lower bound. Therefore, the concept of arena is not able to show the bound is tight. In the work [20], the same authors studied the multicast capacity in homogeneous multihop networks. In this work, the authors used arena to prove the upper bound and the highway system proposed in [30] to prove the lower bound. A gap of at most $O(\sqrt[d]{\log n})$ exist between the upper and lower bound, where $d \geq 2$ is the number of dimension.

To provide a unified model of unicast, multicast and broadcast, [15] introduced (n, m, k) -casting as a generalization of all forms of one-to-one, one-to-many and many-to-many transmissions. The results are also the same as [1], [23], [35]: when the nodes are connected w.h.p, the per-node capacity is

$$C_{m,k}(n) = \begin{cases} \Theta(\frac{\sqrt{m}}{nkr(n)}), \Theta(1) \leq m \leq \Theta(\frac{1}{r^2(n)}) \\ \Theta(\frac{1}{nkr^2(n)}), k < \Theta(\frac{1}{r^2(n)}) \leq m \leq n \\ \Theta(\frac{1}{n}), \Theta(\frac{1}{r^2(n)}) \leq k < m \leq n \end{cases} \quad (9)$$

B. Random Inhomogeneous Networks: Clusters

All of above works are in homogeneous network, in which node are i.i.d. Another class of network is inhomogeneous network in which nodes are distributed in clusters rather than i.i.d. To the best of our knowledge, the starting point of cluster network study is [42], in which Toumpis studied three kinds of networks: asymmetric, cluster and hybrid network.

In a recent work [3], the authors studied the cases of two cluster methods: cluster grid and cluster random. Under the generalized physical model, the authors proved the lower bound. Comparing with the preceding work [10], some of these bounds are tight while others are not.

C. The Impact of the Number of Channels

While many existing standards, such as IEEE 802.11a, 802.11b and 802.15.4 allow for multiple channels, nodes are typically hardware- constrained and have much fewer interfaces. This issue was studied in [39], under a model where nodes were capable of switching their interfaces to any channel. It was shown that given c available channels of bandwidth W/c each, and $1 \leq m \leq c$ interfaces per node, capacity depends only on the ratio c/m . For a random network and the protocol model, $c/m = O(\log n)$, where they showed that capacity is the same as for $m = c$, i.e. $\Theta(\frac{W}{\sqrt{n \log n}})$ bits/s per-flow. Followed by [39], a recent work [25] studied random (c, f) assignment model. In this work, each node is pre-assigned a random subset of f channels out

of c (each having bandwidth W/c), and may only switch on these. This time, the authors proved that when $f = \Omega(\sqrt{c})$, the capacity is still the same as unconstrained case. In conclude, decreasing the number of subchannels available to each node seems harmless to the throughput capacity.

Also, another consideration is the *ultra-wide band* (UWB) technique. In [40], the author find another way to improve the throughput capacity. Intuitively, the interference can be ignored in UWB, which solve the most critical problem that limit the capacity. By using UWB, the throughput of $\Theta(n^{(\alpha-1)/2})$ can be achieved, with a power rate of $\Omega(n^{(1-\alpha)/2})$, where α is the attenuation exponent.

D. Combination of Cellular System: Hybrid Networks

The hybrid networks has also been widely studied, began with the works [47] [46]. In some of the recent results, [16] made a comprehensive analysis on the impact of hybrid networks on the original ad hoc networks. The authors note that while hybrid network can result in favor of a capacity increase because of shorter range higher-rate links and improved spatial reuse, it relies on multi-hop forwarding which is detrimental to the overall capacity. In conclusion, the authors demonstrated that capacity improvement is possible in certain parametric regimes.

Other works studied the order of capacity that can be achieved. For example, [13] discussed the throughput capacity of multicast hybrid using protocol model. Under certain assumptions, when the connectivity of the network is guaranteed, the aggregated capacity is as follows.

$$\Lambda_k(n) = \begin{cases} O\left(\frac{a}{r} \cdot \frac{\sqrt{m}}{k} \cdot W\right) & \text{when } k = O\left(\frac{a^2}{r^2}\right) \\ & \text{and } k = O(m) \\ \Theta\left(\frac{a}{r} \cdot \frac{1}{k} \cdot W\right) & \text{when } k = O\left(\frac{a^2}{r^2}\right) \\ & \text{and } k = \Omega(m) \\ \Theta\left(\frac{r \cdot n}{a} \cdot \frac{\sqrt{m}}{k} \cdot W\right) & \text{when } k = O\left(\frac{a^2}{r^2}\right) \\ & \text{and } k = o(\sqrt{m}) \\ \Theta(W) & \text{when } k = \Omega\left(\frac{a^2}{r^2}\right) \end{cases} \quad (10)$$

where m is the number of base stations and k is the number of destinations per source nodes. Note some of the bound is not tight. This result makes some improvement on the capacity comparing with previous work on studying multicast capacity of ad hoc networks.

The work [26] studied the hybrid networks in a more realistic way. The authors studied the 1-dimensional and 2-dimensional strip hybrid networks implementing a more advanced Gaussian channel model. The 2-dimensional strip model is between the 1- and 2-dimension. The idea is also be extended in a recent work studying surface coverage [5]. Also, the Gaussian channel model is a widely used model in recent years.

There is still another work done under Gaussian channel model [11] [8]. This work considered hybrid extended network, where the ordinary wireless nodes are placed in the square region $A(n)$ with side-length \sqrt{n} according to

a Poisson point process with unit intensity. In this work, the authors proved some complicated results and discovered several thresholds on the number of base stations (BS) m and the number of destinations per source node n_d . Three strategies were adopted according to the m and n_d , i.e. hybrid strategy, ad hoc strategy and BS-based strategy.

E. New Techniques Used: Network Coding, MPR and MIMO

Some of the recent works have used *network coding* [49] to improve the throughput capacity of the ad hoc networks. The results seems pessimistic because all the works have proved that the throughput capacity cannot get an order gain by simply performing network coding in wireless ad hoc networks. [38] for the first time proved in the unicast case, the network coding cannot help improving the throughput capacity comparing with the result by Gupta and Kumar. Later, [28] studied broadcasting in the wireless ad hoc networks in 1- and 2-dimensional areas and proved the gain on the capacity is at most a constant, while the energy consumption can be decreased by at most 3 times. [33] developed the scaling laws of ad hoc networks using network coding. [22] used optimal scheduling and prove the improvement on throughput is a constant between 1 and 2. [14] proved the transport capacity of random ad hoc networks is bounded by a constant factor π , while the energy consumption is reduced in a ignorable scale. And lastly, [6] studied the implementation of network coding in multicast case, with the same result that the throughput gain by introducing network coding is strictly bounded by a constant ratio of 2.

However, other techniques may improve the capacity order such as *multi-packet transmission* (MPT) and *multi-packet reception* (MPR). In [32], the authors for the first time proved that MPR do increase the order of the transport capacity of random ad hoc networks for multi-pair unicast. By performing MPR, the order of unicast capacity will increase $\Theta(\log n)$ and $\Theta(\log(\log n))$ in order sense, corresponding to protocol and physical model. Another important discovery was made on the issue of multicast. In [21], the authors performs MPT and MPR in multicast and get an exciting result. They get a $\Theta(n^2 T^4(n))$ gain on the capacity order comparing to previous results on multicast, where n is the number of nodes and $T(n)$ is the transmission range. When $T(n) \geq \sqrt{(\log n/n)}$ is chosen to guarantee the connectivity in the network, the gain becomes $\Theta\left(\frac{\log^{3/2} n}{\sqrt{n}}\right)$.

Multiple-input multiple-output (MIMO) is a new physical layer technique, which can also used to improve the throughput capacity of wireless ad hoc networks. In [29], the authors proposed a hierarchical cooperative transmission scheme based on the assumption of MIMO. In this work, the throughput capacity per-node successfully reach the order of a constant as the layers of node increased to infinite. Intuitively, more layers means more delay in the data transmission, and it is indeed the case. In [19], the same author studied the throughput-delay trade-off for proposed hierarchical networks. Some other work [50] was done followed by [29].

V. ARBITRARY NETWORKS

All the works mentioned above is dealing with random networks. On the other hand, the node will not be distributed i.i.d. in reality. The study of arbitrary networks assumes the nodes can be placed as we wish. It is the case in some situations like wireless sensor networks.

There are relatively less studies on arbitrary networks. The first work is again by Gupta and Kumar [1], which indicates that the transport capacity of arbitrary network is $\Theta(\frac{W}{\sqrt{n}})$ for each node, and the throughput capacity is also $\Theta(\frac{W}{\sqrt{n}})$ for each node, in both protocol and physical model. The W is the channel capacity and n is the number of nodes.

The results in [1] is considered the capacity when the nodes are optimally placed. There are still other interesting topics on arbitrary networks. For example, what is the capacity of a network when the nodes topology is in the worst case? Moscibroda studied capacity in sensor networks in [34] under physical model and showed that even in the worst case, a sustainable rate of $\Omega(1/\log^2 n)$ can be achieved in every network. Also, they showed the best possible rate in protocol rate is $\Theta(1/n)$.

The most recent work on this is [4], in which the authors studies the capacity of arbitrary topology in wireless networks. However, this work deals more about scheduling algorithm, which is not our interests here.

VI. CONCLUSION AND FUTURE WORK

To conclude our classification of current works on capacity of wireless ad hoc networks, although some of the results are ideal, most of them needs further improvement. In our future work, we will primarily focus on two different topics: one is the extension of multicast network; the other is some new cases on capacity, coverage and connectivity. Some other issues are also in our consideration, such as mobility and network coding. We have read several papers on multicast, capacity, delay, coverage and connectivity. Now we make every endeavor to propose new models or ideas to improve results already achieved by the research community or develop new conclusions. So far although few inspirations occur to us now and then, we haven't develop them quite clearly. Therefore, there exist considerable things for us to carry out.

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