Report1 for Project on CS

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Abstract—-**This report presents the recently introduced model of reconstructing an object or a signal from incomplete frequency samples, compressed sensing, also known as compressive sensing or CS. If x is a vector in R^N (a digital image or signal), now we can have substantially smaller measurements to reconstruct the vector according to CS, than the traditional ways. This method employs nonadaptive linear projections to preserve the structure of the signal;then the signal is recovered from the projections using an optimization process[1],[2],[3].**

Specifically,suppose x has a sparse representation in some orthonormal basis(e.g., wavelet, Fourier) or tight frame (e.g., curvelet, Gabor), we can design $M = O(K\log(N/K))$[4] with a random projection, where K means x is K-sparse in some certain basis.

Because of its revolutionary model, compressive sensing is one of the hottest areas which involves lots of domain such as probability, convex optimization, linear optimization. Countless researchers devote themselves into CS and dramatic discoveries have been worked out. As a new technique for simultaneous sampling, it has been applied from processing and reconstruction of images to channel estimation in communication.

Index Terms—-**Sparse representation, RIP, convex optimization process, random measurement, ℓ_0 norm, ℓ_1 norm, signal recovery, perturbation, single-pixel CS camera, coherent data communication, multi-antenna channels, mobile cooperative network, wireless sensor network.**

I Introduction

Our requirements for the resolution of pictures and videos are more and more demanding ask for denser sampling according to the traditional sampling theory; meanwhile, widely distributed sensor networks, camera arrays and data bases make it unthinkable to preserve or store the data without compression. While the Nyquist rate is so high, it is encouraging that researchers find out a new method, CS, to capture and represent compressible signals at a rate significantly below the Nyquist rate.

This report also presents some newest research achievements in CS, indicating what is the frontier in this area. Also, with highly attention paid to it, it has spawned out lots of exiting applications in image processing, medical diagnosis, and communication.

II What Is CS

A. Backgrounds On Transforms And Compressible Signals

Suppose a real-value, finite-length,one-dimensional, discrete-time signal x, which can be considered as an vector in $\mathbf{R^N}$. Then we can represent x by

$$x = \sum_{i=1}^{N} s_i \psi_i \quad or \quad x = \Psi s,$$

where $\{\psi_i\}_{i=1}^N$ is a basis of $N \times 1$ vectors in $\mathbf{R^N}$ and $\{s_i\}_{i=1}^N$ is the $N \times 1$ column vector. For example, we can represent x from the time domain into the frequency domain by the Fourier transform.

Now, if the signal x is a linear combination of only K basis vectors then we say that x is K-sparse. That is, all of the s_i coefficients but K of them are zero. When there are just a few large coefficients and many small coefficients, we say x is compressible. Thereby, images can be compressed based on some algorithms like JPEG or JPEG 2000. In these algorithms, the K largest coefficients are located and the $(N-K)$ smallest ones are discarded[3].

B. The Compressive Sensing Problem

Unfortunately, the traditional compress framework suffer from some inefficiencies. First, we have to deal with N samples but only K are desired. Second, the locations of the large coefficients must be encoded as well, causing more spaces and complexity. Compressive sensing, on the other hand, directly acquires a compressed signal representation without tackle with all the N samples. We can have M measurements, where $N < M \ll N$[1][2] to process it. Now suppose that we have M vectors denoting $\{\phi_j\}_{j=1}^{M}$, where ϕ_j is one row vector in the $M \times N$ matrix Φ $(= \{\phi_j\}_{j=1}^{M})$. Arrange an $M \times 1$ vector y, make

$$y = \Phi x = \Phi\Psi s = \Theta s,$$

where Θ is an $M \times N$ matrix. The problem is to design a universal Φ to ensure that a reconstruction algorithm is efficient and thus replace the traditional ones.

C. Design Measurement–Φ

If a measurement can be effective, it is suggested to follow a so-called reconstructive isometry property(RIP). For a Φ, if it obeys[4]

$$(1 - \delta_K)\|f\|_{\ell_2}^2 \leq \|\Phi f\|_{\ell_2}^2 \leq (1 + \delta_K)\|f\|_{\ell_2}^2$$

for any K-sparse vector f and some $\delta_K > 0$, then we say that Φ loosely obeys the RIP of order K.

If the RIP holds, then the accurate reconstruction can be obtained from the following program:

$$\min_{\widetilde{x} \in \mathbf{R^N}} \|\widetilde{x}\|_{\ell_0} \quad \mathit{subject\ to} \quad \Phi\widetilde{x} = y(= \Phi x).$$

That is, if we search for the sparsest vector that explains y, we will find x, which is explained in [1]. This method is called minimum ℓ_0 norm reconstruction. However, it is a combinational problem and is impossible to realized.

Surprisingly, minimum ℓ_1 norm reconstruction

$$\min_{\widetilde{x} \in \mathbf{R^N}} \|\widetilde{x}\|_{\ell_1} \quad \mathit{subject\ to} \quad \Phi\widetilde{x} = y(= \Phi x)$$

can exactly recover K-sparse signals and closely approximate compressible signals with high probability using some random measurements and $M \geq cK \log N/K$ [1],[2],[3]. This is a convex optimization problem that can be conveniently solved [1],[2].

Some researchers have designed a few measurement matrices which obey RIP and have encouraging results:

Gaussian measurements. The entries of the $M \times N$ sensing matrix Φ are independently sampled form the normal distribution with mean zero and variance 1/M. *Binary measurements*. The entries of the $M \times N$ sensing matrix Φ are independently sampled from the symmetric Bernoulli distribution $P(\Phi_{ki} = \pm 1/M) = 1/2$.

Fourier measurements. Φ is partial Fourier matrix obtained by selecting M rows uniformly at random, and renormalizing the columns so that they are unit-normed.

Incoherent measurements. Φ is obtained by selecting M rows uniformly at random from an N by N orthonormal matrix U and renormalizing the columns so that they are unit-normed.

Now, some researchers in CS is still focus on designing measure matrices, some of which will be presented in III.

III RECENT RESEARCHES

Due to CS's promising application prospect, lots of researchers have focus on this field and a large amount of encouraging new results have been witnessed.

Proximate QR factorization of measurement matrix. Its a new method designed by FU Ying-hua to enhance the efficiency and the quality of recovered images, see [5].The process is briefly represented as follows:

$$\Phi^T = QR,$$

where Q is an $N \times N$ orthogonal matrix and R is an $N \times M$ upper triangular matrix. So

$$\Phi = R^T Q^T$$

Do not change elements in diagonal line of Q and set others into 0 for elements in diagonal line are far larger than others. We can get a new matrix $\widetilde{\Phi} = \widetilde{R}^T Q^T$. Then $\widetilde{\Phi}$ obeys RIP and enhance the efficiency.

Very sparse projection matrix. This matrix is based on the very sparse random projection. Its proved that this matrix satisfies the necessary condition for CS measurement matrix by the asymptotic normality for very sparse random projection distribution. Owning to its sparsity of structure, the matrix greatly simplifies the projection operation during images reconstruction, which greatly improving the speed of reconstruction, see [6].

Very sparse random projection is a generalized form of sparse random projection.

$$X \sim \begin{bmatrix} \sqrt{s} & 0 & -\sqrt{s} \\ \frac{1}{2s} & 1 - \frac{1}{s} & \frac{1}{2s} \end{bmatrix}$$

Where X obeys sparse random projection distribution when $1 \leq s \leq 3$ and when $s \gg 3$ X obeys very sparse random projection distribution. The very sparse random projection sampling rate is $1/s$, so it enhance the efficiency.

Sparse recovery of positive signals with minimal expansion. In compressed sensing, Φ is often a dense matrix drawn from some ensemble of random matrices but this is a topic focusing on sparse measurement matrices. unlike random measurement matrices (such as Gaussian or Bernoulli), which only guarantee the recovery of sparse vectors with high probability, expander graphs give deterministic guarantees.[7]

Circulant and toeplitz matrices. While most work so far focuses on Gaussian or Bernoulli random measurements Holger Rauhut investigates the use of partial random circulant and Toeplitz matrices in connection with recovery by ℓ_1-minization. In contrast to recent work, in this direction he allows the use of an arbitrary subset of rows of a circulant and Toeplitz matrix. Their recovery result predicts that the necessary number of measurements to ensure sparse reconstruction by -minimization with random partial circulant or Toeplitz matrices scales linearly in the sparsity up to a log-factor in the ambient dimension. This represents a significant improvement over previous recovery results for such matrices.[8]

Besides,Matthew A. Herman and Thomas Strohmer analyze the Basis Pursuit recovery method when observing signals with general perturbations. Their results show that, under suitable conditions, the stability of the recovered signal is limited by the noise level in the observation. Moreover, this accuracy is within a constant multiple of the bestcase reconstruction using the technique of least squares.[9]M. A. Iwen presents a simple deterministic construction for RIP matrices which leads to small deterministic Fourier sampling set constructions. As a consequence, he obtains a deterministic sparse Fourier transform method which is guaranteed to recover a near-optimal sparse Fourier representation (if one exists) for any input signal by reading only a small deterministic subset of its entries.[10]

IV Applications

As a novel technology, CS presents its unique charm to experts attracting their interests to expand more applications by taking these advantages of it. Here, we will introduce various applications of this technique– Compressive Sensing, in processing images, clinical imaging, and especially, in communication issues.

applications of CS in the architecture of images There are many different ways to implement the corresponding architecture.Some typical applications are illustrated as follows.

- One of imaging architectures is based on combining the existing single-pixel CS camera[3] with a Bayer color filter, which enable acquisition of compressive color measurements by employing joint sparsity models in simultaneously recovering the R, G, B channels, see[11][12][13].
- Another application is that using CS completes the recovery of background subtracted images and solves some communication constrained multi-camera computer vision problems. Researchers cast the background substraction as a sparse approximation problem and provide different solutions based on convex optimization and total variation(TV)[11][12][13].
- The last application of the architecture of images is a novel approach based on the acquisition of random projections of the signal without first collecting the pixels.And the achitecture employs a digital micromirror array to perform optical calculations of linear projections of an image onto pseudo-random binary patterns[11][12][13].

The applications on the operations on the images are numerous in the frontier of the field, discussed above are only some glances of them.

applications in medical imaging CS has inspired significant interest because of its potential to reduce data acquisition time.There are two fundamental tents to CS theory: (1)signals must be sparse or compressible in a known basis,and (2)the measurement scheme must satisfy specific mathematical properties with respect to this basis.

- While MR images are often compressible respect to several bases, the second requirement is only weakly satisfied with respect to the commonly used Fourier encoding scheme.Whereas the possibility of improved CS-MRI performance using non-Fourier encoding, which is achieved with tailored spatially-selective RF pulses has good results. Simulation and experimental results show that non-Fourier encoding can significantly reduce the number of samples required for accurate reconstruction, though at some expense of noises ensitivity[14].
- One typical use is the presentation of the flyback 13C 3D-MRSI sequence[15]. High polarization of nuclear spins in liquid state through dynamic nuclear polarization has enabled the direct monitoring of 13C metabolites in vivo at very high signal to noise, allowing for rapid assessment of tissue metabolism.The abundant SNR afforded by this hyperpolarization technique makes high resolution 13C 3D-MRSI feasible.To take advantage of the high SNR available from hyperpolarization,we have applied CS to achieve a factor of 2 enhancement in spatial resolution without increasing acquisition time or decreasing coverage.
- Another application about CS on medical imaging is Kalman Filtered Compressed Sensing(KF-CS)[14], which was proposed to causally reconstruct time sequences of sparse signals, from a limited number of ”incoherent” measurements. The KF-CS is developed for causal reconstruction of medical image sequences from MR data. An important example of this type of problems is real-time medical image sequence reconstruction using MRI, for e.g., dynamic MRI to image the beating heart or functional MRI to image the brains neuronal responses to changing stimuli.

applications on communication Coherent data communication over doubly-selective channels requires that the channel response be known at the receiver. Training-based schemes, which involve probing of the channel with known signaling waveforms and processing of the corresponding channel output to estimate the channel response in practice.Conventional training-based methods, often comprising of linear least squares channel estimators, are known to be optimal under the assumption of rich multipath channels.Numerous measurement campaigns have shown, however, that physical multipath channels tend to exhibit a sparse structure at high signal space dimension (time-bandwith product), and can be characterized with significantly fewer parameters compared to the maximum number dictated by the delay-Doppler spread of the channel.

- One application of CS here proposes sparse channel learning methods for both signal-carrier and multi-carrier probing waveforms that employ reconstruction algorithms based on convex/linear programming.And the use of CS also propose new methods[16][17] for efficient estimation of sparse multi-antenna channels,and show that explicitly accounting for multipath sparsity in channel estimation can result in significant performance improvements when compared with existing training-based methods.

- Another use of CS in communications depends on a mobile cooperative network [18]that is tasked with building a map of the received signal strength to a fixed station.With the help of CS,we can show how the nodes can exploit the sparse representation of the channels spatial variations to build a map of the signal strength with minimal sensing.Someone has proposed a successive interference cancelation method for signal reconstruction based on a considerably incomplete set of measurements.The proposed method is an extension of the existing signal reconstruction strategies but with a considerably better performance.

- The estmation of doubly selective wireless channels within pulse-shaping multicarrier systems (which include OFDM system as a special case) is widely researched by many experts. A pilot-assisted channel estimation technique using the methodology of CS is proposed, see[19].By exploiting a channels delay-Doppler sparity.CS-based channel estimation allows an increase in spectral efficiency through a reduction of the number of pilot symbols that have to be transmitted.And another extension of basic channel estimator that employs a sparsity-improving basis expansion also presents.A framework for optimizing the basis and an iterative approximate basis optimization algorithm has been proposed.

- In large-scale wireless sensor networks, when the nodes are densely deployed and sensor readings are spatially correlated, a compressive data gathering scheme can be proposed to improve its energy efficiency and reduce overhead, as well as to deal with abnormal sensor readings gracefully, see[20]. Generally, it works for when any sparse signals or combinations of a few sparse signals even from different domains.

From these applications discussed above, we can recognize the multiple applications of CS in different careers,such as the compressive imaging,medical imaging, communications and so on.Though the realms are not all the same, the concepts utilized is surprisingly similar.It is the properties of sparsity and incoherence of CS.They both compose the nature of the CS.And along with the advanced development of science and technology,CS will have more chances to be applied to more fields.

V Conclusions and Further Work

Compared to the Shannon and Nyquist sampling theorem, compressive sensing offers a nonlinear sampling model which significantly improve the efficiency of acquisition of data, which can be applied into various fields, along with innovations developed for the time being. This report focus on the primitive survey on CS, which considers only limited aspects; for the next period of project, we will concentrate on untangling the intrigue mathematic theories and algorithms on CS, to find out some interesting results, especially in wireless communication.

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